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Noise and Disturbance Rejection Performance Evaluation on Explicit Model Predictive Control Technique Applied to Inverted Pendulum with Various Test Scenarios

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Abstract

An inverted pendulum on a cart (IPC) system, which is a widely used test environment for controller design due to ease of applicability, has the opportunity to be applied in different fields with nonlinear and under-actuated characteristics. In this study, the performance of the explicit MPC control method has been examined against the noise and disturbances by using two test cases and analysis approaches. Different trajectory tracking, disturbance, and noise situations have been taken into account in the elaborated scenarios. The numerical applications have been performed by the model predictive control toolbox of Matlab®/Simulink®. The advantages and drawbacks of the controller have been discussed in terms of time-domain specifications.

Keywords: IPC, Explicit MPC, Disturbance rejection, Noise attenuation

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Çeşitli Test Senaryoları ile Ters Sarkaca Uygulanan Açık Model Öngörümlü Kontrol Tekniği Üzerinde Gürültü ve Bozucu Bastırma Performans Değerlendirmesi

Öz

Uygulama kolaylığı nedeniyle kontrolör tasarımı için yaygın olarak kullanılan bir test ortamı olan araba üzerinde ters sarkaç (IPC) sistemi, doğrusal olmayan ve düşük harekete geçirilmiş özellikleri ile farklı alanlarda uygulama imkânına sahiptir. Bu çalışmada, açık MPC kontrol yönteminin, iki test durumu ve analiz yaklaşımları kullanılarak gürültü ve bozuculara karşı performansı incelenmiştir. Ayrıntılı senaryolarda farklı yörünge takibi, bozucu ve gürültü durumları dikkate alınmıştır. Sayısal uygulamalar, Matlab®/Simulink®'in model öngörülü kontrol araç kutusu tarafından gerçekleştirilmiştir. Kontrolcünün avantajları ve dezavantajları, zaman alanı spesifikasyonları açısından tartışılmıştır.

Anahtar Kelimeler: IPC, Açık MPC, Bozucu bastırma, Gürültü azaltma

1. INTRODUCTION

Inverted pendulum on a cart (IPC) is an under-actuated nonlinear system with non-minimum phase zero dynamics. Pendulum-based systems are widely used in real world applications. To mention a few; rockets, Segway (a mobile inverted pendulum system), and even bipedal movement modelling can be modelled by IPC. The main task of the IPC is balancing an inverted pendulum on a moving cart. To balance the pole, only horizontal force input on the cart is permitted [1]. A wide variety of control approaches can be examined with IPC due to the aforementioned properties. Further, IPC systems are widely used as a classical control problem for teaching control techniques. Up to date various control methods have been developed for the IPC. Among these, linear quadratic regulator (LQR) approach is one of the fundamental optimal control approaches and it's an improvement to full state control by usage of quadratic cost function in controller optimization [2]. The LQR uses an infinite horizon approach for the optimization process. Kumar et al. [3] presented a modern implication of LQR on IPC systems. LQG is yet another optimal control approach with a Gaussian State estimator. Eide et al. [4] presented a modern example of an LQG controller on an IPC system. Askari et al. [5] presented a model predictive control (MPC) implication on the IPC system. Boubaker [6] presented a nonlinear control approach to the IPC. Even some of the data-driven control approaches have been implemented on the IPC systems. Baciu and Lazar [7] presented a data-driven controller usage on an IPC system.

Disturbance and noise in control systems are the important challenges to overcome. The disturbance is unexpected/undesirable effects on the system caused by external sources. Noise is random distortion in sensor signals. Although disturbance or noise can be ignored under ideal conditions, their effects can be seen in real-life applications. The performance of controllers under these effects is very important for both industrial and research implementations of control systems.

Generally speaking, linear control schemes are designed by considering an infinite period. Those types of controllers can provide excellent performance in well modelled and relatively stable systems. In particular, the LQR is in this category and it can give successful results with low computational cost under fully specified conditions. However, the LQR may not be able to reject disturbances and noise effects in an unpredictable dynamic environment. This dilemma can be overcome by limiting the optimization horizon to a finite range with repetitive re-optimization after each step.

MPC approach is one of the solutions to overcome the aforementioned drawback. One of the most significant benefits of a MPC technique for multivariate systems is its ease of deployment. Further, the MPC can reduce the effects of time delays and deal with constraints systematically. Despite the simple nature of the MPC, the constraints of the system states and control inputs can effectively be overcome. In the MPC, the optimization process is implemented in a finite control horizon with re-optimization after each time step. The general concepts of the MPC approach have been presented in Figure 1. By using this approach, a controller might adapt to unpredicted parameter effects, disturbance effects, and noise effects to produce the required control performance with desired system control parameter ranges. Other significant advantages include its capacity to avoid the influence of time delays due to its nature and the systematic inclusion of system restrictions in the design process.

The application of MPC on an IPC to analyze the effects of input disturbance was studied by Mills et al. [8]. It should be noted that conventional MPC has high computational costs due to its online optimization approach. However, Bemporad et al. [9] presented a linear programming-based explicit MPC solution that reduces optimization cost by converting the online optimization approach to offline optimization with an affine piecewise computation approach. Furthermore, Bemporad presented a chapter about the explicit MPC approach in [10], where the central notion of the explicit MPC is discussed.

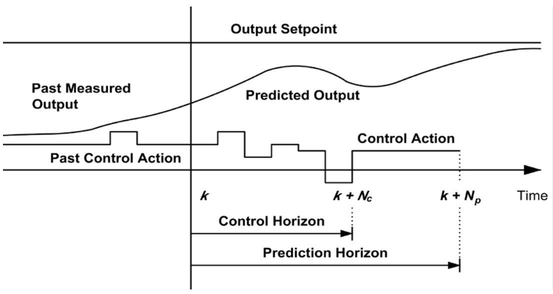

Figure 1. MPC Horizon Scheme (k is the time step representation, N_c is the selected control horizon time-step, and the N_p is the selected prediction horizon time-step)

The linear control strategy is used in the conventional MPC control approach. An IPC, on the other hand, is a nonlinear system. Hence several prominent nonlinear methods are presented in various studies. For example, Jaiwat and Ohtsuka [11] designed nonlinear model predictive control (NMPC) to tackle a double IPC system swing-up problem. Hybrid MPC is another solution for systems with continuous-valued states and discrete-valued states. Patne et al. [12] presented an FPGA application of HMPC on an IPC system, wherein a step-by-step technique for FPGA implementation using the inverted pendulum model is demonstrated. A further solution is Robust MPC (RMPC) to control nonlinear systems. In this context, Tian et al. [13] addressed an RBF-ARX-ERPC approach for the solution of an IPC by using model-based RMPC. Data-driven control is yet another approach to solution of nonlinear control problems. Verhoek et al. [14] presented a data-driven predictive control (DPC) solution on double IPC.

The prediction horizon is a finite time range where the MPC controller tries to predict the outcomes and the effects on the controlled systems. The control horizon is another finite time range where the controller predicts the required control inputs in that time range for each time step to the controlled system and produces desired plant output. The MPC predicts output in a range of prediction horizon and predicts required Δu in the range of control horizon (after that point, Δu is assumed as 0).

The repetitive online optimization process of the MPC approach requires a considerable amount of computational power which limits applications of the MPC controllers. To deal with extensive computational cost requirements, the Explicit MPC approach was presented by Bemporad [9] by using an offline optimization process. In the explicit method, solutions are found across all regions in an offline manner by converting the process calculations to piecewise affine functions which require fewer computation costs. Performance analysis of Explicit MPC applied to IPC with various optimization parameters of trajectory and pole stability without noise and disturbance inputs has been presented by in a recent study [15].

The main motivation of this research is to assess the noise and disturbance rejection capability of the Explicit MPC method applied to an inverted pendulum system. Therefore, various configurations on the positional trajectory, disturbance, and noise have been defined. Then, testing scenarios have been developed for trajectory tracking schemes, disturbance rejection capabilities and measurement noise scenarios.

2. MATERIAL AND METHOD

2.1. Material

2.1.1. IPC Model

The system model represented in Figure 2a has one control input. The controllers have been utilized by consideration of the reference trajectory, the position state feedback, and the pole angle state. The general diagram of the control structure is shown in Figure 2b. As a computation and simulation tool, the Matlab®/Simulink® program, the Control Toolbox, and the MPC Control Toolbox have been used to design and implement the controllers.

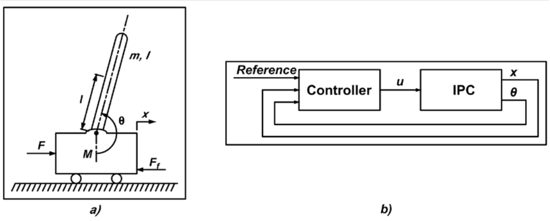

Figure 2. a) Schematic representation of an IPC b) Block diagram of a used closed-loop structure

In Figure 2, *M, m, b, l, I, F* and F_f represent the mass of the cart, the mass of the pendulum, coefficient of friction between the ground and cart, length to the pendulum center of mass (*COM*), mass moment of inertia, force applied to the cart and friction force, respectively. Furthermore, *u*, *x* and *θ* show the control signal, state of cart position and pole angle, respectively. The numerical values of the parameters are given in Table 1.

Table 1. The numerical values of IPC parameters

M	***m***	***b***	***l***
1kg	1kg	10 N/m/sec	0.5 m

The dynamical equations of the IPC [16] have been shown in Equations 1 and 2.

$$(M+m)\ddot{x}+b\dot{x}+ml\ddot{\theta}cos\theta-ml\dot{\theta}^2\,sin\,\theta=F \tag{1}$$

$$(I+ml^2)\ddot{\theta}+mgl\,sin\,\theta=-ml\ddot{x}\,cos\,\theta \tag{2}$$

The dynamical system equations are nonlinear. Hence, those equations have to be linearized for the utilization of the Explicit MPC technique. Toward this goal, the system is linearized by selecting an equilibrium point then a linear approximated model is obtained. During the linearization, the upward position ($\theta = \pi$) has been selected as the equilibrium point. The linearized model of the nonlinear system can be employed for small angular deviation (φ) where φ has been limited to ±20°. The results of the linearized system and the linear governing equations have been given in Equations 3 and 4.

$$\left(I+ml^2\right)\ddot{\phi}-mgl\phi=ml\ddot{x} \tag{3}$$

$$(M+m)\ddot{x}+b\dot{x}-ml\ddot{\phi}=u \tag{4}$$

The state-space model has been implemented for the IPC system. The obtained equations of the linearized IPC system have been shown in Equations 5 and 6. The state-space model has been utilized for the Explicit MPC approach [17].

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\phi}\\ \ddot{\phi}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \phi\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\\ \frac{I+ml^2}{I(M+m)+Mml^2}\\ 0\\ \frac{ml}{I(M+m)+Mml^2}\end{bmatrix}u \tag{5}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u \tag{6}$$

By using the parametric values in Table 1 and Equations 5 and 6, the following state matrices can be obtained. The additional dimensions in *B* and *D* are caused by disturbance.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -10 & 9.81 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -20 & 39.24 & 0 \end{bmatrix}}^{A} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \overbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 2 & 2 \end{bmatrix}}^{B} u \tag{7}$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{D} u \tag{8}$$

2.1.2. Environments and Testing Scenarios

Environment scenarios are created with consideration of regulation, trajectory tracking, disturbance and noise conditions. Two different scenarios have been determined. In theory, movement sequences do not affect an evaluation of controller performance for correctly tuned controllers. But in real-life conditions, nonlinearities and inertial effects may play unpredictable effects, especially in sequences that include start-stop or direction-changing/oscillating conditions. The first trajectory sequence is the classical step reference signal for trajectory tracking. The second sequence provides conditions to analyze oscillating trajectory tracking performance, which provides insights about properties under direction change.

Table 2. Position tracking performance test environment configurations

Sequence		1st sec	6th sec	11th sec	16th sec	Notes
Sequence 1	0	0.2m	0.2m	02.m	0.2m	Step reference of 0.2 m at 1st sec
Sequence 2	0	0.15m	-0.15m	0.15m	-0.15m	Oscillating Trajectory tracking (performance under direction change)

The proposed scenarios and various configuration options have been presented in Table 2. The disturbance is a 1N force change when implemented. The noise is a band-limited white noise with a $1x10^{-9}$ gain option on pole angle readings when utilized.

2.1.3. Explicit MPC

In implicit MPC (classical MPC), the optimization solution for control occurs in an online manner. Online optimization calculations require less memory but higher computational power. In the explicit MPC, the optimization calculations are made offline manner. The explicit MPC pre-solves optimization problems and converts the problem to regions in a piecewise affine manner which is easier to calculate. This approach demands higher memory with a less computational cost which is suitable for industrial applications. In Equation 9, the process has been shown. The complex optimization problem is reduced to arithmetic calculations [10].

$$\min_{z,\epsilon} \sum_{k=0}^{N-1} \frac{1}{2}(y_k - \mathbf{r_k})' Q_y (y_k - r_k) + \frac{1}{2}\Delta u_k' R_\Delta \Delta u_k + (u_k - \mathbf{u_k^r})' R (u_k - \mathbf{u_k^r})' + \rho_\epsilon \epsilon^2 \tag{9a}$$

$$s.t.\, x_{k+1} = Ax_k + Bu_k + B_v \mathbf{v_k} \tag{9b}$$

$$y_k = Cx_k + D_u u_k + D_v \mathbf{v_k} \tag{9c}$$

$$u_k = u_{k-1} + \Delta u_k, k = 0, \dots, N-1 \tag{9d}$$

$$\Delta u_k = 0, k = N_u, \dots, N-1 \tag{9e}$$

$$\mathbf{u_{min}^k} \le u_k \le \mathbf{u_{max}^k}, \boldsymbol{k} = \mathbf{0}, \dots, N_u - 1 \tag{9f}$$

$$\Delta u_{min}^k \le \Delta u_k \le \Delta_{max}^k, \mathrm{k} = 0, \dots, N_u - 1 \tag{9g}$$

$$y^k_{min} - \epsilon V_{min} \leq y^k_{max} + \epsilon V_{max}, k = 0, \dots, Nc - 1 \quad (9h)$$

In Equation 9; N is the prediction horizon, N_u is the control horizon, and N_c is constrain horizon. y_k is output vector, r_k is tracked reference, u^r_k is input reference, and u_k is the reference. R_Δ, Q_y, and R are matrices. v_k is the vector of measured disturbances, y_k is the output vector, r_k is tracked reference, Δu_k is the input increments vector and u^r_k is the input reference. u^k_{min}, u^k_{max}, Δu^k_{min}, Δ^k_{max}, y^k_{min} and y^k_{max} are limits.

2.1.4. Controller-Application of Explicit MPC

The simulation models for the explicit MPC method are constructed by utilizing the Model Predictive Control Toolbox of Matlab/Simulink. The Output Variable (OV) of the positional tracking weight w_x has been examined by a value of 0.75. Trajectory tracking is the primary objective of the study. Lower weight results in a lower cost of positional tracking, which directs the optimization process towards better trajectory tracking. Hence, the w_x can be selected as a lower value. MPC controllers have adequate pole balancing performance. Hence w_θ value is selected as 1.

2.2. Method

In the current study, two different comparisons were made using the Explicit MPC controllers with different control inputs, outputs and two different trajectories: disturbance and noise scenarios. In the first comparison, step trajectory tracking performance of controllers with control input constraints at three different levels (no limit, F=±1, F=±2) were examined. In the second comparison, the oscillating trajectory tracking the performance of controllers with the same three different levels of control input constraints were examined. Both comparisons are examined in scenarios without disturbance and noise effects without control input restrictions as the first steps. The relevant comparison options are shown in Table 3.

Table 3. Configurations for analyses

Analyzing performance effects of control input limit, disturbance, and noise	w_x	Control input limit
1st comparison: by using step trajectory	0.75	F = 0 F = ±1 F = ±2
2nd comparison: by using oscillating trajectory	0.75	F = 0 F = ±1 F = ±2

3. RESULTS AND DISCUSSIONS

The results of the first group of trials have been presented in Figure 3. The figure is based on a step trajectory reference of 0.2 m. 1N disturbance is introduced at the 7th second.

The first line (solid line) represents a pure response of the controller from the IPC without disturbance and noise scenario. The dotted line represents the controller configuration without control input limit under disturbance and noise conditions.

The dashed line represents the controller with limited control input (±1N) and the dash-dot line represents the controller with limited control input (±2N) under disturbance and noise scenarios.

The controller configurations (except F=±1N) have shown adequate trajectory tracking performance. The reference position has been achieved in 4 seconds. The effect of the noise is successfully compensated by the controllers. The controller with limited control output of ±1 was able to track trajectory until the introduction of disturbance. After the disturbance, the controller cannot produce the required control force which causes inadequate trajectory tracking performance due to limited control output level. However, the controller with F=±2N can produce adequate tracking performance with slight misses and overshoots. The misses are caused by the control input production limit. The limit causes response latency which also creates overshoots.

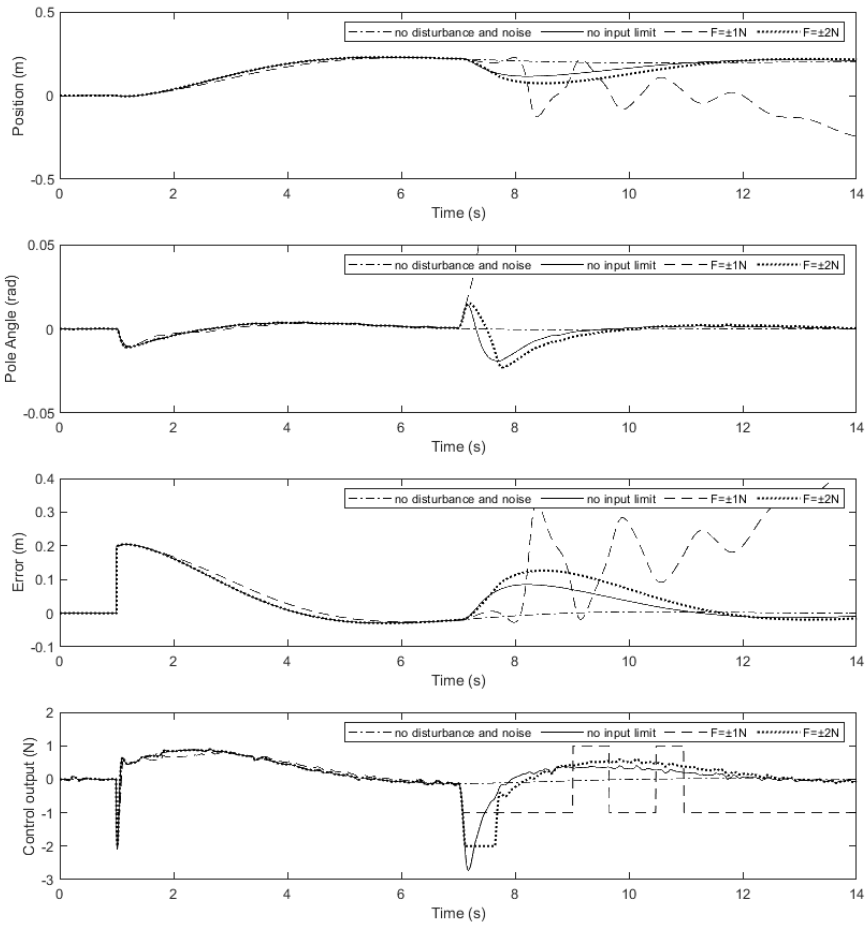

Figure 3. The results of the first group of tests which include a step trajectory (with/without noise and disturbance)

In the second scenario, an oscillating trajectory tracking the performance of explicit MPC controllers have been inspected. The results of the second group of tests have been shown in Figure 4.

Unlike the first scenario, the controller with control input limit ±1N showed inadequate position tracking performance before the introduction of disturbance input. This result is caused by the required control input by the direction change. ±1N limited controller cannot produce the required control signal for the direction change which caused the poor performance on trajectory tracking for the sequent movement. After the introduction of the disturbance, ±1N controller cannot produce a reasonable control input signal for the plant which causes total failure of its tracking performance. The controller with a ±2N limit produced adequate positional tracking performance, even after direction change and disturbance. The ±2N limited controller showed a slight tracking performance cost due to the limit on instantaneous control force limitation which causes a slight amount of overshoot when compared to the limitless controller. But the positional tracking performance differences of ±2N limited and limitless controller can be neglected.

The results of the trials have shown that an Explicit MPC with adequate control input limit can track positional trajectory with pole balance on various trajectory scenarios, disturbance effects, and noise effects.

4. CONCLUSION

Disturbance and noise have detrimental factors in a control system. Disturbance rejection and noise attenuation are some useful properties for the performance assessment of a controller. In this context, an explicit MPC-based controller can work under such undesirable conditions. With this motivation in mind, several Explicit MPC controller configurations of various control output limits have been implemented on the step and oscillating trajectory tracking with disturbance and noise scenarios. The IPC system was selected as the controlled plant due to IPC's under-actuated nonlinear system with non-minimum phase zero dynamics and its wide usage. Most of the controllers (except the controller with a ±1N limit) showed adequate position tracking performances even under disturbance and noise conditions.

The classic and Explicit MPC approaches are based on the linear control approach. To utilize those controllers on a nonlinear system, a linearization process has to be implemented. The linearization process alters the characteristics of a system for linearization reference state ranges. Unfortunately, there can be significant differences between nonlinear and linear characteristics. A linearized model might not reflect the reality of a system. Hence, controlling a nonlinear system might be beneficial for implementations. In a future study, an IPC system can be analyzed without linearization.

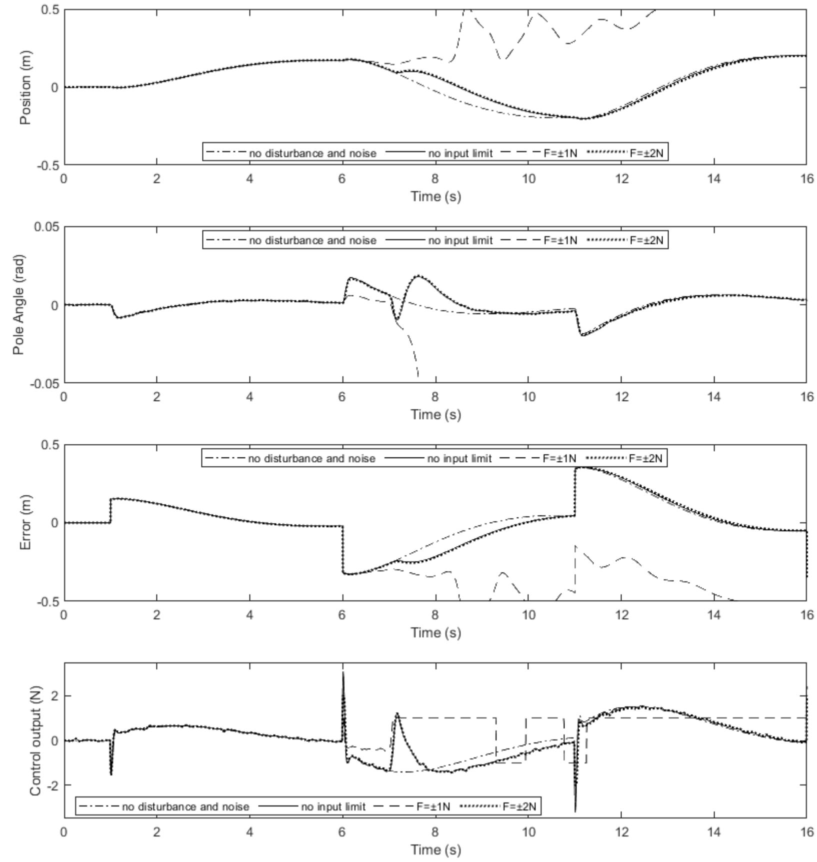

Figure 4. The results of the second group of tests which include an oscillating trajectory (with/without noise and disturbance)

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