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Research Article

Magnetization of a Quantum Dot Superlattice System in the Presence of aRashba Spin-Orbit Coupling Term

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Abstract

In this paper, we have investigated the magnetization and the magnetic susceptibility of a quantum dot superlattice system in the presence of a magnetic field parallel to the superlattice axis and confined in a lateral parabolic quantum well. We took the effect of the Rashba spin-orbit interaction and Zeeman term on the specific heat into account. We have calculated the energy spectrum of the electron in the quantum dot superlattice system. Moreover, we have calculated the magnetization and the magnetic susceptibility dependence on the magnetic field and temperature of a quantum dot superlattice system.

Keywords: Superlattice, Rashba spin-orbit interaction, Quantum dot.

Rashba Spin-Yörünge Etkileşimli Kuantum Nokta Süper Örgü Sisteminin Manyetizasyonu

Öz

Bu makalede, süper örgü eksenine paralel manyetik alan varlığında ve parabolik kuantum kuyusunda sınırlandırılmış kuantum nokta süper örgü sisteminin manyetizasyonunu ve manyetik duyarlılığını araştırdık. Rashba spin-yörünge etkileşiminin ve Zeeman teriminin özgül ısı üzerindeki etkisini dikkate aldık. Kuantum nokta süper örgü sistemindeki elektronun enerji spektrumunu hesapladık. Ayrıca, kuantum nokta süper örgü sisteminin manyetizasyon ve manyetik duyarlılığının manyetik alana ve sıcaklığa bağımlılığını hesapladık.

Anahtar Kelimeler: Süperörgü, Rashba spin-yörünge etkileşimi, Kauntum Nokta

1. Introduction

Low-dimensional systems based on semiconductors have been the object of great interest for many years since then there have been numerous applications based on these systems. Recently, a great deal of attention is being given to the study of transport phenomena in low-dimensional systems. In such systems, transport phenomena are sharply different from transport phenomena in macrosystems. The Landau diamagnetism of an electron gas in superlattices was presented in the paper [1], and it was found that the magnetization of a strongly degenerate electron gas changes its sign depending on the degree of band filling and magnetic field magnitude. In Ref.[2] the influence of interband and intraband transitions on vertical conductance oscillations in superlattices in a strong longitudinal magnetic field is considered. The thermal properties of a system, comprising spinless noninteracting charged particles in the presence of a constant external magnetic field and confined in a parabolic quantum well were studied in Ref.[3].

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The thermodynamic properties of an InSb quantum dot have been investigated in paper [4] in the presence of Rashba spin-orbit interaction and a static magnetic field.

In Ref. [5] the heat capacity used for the electronic system in the quantum dot superlattices, including the Zeeman effect of GaAs, InAs, and InSb was investigated, respectively, by using an isotropic parabolic potential under a magnetic field based on canonical ensemble statistics.

Voskoboynikov et. al. [6] presented a theoretical study of the effect of spin-orbit interaction on the electron magnetization and magnetic susceptibility of quantum dots at the weak magnetic field for degenerate electron gas.

B. Boyacioglu et. all [7] studied the heat capacity and entropy in GaAs quantum wire taking Gaussian potential as confinement in the presence of a magnetic field using the canonical ensemble approach. They found that at high temperatures, entropy is a monotonic increasing function of temperature but decreases with an increase in the magnetic field at a sufficiently low temperature.

In this paper, we have investigated the magnetization of a quantum dot superlattice system in the presence of external magnetic fields and Rashba spin-orbit interaction in the case non-degenerate electron gas.

We consider the thermal properties of an electron system in a quantum dot superlattice structure with a periodic potential of a period along the z-direction by using a cosine shape under the tight-binding approximation V(z)= $\varepsilon_0\left(1-\cos ak_z\right)$. Here ε_0-is the miniband half-width of the superlattice, k_z is the wave vector.

2. Material and Methods

The electron gas in the quantum dot superlattice structure is assumed to be confined in parabolic lateral potential [6]:

$$V_c\left(\rho\right)=\frac{1}{2}m\omega_0^2\rho^2 \tag{1}$$

Where $\hbar\omega_0$ is the characteristic confinement energy, and ρ is the radius vector. A static magnetic field B is applied parallel to the superlattice z-axis.

In an axial magnetic field of the symmetric gauge for the vector potential $\vec{A}=\frac{B\rho}{2}\vec{e_\phi}$ (where ϕ −is the azimuthal angle) the Rashba spin-orbital term in the cylindrical coordinates is given by

$$V_{SO}^{R}\left(\rho,\phi\right)=\sigma_z\alpha\frac{dV_c\left(\rho\right)}{d\rho}\left(-i\frac{1}{\rho}\frac{\partial}{\partial\phi}+\frac{e}{2\hbar}B\rho\right) \tag{2}$$

α is the Rashba spin-orbit coupling parameter, σ_z -is the Pauli z matrix. By including the Zeeman term Hamiltonian can be written as:

$$H=-\frac{\hbar^2}{2m}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}\right)-i\frac{\hbar\omega_c}{2}\frac{\partial}{\partial\phi}+\frac{1}{8}m\omega_c^2\rho^2+\frac{\sigma_z}{2}g\mu_B B+V_c\left(\rho\right)+V\left(z\right)+V_{SO}^{R}\left(\rho,\phi\right) \tag{3}$$

where B-is the applied magnetic field, $\omega_c=\frac{eB}{m}$ is the cyclotron frequency, μ_B -is the Bohr magneton, m-is the electron effective mass, and g -is the spin decoupling. The eigenfunctions and eigenenergies of the Hamiltonian are given by

$$\Psi\left(\rho,\phi,k_z\right)=\frac{1}{\sqrt{2\pi L}}e^{ik_z\phi+il\phi}R\left(\rho\right)\chi_\sigma \tag{4}$$

where

$$R\left(\rho\right)=\frac{\sqrt{2}}{\rho_\sigma}\sqrt{\left(\frac{n!}{n+|l|}\right)}\exp\left(-\frac{\rho^2}{2\rho_\sigma^2}\right)\left(\frac{\rho^2}{\rho_\sigma^2}\right)^{\frac{|l|}{2}}L_n^{|l|}\left(\frac{\rho^2}{\rho_\sigma^2}\right) \tag{5}$$

Where $\rho_\sigma=\frac{\hbar}{m\Omega_\sigma}$, and $L_n^{|l|}\left(\frac{\rho^2}{\rho_\sigma^2}\right)$ the generalized Laguerre polynomial and χ_σ is the spin wave function, L is the z directional normalization lengths

$$\varepsilon_{nl\sigma k_z}=\hbar\Omega_\sigma\left(2n+|l|+1\right)+\frac{l}{2}\hbar\omega_c+\sigma\left(\frac{1}{2}g\mu_B B+lm\alpha\omega_0^2\right)+\varepsilon_0\left(1-\cos ak_z\right) \tag{6}$$

It is convenient to reparametrize these quantum numbers in terms of a pair of integers n_+, n_-, defined as

$$n_+=\frac{2n+|l|+l}{2};n_-=\frac{2n+|l|-l}{2};n_+-n_-=l \tag{7}$$

With $n_\pm=0,1,2,\ldots\infty$. The energy spectrum can be rewritten as

$$\varepsilon_{n_+n_-\sigma k_z} = \hbar\omega_-^\sigma\left(n_- + \frac{1}{2}\right) + \hbar\omega_+^\sigma\left(n_+ + \frac{1}{2}\right) + \frac{\sigma}{2} g\mu_B B + \varepsilon_0\left(1 - \cos ak_z\right) \tag{8}$$

where

$$\omega_+^\sigma = \Omega_\sigma + \frac{\omega_c}{2} + \frac{m\alpha\omega_0^2}{\hbar};\omega_-^\sigma = \Omega_\sigma - \frac{\omega_c}{2} - \frac{m\alpha\omega_0^2}{\hbar}; \tag{9}$$

where $\sigma = \pm 1$ refers to the electron-spin polarization along the z-axis.

$$\Omega_\sigma = \sqrt{\omega_0^2 + \frac{\omega_c^2}{4} + \sigma\alpha\omega_c\frac{m\omega_0^2}{\hbar}} \tag{10}$$

The grand thermodynamic potential of a nondegenerate electron system is given as:

$$\Omega = -\frac{1}{\beta}\sum_{n_+n_-\sigma k_z} e^{-\beta\left(\mu - \varepsilon_{n_+n_-\sigma k_z}\right)} \tag{11}$$

where $\varepsilon_{n_+n_-\sigma k_z}$ -is the energy spectrum of the considered system, $\beta = \frac{1}{k_B T}$ with k_B being the Boltzmann constant, $and\ \mu$ is the chemical potential of the electron gas. We pass from the summation concerning kz to the integration over k_z and after summation for n_+, n_- equation (11) takes the form

$$\Omega = -\frac{LI\left(0,\beta\varepsilon_0\right)e^{\beta\left(\mu-\varepsilon_0\right)}}{\beta a}\left(\frac{e^{\beta\frac{\mu_B}{2}gB}}{4\sinh\left(\beta\frac{\hbar\omega_+^+}{2}\right)\sinh\left(\beta\frac{\hbar\omega_-^+}{2}\right)} + \frac{e^{-\beta\frac{\mu_B}{2}gB}}{4\sinh\left(\beta\frac{\hbar\omega_+^-}{2}\right)\sinh\left(\beta\frac{\hbar\omega_-^-}{2}\right)}\right) \tag{12}$$

where $I(0, \beta\epsilon_0)$ is the modified Bessel function zero-order, and L is the length along the z-direction. We can find the total number of particles N of the system as

$$N = -\left(\frac{\partial\Omega}{\partial\mu}\right)_{V,T} = -\frac{LI\left(0,\beta\varepsilon_0\right)e^{\beta\left(\mu-\varepsilon_0\right)}}{a}\left(\frac{e^{\beta\frac{\mu_B}{2}gB}}{4\sinh\left(\beta\frac{\hbar\omega_+^+}{2}\right)\sinh\left(\beta\frac{\hbar\omega_-^+}{2}\right)} + \frac{e^{-\beta\frac{\mu_B}{2}gB}}{4\sinh\left(\beta\frac{\hbar\omega_+^-}{2}\right)\sinh\left(\beta\frac{\hbar\omega_-^-}{2}\right)}\right) \tag{13}$$

The chemical potential of a non-degenerate electron gas in a magnetic field from (13), we obtain

$$\mu = \frac{1}{\beta}\ln\left[\frac{Na}{L}\frac{e^{\beta\varepsilon_0}}{I(0,\beta\varepsilon_0)}\left(\frac{e^{\beta\frac{\mu_B}{2}gB}}{4\sinh\left(\beta\frac{\hbar\omega_+^+}{2}\right)\sinh\left(\beta\frac{\hbar\omega_-^+}{2}\right)}+\frac{e^{-\beta\frac{\mu_B}{2}gB}}{4\sinh\left(\beta\frac{\hbar\omega_+^-}{2}\right)\sinh\left(\beta\frac{\hbar\omega_-^-}{2}\right)}\right)^{-1}\right] \quad (14)$$

The magnetization of an electron gas M can be found in the form of the grand thermodynamic potential as follows [9]:

$$M = -\frac{1}{V}\left(\frac{\partial\Omega}{\partial B}\right)_{\mu,T} \quad (15)$$

The magnetic susceptibility

$$\chi = \frac{\partial M}{\partial B} \quad (16)$$

3. Results and Discussion

We introduce dimensionless parameters $x = \frac{\omega_c}{\omega_0}$ the quantity of the magnetic field strength, which $\xi = \frac{2}{\beta\hbar\omega_0}$ represents the temperature measured, $z = \frac{2\varepsilon_0}{\hbar\omega_0}$ to represent the miniband half-width, and $\lambda = \alpha\frac{m\omega_0}{\hbar}$ to represent the Rashba spin-orbit coupling. For our calculation we consider the parameter corresponding to InSb materials:$\mathrm{m} = 0.014\mathrm{m}_0$, where m_0 is the free electron mass, and $\varepsilon_0 = 1\mathrm{meV}$[8],g = 3.2 ,$\alpha = 500\mathrm{A}^2$, $\hbar\omega_0 = 0.025\mathrm{eV}$ are taken from the literature [8].

In Fig.1 we plot the reduced magnetization and the reduced magnetic susceptibility as a function of the reduced magnetic field x for the fixed value of the dimensionless parameter ξ=0.5, and the dimensionless Rashba parameter λ=0;0.022. As seen from Fig. 1 in InSb type quantum dot superlattice, the magnetization and the magnetic susceptibility change from a negative value (diamagnetic) to a positive value (paramagnetic) with magnetic field value λ=0.022. For parameter λ=0 the magnetic susceptibility and magnetization take a negative value. These results are consistent with the results in the literature [1].

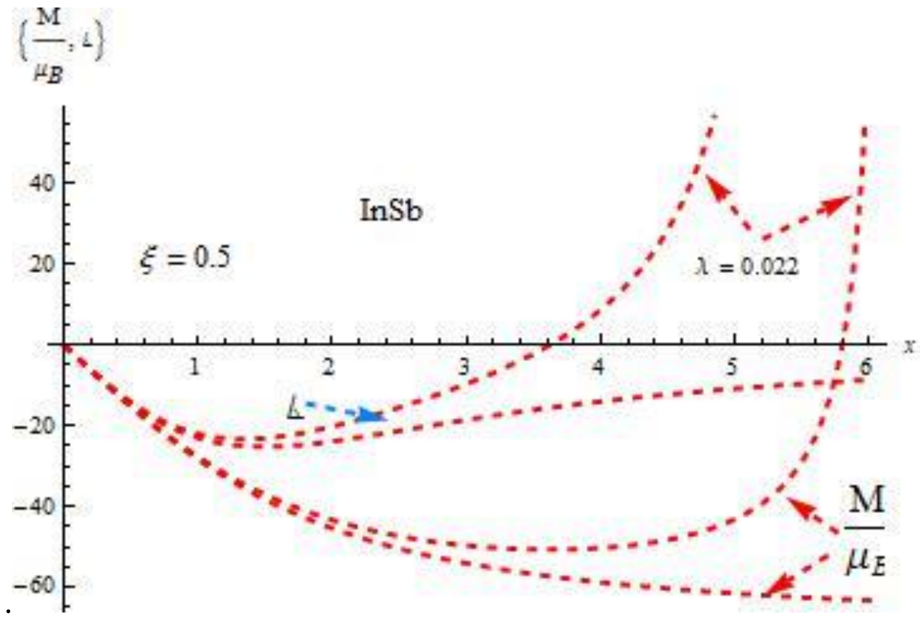

Fig.1.The reduced magnetization and the reduced magnetic susceptibility as a function of the reduced magnetic field x for a fixed value of the dimensionless parameter ξ=0.5, and the dimensionless Rashba parameter λ=0;0.022.$L = \frac{\chi}{\chi_0}, \chi_0 = \frac{N\mu_B}{B}$.

Fig.2 shows the behavior of reduced magnetic susceptibility and reduced magnetization versus the temperature for two values of Rashba parameters, at fixed reduced magnetic field x=0.5. It is seen from the figure that the system has diamagnetic behavior at the dimensionless Rashba parameter λ=0. For the dimensionless Rashba parameter λ=0.022, but with increasing the temperature the susceptibility shows a transition from a negative value (diamagnetic) to a positive value (paramagnetic) at a fixed magnetic field.

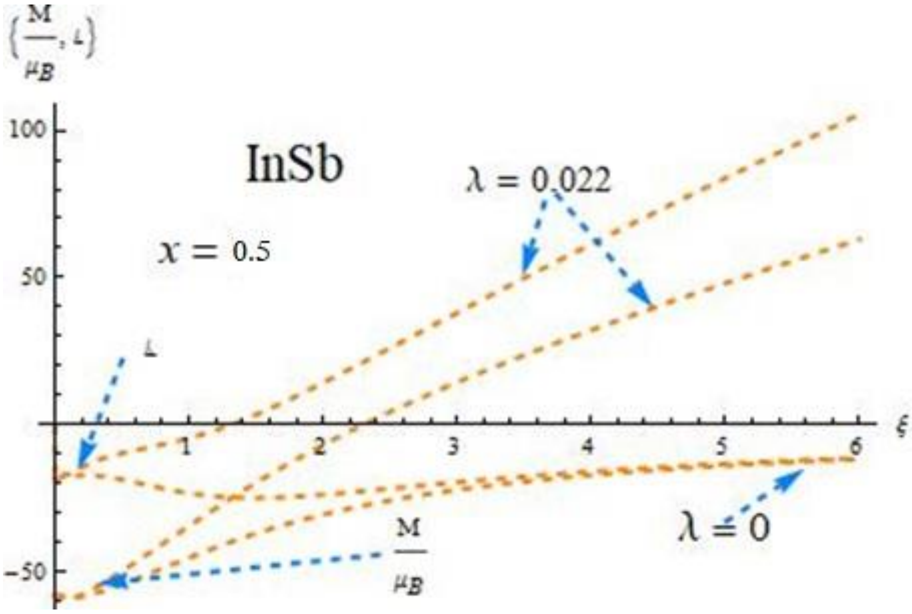

Fig.2. The reduced magnetic susceptibility and reduced magnetization versus the temperature for two values of Rashba parameters λ=0;0.022, at fixed reduced magnetic field x=0.5. $L = \frac{\chi}{\chi_0}, \chi_0 = \frac{N\mu_B}{VB}$.

4. Conclusions

We have calculated the magnetic susceptibility and magnetization of the quantum dot superlattice with Rashba spin-orbit coupling in the magnetic field for non-degenerate electron gas. Canonical formalism is used to calculate magnetization. We have investigated the dependence of the magnetic susceptibility and magnetization on the magnetic field and the temperature in quantum dots superlattice. As a function of temperature, magnetic susceptibility, and magnetization change signs.

Ethics in Publishing

There are no ethical issues regarding the publication of this study.

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