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Evaluation of the Probability Teaching-Learning Process Based on Mathematics Teachers' Views*

Olasılık Öğretme-Öğrenme Sürecinin Matematik Öğretmenlerinin Görüşlerine Dayalı Olarak Değerlendirilmesi

Selçuk Fırat[1], Ramazan Gürbüz[2]

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Abstract: In this study, it is aimed to evaluate the probability teaching-learning process based on the views of mathematics teachers. In the research case study which is one of the qualitative research methods was used. The participants of the study consisted of eight middle and eight high school mathematics teachers and 66 students of these teachers were determined by purposive sampling. The data of the research were collected in three stages by using semi-structured interview forms and probability problems. The data obtained from the study were analyzed by open coding and axial coding methods. According to the results obtained from the first and second stages of the research, most of the teachers stated that the time allocated for the probability teaching is not enough to perform conceptual learning. High school teachers also point out that different outcomes at different levels are causing problems in the teaching of probability. While middle school teachers stressed the need to begin at an earlier age to probability teaching, high school teachers often said that they had to start with the high school level. Teachers stated that they used routine question types in the probability teaching process. In line with their views, it was determined that they reached correct solutions more than non-routine ones. Moreover, teachers made more realistic predictions about their students' solutions to routine problems than non-routine problems.
Keywords: Probability, Probability Teaching-Learning Process, Middle School Teachers, High School Teachers.

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Öz: Bu çalışmada olasılık öğretme-öğrenme sürecinin matematik öğretmenlerinin görüşlerine dayalı olarak değerlendirilmesi amaçlanmıştır. Nitel araştırma yöntemlerinden durum çalışması deseni ile gerçekleştirilmiş olan çalışmada katılımcılar amaçsal örneklem yolu ile belirlenen sekiz ortaokul, sekiz lise matematik öğretmeni ve bu öğretmenlerin 66 öğrencisinden oluşmaktadır. Araştırmanın verileri yarı yapılandırılmış görüşme formları ve olasılık problemleri kullanılarak üç aşamada toplanmıştır. Araştırmadan elde edilen veriler, açık kodlama ve eksensel kodlama yöntemleri kullanılarak analiz edilmiştir. İlk iki aşamadan elde edilen verilerin analizi sonucunda öğretmenlerin çoğunluğunun olasılık konusuna ayrılan sürenin kavramsal öğrenmeyi gerçekleştirmek için yeterli olmadığı görüşünde oldukları belirlenmiştir. Lise öğretmenleri, kazanımların dağınık olmasından memnun olmadıklarını ve bu durumun olasılık öğretiminde sorunlara neden olduğunu belirtmişlerdir. Ortaokul öğretmenleri, olasılık konusunun öğretiminin daha küçük yaşlardan başlaması gerektiğini vurgulamışlar ancak lise öğretmeleri ise genel olarak lise seviyesinden başlaması gerektiğini belirtmişlerdir. Öğretmenler olasılık öğretim sürecinde rutin problem tiplerini kullandıklarını belirtmişler ve bu görüşleriyle paralel olarak bu sorularda daha fazla doğru çözüme ulaşmışlardır. Ayrıca öğretmenler öğrencilerinin çözümleri hakkında rutin olmayan problemlere göre rutin problemlerin çözümünde daha gerçekçi tahminlerde bulunmuşlardır.
Anahtar Kelimeler: Olasılık, Olasılık Öğretme-Öğrenme Süreci, Ortaokul Matematik Öğretmenleri, Lise Matematik Öğretmenleri.

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[1] Sorumlu Yazar: Dr. Öğr. Üyesi Selçuk Fırat, Adıyaman Üniversitesi, Eğitim Bilimleri, selcukfirat@adiyaman.edu.tr, ORCID: https://orcid.org/0000-0002-2935-2929

[2] Prof. Dr. Ramazan Gürbüz, Adıyaman Üniversitesi, Matematik ve Fen Bilimleri Eğitimi, rgurbuz@adiyaman.edu.tr, ORCID: https://orcid.org/0000-0002-2412-5882

1. INTRODUCTION

In the 21st century, it is important to raise individuals who have higher-level thinking skills instead of individuals who readily obtain information. The subject of probability includes factors such as risk and chance that will shape the mathematics of the future, which belong to the ability to construct the knowledge that individuals of today's world should have. While Shaughnessy (1992) stated the importance of probability as "Perhaps no other subject in mathematics is more important than probability and statistics for students", Bagehot (1956) stated that "life is a school of possibility" so that probability finds its place in every aspect of our lives. Probability is an ancient mathematical discipline that deals with calculating the probability of various events (HodnikČadež & Maja Škrbec, 2011). In addition, this subject is very important in terms of developing creative thinking skills and probability-based thinking skills that are one of the most important purposes of mathematics (Gürbüz, 2007). In our daily lives, we consciously or unconsciously use probability. Because it is a part of our daily vocabulary that we use frequently and fluently (Njenga, 2010). Expressions such as definitely, probably, and most likely are concepts that belong to the field of probability that we often use. Conditional probability, proportional reasoning, random variables, and expectation are the probability concepts that enable us to understand situations in our daily life (Batanero, 2020; Batanero, Chernoff, Engel, Lee & Sánchez, 2016, Ingram, 2022). Thus, probability affects the way we interpret the world around us and shapes many of the critical decisions in our lives (Njenga, 2010). Therefore, we live in a society where probabilistic skills are required to survive (Taylor, 2011). For this reason, teaching probability and gaining probability skills are considered important for societies. According to Fishbein and Gazit (1984), the reason for teaching probability is to decide between uncertain situations (critical interpretation), predict, solve problems and develop different thinking methods from deterministic thinking.

Due to the importance of the subject of probability, there are many studies on its teaching and learning. Jones, Langrall, and Mooney (2007) mentioned the need to understand the theoretical nature of probability, its different perspectives, and especially the relationship between its theoretical basis and experimental applications. For this reason, it is important to learn the nature of probability first in probability teaching. Bryant and Nunes (2012) emphasized that individuals should first understand the four different aspects of events and the order in which they occur since probability is a difficult-to-understand and highly complex concept. These requirements are; randomness, determining the sample space, comparing and quantifying probabilities, and understanding the correlation (or relationships between events).

With the realization of the importance of probability both inside and outside of school, many countries have started to add probability to mathematics curriculum at every educational level (Batanero, Godina & Roa, 2004; Paul & Hlanganipai, 2014). Students are constantly confronted with the concept of uncertainty in economic, meteorological, biological, and political environments, and in social activities such as games or sports, and they need to understand the concept of randomness in order to make decisions in these situations. This requirement has enabled probability to be included in the curriculum from primary school to high school (Batanero, 2020). The first important curriculum in probability was realized in 1965 with the preparation of SMSG (School Mathematics Study Group) texts for primary schools (Jones, 1974). SMSG's work includes attempts to develop concepts of probability with textbooks, activity-oriented games, and special materials. In Turkey, although the probability has been included in the high school curriculum since the 1960s, it has not attracted much attention (Bulut, 1994). The subject of probability, which was previously only at the high school level in the curriculum, later started to take place at the 8th and 10th grade levels with the revisions made in the curriculum (MoNE, 1990). By including the subject of probability in the 6th, 7th and 8th grades in the curriculums renewed in 2005 and 2009, the necessity of giving the subject of probability from an early age was revealed. With the revision made in the mathematics curriculum in 2013, the issue of probability started from the 8th grade and was distributed to the 9th, 10th and 12th grades at the high school level (MoNE, 2013a; 2013b).

Since the subject of probability is included in the curriculum, there are disagreements about which grade level to start from. Some researchers argue that it is difficult to teach probability from an early age. Piaget and Inhelder (1975), one of the pioneers of this view, stated that a child entering the concrete-operational period cannot distinguish specific and random predictions or clearly state his predictions, taking into account his experiences from previous similar situations. However, Fishbein (1975) suggested that students at an early grade level like third grade have probabilistic thinking.

It is seen that there is a need for studies on the effective implementation of probability teaching because, it is considered an important topic and it is one of the issues that both teachers and students experience difficulties (Bryant & Nunes, 2012; Çakmak & Durmuş, 2015; Gürbüz, 2007). The main difficulties in teaching probability concepts; lack of appropriate teaching material (Gürbüz, 2006; Pijls, Dekker & Van Hout-Wolters, 2007), students' readiness level (Ben-Hur, 2006; Jones, 2005; Lee, 2006), students' negative attitude (Bulut, 2001; Memnun, 2008), students' age and misconceptions arising from various reasons (Fischbein & Schnarch, 1997; Gürbüz & Birgin, 2012; Liu & Thompson, 2007; Talawat, 2015). Apart from these difficulties, the students' insufficient readiness in subjects such as fractions, permutations, combinations and sets, which are prerequisites for probability, is also reflected in teaching as a problem. For example, some studies have stated that the difficulty of high school students in learning the concepts of combination and permutation is a very important problem in probability lessons (Ben-Hur, 2006; Fischbein, 1975; Jones, 2005; Lee, 2006). Teachers also experience difficulties for reasons such as insufficient preparation and knowledge about probability (Jacobbe & Horton, 2010; Liu & Thompson, 2007), misconceptions (Stohl, 2005; Talawat, 2015), and lack of experience (Jendraszek, 2008). As a result, it can be said that the difficulties encountered in teaching probability stem from the teacher, the student, the curriculum and the nature of the subject.

It was revealed that there are many difficulties encountered in teaching probability and teachers have important duties in overcoming these difficulties. It is important to understand how teachers can teach probability concepts in order to understand the situations related to the teaching and learning of probability (Stohl, 2005). For this reason, teachers' views should be investigated in order to have a broad lens to interpret a teacher's probability knowledge (Njenga, 2010). From this point of view, the problem of this study is to evaluate the probability teaching-learning process in line with the opinions of the teachers who are the implementers of the curriculum and try to understand the process from different angles.

Based on this, answers were sought for the following sub-problems:

1. What are the teachers' views on the mathematics lesson curriculum in terms of probability?

2. What are the teachers' views on the process of learning-teaching the subject of probability?

3. What are the teachers 'views on the solutions to probability problems and their students' situations of solving these problems?

2. METHOD

2.1. Research Model

In this study, case study design, which is one of the qualitative research methods, was used. Such studies are an inquiry strategy in which researchers investigate a program, event, activity, process, or one or more people in depth (Creswell, 2009). Therefore, a qualitative research method was adopted regarding the difficulties experienced by teachers in the teaching process of probability, the solutions they created for these difficulties, and how students learned about probability.

The research was carried out in three stages to understand the situation in detail. In the first stage, teachers' opinions on the probability teaching-learning process were investigated. As a result of the evaluation of the data obtained from the first stage, it was decided to realize the second stage. At this stage, probability problems were prepared based on the opinions of the teachers in the first stage interviews, and the teachers' situations of solving these problems and their opinions on the solutions to the problems were examined. In addition, at this stage, the opinions of teachers' regarding their students' solutions to these problems were also taken. In the third stage of the study, these problems were applied to the students of two teachers in the study group, and the student responses were analyzed comparatively with the teachers' opinions. Therefore, it was tried to collect data on different dimensions of the research problem by examining the opinions of teachers working at both middle and high school levels, as well as the ways of solving probability problems of students.

2.2. Study Group

In accordance with the nature of the qualitative research, the study group of the research was determined by purposeful sampling method, because purposeful sampling helps to examine information-rich cases in depth and detail (Patton, 2002). The study group of the research consists of 16 mathematics teachers working in middle and high schools in a city in the Southeastern Anatolia Region of Turkey. By including both middle and high school teachers in the study, it was aimed to better evaluate the teaching learning process of probability subject in terms of all grade levels in the maths curriculum in Turkey. The demographic information of the teachers who took part in the study is as in Table 1:

Table 1.
Demographic Characteristics of the Participants

	Gender	Level of School	Length of Service	Teaching Time of Probability
M 1	Male	Middle	7 years	4 years
M 2	Male	Middle	13 years	7 years
M 3	Female	Middle	8 years	3-4 years
M 4	Male	Middle	11 years	7 years
M 5	Female	Middle	15 years	15 years
M 6	Male	Middle	7 years	6 years
M 7	Male	Middle	17 years	10 years
M 8	Male	Middle	5 years	5 years
H1	Male	High	17 years	17 years
H2	Male	High	16 years	16 years
H3	Male	High	10 years	7 years
H4	Female	High	16 years	8 years
H5	Male	High	17 years	17 years
H6	Male	High	40 years	10 years
H7	Male	High	15 years	13 years
H8	Male	High	8 years	8 years

39 middle school and high school students, who were selected purposefully among the students in the study group M3 and H2 taught mathematics in the last year, also participated in the study. The reason why M3's students were selected from the 8th grade is that the subject of probability is only in the 8th grade at the middle school level. The reason why H2's students were selected from the 10th grade is that the selected questions are suitable for the 10th grade in line with the outcomes in the curriculum and the majority of the teachers stated that the problems are appropriate for the 10th grade.

2.3. Data Collection Tools

Semi-structured interview protocol and routine / non-routine probability problems were used as data collection tools in the study. In the study, separate protocols were prepared for the first / second stage, and the middle / high school levels. While planning the interview process, the interview protocol consisting of semi-structured open-ended questions suggested by Creswell (2009) was prepared in accordance with the purpose of the research. In the first stage, there are 8 open-ended interview questions (Table 2).

Table 2.

Examples of Semi-Structured Interview Questions

Sub problems	First stage	Second stage
How do teachers evaluate the probability of learning subject in the curriculum?	How do you evaluate these outcomes in terms of applicability in the classroom? Explain.	
What are the teachers' opinions about the learning-teaching process about probability?	What difficulties do you have while teaching the topic of probability? Explain the reasons.	
What are the teachers' views on the solutions to probability problems?		What do you think the correct solution to this problem is? What are the possible solutions your students can give to this question?

The second stage interview protocol consists of two parts. While the first part includes semi-structured interview questions to be asked to teachers, the second part includes open-ended probability problems consisting of routine and non-routine problems. One of the non-routine probability problem asked in the second stage is as follows:

Buğra designs a game to reach the caves through a road covered with trees. At the beginning of the game, players will choose either cave A or cave B. Then the players will walk to the caves using random paths from the starting point. If the player reaches the cave he chose at the beginning of the game, he will win the game. Which cave do you think is most likely to reach?

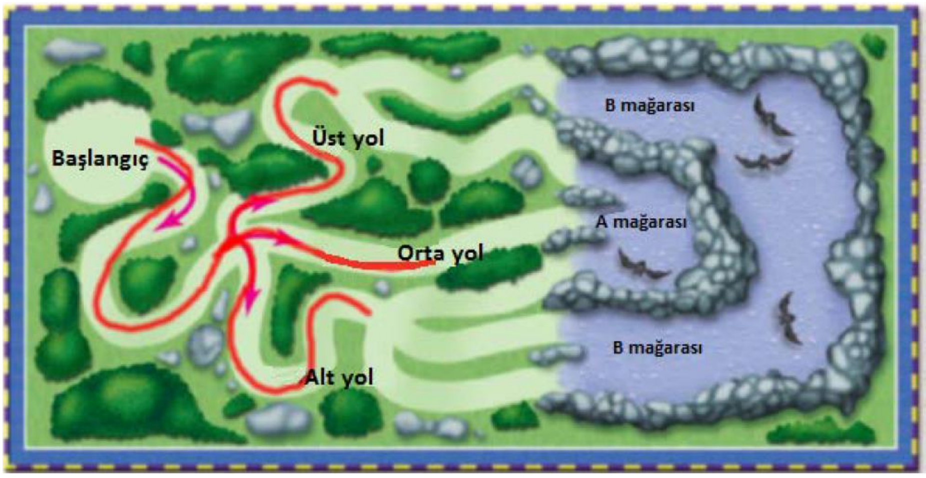

Figure 1. Cave Problem (Retrieved from http://www.kentschools.net/ccarpenter/files/2009/09/3.1-notes.pdf)

Due to the flexible nature of qualitative research, while preparing probability problems, decisions were made in the process and the first stage evaluations of the teachers were taken into consideration. In the first stage, the teacher stated that they solved certain types of questions for the exam. In addition, there are many teachers who mentioned that they frequently solved the questions of national exams in the past years in their lessons. So, one of the questions in the second stage is the exam question asked in previous years. Besides, the problems were determined as routine and non-routine probability problems, since the teachers stated that they reached the result by following certain process steps. The determined problems were examined by an expert in mathematics education and necessary corrections were made.

2.3.1. Data Collection Process

The interviews were held in environments where teachers felt comfortable and when they made an appointment themselves. Since the teachers stated that they did not feel comfortable when the video was recorded, the interviews were recorded using a tape recorder and note-taking techniques. After the protocols were prepared, pilot studies were conducted (Yin, 2003). Possible problems were identified as the result of the pilot study, necessary questions were corrected, and the average duration of the interview was planned (First stage = 65 minutes, Second stage = 50 minutes).

2.4. Data Analysis

The data obtained from the interviews were analyzed using open coding and axial coding methods based on grounded theory (Strauss & Corbin, 1990; Strauss & Corbin, 1998). In this coding method, the coding process is formed around the axis of a category, and categories are associated with each other at the level of features and dimensions (Strauss & Corbin, 1998). The data were analyzed using the Nvivo 11 program. At the beginning of the analysis process, row analysis was performed on the data obtained by the open coding method, and comparisons were made between the data. At this stage, the data obtained from the interviews in the first stage were analyzed comparatively. Besides, researchers' notes were used where necessary. As a result of these comparisons, the first codes were produced. Data were analyzed by two separate coders in order to ensure encoder reliability on the obtained data. The coder reliability of the study was found by dividing the number of codes agreed upon by the total number of codes (Bakeman & Gottman, 1997). In the analysis, 86% (131/153) consensus was reached among the researchers. In the second round, by comparing which codes are consistent, the importance of each code, the corrections in the codes and the coding made by the researchers and the codes with disagreement were examined by an expert mathematics educator and a consensus was reached.

In the open coding process, while codes were determined with comparative analysis, data were assigned to the relevant codes. In the following process, the relationships between the axial coding method and common codes were determined and categories were created as a result of the relationships. After the first codes were determined, the comparative analyzes were continued and the categories were determined. The analysis process ended when there was no new data or coding. In order to analyze the data obtained from the second stage of the research, firstly, the answers given by the teachers to probability problems were examined. Also, teachers' views on the solutions to the problems and students' opinions were analyzed and presented in the findings. In order to determine the consistency of the data obtained from the student answers with the teachers' predictions, the percentages were calculated by evaluating the correct answer / wrong answer. The solutions of the students were examined in the context of whether the actions they performed and / or the formulas they used were compatible with the opinions of the teachers.

2.5. Credibility and Transferability

The model of the research, the participants, the data collection and analysis process, the limitations and the analytical generalizations reached are expressed in detail to ensure the transferability of the research (Erlandson, Harris, Skipper & Allen, 1993; cited in Yıldırım & Şimşek, 2013: 304). The interviews were carried out in two stages to increase the data source to ensure credibility in the research. Also, data were collected from students in the third stage. Besides, a case study protocol was developed first. The purpose of this protocol is to clearly state the process followed in the research and to develop the research gradually. The aim of this process is to ensure that a researcher later achieves the same results if he follows the procedures described by a previous researcher and reruns the same case study (Yin, 2003). While preparing the interview protocol, a conceptual framework about probability was created by examining the relevant literature. During the data analysis process, pattern matching, explanation of these matches, and counter explanations were also provided by open coding (Corbin & Strauss, 1990).

Another process used to ensure the credibility of the research is the use of member checking. Creswell (2002) suggested member checking, in which the final report was brought back to the participants for certain statements or themes to ensure the accuracy of the findings, and that the findings were verified by these participants. As a result of the analysis of the data obtained in this study, a report consisting of the themes reached and some explanations were presented to a middle school and a high school teacher. Two participants read this report and expressed their thoughts in writing by evaluating the adequacy of the analyzes made in the research to reflect their own situation and whether the results were related to their lives. These views are as follows:

M3: It is seen that middle school mathematics teachers have similar thoughts about each sub-problem of the study. I stated that it would be beneficial to explain the subject of probability by giving examples from daily life in the teaching process for students to comprehend the subject. The findings of the research also revealed this. In the course, solving questions from easy to difficult, was specified as a common practice of the other participants in the study. Dice, money, and bag samples are generally used to concretize the probability. I use unit assessment exams and written exams as measurement tools. This practice is consistent with the research results. One of the main points about the problems encountered in probability teaching is the problems arising from the education system. In particular, exam anxiety makes conceptual learning difficult for students about probability and causes their learning to remain at the level of knowledge. It was a study that accurately revealed what teachers experienced during the process.

H7: The results obtained from the categories and themes obtained in the research correspond exactly to the realities we experience in schools in probability teaching. Moreover, the study seems very good in reflecting the situation under investigation regarding the probability. The themes obtained in the first sub-problem of the research, about the subject of learning probability match with the answers I gave. I mentioned in our interview with the researcher, the class-level difficulties of probability and the difficulties inherent in the spiral curriculum which is one of the findings obtained in the study. The determination of teachers' difficulties in non-routine problems, which is one of the themes obtained in the second sub-problem of the research, regarding the applications of teachers in the probability teaching process is consistent with my explanations. In addition, I emphasized that teachers prioritize operational skills rather than conceptual learning at the undergraduate level and have problems arising from lack of experience, and it was a category obtained from the study. As a result, based on teacher evaluations, it has been a study that describes the situation in our country regarding probability teaching very well.

When the reports of M3 and H7 are examined, it shows that the findings obtained from the research reflect the real-life situations of the teachers. This contributes to the credibility of the study.

2.6. Ethical permission to research

In this study, all the rules stated to be followed within the scope of " The Higher Education Institutions Scientific Research and Publication Ethics Directive" were followed. None of the actions specified under the title of "Actions Contrary to Scientific Research and Publication Ethics", which is the second part of the directive, were not carried out.

Name of the committee that made the ethical evaluation: Adıyaman Provincial Directorate of National Education
Date of ethical review decision: 29.03.2017
Ethics assessment document issue number: E.4181594

3. FINDINGS

3.1. Findings and comments on the first sub-problem

The first sub-problem of the study was "What are the teachers' views on probability in the mathematics curriculum?". In line with the expressions of the teachers about the curriculum, four categories were determined as outcomes, grade level, time and spiral curriculum (Table 3).

Table 3.
Teachers' views on the curriculum in terms of probability

Categories		Participants	Sample Opinions
Outcomes	Positive Opinions	M5, M7, H8, M3, M8	Positive Opinion: Students could be confused about dependent and independent events, but after these topics were removed, the current outcomes are very good.
	Negative Opinions	M3, H8, H5, H1	
	Suggestions	H7, H1, M8, H5, M5, H4	
Spiral Curriculum	Negative Opinions	H1, H2	Negative Opinion: "Permutation and probability were divided. The only undivided subject combination. It is a mistake to expect a child who has passed from the 10th to the 12th grade to remember the subject of permutation, because it takes about 2 years after learning the subject.
	Suggestions	M3, H7	
Grade Level	Positive Opinions	M1, M5, H7	Suggestion: I think it is sufficient if the subject is simplified and only included in the 10th grade curriculum. I already believe that it will not take up too many pages in textbooks. I also think that the student will apply it more easily when there are questions that can be explained with daily life in the content of the subject.
	Negative Opinions	H4, H8	
	Suggestions	M4, M5, M3, H3, H4, H5, H6	
Time	Positive Opinions	M5, M8, M7, M4, H1, H4, H7	Positive Opinion :"In the previous version of the curriculum, we were giving almost everything. There wasn't enough time. We had to give everything, the range of questions was very wide."
	Negative Opinions	M1, M3, H2, H3, H5, H8	
	Suggestions	M3, H2, H3	

When the opinions of the teachers about the curriculum on probability are examined, it was determined that middle school teachers generally have positive opinions about the outcomes and time. High school teachers, on the other hand, stated negative opinions that the outcomes are intense and the time is not enough. Similarly, H1 and H2 expressed a negative opinion about the disorganized outcomes in the probability program. H1 said, *"Permutation and probability were divided. The only subject that didn't divide is combination. It is a mistake to expect the child who is in the 10th to the 12th grade to remember the permutation because it's been about 2 years"*. In general, teachers stated that the time allocated to probability is not sufficient to realize conceptual learning. Middle school teachers emphasized that, probability subject should start at an earlier grade level, while high school teachers generally said that it should start from high school. However, some teachers stated that having the subject of probability in the higher grades caused students to have difficulties due to lack of prior knowledge.

3.2. Findings and comments on the second sub-problem

The second sub-problem of the study was "What are the teachers' views on the teaching-learning processes of the probability?". When probability teaching practices were examined, planning, teaching process and measurement-evaluation categories were created in line with the data obtained (Table 4).

Table 4.
Teachers' probabilistic teaching practices

Categories	Codes	Coding Teachers
	Teaching based on question-answer method	H2, H4, H5, H8, M8, M7, M3,
Teaching process	Teaching in relation to daily life	H1, H3, H4, M1, M1, M2, M3, M5, M8, M7, 04
	Teaching according to individual differences	M1, M8, M7, M3, H6
	Teaching by concretizing	M1, M3, M3, H1, H8

Table 4. Continued

Teachers' probabilistic teaching practices

Categories	Codes	Coding Teachers
	Performance evaluation	M3, M8, M7, M3, H3, H4, H5, H8, H7
Measurement-Evaluation	Homework	M3, H2
	Result evaluation	M2, M3, M5, M7, H1, H2, H6

Teachers' implementations in the probability teaching process: When examining how they conducted the probability lesson, most of the teachers stated that they explained the probability by associating it with daily life. Teachers first mentioned that they increase students' motivation by mentioning the contribution of probability in daily life in order to attract attention to the lesson. The majority of teachers agree that students will get their attention if they realize that they can use in their daily lives. For this reason, it was determined that they generally started the lesson with examples of using probability in daily life. M6 stated that; *"I say that we can probably meet everywhere in daily life, for example, the probability of their football teams to be the first or the probability of relegation. After giving an example like this, I started to probability subject"*. In addition, teachers stated that they mostly used the question-answer method to explain the probability with questions from simple to complex. The fact that teachers make use of the questions while teaching the lesson at both school levels may suggest that they give general exam-oriented narration and give priority to gaining operational skills. M3 stated that teachers generally used the questions to support their anxiety about gaining operational skills. H7 stated that he made use of simple to complex questions and evaluated non-routine problems as complex. This finding explains that teachers have difficulties with non-routine problems in the second stage.

It is concluded that they generally perceive concretization as giving examples from daily life and using concrete materials in the lesson. The examples that teachers give to connect with daily life while teaching the probability subject usually include football matches, dice, money and bags. This situation also shows that the teachers actually do not go beyond the standard patterns in the examples they give from daily life same as in the questions they ask in the lessons. H1 stated that he believed using probability in situations they constantly encounter in daily life would be more effective in learning instead of giving standard examples. It was determined that M2, who thinks that probability is the situations encountered in daily life, also performs his teaching in the same parallel. M2 stated that he tried to give students the knowledge that probability could offer more than one solution to problems encountered in daily life.

Finally, when the expressions of the teachers who stated that they used materials to make the subject concrete were examined, it was determined that the teachers used tools such as bags, dice, money and paper in a similar way. A striking situation in the data obtained is the expression of H1 is that H1 uses dice and money as materials, although he thinks that examples such as money and dice will not be useful in terms of concretization in students' learning. M7 stated that he used the applications on the smart board as the material, there are too many visual events on the smart board and it grasps 95 percent of the probability. He mentioned that he used a computer program as a material to achieve the real result by doing lots of experiments in M2.

Measurement and evaluation practices of teachers: When examining how teachers evaluate probability teaching, it is seen that teachers generally make result-oriented measurements and evaluations. Most of the teachers who make result-oriented measurement and evaluation stated that they use end-of-unit exams as a measurement tool. When the opinions of the teachers were examined, it was determined that they made result-oriented evaluations, but they were not very satisfied with this situation. Teachers actually think that the knowledge of students cannot be measured by only result evaluation. For example, H5 expressed how he evaluates the learning of the student as; *"It can be distinguished by the child's stance in class, even by his self-confidence. In other words, attending the class in the classroom, being self-confident, participation in the lesson, the* *power of interpretation, the desire to solve, and the desire to speak, we can very easily derive its consequences".* However, in practice, this situation is changing. M3 stated that the reason for this situation is the examination system in Turkey. This situation shows that national tests affect the measurement and evaluation techniques used by teachers.

In line with the expressions of the teachers, it was determined that the problems encountered in probability teaching have four main sources. While the teachers stated that these problems mostly stem from the education system, they stated that this situation caused students and teachers to acquire operational skills and to realize rote learning. This situation leads to inadequacy in conceptual learning. Especially middle school teachers stated that students had difficulty in learning because of the abstract nature of the probability. There is a relationship between the problems that teachers encounter in probability teaching and their suggestions. For example; they suggested conceptual learning because they had problems with learning and teaching towards operational skills. They suggested making use of concrete materials and examples regarding the problems of the subject being abstract and students' inability to concretize.

Findings and comments on the third sub-problem

The third sub-problem of the study was "What are the teachers 'views on the solutions to probability problems and their students' situations of solving these problems?". For this purpose, teachers' views on solving problems, solutions to probability problems, and their views on their students' solutions to these problems were examined. In addition, a designated high school and a middle school teacher were asked to give the same probability problems to their students and solve them. In this way, in line with the teachers' evaluations, the students' mistakes in solving probability problems and teachers' situations of identifying the sources of these errors were also examined comparatively. When the reasons for teachers to make mistakes in questions containing figures were examined, it was determined that this was due to their interpretation directly on the figure without taking any action. For example, the solution way of H1 in Question 1 is as follows: "*You have many choices to reach cave B. But to cave A you have only one choice to reach. Therefore, it will be easier for you to reach cave A*". The ways of solving the questions contradict with the teachers stating that they are trying to gain operational skills. Because it is seen that teachers try to solve the questions by interpreting the figure while they are expected to take action while solving them. Since this question is different type, some teachers think that students will not be able to understand this question, and therefore they will tend to use their rote knowledge. For example, H1 predicted that 70% of the students would answer the question of probability of the roads leading to the cave incorrectly, and stated that the reason for this was not because of the students' lack of operational skills, but because they did not understand the question. The teachers mentioned that the problem could be solved easily, but they think that a small part of the students can be answered correctly, and this is due to the fact that it is a question designed in a different style. H4, on the other hand, stated that students will choose the shortest path, and evaluate the problem in terms of figures. Teachers' mentions about the complexity of non-routine problems supports this finding. Another finding obtained from the study is that most of the teachers do not remember the national exam question, and cannot give correct answers, although they mention the negative effects of this situation by stating that they do exam-oriented teaching in their opinions.

When the expressions of the teachers in these three questions were examined in general, it was determined that there were some contradictions. While the teachers stated that questions including figures, students could not reach a solution due to their carelessness and not understanding the figure, they suggested solving them by transferring them to the figure in other questions. Teachers also mentioned that there are constantly used question patterns. One of the questions that teachers express as mold questions is about the probability of drawing cards from the bag and is aimed at teaching the concept of conditional probability. Teachers understood this question more comfortably and offered solutions. In addition, teachers except H1 predicted that the majority of students would answer correctly. It has been determined that middle school teachers, like high school teachers, have a prejudice against non-routine problems. In line with their views, teachers came up with more correct solutions to routine problems based on specific formulas and operational skills. Although the teachers were teaching exam-oriented, most teachers could not remember the general exam question, which is a routine problem similar to high school teachers. In addition, some teachers could not reach the correct solution (M1, M2, M5, M7). Teachers stated that their students also think exam-oriented and do not deal with problems other than the question types that will appear in the national exam.

As a result, it can be said that both middle school and high school teachers similarly tend to follow certain procedures in probability questions. It has been determined that teachers do not look positively at different question types other than routine problems. From the comments they made to the questions given by the researcher, it was determined that the questions they used during the lesson were also routine problems with certain limits. This situation is reflected in the students' evaluations of their solutions. Therefore, these opinions of teachers can affect the whole teaching process from the first to the last stage of the lesson. In the data collected on how teachers perform their teaching, they emphasized that they mostly explain the probability with the help of questions and make result-oriented evaluations. It was concluded that they did not use the questions that prompted students to think differently in their lessons.

Teachers frequently mentioned the anxiety of acquiring operational skills and emphasized that conceptual learning should take place. However, in the second stage, when the teachers were asked to specify the probability concepts contained in the questions, it was observed that they did not want to specify the concepts. The fact that teachers have this approach may cause students to turn to procedural skills. As a result, it was determined that middle school and high school teachers felt more comfortable both in solving routine problems and in their views. This situation was also determined by the correct answers given to the questions.

In order to determine the consistency of all these evaluations of the teachers regarding their students' solving probability problems, the randomly selected students of H6 and M3 were also asked to solve them by giving probability problems. The comparison of H6 and M3's evaluations and student answers regarding the solutions of probability problems is as follows:

M3 has been in the profession for 15 years. He stated that he taught the subject of probability throughout his professional life. Relating the subject of probability with examples from daily life, M3 stated that positive or negative situations that may occur in daily life are possible. He stated that he first learned the subject of probability in middle school at a simple level, and he liked it very much because of his teacher's interest. However, he stated that he developed a negative attitude towards the subject of probability due to the difficulty of the subjects at the high school level and the inability to fully benefit from the high school teacher, unlike his middle school teacher. For this reason, he stated that he had problems in high school. He had problems regarding dependent-independent events and discrete-non-discrete events and these problems continue in his teaching profession. He solved this problem by working on his own. It was determined that he taught the subject through questions, and stated that it is appropriate to simplify the middle school probability curriculum. He explained the reason for this by removing the difficult questions from the books.

The first problem posed to middle school teachers is a non-routine cave question. M3 gave an incorrect answer by calling the answer to the first problem as cave B. He stated that the reason for his answer as cave B is that the cave is bigger. This situation may be an indication that M3 evaluates the question through the figure without taking action and does not fully understand the desired situation. Because it is not the size of the caves that one should pay attention to in order to make the right solution to the question, but the roads and connections to the cave. The problem is about the concept of the probability of an event. M3 stated the concept contained in the problem as equal probability. This situation can be associated with the lack of conceptual knowledge about probability. M3 stated that this problem is appropriate for the 8th grade and stated that 60-70% of the students will solve this question correctly. What M3 means from the correct solution to the problem is his own answer, cave B. When asked the same problem to 27 students, whom M3 taught probability, 9 students answered B cave, 16 students answered A cave and 2 students said that the probability of reaching both caves is equal. M3 then tried to solve it by calculating the probability of reaching each cave. As a result of his calculations, he claimed that the answer was again cave B. In his second evaluation, he stated that his students could not make a solution using such a procedure and that they would solve it only by interpretation. He stated in the first stage that the students' operational skills were developed and contradicted his answer. When the answers of the students were examined, it was determined that there was only one person who tried to solve by processing in line with the estimation of M3. It is seen that other students try to reach the result by evaluating the figure without making a process while solving the problem. For example; one of M3's students chose cave A because it looks easy on the figure. When the students' answers were examined, it was seen that only 2 people answered the question in this way. Therefore, it can be said that M3 is insufficient in detecting students' mistakes for this question. When M3's opinions about the sources of errors that students could make were examined, he stated that the students' possible incorrect answers stemmed from their attempts to solve the question by commenting and the lack of theoretical probability knowledge. He stated that this problem could not be solved with the current 8th grade outcomes in M3 curriculum. However, for the solution of the problem, there is no need for probability outcomes at a higher grade than the 8th grade, and this question can be solved by any student who knows the probability of an event. Although M3 could not fully explain the dependent-independent event concepts, which is one of the solution ways, he tried to use his formulas and could not propose an alternative perspective for the solution of the problem. Therefore, he had difficulties in determining the possible correct and incorrect solutions that the students could give. It is thought that M3, who stated that he can correct the mistakes made on this subject by solving similar questions during teaching, cannot go beyond providing students with operational skills in the context of the current question. Moreover, M3 stated that the time allocated to the subject of probability was not sufficient for conceptual learning and he carried out teaching aimed at gaining operational skills. However, in accordance with the data obtained from the second stage, it can be said that teaching operational skills is not only due to insufficient time but also to his own learning.

H2 has 16 years of professional life. He has taught the subject of probability since the first year he started teaching. Considering probability as a very important issue, H2 stated that probabilities take place in every moment of daily life. He said that people with a numerical mindset can use probability more effectively. Although it has a structure that can be used widely in verbal branches such as law, lawyers cannot use it effectively. In short, H2 stated that probability can be used in all areas of life. Stating that he first took the probability course in high school, he had difficulty while taking the probability course at the university level and that he overcame this situation by working on his own. H2 evaluated the probability for students based on the basis of questions. He attributed the students' difficulties with this subject to their inability to solve the questions, and he stated that the factor that made the subject difficult was the complex probability questions that he saw as unnecessary. H2 was asked about which probability concept this question includes, and he stated "I mean, there is not necessarily such a concept, so I concentrate on solving the question directly". He revealed that he only teaches problem-solving focused lessons through his answer. He also stated that he knows the general mistakes that students will make in solving the questions, and this gives him an advantage during the teaching process. Stating that probability should be given from the primary school level, H2 also criticized the curriculum. He stated that the current probability curriculum was heavy and scattered and the time allocated to probability was insufficient.

The cave question asked to middle school teachers was also asked to high school teachers. H2 said, *"There are different choices here, like choosing probability and reaching the sample space, so I didn't understand what you meant"* about which probability concept the problem contains. It was determined that he knew the calculations of the probability of an event required for the solution of the problem, but could not relate it to the concepts of probability. In the solution of the question, he made calculations but he could not reach a solution. In this way, he used dependent-independent event formulas. When his views on the students' solutions to this problem were examined, he said that over 50% of the 10th grade students would give correct answers. When the answers of the students were examined, 20 students answered "cave A", 11 students "cave B" and 2 students said "the probability of going to both caves is equal". These results are consistent with the estimation of H2, but when the solutions are examined, it is seen that most of the students have reached the correct answer based on interpretation or with the wrong strategy without taking action. 2 students gave the correct answer, although they made wrong calculations. He stated that for the possible wrong solutions that the students will make, they will try to reach the result quickly, so they can give wrong answers based on the operation error. When student solutions were examined, no such error was encountered. When the solutions ways of students who gave wrong answers are examined, it is seen that there are errors arising from misinterpretation of the shape and lack of knowledge about probability rather than operational errors. For example; one student chose cave B by calculating the probability of reaching cave B as 10/9. This student has a lack of knowledge that the probability value cannot be greater than 1. One of the mistakes made by the students is that while considering only the entrances of the caves on the figure, they do not take into account the routes from the starting point to the cave entrances. For this reason, while calculating the probability of reaching the caves, some students count the entrances of caves A and B or calculate according to the size of the cave.

As a result of the analysis of the obtained data, it is seen that teachers explain probability with the help of questions and do not provide learning environments that will enable students to look at probability from a broader perspective. Also, it was determined that they quickly reached a solution to the probability questions following certain process steps and experienced problems in the non-routine probability problem. It was determined that the teachers 'views on the students' solutions were similar to their own solutions. Teachers could not suggest a way other than their own solutions in solving the problems. In addition, when the students' answers were examined, it was determined that the teachers' predictions did not fully reflect the students' answers.

4. DISCUSSION AND RESULTS

The aim of this study is to evaluate the teaching-learning process of probability based on teachers' opinions. For this purpose, the research was carried out in three stages with mathematics teachers and students. According to the results, it was determined that middle school teachers generally have positive opinions about the content of outcomes and course length. High school teachers, on the other hand, gave negative opinions about the intensity of outcomes and insufficient time. As for which grade level the probability subject should start, middle school teachers emphasized that it should start at an earlier age, while high school teachers generally stated that it should start from high school. However, some teachers emphasized that the inclusion of the subject of probability in the higher grades caused students to have difficulties due to lack of prior knowledge. Piaget and Inhelder (1975) also stated that a student in the concrete operational period cannot distinguish between certain and random situations, considering the experiences of previous similar situations. Fishbein (1975), on the other hand, stated that unlike Piaget and Inhelder's study, traces of probabilistic thinking were encountered even in lower grades such as the third grade. HodnikČadež and MajaŠkrbec (2011) supported this opinion and found that children at the age of 4-5, and first-year students were assigned tasks related to various concepts of probability and that children reached the goal earlier. However, there are studies indicating that complex probability issues are not understood by very young students (Hawkins & Kapadia, 1984; Garfield & Ahlgren, 1988). In the present study, the majority of teachers also think that lower-grade students have difficulty in understanding the probability. However, some teachers at the middle and high school levels stated that more meaningful learning will occur in the future if the probability is started to make feel very simple at the lower grade levels.

The teaching process and measurement-evaluation categories were created when teachers' teaching practices on probability were examined. Teachers stated that they generally benefited from the question solving technique during the lesson. In line with the expressions of the teachers, it can be said that they performed teacher-centered teaching. Chong, Chong, Shahrill and Abdullah (2017) also emphasized that although the use of student-centered innovative approaches is recommended today, teaching continues to be teacher-centered. In the present study, teachers also mentioned the importance of concretization and stated that they benefited from examples and materials related to daily life in order to concretize the subject in lessons. Teachers generally consider concretization as the use of concrete examples and materials. Watson (2001) also revealed that teachers made use of concrete materials and technology in teaching probability.

The majority of the teachers stated that they measure and evaluate in accordance with their narrative style in a result-oriented manner. They attributed this situation to the education system and the necessity of exam-oriented education. Therefore, they mentioned that they teach students to gain the ability to solve questions that may arise in national exams. According to the results, it can be said that probability teaching is carried out to improve the students' operational skills. The fact that teachers talk about the limited use of materials in their opinions about the problems arising from the teacher also supports this situation. Similarly, it has been stated in some studies that the subjects are generally carried out in teacher-centered classroom environments and the lack of appropriate teaching materials negatively affects probability teaching (Gürbüz, 2006; Pijls et al., 2007).

When the literature is examined, it is seen that the difficulties encountered in learning and teaching the subject of probability are stated in many studies (Batanero & Serrano, 1999; Fischbein & Schnarch, 1997; Garfield & Ahlgren, 1988). In this study, it was determined that the problems encountered in probability teaching stem from the education system, the nature of the subject, the student and the teacher. In addition, there are many studies that indicate the sources of difficulties in teaching probability as teachers and students (Gürbüz, 2006; Borovcnik & Kapadia, 2010; Bulut, 2001; Çakmak & Durmuş, 2015; Fischbein & Schnarch, 1997; Garfield & Ahlgren, 1988; Gürbüz & Birgin, 2012; Pijls et al., 2007).

According to the results of this study, among the problems encountered in probability teaching, it is seen that the problems caused by the teacher are grouped into the categories of narration, lack of knowledge, lack of experience, teaching for memorization, anxiety about gaining operational skills, having prejudices, and not being able to concretize the subject. Based on these opinions, the basis of the education system for memorization was determined as the anxiety of gaining operational skills. Middle school teachers stated that teachers have more anxiety about gaining operational skills than high school teachers and this situation causes difficulties in teaching probability. This situation is thought to be due to the fact that middle school teachers only explained the subject of probability in the 8th grade when the national exam was held. Similar to this study, Batanero and Diaz (2012) investigated the reasons why probability teaching is difficult for mathematics teachers and they found that some of the difficulties are teachers' behaviors, beliefs, probability knowledge and professional experiences. Gürbüz and Birgin (2012) also stated that teachers' misconceptions and lack of theoretical knowledge negatively affect probability teaching.

Another important result obtained from this study is the students' result-oriented thinking and their tendency to learn probability operationally. Noddings, Gilbert-MacMillan, and Lutz (1980) also determined that students tend to follow the formula or procedure in their mathematical problems without understanding the problem. Garfield (2001) stated that although students can learn probability rules / procedures and find the correct answer in math tests, the same students often have misunderstandings in basic ideas / concepts. The opinions of the teachers about the solutions of the students in the second stage are in this direction. Teachers believe that students will be more successful in routine problems that can be solved with certain rules and formulas. This may be an indication that students have operational knowledge. Students with operational skills are able to memorize well-structured problems with familiar formulas and steps to follow, but rarely explain what the rationale is or how concepts are applied in new situations (Garfield & Ahlgren 1988). It is thought that seeing the subject of probability as a sequence of operations may trigger students to perceive the subject abstract and have difficulties in concretizing. Andrew (2009) also mentioned the importance of concrete experiences for students to better understand the content.

In order to find an answer to the third sub-problem of the study, firstly, teachers' opinions about their own solutions and teachers' opinions about their students' solutions to probability problems were investigated. As a result of the analysis of the data obtained, it was determined that teachers had difficulties in solving non-routine problems and they evaluated such problems as complex and came up with more correct solutions to routine problems based on specific formulas and operational skills. Stohl (2005) also concluded that according to the results obtained in his study, teachers mostly consider the calculation of theoretical probabilities as a use of procedures and deal with probability problems from a deterministic perspective. Therefore, in this study, it can be said that teachers approach to the problems from a deterministic perspective while searching for solutions. This result contradicts Fishbein and Gazit (1984), and HodnikČadež and Škrbec (2011) stating that the purpose of probability teaching is to develop different thinking skills from a deterministic point of view. Therefore, these evaluations of teachers may show that they do not teach except for certain question types. Results about how teachers carried out their teaching confirm this situation. Another important result is that most teachers do not remember the probability problems asked in general exams in our country, although teachers do not go beyond certain question patterns and stated that they teach exam-oriented. In the study of Kurt Birel (2017), it was determined that middle school mathematics teachers mostly have computational minds while solving probability problems and therefore their operational knowledge level was higher. As a result of the study, it was emphasized that teachers' conceptual knowledge about probability should be developed. In line with the findings obtained in this study, it can be said that teachers have computational minds.

The results of the current study show that most mathematics teachers execute probabilistic procedures using probability formulas. However, teachers did not use the probability concepts in the problems given in the second stage and could not associate these concepts with the problems. Ball (1990) also concluded that pre-service teachers had a procedural understanding arising from their view of mathematics as a set of rules / procedures and could not explain the reason for a particular phenomenon or principle. According to the results obtained from the current study, teachers think that in parallel with their own solutions, their students will be more successful in routine problems that can be solved with operational steps based on certain formulas. In the answers given by the students to the problems, similar results were obtained with their predictions. However, teachers could not make accurate predictions about the students' solutions. Swenson (1997) also found in his study that middle school teachers have traditional views on probability learning and teaching and have deficiencies in knowing students' probabilistic concepts / misconceptions. These findings are similar to the findings of the present study.

Therefore, it can be said that the teachers evaluate their students in line with their own knowledge and they transfer this knowledge to their students. Similarly, Talawat (2015) found that teachers 'probabilistic misconceptions and their understanding of probability affect their students' understanding of probability and their learning. Haller (1997) also determined that middle school teachers' probability knowledge has an effect on teachers' mistakes or misconceptions in their lessons and that they affect their ability to benefit from student questions / answers. Therefore, it is seen that teachers' knowledge is related to many elements ranging from how they learn mathematics, how they reflect their views, students' mathematical knowledge, the roles of the school, and its goals (Thompson, 1992). Similarly, some studies have shown that there is a relationship between teaching practices and student learning (Richardson, Anders, Tidwell & Lloyd, 1991; Staub & Stern, 2002). Findings from the current study also support this relationship.

It is thought that this study will contribute to the curriculum development studies with the evaluation of the mathematics course curriculum in terms of probability. In addition, in line with the data obtained from the study, it is thought that this study will shed light on other studies to be conducted in order to better understand the reasons for the difficulties encountered in the process and find solutions to these difficulties for better probability teaching.

It can be suggested that training in-service teachers to improve their pedagogical content knowledge and providing prospective teachers with active learning environments where they can learn without memorization. Moreover, projects can be carried out and workshops can be organized to ensure that teachers have different perspectives on probability, simplifying the high school outcomes and increasing the time allocated to probability can be suggested.

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GENİŞLETİLMİŞ ÖZET

1. GİRİŞ

Olasılık günümüzde hayatımızın vazgeçilmez bir unsuru olmasıyla birlikte tarihi çok eskilere dayanmaktadır. Şans oyunlarının fiziksel olarak gözlenmesiyle karşımıza çıkan olasılık teorisinin başlangıcı 17. yüzyıla kadar uzanmaktadır. Njenga (2010), olasılık ile ilgili kelimeleri günlük yaşantımızda sıklıkla ve akıcı şekilde kullandığımızı belirtmiştir. Farkında olarak ya da olmadan yararlandığımız olasılık, literatürde gerçekliği modellemek ve simüle etmek için matematik ve gerçek hayat arasında bir bağlantı sağlayabilen araç olarak tanımlanmıştır (Borovcnik, 2008). Olasılık alanına ait olan kesinlikle, büyük olasılıkla, muhtemelen gibi kelimeler sıklıkla kullandığımız kavramlardır. Batanero, Chernoff, Engel, Lee ve Sánchez (2016), koşullu olasılık, orantısal muhakeme, rastgele değişkenler ve beklenti gibi kavramların günlük hayatımızdaki durumları anlamayı sağlayan olasılık bilgileri olduğunu belirtmişlerdir. Buradan hareketle olasılık, etrafımızda olan olayları yorumlama biçimimizi ve sonuç olarak hayatımızda alacağımız kararları etkilemektedir (Njenga, 2010). Taylor (2011)'da, olasılıksal yeteneklerin hayatımızı sürdürmek için gerekli olduğunu belirtmiştir. Bu nedenle olasılığın öğretimi ve olasılık becerilerinin kazandırılması toplumlar açısından önemli görülmektedir.

Olasılık konusu insan hayatında bu kadar önemli bir yere sahip olmasına rağmen, öğreniminde ve öğretiminde zorluklar yaşanmaktadır. Yapılan çalışmalar doğrultusunda olasılık öğretiminde karşılaşılan zorlukların öğretmenden, öğrenciden, öğretim programından ve konunun doğasından kaynaklandığı söylenebilir. (Ben-Hur, 2006; Bulut, 2001; Fischbein & Schnarch, 1997; Fırat, 2018; Gürbüz, 2006; Jones, 2005; Lee, 2006; Liu ve Thompson, 2007; Memnun, 2008; Pijls, Dekker & Van Hout-Wolters, 2007; Talawat, 2015). Olasılık kavramları hayatımızı etkileyen hemen her noktada karşımıza çıkmakta olduğundan, bu konunun etkili bir şekilde öğretilmesi önem arz etmektedir. Literatürde yer alan zorlukların aşılması noktasında da en büyük görev öğretmenlere düşmektedir. Öğretmenlerin matematik öğretim sürecinde etkin bir rol almalarından dolayı olasılık öğretim sürecinin öğretmenler açısından araştırılmasının önemli olduğu düşünülmektedir. Buradan hareketle bu araştırmada öğretim programının uygulayıcısı olan, öğrencilerin öğrenmelerinden sorumlu olan, öğretim sürecini yürütmek için kararlar veren öğretmenlerin görüşleri doğrultusunda olasılık öğretme-öğrenme sürecini değerlendirmek ve süreci farklı açılardan anlamaya çalışmak amaçlanmıştır.

2. YÖNTEM

Bu araştırmada olasılık konusunun öğretim sürecinin matematik öğretmenlerinin görüşlerine dayalı olarak değerlendirilmesi amaçlanmıştır. Araştırmada nitel araştırma yöntemlerinden durum çalışması kullanılmıştır. Durum çalışması, sınırlı bir sistemin derinlemesine betimlenmesi ve incelenmesini içermektedir (Merriam, 2013). Araştırmanın çalışma grubu amaçsal örnekleme yöntemi ile belirlenmiştir. Araştırmanın çalışma grubunu Güneydoğu Anadolu Bölgesi'nde bulunan bir ilde ortaokul ve liselerde görev yapan 8 ortaokul, 8 lise olmak üzere 16 matematik öğretmeni oluşturmaktadır. Ayrıca bir ortaokul öğretmeninin 8. sınıfta öğrenim gören 27 öğrencisi ve bir lise öğretmeninin 10. sınıfta öğrenim gören 39 öğrencisi de çalışmada yer almışlardır. Çalışma grubu belirlenirken araştırmanın amacı doğrultusunda öğretmenlerin ortaokul veya lise düzeyinde olasılık konusunu son bir yılda anlatmış olmalarına ve öğretmenlik deneyimlerinin 5 yıldan fazla olmasına dikkat edilmiştir.

Araştırmanın verileri yarı-yapılandırılmış görüşme protokolü ve rutin/rutin olmayan olasılık problemleri ile toplanmıştır. Çalışmanın ilk aşamasında ortaokul ve lise öğretmenleriyle olasılık konusunun öğretme-öğrenme sürecine ilişkin yarı-yapılandırılmış görüşme tekniği kullanılarak görüşmeler gerçekleştirilmiştir. İkinci aşamada ise veriler yarı yapılandırılmış görüşme soruları ve olasılık problemleri ile toplanmıştır. Öğretmenlerin görüşleri doğrultusunda iki öğretmenin öğrencilerine olasılık problemleri uygulanmış ve öğrencilerin soruları çözerken kullandıkları yol ve yöntemler de incelenmiştir. Elde edilen verilerin analizinde, gömülü teoriyi esas alan açık kodlama ve eksensel kodlama yöntemleri kullanılmıştır (Strauss ve Corbin, 1990).

3. BULGULAR, TARTIŞMA ve SONUÇLAR

Araştırma sonuçlarına göre ortaokul öğretmenleri kazanımların içerikleri ve konuya ayrılan süre ile ilgili olumlu görüşlere sahipken, lise öğretmenleri ise kazanımları yoğun buldukları ve öğretimi gerçekleştirmek için gereken sürenin yetmediğini belirtmişlerdir. Öğretmenlerin çoğu olasılık öğretiminde kavramsal öğrenmenin gerçekleştirilebilmesi için mevcut programda ayrılan sürenin yetersiz olduğundan bahsetmişlerdir. Ortaokul öğretmenleri olasılık konusunun daha alt sınıf seviyelerinden başlayarak verilmesi gerektiğini vurgulamışlar, lise öğretmeleri ise konunun lise seviyesinden başlaması gerektiğini belirtmişlerdir. Piaget ve Inhelder (1975) çalışmalarında olasılık ile ilgili sezgisel düşüncenin yaşla birlikte daha güçlendiğini belirtmişlerdir. Olasılığın anlaşılması için gerekli görülen rastgelelik kavramının anlaşılmasının 9-12 yaşlarında gerçekleştiğini ve daha küçük yaşlardaki çocukların problemleri sezgisel olarak çözdüklerini ifade etmişlerdir. Munisamy ve Doraisamy (1998) de çalışmalarında üst sınıf seviyelerinde olasılık konusunun daha iyi anlaşıldığını ortaya koymuşlardır. Öğretmenlerin derste en çok soru çözme tekniğinden faydalandıkları görüşlerinden hareketle öğretmen merkezli öğretim gerçekleştirdikleri sonucuna ulaşılmıştır. Benzer şekilde Chong vd., (2017) de öğretim süreçlerinde öğrenciyi merkeze alan yenilikçi yaklaşımların önerilmesine rağmen öğretmen merkezli öğretime devam edildiğini belirtmişlerdir. Öğretmenler rutin olmayan problemleri karmaşık olarak gördüklerini belirtmişler ve görüşleriyle paralel olarak standart işlem adımlarıyla çözülen rutin problemlerde rutin olmayan problemlere göre daha fazla doğru çözüme ulaşmışlardır. Elde edilen sonuçlar öğretmenlerin çoğunun olasılıksal prosedürleri olasılık formüllerine dayalı olarak yürüttüklerini göstermiş olmasına rağmen öğretmenler ikinci aşamada verilen olasılık problemlerinde olasılık kavramlarını kullanamamış ve problemlerle ilişkilendirememişlerdir. Benzer şekilde Ball (1990) da öğretmen adaylarının matematiği kurallar kümesi olarak gördükleri ve olayların nedenini açıklamada zorlandıkları sonucuna ulaşmıştır. Öğretmenlerin öğrencilerinin olasılık problemlerine verecekleri cevapları doğru tahmin ettikleri ama çözüm yolları konusunda doğru tahminlerde bulunamadıkları belirlenmiştir. Benzer şekilde Swenson (1997) de çalışmasında ortaokul öğretmenlerinin olasılık öğretimine ilişkin geleneksel görüşlere sahip olduklarını belirlemiş ve öğrencilerinin konu ile ilgili sahip oldukları kavramları ve kavram yanılgılarını tespit etme konusunda eksiklikleri olduğu sonucuna ulaşmıştır.

ETHICAL PERMISSION TO RESEARCH

In this study, all the rules stated to be followed within the scope of " The Higher Education Institutions Scientific Research and Publication Ethics Directive" were followed. None of the actions specified under the title of "Actions Contrary to Scientific Research and Publication Ethics", which is the second part of the directive, were not carried out.

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CONTRIBUTION OF RESEARCHERS

The contribution rate of the 1st author to the research is 50%, and the contribution rate of the 2nd author to the research is 50%.

Author 1: Determination of the method, research design, validity and reliability studies, data analysis, reporting, writing the introduction, conclusion parts.

Author 2: Designing of the study, supervisor.

CONFLICT OF INTEREST

There is no conflict of interest in the research.