An Application of Infinite Programming on Desirability Functions

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Abstract

When assessing the quality of a system or product, it is necessary to take all responses into account and optimize them in a concurrent manner to find the factor levels that satisfy the overall system, process, or product properties to solve the robust design problem. This problem can be solved as a multi-response optimization problem. There are many methods suggested to solve this problem based on different disciplines like multi-objective optimization. In this study, we improve the theory of nondifferentiable desirability functions' optimization for which the so-called gradient-based methods are not useful. In this study, we propose an infinite programming approach for nondifferentiable desirability functions including more than one nondifferentiable point. We employed DNLP model of GAMS/BARON which is a nondifferentiable solver, however, the solution of more than one nondifferentiable point problem is resulted as infeasible. We also tested MATLAB/NOMAD which is a derivative-free solver for MINLP problems, however, MATLAB/NOMAD also did not succeed and could not solve this nondifferentiable problem. Lastly, we use a genetic algorithm that is implemented under MATLAB and it also cannot find a feasible solution. We use an example that is solved by different desirability functions approaches before and show that the desirability functions approach with more than one nondifferentiable point is a good alternative to the ones in the literature. We present the conclusion and future studies at the end of the paper.

1. Introduction

During 1980s, Taguchi introduced the notion of robust design to the quality engineering world [1, 2]. Robust design methodology finds the control factors that give the best setting of responses which corresponds to and aims at variance reduction and insensitivity to noise factors [3]. In this study, we are interested in static systems with multiple responses where the target information of the responses are static [4–8]. From the design of the experiment, the quality data are used to obtain response models and then optimized concurrently by means of desirability functions [9]. Although other multi-objective optimization methods based on utility functions with normed distances are possible to solve a multi-response optimization problem, desirability functions are highly preferred because of their simplicity and ease of implementation with less mathematical and statistical knowledge [10, 11]. Desirability functions are improved in the literature, to solve the dynamic robust design problems which is an advantage of them over methods like loss functions and response surface methodology [4, 12].

Desirability functions first introduced by [13] in 1965, have the disadvantage of lack of shape flexibility. Derringer and Suich suggested new desirability functions with more shape alternatives, however, have the disadvantage of being nondifferentiable when the two-sided individual desirability are included in the problem [14, 15]. Castillo et. al. [16] suggested smoothing the nondifferentiable points by polynomial approximations and then combine them to have an objective function that is differentiable. This method is reported to have the disadvantage of requiring considerable algebra [16]. There are other desirability functions examples in the literature all reported to have disadvantages related with producing global optimal, requiring sophisticated algebra knowledge for being able to use and less shape flexibility. In this study, we improve the method of [16] and propose a method that is easy to use and has shape flexibility which make it implement successfully.

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In [17], we explained the topological structure of desirability functions which give rise to different forms of desirability functions in the literature. We combine this knowledge with the model we present in [18], and propose infinite programming to use in modeling individual desirability functions with infinitely many nondifferentiable points. We propose to have an infinite number of nondifferentiable point in an individual desirability function to have a smooth individual desirability function instead of using powers over desirability functions to have a flexible desirability function. Since we need the constraints of each individual desirability functions being between 0 and 1 in our optimization problem, we face infinity both in the objective and the constraint. This makes the problem infinite programming problem. We employ binary integer variables to represent the function pieces that lie in between the nondifferentiable points. This makes the problem nondifferentiable MINLP in which the number of function pieces can be chosen as infinite. In [17], we explained that these function pieces are min-type continuous selection functions and can be selected either linear or nonlinear. In this study, we choose linear min-type functions for computational purposes, however, it is possible to choose them quadratic or other nonlinear forms.

To solve this infinite programming problem, we employ DNLP model of GAMS/BARON [19, 20] and MINLP model [21] with MAT-LAB/NOMAD [22] and genetic algorithm of MATLAB [23]. GAMS/BARON finds no feasible solution and MATLAB/NOMAD produce the exceeded iterations output. However, genetic algorithm of MATLAB finds an alternative solution to the solution reported in the literature. We understand that this highly nondifferentiable problem can not be solved by two well-known nondifferentiable solver but only genetic algorithm of MATLAB. We show our implementation on a well-known example from the multi-response optimization literature.

In Section 2, we present our optimization model that enables the user to choose any shape of a desirability function by employing necessary number of nondifferentiable points. In Section 3, we solve a problem including two-sided desirability functions which [16] used and solved. Lastly, we present outlook and future work of the study.

2. Infinite Programming For Desirability Functions

We have studied nondifferentiable individual desirability functions in [9]. All the formulations and classical desirability functions are based on [14]. In this study, we take the binary variables that represent individual desirability functions over finite pieces of the function in the numerical example. Here, it is possible to employ an infinite number of function pieces in obtaining individual desirability function $d_j^Y(\mathbf{x}, \mathbf{z}_j)$. We calculate each of the function pieces by using the method given in [16]:

$$d(Y(\mathbf{x})) = a_\zeta + b_\zeta Y \qquad y_\zeta < Y \le y_{\zeta+1} \qquad (\zeta = 1, 2, \ldots, \zeta_j)$$

where $b_\zeta = (d_{\zeta+1} - d_\zeta)/(y_{\zeta+1} - y_\zeta)$ and $a_\zeta = d_{\zeta+1} - b_\zeta y_{\zeta+1}$.

An individual DF $d_j^Y(\mathbf{x}, \mathbf{z}_j)$ including $\zeta_j - 1$ many nondifferentiable points can be expressed as follows for $\mathbf{z}_j = (z_{j1}, z_{j2}, \ldots, z_{j\zeta_j})^T$ with $\sum_{\zeta=1}^{\zeta_j} z_{j\zeta} = 1$ ($\zeta = 1, 2, \ldots, \zeta_j$; $j = 1, 2, \ldots, m$):

$$d_j^Y(\mathbf{x}, \mathbf{z}_j) = \sum_{\zeta=1}^{\zeta_j} z_{j\zeta} d_{j\zeta}^Y(\mathbf{x}) \quad (j = 1, 2, \ldots, m).$$

Hence, the overall problem ($\mathscr{P}$) turns into a minimization problem:

$$
\begin{aligned}
\text{maximize} \quad & \left(\prod_{j=1}^{m} d_j^{Y,}(\mathbf{x}, \mathbf{z}_j)\right)^{(1/m)} \\
\text{subject to} \quad & \mathbf{x} \in [\mathbf{l}_x, \mathbf{u}_x], \\
& 0 \le d_j^{Y,}(\mathbf{x}, \mathbf{z}_j) \le 1 \quad (\zeta = 1, 2 \ldots \zeta_j; j = 1, 2, \ldots, m), \\
& \sum_{\zeta=1}^{\zeta_j} z_{j\zeta} = 1 \ (\zeta = 1, 2, \ldots, \zeta_j; \ j = 1, 2, \ldots, m), \\
& z_{j\zeta} \in \{0, 1\} \quad (\zeta = 1, 2 \ldots \zeta_j; j = 1, 2, \ldots, m)
\end{aligned}
\tag{2.1}
$$

where weights of the desirability functions are given equal ($w_j \ge 0$ ($j = 1, 2, \ldots, m$)) and $\sum_{j=1}^{m} w_j = 1$ in the problem. We note that each function $d_{j\zeta}^Y : \mathbb{R}^n \to \mathbb{R}$ is assumed to be a C^2-function. Here, $z_{j\zeta}$ is the binary variables for $d_{j\zeta}^Y$ of d_j^Y. In this problem, we assume $\zeta_j \ge 1$ which gives our semi-infinite constraint. Overall desirability function $D^{\mathbf{Y}}(\mathbf{x}, \mathbf{z})$ is the geometric mean of the individual desirability functions $d_j^{Y,}(\mathbf{x}, \mathbf{z}_j)$.

3. Numerical Example and Results

In this section, we solve wire bonding process optimization problem which is presented in [16] with 3 response models given in [9] according to the proposed method. In this problem, the overall desirability function $D^{\mathbf{Y}}(\mathbf{x}, \mathbf{z}) = D(\mathbf{y}, \hat{\mathbf{z}}) = D(\mathbf{Y}(\mathbf{x}), \hat{\mathbf{z}})$ with $y_j = Y_j(\mathbf{x})$ ($j = 1, 2, 3$) is used. It is obtained by calculating the geometric mean of the individual desirability functions with equal weights. In this study, for achieving the more than one nondifferentiable case, we use three pieces on the left sides of the individual desirability functions and two pieces on the

right. We use the binary variables $z1(j), z2(j), z3(j)$ to show the active piece of the individual desirability functions where the responses of the problem are $\mathbf{Y}(\mathbf{x}) = (Y_1(\mathbf{x}), Y_2(\mathbf{x}), Y_3(\mathbf{x}))$ and all the desirability functions of the problem are two-sided.

In this study, we choose three solvers for solving our infinite programming model of desirability functions including 3 nondifferentiable points. These solvers are well-known in solving successfully the nonlinear programming problems. Although, we fail to achieve feasible solution, in multi-response optimization this can be acceptable as a deep solution.

We added the nonlinear constraints of individual desirability functions being 0 and 1 to the model on which GAMS/BARON is implemented by using a DNLP model. While implementing this DNLP model under GAMS/BARON, we add the following constraint to the problem (2.1) for defining binary variables $z_{j\zeta}$

$$z_{j\zeta} - (z_{j\zeta})^2 = 0.$$

We give the details of the optimization model in appendix A. This model is resulted as infeasible possibly because the highly nondifferentiability of the model.

We also implemented MATLAB/NOMAD on the same problem as a MINLP model, however, MATLAB/NOMAD produce the exceeded iterations output (see Appendix B) and finds no solution.

Lastly, we run genetic algorithm that is implemented under MATLAB and we there is no feasible solution. The results are given in Table 3.1.

Table 3.1: Optimal solution of the Wire Bonding Process Optimization problem with 3 responses with genetic algorithm with MATLAB [23].

Method	$(x_1^0, x_2^0, x_3^0, x_4^0, x_5^0, x_6^0, x_7^0, x_8^0, x_9^0,)$ $(x_{10}^0, x_{11}^0, x_{12}^0, x_{13}^0, x_{14}^0, x_{15}^0, x_{16}^0, x_{17}^0, x_{18}^0)$	
1	$(0.9999, 0.7611, 0.7288, 0.1193, 0.0371, 0.7690, 0.0262, 0.0484, 0.5051, 0.0947)$ $(0.2157, 0.0430, 0.1415, 0.2029, 0.0903, 0.6877, 0.0123, 0.0067)$	
	$(d_1(y_1^*), d_2(y_2^*), d_3(y_3^*), d_4(y_4^*))$	D^*
1	$(0.5121, 0.9009, 0.5330)$	0.2459

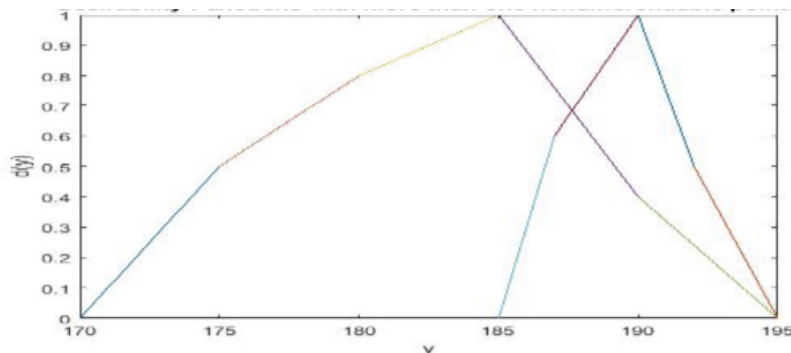We give in Figure 3.1 the desirability functions with a number of nonsmooth points.

Figure 3.1: Desirability Functions of the example problem.

4. Conclusion and Future Outlook

We show an application of the method for desirability functions including more than one nondifferentiable points. This was suggested in [16] and we achieved this by using infinite programming. We develop the optimization problem with a nondifferentiable model using well-known optimization solvers for this problem. Since our problem is highly nondifferentiable, both solvers, i.e., GAMS/BARON, MATLAB/NOMAD and genetic algorithm of MATLAB fail to find a feasible solution or global but produce a infeasible solution or exceed iterations.

Although the model we develop with infinite programming presents shape alternatives to the users of desirability functions of Derringer and Suich [14, 15], since the current nondifferentiable solvers is not successful on this problem, we suggest to the researchers and scientists to use gradient-based methods in combination with the smoothing technique of [16] for desirability functions with more than one nondifferentiable points or the methodology we suggest with genetic algorithm of MATLAB. However, our study is a theoretical advantage in the field of desirability functions including nondifferentiable points. This suggests to use the modeling of infinite programming and different solvers. Since this study is an emerging field in the multi-response optimization field, it is possible to achieve better results in the future.

In the future, it is possible to use semi-infinite programming approach on this infinitely many nondifferentiable problem study within desirability functions where the objective is based on an infinite number of pieces, however, the constraint given by individual desirability functions of being between 0 and 1 is replaced by the bounds of the responses. Also, exponential desirability functions can be studied with the approach we propose here including infinite programming [25]. We note this highly nondifferentiable optimization problem we present is not signomial optimization problem [24] since the decision variables may take negative values also.

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APPENDIX A

```
$title THE WIRE BONDING PROBLEM - 3 Response

Sets
i index of factors /1*3/
j index of responses /1*3/
l index convex/1*2/;

Variable
x(i)  factors
D     objective function
Y(j)  responses
z11(j)
z12(l)
z21(j)
z22(l)
z31(j)
z32(l);
```

```
z11.lo(j)=0;
z11.up(j)=1;
z12.lo(l)=0;
z12.lo(l)=1;
z21.lo(j)=0;
z21.up(j)=1;
z22.lo(l)=0;
z22.up(l)=1;
z31.lo(j)=0;
z31.up(j)=1;
z32.lo(l)=0;
z32.up(l)=1;

x.lo(i)=-1 ;
x.up(i)=1;

TABLE yDATA(j,*) Response Bounds

           A          B          C         D         E         F         G

    1   174.93333      0.00      23.375    3.625    -19.00     0.00      0.00
    2   154.85714      8.5       30.625    7.875     0.00    -12.85714  11.25
    3   140.23333      5.3438    18.25    19.5938    0.00      0.00      0.00 ;


Equations
COST                     objective function
binarcons11              constraint for binary variable
 binarcons21
binarcons31
mipcons11(j)
mipcons12(l)
mipcons21(j)
mipcons22(l)
mipcons31(j)
mipcons32(l)
subfunc(j)
descons1
descons2
descons3
descons4
descons5
descons6
;

COST.. D=e=((((z11('1'))*((0.5-((0.5/(187-185))*187))+(0.5/(187-185))*Y('1'))
+(z11('2'))*((0.8-(((0.8-0.5)/(188-187))*188))+((0.8-0.5)/(188-187))*Y('1'))
+(z11('3'))*((1-(((1-0.8)/(190-188))*190))+((1-0.8)/(190-188))*Y('1')))
+((z12('1'))*(0.4-(((0.4-1)/(193-190))*193)+((0.4-1)/(193-190))*Y('1'))
+((z12('2'))*((0-(((0-0.4)/(195-193))*195))+((0-0.4)/(195-193)*Y('1')))))
 *(((z21('1'))*((0.5-((0.5/(187-185))*187))+(0.5/(187-185))*Y('2'))
+(z21('2'))*((0.8-(((0.8-0.5)/(188-187))*188))+((0.8-0.5)/(188-187))*Y('2'))
+(z21('3'))*((1-(((1-0.8)/(190-188))*190))+((1-0.8)/(190-188))*Y('2')))
+((z22('1'))*((0.4-(((0.4-1)/(193-190))*193))+((0.4-1)/(193-190))*Y('2'))
+(z22('2'))*((0-(((0-0.4)/(195-193))*195))+((0-0.4)/(195-193)*Y('2')))))
 *(((z31('1'))*((0.5-(((0.5/(175-170)))*175))+(0.5/(175-170))*Y('3'))
+(z31('2'))*((0.8-(((0.8-0.5)/(180-175))*180))+((0.8-0.5)/(180-175))*Y('3'))
+(z31('3'))*((1-(((1-0.8)/(185-180))*185))+((1-0.8)/(185-180))*Y('3')))
+((z32('1'))*((0.4-(((0.4-1)/(190-185))*190))+((0.4-1)/(190-185))*Y('3'))
+(z32('2'))*((0-(((0-0.4)/(195-190)))*195)+((0-0.4)/(195-190)*Y('3'))))))**(1/3);

binarcons11..  sum((j),(z11(j)))+sum((l),(z12(l))) =e= 1 ;
binarcons21..  sum(j,(z21(j)))+sum(l,(z22(l))) =e= 1 ;
binarcons31..  sum((j),(z31(j)))+sum(l,(z32(l))) =e= 1 ;
```

```
mipcons11(j)..(z11(j)-sqr(z11(j)))=e=0;
mipcons12(l)..(z12(l)-sqr(z12(l)))=e=0;
mipcons21(j)..(z21(j)-sqr(z21(j)))=e=0;
mipcons22(l)..(z22(l)-sqr(z22(l)))=e=0;
mipcons31(j)..(z31(j)-sqr(z31(j)))=e=0;
mipcons32(l)..(z32(l)-sqr(z32(l)))=e=0;

subfunc(j)..  (yDATA(j,'A')+yDATA(j,'B')*x('1')+yDATA(j,'C')*x('2')
+yDATA(j,'D')*x('3')+yDATA(j,'E')*x('2')*x('3')+
yDATA(j,'F')*x('1')*x('1')+yDATA(j,'G')*x('1')*x('2')) =e= Y(j);

descons1..((z11('1'))*((0.5-(0.5/(187-185))*187)+(0.5/(187-185))*Y('1'))
+(z11('2'))*((0.8-((0.8-0.5)/(188-187))*188)+((0.8-0.5)/(188-187))*Y('1'))
+(z11('3'))*((1-((1-0.8)/(190-188))*190)+((1-0.8)/(190-188))*Y('1')))
+((z12('1'))*(0.4-((0.4-1)/(193-190))*193+((0.4-1)/(193-190))*Y('1'))
+(z12('2'))*((0-((0-0.4)/(195-193))*195)+((0-0.4)/(195-193)*Y('1'))))=l=1;

descons2.. ((z21('1'))*((0.5-(0.5/(187-185))*187)+(0.5/(187-185))*Y('2'))
+(z21('2'))*((0.8-((0.8-0.5)/(188-187))*188)+((0.8-0.5)/(188-187))*Y('2'))
+(z21('3'))*((1-((1-0.8)/(190-188))*190)+((1-0.8)/(190-188))*Y('2')))
+((z22('1'))*(0.4-((0.4-1)/(193-190))*193+((0.4-1)/(193-190))*Y('2'))
+(z22('2'))*((0-((0-0.4)/(195-193))*195)+((0-0.4)/(195-193)*Y('2'))))=l=1;

descons3..((z31('1'))*((0.5-(0.5/(175-170))*175)+(0.5/(175-170))*Y('3'))
+(z31('2'))*((0.8-((0.8-0.5)/(180-175))*180)+((0.8-0.5)/(180-175))*Y('3'))
+(z31('3'))*((1-((1-0.8)/(185-180))*185)+((1-0.8)/(185-180))*Y('3')))
+((z32('1'))*(0.4-((0.4-1)/(190-185))*190+((0.4-1)/(190-185))*Y('3'))
+(z32('2'))*((0-((0-0.4)/(195-190))*195)+((0-0.4)/(195-190)*Y('3'))))=l=1;

descons4..((z11('1'))*((0.5-(0.5/(187-185))*187)+(0.5/(187-185))*Y('1'))
+(z11('2'))*((0.8-((0.8-0.5)/(188-187))*188)+((0.8-0.5)/(188-187))*Y('1'))
+(z11('3'))*((1-((1-0.8)/(190-188))*190)+((1-0.8)/(190-188))*Y('1')))
+((z12('1'))*(0.4-((0.4-1)/(193-190))*193+((0.4-1)/(193-190))*Y('1'))
+(z12('2'))*((0-((0-0.4)/(195-193))*195)+((0-0.4)/(195-193)*Y('1'))))=g=0;

descons5.. ((z21('1'))*((0.5-(0.5/(187-185))*187)+(0.5/(187-185))*Y('2'))
+(z21('2'))*((0.8-((0.8-0.5)/(188-187))*188)+((0.8-0.5)/(188-187))*Y('2'))
+(z21('3'))*((1-((1-0.8)/(190-188))*190)+((1-0.8)/(190-188))*Y('2')))
+((z22('1'))*(0.4-((0.4-1)/(193-190))*193+((0.4-1)/(193-190))*Y('2'))
+(z22('2'))*((0-((0-0.4)/(195-193))*195)+((0-0.4)/(195-193)*Y('2'))))=g=0;

descons6..((z31('1'))*((0.5-(0.5/(175-170))*175)+(0.5/(175-170))*Y('3'))
+(z31('2'))*((0.8-((0.8-0.5)/(180-175))*180)+((0.8-0.5)/(180-175))*Y('3'))
+(z31('3'))*((1-((1-0.8)/(185-180))*185)+((1-0.8)/(185-180))*Y('3')))
+((z32('1'))*(0.4-((0.4-1)/(190-185))*190+((0.4-1)/(190-185))*Y('3'))
+(z32('2'))*((0-((0-0.4)/(195-190))*195)+((0-0.4)/(195-190)*Y('3'))))=g=0;

MODEL xz /ALL/;

*option domlim=10;
option dnlp=baron;

SOLVE xz USING dnlp MAXIMIZING D;

DISPLAY x.l,Y.l,z11.l,z12.l,z21.l,z22.l,z31.l,z32.l,D.l;
```

APPENDIX B

```
clc
fun=@(x)-((((x(4))*((0.5-((0.5/(187-185))*187))+(0.5/(187-185)))...
*(174.9333+23.3750*x(2)+3.6250*x(3)-19.0000*x(2)*x(3)))...
+(x(5))*((0.8-(((0.8-0.5)/(188-187))*188))+...
((0.8-0.5)/(188-187))*(174.9333+23.3750*x(2)+3.6250*x(3)-19.0000*x(2)*x(3)))...
+(x(6))*((1-(((1-0.8)/(190-188))*190))...
+((1-0.8)/(190-188))*(174.9333+23.3750*x(2)+3.6250*x(3)-19.0000*x(2)*x(3))))...
```

```
+(((x(7))*(0.4-(((0.4-1)/(193-190))*193)...
+((0.4-1)/(193-190))*(174.9333+23.3750*x(2)+3.6250*x(3)-19.0000*x(2)*x(3)))...
+(x(8))*((0-(((0-0.4)/(195-193))*195))...
+((0-0.4)/(195-193)*(174.9333+23.3750*x(2)+3.6250*x(3)-19.0000*x(2)*x(3)))))))...
 *(((x(9))*((0.5-((0.5/(187-185))*187))...
+(0.5/(187-185))*(154.8571+8.5000*x(1)+30.6250*x(2)+...
7.8750*x(3)-12.8571*x(1)^2+11.2500*x(1)*x(2)))...
+(x(10))*((0.8-(((0.8-0.5)/(188-187))*188))...
+((0.8-0.5)/(188-187))*(154.8571+8.5000*x(1)+30.6250*x(2)...
+7.8750*x(3)-12.8571*x(1)^2+11.2500*x(1)*x(2)))...
+(x(11))*((1-(((1-0.8)/(190-188))*190))...
+((1-0.8)/(190-188))*(154.8571+8.5000*x(1)+30.6250*x(2)...
+7.8750*x(3)-12.8571*x(1)^2+11.2500*x(1)*x(2))))...
+((x(12))*((0.4-(((0.4-1)/(193-190))*193))+...
((0.4-1)/(193-190))*(154.8571+8.5000*x(1)+30.6250*x(2)+...
7.8750*x(3)-12.8571*x(1)^2+11.2500*x(1)*x(2)))...
+(x(13))*((0-(((0-0.4)/(195-193))*195))+...
((0-0.4)/(195-193)*(154.8571+8.5000*x(1)+30.6250*x(2)...
+7.8750*x(3)-12.8571*x(1)^2+11.2500*x(1)*x(2))))))...
 *(((x(14))*((0.5-(((0.5/(175-170)))*175))+...
(0.5/(175-170))*(140.2333+ 5.3437*x(1)+18.2500*x(2)+19.5938*x(3)))...
+(x(15))*((0.8-(((0.8-0.5)/(180-175))*180))+...
((0.8-0.5)/(180-175))*(140.2333+ 5.3437*x(1)+18.2500*x(2)+19.5938*x(3)))...
+(x(16))*((1-(((1-0.8)/(185-180))*185))+...
((1-0.8)/(185-180))*(140.2333+ 5.3437*x(1)+18.2500*x(2)+19.5938*x(3))))...
+(((x(17))*((0.4-(((0.4-1)/(190-185))*190))+...
((0.4-1)/(190-185))*(140.2333+ 5.3437*x(1)+18.2500*x(2)+19.5938*x(3)))...
+(x(18))*((0-(((0-0.4)/(195-190)))*195)...
+((0-0.4)/(195-190)*(140.2333+ 5.3437*x(1)+18.2500*x(2)+19.5938*x(3))))))))^(1/3);


%x0 = [0.0920 1.0000 0.8170 1 1 1];
x0 = [-1 -1 -1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1];
%x0 = [0 0 0 0 0 0];
lb = [-1;-1;-1;-1;-1;-1;-1;-1;-1;-1;-1;-1;-1;-1;-1;-1;-1;-1];
ub = [1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1];

nlcon = @(x)[(((x(4))*((0.5-((0.5/(187-185))*187))...
+(0.5/(187-185))*(174.9333+23.3750*x(2)+3.6250*x(3)-19.0000*x(2)*x(3)))...
+(x(5))*((0.8-(((0.8-0.5)/(188-187))*188))...
+((0.8-0.5)/(188-187))*(174.9333+23.3750*x(2)+3.6250*x(3)-19.0000*x(2)*x(3)))...
+(x(6))*((1-(((1-0.8)/(190-188))*190))+((1-0.8)/(190-188))*...
(174.9333+23.3750*x(2)+3.6250*x(3)-19.0000*x(2)*x(3))))...
+((x(7))*(0.4-(((0.4-1)/(193-190))*193)+((0.4-1)/(193-190))...
*(174.9333+23.3750*x(2)+3.6250*x(3)-19.0000*x(2)*x(3)))...
+(x(8))*((0-(((0-0.4)/(195-193))*195))+((0-0.4)/(195-193)*...
(174.9333+23.3750*x(2)+3.6250*x(3)-19.0000*x(2)*x(3)))))...

(((x(9))*((0.5-((0.5/(187-185))*187))+(0.5/(187-185))*...
(154.8571+8.5000*x(1)+30.6250*x(2)+7.8750*x(3)-12.8571*x(1)^2+11.2500*x(1)*x(2)))...
+(x(10))*((0.8-(((0.8-0.5)/(188-187))*188))+((0.8-0.5)/(188-187))*...
(154.8571+8.5000*x(1)+30.6250*x(2)+7.8750*x(3)-12.8571*x(1)^2+11.2500*x(1)*x(2)))...
+(x(11))*((1-(((1-0.8)/(190-188))*190))+((1-0.8)/(190-188))*...
(154.8571+8.5000*x(1)+30.6250*x(2)+7.8750*x(3)-12.8571*x(1)^2+11.2500*x(1)*x(2)))...
+(((x(12))*((0.4-(((0.4-1)/(193-190))*193))+((0.4-1)/(193-190))*...
(154.8571+8.5000*x(1)+30.6250*x(2)+7.8750*x(3)-12.8571*x(1)^2+11.2500*x(1)*x(2)))...
+(x(13))*((0-(((0-0.4)/(195-193))*195))+((0-0.4)/(195-193)*...
(154.8571+8.5000*x(1)+30.6250*x(2)+7.8750*x(3)-12.8571*x(1)^2+11.2500*x(1)*x(2)))))))...

 (((x(14))*((0.5-(((0.5/(175-170)))*175))+(0.5/(175-170))*...
(140.2333+ 5.3437*x(1)+18.2500*x(2)+19.5938*x(3)))...
+(x(15))*((0.8-(((0.8-0.5)/(180-175))*180))+((0.8-0.5)/(180-175))*...
(140.2333+ 5.3437*x(1)+18.2500*x(2)+19.5938*x(3)))...
+(x(16))*((1-(((1-0.8)/(185-180))*185))+((1-0.8)/(185-180))*...
(140.2333+ 5.3437*x(1)+18.2500*x(2)+19.5938*x(3)))...
+((x(17))*((0.4-(((0.4-1)/(190-185))*190))+((0.4-1)/(190-185))*...
(140.2333+ 5.3437*x(1)+18.2500*x(2)+19.5938*x(3)))...
```

```
+(x(18))*((0-(((0-0.4)/(195-190)))*195)+((0-0.4)/(195-190)*...
(140.2333+ 5.3437*x(1)+18.2500*x(2)+19.5938*x(3))))))

x(4)+x(5)+x(6)+x(7)+x(8)
x(9)+x(10)+x(11)+x(12)+x(13)
x(14)+x(15)+x(16)+x(17)+x(18)];

cl=[0 0 0 0 0 0 0 0 0];
cu=[1 1 1 1 1 1 1 1 1];

xtype='CCCBBBBBBBBBBBBBBBB';
opts=optiset('solver','nomad','display','iter')
Opt=opti('fun',fun,'bounds',lb,ub,'nl',nlcon,cl,cu,'xtype',xtype,'options',opts)
[x,fval,exitflag,info] = solve(Opt,x0)
```