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MULTI-SCALE SAMPLE ENTROPY ANALYSIS OF THE TURKISH STOCK MARKET EFFICIENCY

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ABSTRACT

This study evaluates the market efficiency of the market index and five main sector indices in the Turkish stock market: BIST 100 (XU100), BIST Industrials (XUSIN), BIST Services (XUHIZ), BIST Transportation (XULAS), BIST Financials (XUMAL), and BIST Technology (XUTEK) for the pre-and post-COVID-19 pandemic, covering the period from January 2017 to July 2022. Market efficiency is calculated using a multiscale entropy-based method for the scales of 1 to 20 business days. Entropy can yield a relative degree of efficiency, by contrast with previous methods that dealt with the efficiency question in all-or-nothing form. On a daily scale, during the pre-COVID-19 pandemic period, the XUHIZ, XU100 and XULAS indices exhibit the highest efficiency. However, in the post-COVID-19 pandemic period, the XUMAL and XU100 indices demonstrate the highest efficiency. The findings suggest that the efficiency of all indices has declined due to the COVID-19 pandemic, with the XULAS index showing the most significant decrease in informational efficiency. A general tendency of reduced informational efficiency levels is found as the time scale increases in both periods. Therefore, the indices are partially efficient for certain time scales, indicating that they are not fully efficient.

Keywords: Sample Entropy, Market Efficiency, COVID-19, Borsa Istanbul

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TÜRKİYE PAY PİYASASI ETKİNLİĞİNİN ÇOK ÖLÇEKLİ SAMPLE ENTROPİ İLE ANALİZİ

ÖZ

Bu çalışma, Türkiye borsasında piyasa endeksi ve beş ana sektör endeksinin piyasa etkinliğini değerlendirmektedir. Çalışmada kullanılan endeksler sırasıyla BIST 100 (XU100), BIST Sinai (XUSIN), BIST Hizmetler (XUHIZ), BIST Ulastırma (XULAS), BIST Mali (XUMAL), BIST Teknoloji (XUTEK) olup, calışma dönemi 01/2017-07/2022 tarihleri arasını kapsamaktadır. Veri seti iki ayrı döneme ayrılmış olup, çalışmada COVID-19 pandemisi öncesi ve sonrası için piyasa etkinliği arastırılmıştır. Piyasa etkinliği, 1 ila 20 işlem günü arasındaki ölçekler için çok ölçekli entropi yaklasımlı bir yöntemle hesaplanmaktadır. Entropi, piyasa etkinliği problemini ya hep ya hiç biçiminde ele alan önceki yöntemlerin aksine, göreceli bir etkinlik derecesi sağlayabilmektedir. Günlük ölçekte piyasa etkinliği tahmin edildiğinde, COVID-19 pandemi öncesi dönemde en yüksek etkinliğe XUHIZ, XU100 ve XULAS endeksleri sahipken, COVID-19 pandemi sonrası dönemde XUMAL ve XU100 endeksleri en yüksek etkinliğe sahip olduğu tespit edilmiştir. Sonuclar, COVID-19 salgını nedeniyle tüm endekslerin etkinliğinin düştüğünü ve XULAS endeksinin bilgi etkinliginin en fazla düşüşe sahip oldugunu göstermektedir. Her iki periyotta da zaman ölçeği arttıkça bilgi etkinligi seviyelerine ilişkin genel bir azalma eğilimi bulgulanmıştır. Bu nedenle, endeksler belirli zaman ölçekleri için tam olarak etkin olmayıp kısmi etkinliğe sahiptir.

Anahtar Kelimeler: Sample Entropi, Piyasa etkinliği, COVID-19, Borsa Istanbul

1. INTRODUCTION

The efficient market hypothesis (EMH) has been a highly contentious subject in finance theory. In its weak form (WEMH), the hypothesis claims that past market prices are fully reflected in asset prices, and thus no investment strategy can consistently outperform the market. The key assumption of weak form efficiency is based on the theory that price movements cannot be exploited due to the randomness of stock prices. In particular, daily price variations are assumed to be completely independent of each other, and there is no indication of price momentum.

Supporters of the EMH generally tolerate short serial correlations but reject long serial correlations. There is a substantial body of literature dedicated to exploring long-range dependence and informational efficiency. The presence of long memory in asset returns has been demonstrated since Mandelbrot's groundbreaking paper (Mandelbrot, 1971). However, it should be noted that in real-life applications, the existence of long-range dependencies presents several issues, apart from violating the WEMH. In the event that a series has a long memory, derivative pricing techniques like Black-Scholes may not be accurate, and tests based on the Capital Asset Pricing Model (CAPM) may not be appropriate.

A great deal of scientific literature on the efficient market hypothesis (EMH) in the Turkish stock market has been devoted to proving that prices follow a random walk behavior by examining the predictability of security returns based on historical price movements. They found mixed evidence. Yucel (2016) used unit root tests on 22 BIST indices and discovered that all indices were weak-form efficient. Karademir and Evci (2020) examined the validity of the efficient market hypothesis in BIST 100 and its sub-indices by using unit root tests. The investigation results revealed that the weak form efficiency was valid. Eyuboglu and Eyuboglu (2020) discovered that 13 indices were weak form efficient, while 9 indices were not efficient. Ildirar and Dalli (2021) examined the efficiency of the banking sector by employing unit root tests and uncovered that 11 banks were weak-form efficient. Altuntas et al. (2022) found that while BIST services index is weak-form efficient, the BIST Finance index is not.

In general, empirical findings in the literature remain controversial, as they have focused on analyzing whether price patterns can be exploited to outperform the market. Considering the complexity of the problem, new results should be geared toward not only demonstrating whether the Turkish market is efficient but also providing a quantitative indicator of the informational efficiency of the market. Additionally, assessing temporal changes in efficiency and their connection to socioeconomic factors is crucial. The entropy-based procedure offers deeper insights by calculating the degree of efficiency over time, and this approach has been employed in various markets (Alvarez-Ramirez et al. 2012; Oh et al. 2007; Ortiz-Cruz et al. 2012; Wang et al. 2012; Wang and Wang, 2021).

In this paper, multiscale entropy method is used to inspect the informational efficiency of the Borsa Istanbul composite index (BIST 100) and five main sector indices: BIST Industrials

(XUSIN), BIST Services (XUHIZ), BIST Transportation (XULAS), BIST Financials (XUMAL), and BIST Technology (XUTEK). Unlike previous techniques that addressed the efficiency question in an all-or-nothing manner, entropy can yield a relative degree of efficiency. Another goal of the paper is to investigate the Turkish market efficiency and analyze its responses to the Covid-19 pandemic. Thus, the data set is divided into two subsets: the first one contains before the pandemic, and the second one covers the pandemic and subsequent period. We believe this is the first attempt to assess the degree of Turkish market efficiency rather than simply addressing an all-or-nothing issue in keeping with earlier propositions. Overall, the empirical results indicate that the indices are found to be more efficient for shorter time scales (about days) compared to longer (up to twenty days) time scales, and the Covid-19 pandemic period demotes the Turkish market efficiency greatly.

The paper is arranged as follows. The following section explains the methodology used to analyze the indices' returns. Section 3 presents the data and parameter selection. Then, Section 4 shows the main empirical results. Finally, Section 5 concludes the paper.

2. METHODS

2.1. Sample Entropy

Sample entropy (SampEn) is an information-theoretical concept for quantifying the degree of regularity and predictability in time-series data. The method, developed by Richman and Moorman (2000), was originally used to analyze the short and noisy signals encountered in biomedicine. The SampEn measure yields a non-negative number to a time series, with higher scores indicating a greater degree of obvious process randomness or serial disorder, while lower results suggests more observations of clearly identifiable features or patterns in the data. That is, a time series with many repetitive patterns has a relatively low SampEn score; more complicated processes (i.e., less predictable) have a higher value. Therefore, the SampEn can be used as a measure of the market efficiency.

Let m represent the embedding dimension of two segments in a sequence to be compared, and let r denote the similarity threshold for accepting matches. Given a time series $\boldsymbol{x} = \{x_1, x_2, \ldots, x_N\}$ of N points, following are the steps for computing sample entropy:

Step 1. Form m-dimensional template vectors $x_m(i)$, sequence of vectors from 1 to N-m+1

$$x_m(i) = \{x_{i+k}: 0 \le k \le m-1\},\ \ 1 \le i \le N-m$$

Step 2. For each $x_m(i)$, the distance between two such vectors $x_m(i)$, and $x_m(j)$, is computed by employing the Chebyshev distance:

$$d(x_m(i), x_m(j)) = \left|\left|x_m(i), x_m(j)\right|\right| = max\left\{\left|x_{i+k} - x_{j+k}\right|: 0 \le k \le m-1\right\}, 1 \le i, j \le N-m, j \ne i$$

Step 3. Give the threshold parameter r, let $n_i^m(r)$ represent the number of matches for $x_m(i)$ within a distance r. $C_i^m(r)$ is expressed as follows:

$$C_i^m(r) = \frac{n_i^m(r)}{N-m+1} \tag{1}$$

Step 4. Compute

$$C^m(r) = \frac{1}{(N-m+1)} \sum_{i=1}^{N-m+1} \ln C_i^m(r) \tag{2}$$

Step 5. Raise the dimension from m to m+1, then replicate the above procedure to get $C^{m+1}(r)$.

Step 6. Sample entropy is computed as:

$$SampEn(m, r) = -ln\frac{C^{m+1}(r)}{C^m(r)} \tag{3}$$

2.2. Multiscale Sample Entropy

Given a time series $x = \{x_1, x_2, \ldots, x_N\}$ of N points, the algorithm for MSE estimation is described as follows

Step 1. Create a series of consecutive coarse-grained series based on a scale factor **s**. The coarse-grained time series $\boldsymbol{y^s}$ is described as:

$$y_j^s = \frac{1}{s}\sum_{i=(j-1)s+1}^{js} x_i\,,\ \ 1 \le j \le \frac{N}{s} \tag{4}$$

where N/s refers the length of the coarse-gained time series. When scale factor s=1, coarse-grained time series is the original series $\boldsymbol{x}$.

Step 2. Compute the SampEn value for each coarse-grained series.

2.3. Relative Informational Efficiency Index

According to the EMH, a weakly efficient market should have asset prices that are random walks and returns that are Gaussian white noise. As a result, the entropy of Gaussian white noise is employed to measure the entropy of asset prices in efficient markets. For each time scale s, relative market efficiency is defined as follows according to (Ortiz-Cruz et al., 2012):

$$I_{IME(s)} = \frac{MSE(s)}{\beta(s)} x100\% \quad (5)$$

where $\beta(s)$ represents the upper bound for Gaussian white noise entropy based on 10,000 Monte Carlo simulations. Market efficiency is only fractional if the entropy of asset return is below that of Gaussian white noise, or $I_{IME(s)} < 100\%$.

3. DATA AND PARAMETER SELECTION

The dataset used for this study is composed of historically daily closing prices of the Borsa Istanbul composite index (XU100) and five main sector indices: Services (XUHIZ), Transportation (XULAS), Industrials (XUSIN), BIST Financials (XUMAL), and Technology (XUTEK). The data covers the period from 2 January 2017 to 22 July 2022, and includes 1392 trading days. The dataset is divided into two periods by the COVID-19 pandemic: the first period from 2 January 2017 to 31 December 2019[1], and the second period from 2 January 2020 to 22 July 2022. To account for the non-stationarity, the daily closing prices are converted to logarithmic returns for each index as follows:

$$R_i(t) = \log P_i(t) - \log P_i(t-1) \quad (6)$$

where $R_i(t)$ denotes the logarithmic return of i-th stock at time t, and $P_i(t)$ and $P_i(t-1)$ define the closing price of the i-th stock on day t and t-1, respectively.

The embedding dimension m is set to 2, and the tolerance range r is set to 0.25σ, where σ is the standard deviation of the return series following (Richman and Moorman, 2000; Wang et al., 2012). As for the time scale *s*, we set it up to 20 business days, which equals four weeks and

[1] It is the date on which China notified the World Health Organization (WHO) of the first COVID–19 case.

includes the efficiency indexes on a daily basis (1 day), weekly basis (5 days), and monthly basis (20 days) separately.

4. RESULTS

Figure-1 and Figure-2 illustrate the multiscale results of SampEn and relative informational efficiency for scales up to 20 days for both Period I and Period II. Using a coarse-graining filter, we observe that an increase in time scale corresponds to a decrease in the sample entropy, indicating that pattern diversity is decreasing for both periods. In some cases, as in Period II, major local maximums are found for scales higher than two weeks. As an example, all indices in Period II exhibits a local maximum at 10 business days, which implies that the long-run price formation was greatly influenced by information flows aggregated at biweekly time-scales. According to Gulko (1999), it is more challenging to forecast the price behavior as there are more price patterns (maximum entropy). Thus, lower entropy values of indices indicate less information richness and more predictable prices at longer time scales for both periods. The results are consistent with previous studies on financial markets that have found that the complexity of prices is scale-dependent (Alvarez-Ramirez et al., 2012; Ortiz-Cruz et al., 2012; Wang and Wang, 2021).

The bottom plots in Figure-1 and Figure-2 present the relative informational efficiency across different time scales. In general, the findings indicate that the informational efficiency levels decline as time scales increase for both periods. For Period I, as measured on a daily scale, the XU100, XUHIZ, and XULAS indices have the highest efficiency at 84%, followed by XUSIN (83%), XUMAL (%81), and XUTEK Index, which is the least efficient at 69%. On the other hand, for Period II, as estimated on a daily scale, the XUMAL index has the highest efficiency at 75%, followed by XU100 (72%), XUHIZ (71%), XUSIN (68%), XULAS (67%), and XUTEK Index, which is the least efficient at 67%.

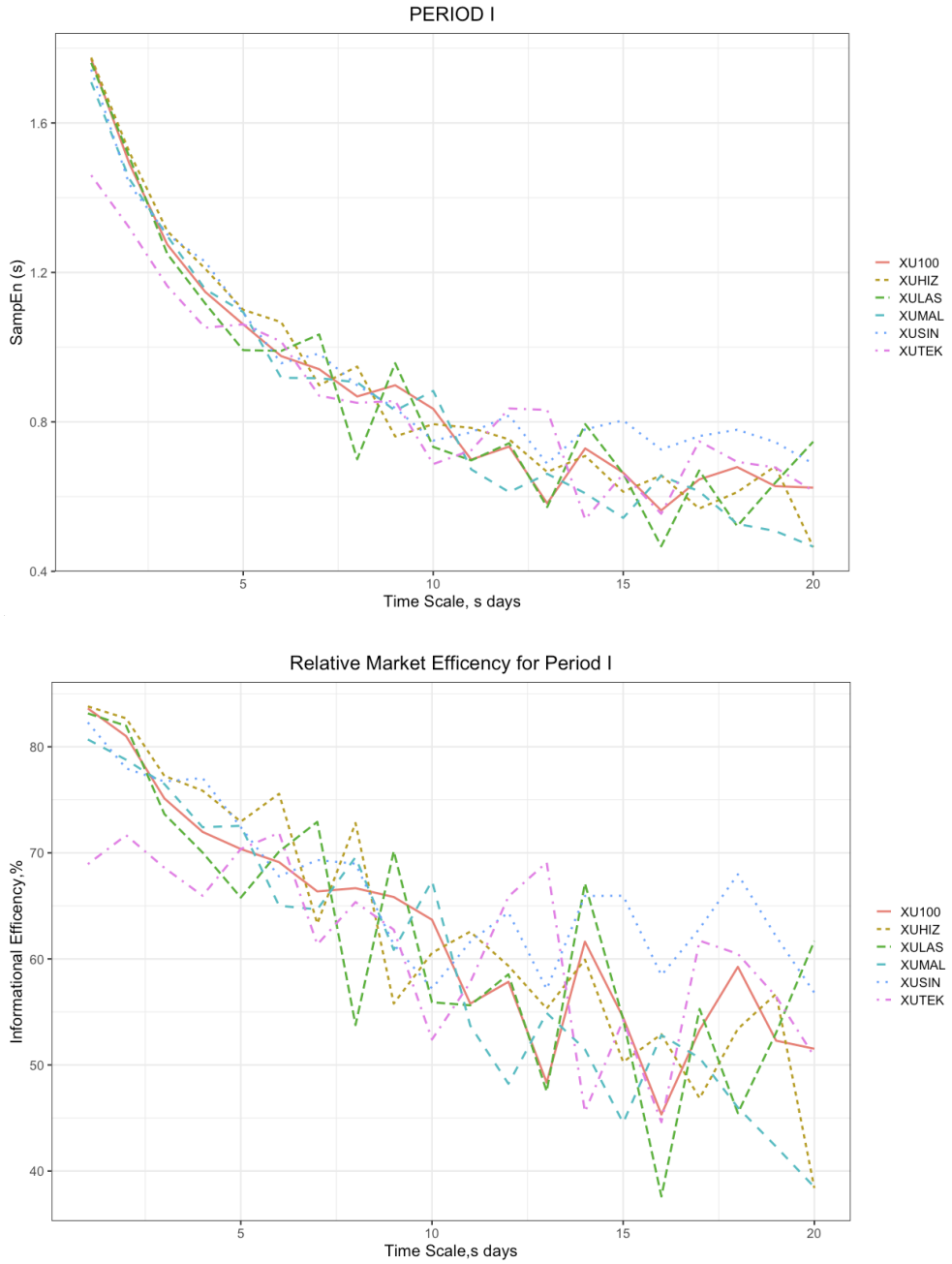

Figure 1. MSE and relative informational efficiency index across time scales for period-1

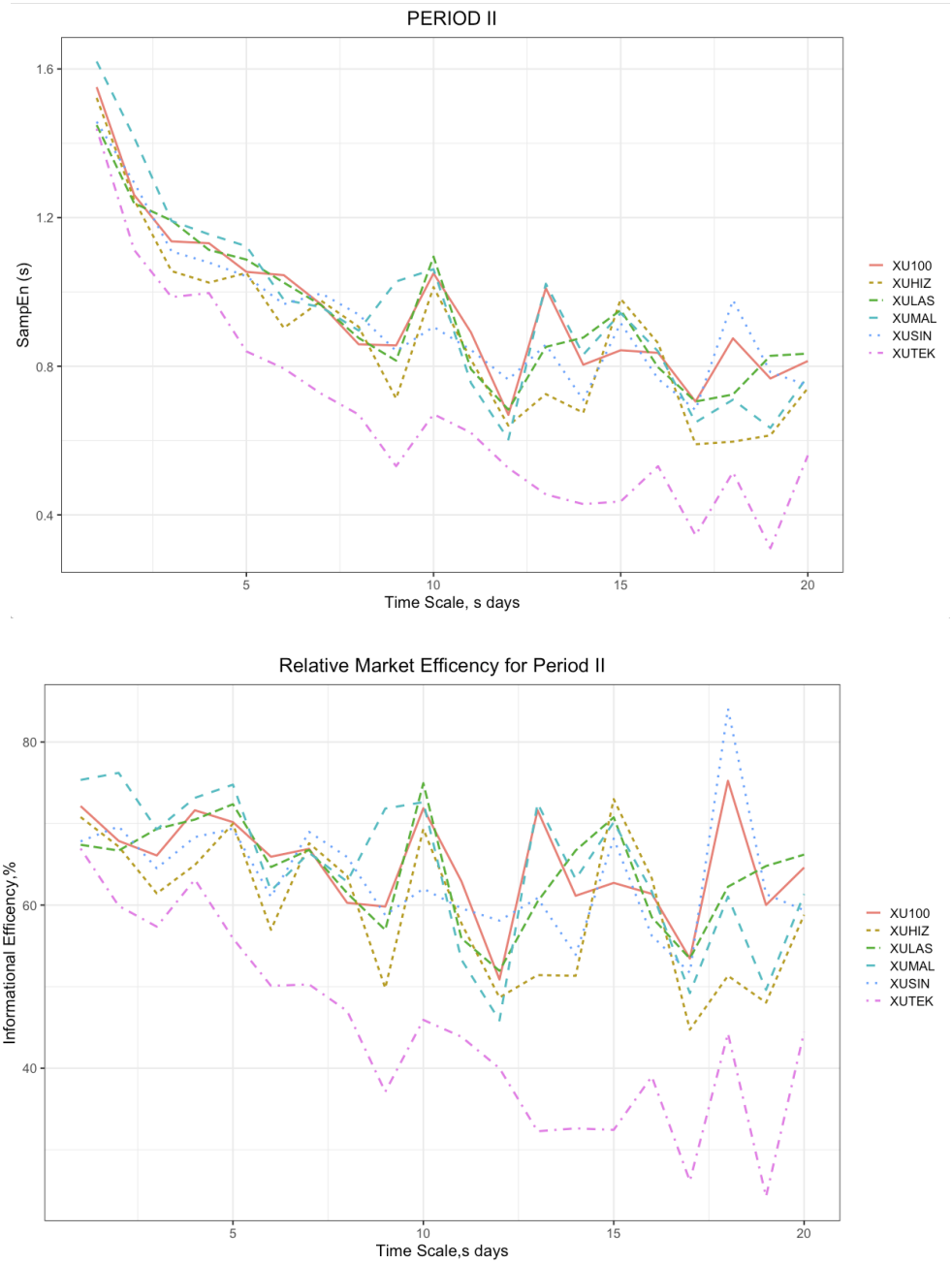

Figure 2. MSE and relative informational efficiency index across time scales for period-2

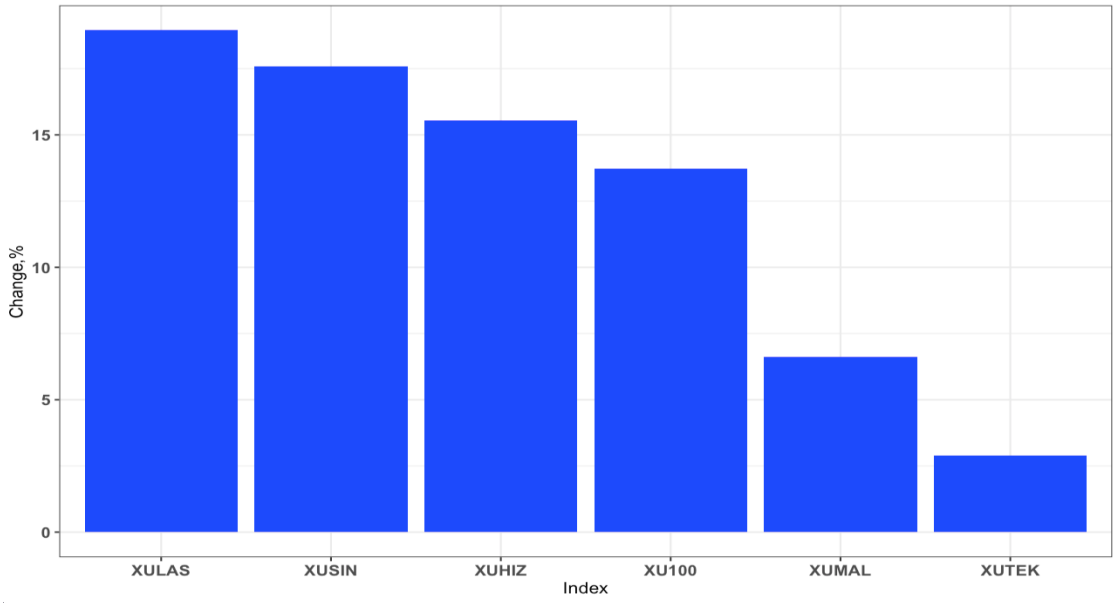

Figure 3. Change of relative informational efficiency index in percentage

A comparison of the relative informational changes of each index during these two periods is presented in Figure 3. It can be seen in the figure that informational efficiency values across all indices decreased during Period II. The XULAS index experienced the greatest decline in informational efficiency with a decrease of 18.94%. This was followed by a 17.6 % decrease in XUSIN, a 15.53% decrease in XUHIZ, and a 13.73% decrease in XU100. On the other hand, both XUMAL and XUTEK suffered the slightest decreases, falling by 6.62 % and 2.91 % respectively. Therefore, the impact of the pandemic on indices is different. It has been observed that while the informational efficiency of some indices has decreased significantly, others have experienced more modest declines.

In light of these results, it seems unlikely that the returns of indices can be explained by a random walk behavior, as they present a decreased number of entropy patterns as a function of time-scale for both periods. Furthermore, the indices are not fully efficient at certain time scales; therefore, they are partially efficient due to departures from 100%.

5. CONCLUSION

This paper assesses the market efficiency of the market index and five main sector indices in the Turkish stock market: XU100, XUSIN BIST Industrials (XUSIN), BIST Services (XUHIZ), BIST Transportation (XULAS), BIST Financials (XUMAL), BIST Technology (XUTEK) during the pre-and post-COVID-19 pandemic, covering the period from January 2017 to July 2022. Market efficiency is assessed using a multiscale entropy-based informational efficiency index that utilizes the relative informational efficiency index across a range of time scales from 1 to 20 business days.

The outbreak of the COVID-19 pandemic has had a significant impact on global financial markets. While several studies have investigated different aspects of its effects on the Turkish stock market, there is a lack of research on market efficiency during the pandemic and subsequent periods. This paper addresses the gap in the existing literature by analyzing the time-varying informational efficiency of the Turkish stock market.

We compare the time-varying information efficiency of all indices by splitting data sets into two periods as the pre-and post-COVID-19 pandemic. Our findings indicate that the COVID-19 pandemic has resulted in a decrease in efficiency across all indices, which aligns with the findings of previous studies by (Alijani et.al., 2021; Choi, 2021). As estimated on a daily scale, during the pre-COVID-19 pandemic period, the XUHIZ, XU100 and XULAS indices have the highest efficiency, while in the post-COVID-19 pandemic period, the XUMAL and XU100 indices have the highest efficiency. It appears that the efficiency of all indices has declined due to the COVID-19 pandemic, with the XULAS index demonstrating the greatest decrease. The results indicate that the returns of the indices cannot be explained by a random walk model, as they show a reduced number of entropy patterns as a function of the time scale for both periods. Additionally, the indices are not completely efficient at particular time scales; thus, they are partially efficient due to deviations from 100%. The findings presented in this paper provide valuable insights to fund managers, institutional investors, and individuals who are interested in making informed decisions regarding asset allocation during periods of economic uncertainty.

ETHICAL DECLARATION

In the writing process of the study titled “Multi-Scale Sample Entropy Analysis of the Turkish Stock Market Efficiency”, there were followed the scientific, ethical and the citation rules; was not made any falsification on the collected data and this study was not sent to any other academic media for evaluation.

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