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Solving the Viscous Burger's Equation Using the Hopf-Cole Transform

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Abstract

In this study, the non-linear Burger's equation is handled and linearization is provided by using the Hopf-Cole transform. Burger!s equation is the convection-diffusion equation. Examines problems associated with the behavior of solutions of partial differential space-time systems in which the highest order terms occur linearly with small coefficients. These problems arise from physical applications and especially from modern fluid dynamics (small viscosity, low thermal conductivity, compressible fluids). Small viscosity values in the equation form the solution of the viscous Burger's equation with the Hopf-Cole transformation method. This solution shows the shock formation and the way the shock is dispersed due to viscosity during movement. By applying the Hopf-Cole transform, the system is turned into a Cauchy problem, boundary conditions are created and solutions are made, and moving wave solutions are obtained. The study of these moving waves has an important place in fluid dynamics, solitary wave solutions found will open new horizons by using similar techniques in mathematical physics (2+1) dimensional and (3+1) dimensional nonlinear PDE types and will ensure that exact or real solutions are obtained.

Keywords: Nonlinear equation, Hopf-Cole transform, viscosity, analytical solution, linear system.

Hopf-Cole Dönüşümünü Kullanarak Viskoz Burger Denklemini Çözme

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Özet

Bu çalışmada, lineer olmayan Burger denklemi ele alınmış, Hopf-Cole dönüşümü kullanılarak, denklemi doğrusallaştırma sağlanmıştır. Burgers denklemi, konveksiyon-difüzyon denklemi olup en yüksek mertebeden terimlerin küçük katsayılarla doğrusal olarak oluştuğu kısmi diferansiyel uzay-zaman sistemlerinin çözümlerinin davranışıyla bağlantılı problemleri inceler. Bu problemler, fiziksel uygulamalardan ve özellikle modern akışkan dinamiğinden (küçük viskozite, küçük ısı iletkenliği, sıkıştırılabilir akışkanlar) kaynaklanmaktadır. Denklemdeki küçük viskozite değerleri Hopf-Cole dönüşümü yöntemi ile viskoz Burger denkleminin çözümünü oluşturur. Bu çözüm, şok oluşumunu ve hareket esnasında viskozite nedeniyle şokun dağılma şeklini gösterir. Hopf-Cole dönüşümün uygulanmasıyla sistem Cauchy problemi haline getirilerek buna bağlı sınır koşulları oluşturulup çözüm yapılır ve hareketli dalga çözümleri elde edilir. Hareket eden bu dalgaların incelenmesi, akışkanlar dinamiğinde önemli bir yer tutar. Bulunan soliter dalga çözümleri, Matematiksel fizikte (2+1) boyutlu ve (3+1) boyutlu doğrusal olmayan PDE türlerinde benzer teknikleri kullanmakla yeni ufuklar açacaktır ve kesin ya da gerçek çözümler elde edilmesini sağlayacaktır.

Anahtar Kelimeler: Doğrusal olmayan denklem, Hopf-Cole dönüşümü, viskozite, analitik çözüm, doğrusal sistem.

1. Introduction

Evolution equations have an important place in real world problems. Solion theory sheds light on physical phenomena such as liquid crystals [1], plasma [2], fluid dynamics [3], nonlinear wave propagation [4]. In this study, the evolution equation modeling the diffusion phenomenon is taken into account. Classical Hopf-Cole transformation is applied to solve the nonlinear diffusion equation. There are two nonlinear generalizations of the diffusion equation. The first one is the second order nonlinearity equation and the other is the Burger's equation. Second order nonlinearity equation, it is associated with plasma and acoustic phenomena. Modeling the weak viscous fluid motion by H. Bateman [5] was used. This non-linear partial differential equation is the equation used in turbulence studies [6]. Burger's equation it can be considered as a simplified version of the Navier -Stokes equation.

The Burger's equation is a semi-linear parabolic partial differential equation. Fluid mechanics has a great place in applications. The Burger's equation is similar to the Navier-Stokes equation. The occurrence of nonlinear terms and high-order derivatives with small coefficients is equivalent in both. The Burger's equation is one of the rare nonlinear partial differential equations that can be fully solved for any initial and boundary conditional problem.

In addition, this equation is reduced to the gas dynamic momentum equation, which is used as some numerical scheme test problem. The main difficulty encountered here is the small viscosity values in the Burger's equation in the numerical solution or the large Reynolds numbers. Miller [7], He has obtained important results for the Burger's equation by creating a predictive-corrective method to solve this problem. Recently, a new finite element method for the solution of the Burger's equation is studied. In this method, the space-time relationship and its properties are studied. In addition, numerical solutions have been successfully applied in the finite element method which includes a time- dependent parameter [8]. Öziş et al. [9] using the Galerkin method, they successfully applied the finite element approach for second order spline interpolation functions and studies have shown the applicability of complementary variable elements to nonlinear equations; in particular, this case has been investigated for the Burger's equation and the static state of a special test, the Burger's equation, has been solved numerically [10-16]. One of the methods used to solve the viscous Burger's equation is the Cole-Hopf transformation method. It has opened new horizons for using similar techniques in the solution of other high-grade PDEs.

This study will explain the contributions of the solution of viscous Burger's equation in terms of clarifying some physical problems. The Cole-Hopf transformation helps us solve a high-grade PDE. The viscous Burger's equation was discussed in 1940, Hopf [17] in 1950, and Cole [18] and 1951. In these studies, they specifically applied the Cole-Hopf transformation method to the viscous Burger's equation and obtained new results. Although the Burger's equations are known as a simple turbulence model in the literature, with the method to be used, the results obtained will provide important facilities for examining the equations that can be converted to Viscose Burger's equation and Cole transformation. Physically, in fluid dynamics, there is a different problem, called the Stokes Problem, in which an infinite plate with liquid is activated at a constant speed. In fact, this problem only permits the study of a dispersion caused by viscous friction. In the studies, however, it has been suggested that the fluid layers are activated only by viscous friction. In this paper, the initial value problem, which we will discuss, is the Burger's equation that remains in the domain of infinite dimensional space. We examine the analytical solution for this problem. Then, with the help of the

Cole-Hopf transformation, we solve the diffusion equation obtained from the Burger's equation by means of open and implicit finite difference schemes. In addition, with the help of the solution data of the diffusion equation, we reach the desired solution by using the Hopf-Cole transformation for the Burger's equation. In short, with the Hopf-Cole transformation, we are able to solve such equations more easily. In the viscous Burger's equation we use, v is the viscosity velocity field (v> 0), t time, y is the direction normal to the plate, and α is an optional constant. Using the term viscosity in the Burger's equation is effective in reducing the amplitude u over time [19]. This has new and different consequences. (In Stokes first problem, α = 0 is taken. There are other cases where α> 0).

2. Materials and Methods

In this study, Hopf-Cole transform is used to linearize the nonlinear Burger's equation given under some initial and boundary conditions. This method has also been applied to the solution of some similar parabolic partial differential equation systems.

2.1. Burger's Equation and Hopf-Cole Transformation

Consider the Burger's equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = v\frac{\partial^2 u}{\partial x^2} , \tag{2.1}$$

where $t \geq 0$ is a time variable, $x \in [0,1]$ is a space variable, $u(x,t)$ is the state and $v > 0$ is the viscosity parameter.

The initial and boundary conditions are

$$u\,(x,0) = u_0(x), \tag{2.2}$$

$$u\,(0,t) = u\,(1,t) = 0\,. \tag{2.3}$$

The region of this problem is $D = \{(x,t): 0 < x < 1,\ 0 < t < T\}$. Let's assume $u_0(x)$ in this article for convenience. We can extend our method here to any initial value of $u_0(x)$.

For the Burger's equation

$$u = -2v\frac{\phi_x}{\phi} , \tag{2.4}$$

transformation is known in the literature as the Hopf-Cole transformation [20]. We perform the transformation in two steps. First, let us assume $u = \varphi_x$. If it is written in (2.4) the Burger's equation (2.1), obtained

$$\varphi_{xt} + \varphi_x\varphi_{xx} = v\varphi_{xxx}. \tag{2.5}$$

If both sides of this expression are integrated according to x, located

$$\int \varphi_{xt}\,dx + \int \varphi_x\varphi_{xx}dx = v\int \varphi_{xxx}dx ,$$

$$\varphi_t + \frac{1}{2}(\varphi_x)^2 = v\varphi_{xx}.$$

Then we make the transformation $\varphi = -2vIn\phi$, the equation corresponds to the heat equation dependent on the $\phi = \phi(x,t)$

$$\phi_t = v\phi_{xx}. \tag{2.6}$$

The initial value condition $\phi(x,0)$ in this heat problem is the kernel function $H(x,t)$,

$$\phi(x,t) = \int_{-\infty}^{\infty} H(x-y,t)\ \phi(y,0)dy \tag{2.7}$$

$$H(x,t) \equiv \frac{1}{\sqrt{4\pi vt}} e^{\frac{-x^2}{4vt}}. \tag{2.8}$$

Considering $\varphi\ (x,t)$ by $\varphi\ (0,t) = 0$, the solution function of this initial conditional problem is as follows

$$\varphi\ (x,t) = -2\ vIn\left[\int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi vt}} e^{\frac{-(x-y)^2}{4vt} - \frac{1}{2v}\int_0^y u(z,0)dz}\ dy\right]. \tag{2.9}$$

Thus, the solution function $u = \varphi_x$ of the Burger's equation also corresponds to

$$u\ (x,t) = \frac{\int_{-\infty}^{\infty} \frac{x-y}{t} e^{\frac{-(x-y)^2}{4vt} - \frac{1}{2v}\int_0^y u(z,0)dz} dy}{\int_{-\infty}^{\infty} e^{\frac{-(x-y)^2}{4vt} - \frac{1}{2v}\int_0^y u(z,0)dz} dy}\ . \tag{2.10}$$

From here, we get the following result; The Hopf-Cole transform is a way of solving the initial value problem of the Burger's equation.

2.2. Hopf-Cole Transformation

Consider the linearity of the Hopf-Cole transform. Let $\tilde{u}(x,t)$ be a solution to Burger's equation

$$\tilde{u}_t + \frac{1}{2}(\tilde{u}_x)^2 = v\tilde{u}_{xx}\ ,$$

$$\tilde{\varphi}_x = \tilde{u},\ \tilde{\varphi}\ (x,t) = -2vIn\tilde{\phi}(x,t),$$

and $\tilde{u}\kappa_t + (\tilde{u}\kappa)_x = v\kappa_{xx}$ be provided.

Hopf-Cole transformation was used to solve the linearized Burger's equation and the steps to be applied for this were explained above. The normal Hopf-Cole relation is

$$Y + \tilde{\varphi} = -2vIn(\tilde{\phi} + \varepsilon),$$

the linearized Hopf-Cole relationship becomes this shape

$$Y = -2v\frac{\varepsilon}{\tilde{\phi}}\,.$$

With these operations, we also reach the heat equation $\varepsilon_t = v\varepsilon_{xx}$ using the ε function, and from there we obtain the following solution of the initial value problem of the Burger's equation,

$$Y(x,t) = \frac{\int_{-\infty}^{\infty}\frac{1}{\sqrt{4\pi vt}} e^{\frac{-(x-y)^2}{4vt}}\ \tilde{\phi}(y,0)Y(y,0)dy}{\phi(x,t)}, \tag{2.11}$$

$$\varepsilon(x,t) = -\frac{1}{2v}\int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi vt}} e^{\frac{-(x-y)^2}{4vt}}\ \tilde{\phi}(y,0)Y(y,0)dy, \tag{2.12}$$

$$\kappa(x,t) = Y_x(x,t).$$

By applying the Hopf-Cole transform, our problem turns into the following Cauchy problem

$$\phi_t = v\phi_{xx}\ ,$$

$$\phi\ (x,0) = e^{\frac{1}{2v}\int_0^x u(z,0)dz}. \tag{2.13}$$

For $u_0(x) = sinx$, the starting condition of the new problem is,

$$\phi\ (x,0) = e^{\frac{1}{2v}\int_0^x sinzdz} = e^{\frac{cosx}{2v}}\ . \tag{2.14}$$

Let's choose the boundary condition of u as follows

$$u(0,t) = 0,\quad u(2\pi,t) = 0\ .$$

Considering the following equation with this condition (2.15) is obtained

$$u = -2v\frac{\phi_x}{\phi},$$

$$\frac{\phi_x(0,t)}{\phi} = \frac{\phi_x(2\pi,t)}{\phi} = 0 \,. \qquad (2.15)$$

Integrating (2.4) with respect to x we obtain the equivalent relation

$$\phi\ (x,t) = C(t)e^{\frac{1}{2v}\int_b^\delta u dx},$$

where b is an arbitrary constant, and $C = \phi(b,t)$. Without loss of generality b may be normalized to the value zero. The initial value is simply related to

$$u\ (x,0)\ = u_0(x)$$

Then,

$$\phi\ (x,0)\ = \phi_0(x) = C_0(t)e^{\frac{1}{2v}\int_b^\delta u_0 dx}.$$

Otherwise, a singularity occurs at every u point. ϕ never disappears unless x and t are set to zero. (2.15) from,

$$\phi_x(0,t) = 0,\ \ \phi_x(2\pi,t) = 0.$$

With Neumann boundary conditions, diffusion equation as initial boundary value problem, the following problem arises

$$\phi_t = v\phi_{xx},$$
$$\phi(x,0) = e^{\frac{cosx}{2v}},$$
$$\phi_x(0,t) = 0 = \phi_x(2\pi).$$

2.3. Analytical Solution of Burger's Equation

Let (2.1) Burger's equation be given under (2.2) - (2.3) initial and boundary conditions. With the Hopf-Cole transformation (2.4) we obtain the analytical solution of (2.6) corresponding to the heat equation in [21]. The initial condition of $\phi(x,t)$, is obtained by substituting in $u_0(x) = \ sinx$ condition equation (2.14). Using the Hopf-Cole transform of (2.4) and (2.2) - (2.3) boundary conditions,

$$\phi_x\ (0,t)\ =\ \phi_x\ (1,t)\ =\ 0\,.$$

$\phi(x,t)$, being the solution of the heat equation (2.6),

$$\phi(x,t) = \psi(t)\omega(t),$$
$$\frac{\partial\phi}{\partial x} = \psi'(x)\ \psi'(x)\omega(t),$$
$$\frac{\partial^2\phi}{\partial x^2} = \ \psi''(x)\omega(t),$$
$$\frac{\partial\phi}{\partial t} = \ \psi(x)\omega'(t).$$

If the equations are written in the heat equation (2.6) obtained.

$$\psi(x)\omega\,'(t)\ = v\,\psi''(x)\omega(t)$$

If necessary arrangements are made

$$\frac{\psi''}{\psi} = \frac{1}{v}\frac{\omega'}{\omega} = -\lambda^2.$$

Here, λ is fixed. From this last equation, two ordinary differential equations are written as,

$$\psi'' + \lambda^2\psi = 0,$$
$$\omega' + v\lambda^2\omega = 0,$$

and the solutions are,

$$\psi\ =\ A_1 cos\ (\lambda x)\ + B_1\ sin\ (\lambda x),$$
$$\omega = \mathrm{a}\ exp\ (-\lambda^2 vt).$$

Thus, $\phi(x,t)$,

$$\phi\ (t) = (A_1 \cos\ (\lambda x)\ + B_1\ \sin\ (\lambda x))(\mathrm{a}\ \exp\ (-\lambda^2 vt)),$$

found as. Where $A\ =\ aA_1,\ B\ =\ aB_1$. Here, if the derivative of the function with respect to x is taken,

$$\frac{\partial\phi\ (x,t)}{\partial x} =\ \exp\ (-\lambda^2 vt)(-A\lambda \sin(\lambda x)\ + B\lambda\ \cos\ (\lambda x)).$$

If the left boundary condition is applied to this expression, has

$$\frac{\partial\phi\ (0,t)}{\partial x} = B\lambda\ \exp(-\lambda^2 vt) = 0.$$

This requires $B = 0$. So, it is obtained that

$$\phi\ (x,t) = (A\ \cos(\lambda x))\ \exp\ (-\lambda^2 vt)).$$

Thus, $\phi_x\ (x,t)$,

$$\frac{\partial\phi\ (x,t)}{\partial x} = \exp(-\lambda^2 vt)\ \left(-A\lambda \sin(\lambda x)\right),$$

and if the boundary condition is also used, we get

$$\frac{\partial\phi\ (1,t)}{\partial x} =\ -A\lambda \exp\ (-\lambda^2 vt) \sin(\lambda) = 0.$$

If it is $\sin(\lambda) = 0$, it must be $\lambda = n\pi\ (n = 1,2, \dots)$.Thus, $\phi\ (x,t)$ obtained

$$\phi\ (x,t) = A_n \exp\ (-n^2\pi^2 vt)\ \cos\ (n\pi x). \tag{2.16}$$

This equation provides boundary conditions. The initial condition must also be provided separately.

$$\phi\ (x,t) = A_0 + \textstyle\sum_{n=1}^{\infty} A_n\ exp(-n^2\pi^2 vt) \cos(n\pi x), \tag{2.17}$$

where A_0, A_n are Fourier coefficients,

$$A_0 = \int_0^1 \phi_0\ (x) dx = a_0 \int_0^1 exp\ \left(-\ (2v\pi)^{-1}(1 - cos\pi x)\right) dx,$$

$$A_n\ = 2a_0 \int_0^1 exp\ (-\ (2v\pi)^{-1}(1 - cos\pi x)) cos\pi x dx\ .$$

Thus, using (2.17) in equation (2.2), we get the analytical solution of the problem, the Hopf-Cole transform, as

$$u(x,t)\ = 2\pi v \frac{\sum_{n=1}^{\infty} nA_n exp\ (-n^2\pi^2 vt)\ \ \sin\ (n\pi x)}{\sum_{n=0}^{\infty} A_n exp\ (-n^2\pi^2 vt)\ \ \cos\ (n\pi x)}. \tag{2.18}$$

2.4. The Burger's Equation Traveling Wave Solution

Burger's equation is a simple turbulence model. This nonlinear equation creates a model for diffusivity waves in fluid dynamics; uu_x means nonlinear convection and vu_{xx} means diffusion in [22-23]. By obtaining the traveling wave solution of equation (2.1), we can observe the semi-parabolic nature of the equation. With c being a constant,

$$u(x,t) = \varphi(z),\ z = x - ct, \tag{2.19}$$

let's investigate the traveling wave solution. Where c is the wave velocity, $u(x,t)$ indicates the form of the wave. (2.19) is written in the following equation

$$X = \xi_1(x) \frac{\partial u}{\partial x_1} + \xi_2(x) \frac{\partial u}{\partial x_2} + \cdots + \xi_n(x) \frac{\partial u}{\partial x_n},$$

if we write $\varphi'(z)\ =\ \frac{d\varphi}{dz}$,

$$-c\varphi\ ' + \varphi\varphi\ '\ - v\varphi\ ''\ = 0, \tag{2.20}$$

ordinary differential equations are obtained. On the other hand, $\varphi\varphi\ ' = \frac{1}{2}(\varphi^2)'$ where (2.20) is integrated, with M being the integral constant,

$$-c\varphi + \frac{1}{2}\varphi^2 - v\varphi' = -\frac{M}{2}\ , \tag{2.21}$$

and from here

$$\varphi' = \frac{1}{2v}(\varphi^2 - 2c\varphi + M), \tag{2.22}$$

obtained. After this step (2.22) to solve the equation and c, M to determine the constants, $z_1 \neq z_2$ is

$$\lim_{z\to-\infty} \varphi(z) = z_1, \tag{2.23}$$

$$\lim_{z\to\infty} \varphi(z) = z_2, \tag{2.24}$$

we will have to assume boundary conditions. Now, let's accept it is $z_1 > z_2$, (2.23) and (2.24) is applied to the equation (2.22) and

$$z_1{}^2\text{-}2cz_1 + M = 0,$$
$$z_2{}^2\text{-}\ 2cz_2 + M = 0, \tag{2.25}$$

the system of algebraic equations is obtained. Thus the solution of (2.25),

$$c = (z_1 + z_2)/2, \qquad M = z_1 z_2\ . \tag{2.26}$$

Then $c^2 > M,\ z_1 = c + \sqrt{c^2 - M},\ z_2 = c - \sqrt{c^2 - M}$ is taken equation (2.22) takes the form

$$\varphi' = \frac{1}{2v}(\varphi - z_1)(\varphi - z_2). \tag{2.27}$$

Here,
$\varphi' = 0$ for $\varphi = z_1$ or $\varphi = z_2$, $\varphi' < 0$ for $z_1 < \varphi < z_2$ and $\varphi' > 0$ for $\varphi > z_1$ and $\varphi < z_2$ equation (2.27) , $\varphi = z_1$ and $\varphi = z_2$ have critical points.

$$F(\varphi) = \frac{1}{v}(\varphi - z_1)(\varphi - z_2),$$
$$F'(\varphi) = \frac{1}{2v}(2\varphi - (z_1 + z_2)).$$

If $F'(z_1) < 0$ then $\varphi = z_1$ point is the unstable critical point, $F'(z_2) > 0$ $\varphi = z_2$ point stable critical point. (2.27) is an equation divided into variables

$$\frac{1}{z_1 - z_2}\left(\frac{1}{\varphi - z_1} - \frac{1}{\varphi - z_2}\right) d\varphi = \frac{1}{2v} dz, \tag{2.28}$$

is found. For $z_1 < \varphi < z_2$, (2.28) is integrated

$$u(x,t) = \varphi(z) = z_2 + \frac{z_1 - z_2}{1+exp\left[\frac{z_1 - z_2}{2v}(x-ct)\right]}\ ,\ \ c = \frac{z_1 + z_2}{2}\ . \tag{2.29}$$

(2.29) is the solution of equation (2.1). Also, if we rearrange (2.29) using the formula

$$\frac{1}{1+e^x} = \frac{1}{2}\left[1 - \tanh\left(\frac{x}{2}\right)\right],$$

$$u(x,t) = \varphi(z) = \frac{1}{2}(z_1 + z_2) - \frac{1}{2}(z_1 - z_2)tanh\left[\frac{(z_1 - z_2)(x-ct)}{4v}\right]\ ,\ \ c = \frac{z_1 + z_2}{2}\ , \tag{2.30}$$

obtained. For a special case, (2.30) $z_2 = 0$ is the result of the acquisition,

$$u(x,t) = \frac{z_1}{2}\left[1 - tanh\left[\frac{z_1}{4v}\left(x - \frac{z_1}{2}t\right]\right]\right], c = \frac{1}{2}z_1.$$

which is called the Taylor shock profile. On the other hand, if $(z_1 - z_2)/2v > 0$ then from equation (2.29), z is large positive values for $\varphi(z) \to z_1$ and for large negative values of z, $\varphi(z) \sim z_2$, $\varphi(0) = \frac{1}{2}(z_1 + z_2)$ and for every z, $\varphi'(z) < 0$. So, as long as it is $z_1 > z_2$,

The wave profile from z_1 to z_2 constant $u(x,t) = \varphi(z)$ is monotonous. The diffusion coefficient ν has a significant effect on the waveform. The term diffusion in the equation means that the current wave profile gradually changes shape and prevents it. Diffusion causes the absence of the term breaks the waveform and creates a shock it happens. The weak diffusion effect (small ν), $\varphi(z)$ sharp (upright). In addition, diffusivity the effect is greater and this means that the diver has a narrow oblique. In convection and diffusion (2.29), exactly balanced. To summarize, φ, given by (2.30) function and $c = \varphi(0) = \frac{1}{2}(z_1 + z_2)$. If ν → 0, $u(x,t,v)$ function will approach the shock dissolve $u(x,t,v) = \varphi(x - ct)$.

3. Results and Discussion

Different methods are being studied for the solution of Burger's equation. The main thing in these methods is the space-time relationship and its properties. This situation was examined for Burger's equation and the static version of Burger's equation, which is a special test, was solved.
The Hopf-Cole transform helps us solve a high-order PDE. It is one of the methods used to solve the viscous Burger's equation. It has opened new horizons for using similar techniques to solve other high quality PDEs. This study will explain the contribution of the solution of the viscous Burger's equation in terms of clarifying some physical problems.

4. Conclusions

Burger's equation, which has great importance in fluid mechanics applications, is a semi-linear parabolic partial differential equation. It is in the style of the Navier-Stokes equation. This equation is one of the rare nonlinear partial differential equations that can be solved exactly for any initial and boundary condition problem.
Small viscosity values in the equation form the solution of the viscous Burger's equation by the Hopf-Cole transformation method. It will open new horizons for using similar techniques to solve other high quality PDEs. This study will explain the contribution of the solution of the viscous Burger's equation in terms of clarifying some physical problems. Although the Burger's equations are known as a simple turbulence model in the literature, the results obtained with the method to be used will provide important convenience in the examination of the equations that can be converted to the viscose Burger's equation and the Hopf-Cole transform.

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