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Numerical determination of the production rate and cumulative production in the constant pressure outer boundary condition

Sabit basınçlı dış sınır şartı durumunda üretim hızının ve kümülatif üretimin sayısal belirlenmesi

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Abstract

The flow regime is identified as a steady-state flow if the pressure at every location in the reservoir remains constant. In this work, we have determined the well production rate and cumulative production in a circular reservoir using the Finite Element Method for the condition of constant pressure outer boundary. The reservoir was divided into 4 smaller part known as finite element. These parts were analyzed and later assembled to form the domain of the reservoir. The analysis was done with the assumption that before the well begins production, there was uniform distribution of pressure all through the reservoir. The results obtained from the production rate analysis shows that the dimensionless production rate decreases significantly and later becomes uniform because the withdrawn fluid has been completely replaced. This condition remains throughout the entire life of the reservoir presumably. Also, the result shows that there is a uniform increase in the dimensionless cumulative production as time increases. The result obtained in this work was compared with the results obtained by previous researcher. The comparison shows a strong positive correlation between the two methods with a maximum percentage error of 0.1711 and 0.1864 and a minimum percentage error of 0.0001 and 0.0122 for dimensionless production rate and dimensionless cumulative production respectively. Also, result from previous researcher only state the production rate and cumulative production of the reservoir at a particular time but this work predicts the production rate and cumulative production in the entire reservoir at the same time.

Keywords: Diffusivity equation, Boundary condition, Cumulative production, Production rate, Finite element method.

Özet

Rezervuarın her noktasındaki basınç sabit kalırsa, akış rejimi kararlı durum akışı olarak tanımlanır. Bu çalışmada, dairesel bir rezervuarda kuyu üretim hızı ve kümülatif üretim, sabit basınç dış sınır koşulu için Sonlu Elemanlar Metodu kullanılarak belirlendi. Rezervuar, sonlu eleman olarak bilinen 4 küçük parçaya bölünmüştür. Bu parçalar analiz edildi ve daha sonra rezervuarın alanını oluşturmak için birleştirildi. Analiz, kuyu üretime başlamadan önce, tüm rezervuar boyunca eşit basınç dağılımı olduğu varsayımıyla yapıldı. Üretim hızı analizinden elde edilen sonuçlar, boyutsuz üretim hızının önemli ölçüde azaldığını ve daha sonra çekilen sıvı tamamen değiştirildiği için üniform hale geldiğini göstermektedir. Bu durum muhtemelen rezervuarın tüm ömrü boyunca devam eder. Ayrıca sonuç, zaman arttıkça rezervuardaki boyutsuz kümülatif üretimde tek tip bir artış olduğunu göstermektedir. Bu çalışmada elde edilen sonuç, Christine tarafından elde edilen sonuçlarla karşılaştırıldı. Karşılaştırma, boyutsuz üretim hızı ve boyutsuz kümülatif üretim için sırasıyla 0.1711 ve 0.1864 maksimum yüzde hatası ve 0.0001 ve 0.0122 minimum yüzde hatası ile iki yöntem arasında güçlü bir pozitif korelasyon gösterir. Ayrıca Christine çözümleri, yalnızca belirli bir zamanda rezervuarın üretim hızını ve kümülatif üretimini belirtir, ancak bu çalışma aynı anda tüm rezervuardaki üretim hızını ve kümülatif üretimi tahmin eder.

Anahtar kelimeler: Yayılma denklemi, Sınır koşulu, Birikimli üretim, Üretim hızı, Sonlu elemanlar yöntemi.

1. Introduction

The finite element technique has over the years been so well established that today it's seems to be one of the best methods for analysing the efficiency of a wide variety of practical problems. In fact, the method has become one of the research areas for applied mathematicians [1]. The basic idea in the finite element method is actually to find the

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solution of a complicated problem by replacing it with a simpler one [2]. For many years now, the finite element method has been considered to be a numerical, and mathematically well defined, discretization method for simulating and analysing a wide variety of boundary value problems. Finally, the finite element method is a very versatile method and has found applications in many engineering problems. Today, there are over 100000 engineers that make use of the finite element method [3].

The semi-analytical techniques that have found wide application in researches in fluid dynamics were the similarity approach, the perturbation methods, and the integral methods (all for the viscous boundary layer calculations) and the methods of characteristics (for inviscid compressible flow simulations) [4]. The fluid flow in reservoir or in porous medium has been a great interest of physicists, engineers and hydrologists who tried to predict the behaviours of compressible and incompressible fluids. They have designed several experiments so as to validate the implementation of their proposed correlations [5]. The basic equation for predicting pressure distribution in a reservoir is the diffusivity equation. Several methods have proposed to solve the diffusivity equation including numerical and analytical approaches.

The diffusivity equation has been solved in dimensionless form [6]. Chakrabarty with some other researchers provided a quantitative analysis of the effects of neglecting the quadratic gradient term on solving the diffusion equation governing the transient state [7]. It should be noted that among the flow regimes in reservoir, the transient flow was the most significant state upon which such important characteristics such as permeability, reservoir capacity, and skin factor can be determined using well test analysis [6], [8]. Barreto and Peres developed a nonlinear hydraulic diffusivity equation that governs the flow of compressible fluids in porous media. In their study, a general solution that properly accounts for both fluid property behaviour and variable rate was presented . The proposed solution, which was derived from the Green's-function method by recasting the effect of the viscosity-compressibility product variation as a nonlinear source term, can handle variable gas rate for several well-reservoir geometries of practical interest [9].

Couto and Marsili presented the application of the integral transform technique in the development of a general analytical solution for the multidimensional hydraulic diffusivity equation. The solution methodology dealt directly with time-dependent well rates and boundary conditions, sparing the use of the superposition principle [10]. Wu and Li generalized mathematical framework model and numerical approach for unconventional-gas-reservoir simulation. The model and numerical scheme were based on generalized flow models with unstructured grids [11].

In the analysis of the diffusivity equation, large computation times occur because the solution involves the infinite series. Each term of the series requires evaluation of exponentials and Bessel functions and the series itself was sometimes slowly convergent and that inaccuracies can result from lack of computer precision or from the use of improper methods of numerical computation [12]. Therefore, they presented a computationally efficient and accurate new methodology in differential quadrature analysis of diffusivity equation to overcome these difficulties. The methodology would overcome the difficulties in boundary conditions implementations of second order partial differential equations encountered in such problems. A new mathematical technique called the Homotopy Analysis Method (HAM) has been used to solve the radial diffusivity equation for slightly compressible fluid [12].

A comparison between analytical and numerical solution for both linear and nonlinear diffusivity equation has been performed at wellbore radius, and secondly, a comprehensive sensitivity analysis was done to find the significant physical properties of reservoir which influence the pressure drop. Also a comprehensive analysis of parameters such as depletion time, reservoir radius and production rate to find where the pressure differences of linear and non-linear diffusivity equations were significant [13].

In all the literature reviewed so far, none of the methods has been able to look holistically at the dimensionless cumulative production and production rate in the constant pressure outer boundary condition using the finite element method, hence, the need for this work.

2. Methodology

The diffusivity equation can be used to determine the flow properties in a circular reservoir. This equation is as shown in eq. (1) below.

$$\frac{\partial^2 P_D}{\partial r_D{}^2} + \frac{1}{r_D}\frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D} \tag{1}$$

and the initial and boundary conditions become:

1. Dimensionless initial condition:

Uniform pressure in the reservoir $P_D(r_D, t_D = 0) \leq 0$ (2)

2. Dimensionless Inner Boundary Condition:

Constant rate at the well $\frac{\partial P_D}{\partial r_D}(1, t_D) = 1$ (3)

3. Dimensionless Outer Boundary Conditions:
   a. "Infinite Acting" Reservoirs

No reservoir boundary $P_D = (r_D \to \infty, t_D) = 0$ (4)

   b. "No Flow" boundary:

No flux across the reservoir $\frac{\partial P_D}{\partial r_D}(r_{eD}, t_D) = 0$ (5)

   c. Constant Pressure Boundary:

Constant pressure outer boundary $P_D(r_{eD}, t_D) = 0$ (6)

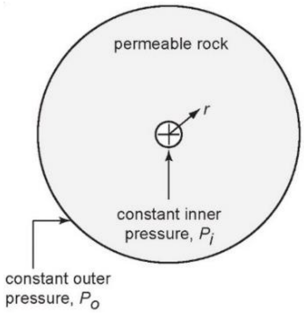

Figure 1. Constant pressure outer boundary

Eq. (1) can also be written in a condensed form as:

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial P_D}{\partial r_D}\right)=\frac{\partial P_D}{\partial t_D} \tag{7}$$

In the analysis involving Finite Element method, the governing equation can only be solved if it is in order one. But the governing equation for the diffusivity equation is in order two, so, the need to weaken the governing equation to order one. This is followed by the introduction of the interpolation functions to enable us derive the finite element model. This model is used to generate the elemental matrices and finally assembled to represent the entire domain of the reservoir. The assembled matrix cannot be solved directly. But with the introduction of either the boundary condition or initial conditions or a combination of both the initial and boundary conditions, the nodal values of the parameter can be determined . These procedures were followed and eq. (7) becomes:

$$\left[K_{ij}^e\right]\left\{P_D\right\}+\left[M_{ij}^e\right]\left\{\dot{P}_{Dj}\right\}=\left\{Q_i^e\right\} \tag{8}$$

Eq. 8 is the developed finite element model of the diffusivity equation under the unsteady state flow regime.

where $K_{ij}^e=\int_{r_{DA}}^{r_{DB}} r_D\frac{d\psi_i^e}{dr_D}\frac{d\psi_j^e}{dr_D}dr_D$ (9)

$$M_{ij}^e=\int_{r_{DA}}^{r_{DB}} r_D\psi_i^e\psi_j^e dr_D \tag{10}$$

Using Quadratic Lagrange Interpolation functions for a quadratic element:

$$\psi_1(r_D)=\frac{1}{h_e^2}(h_e+r_{DA}-r_D)(h_e-2r_D+2r_{DA}) \tag{11}$$

$$\psi_2(r_D)=\frac{4}{h_e^2}(r_D-r_{DA})(h_e+r_{DA}-r_D) \tag{12}$$

$$\psi_3(r_D)=\frac{-1}{h_e^2}(r_D-r_{DA})(h_e-2r_D+2r_{DA}) \tag{13}$$

2.1.Time approximation

Since the problem is a time dependent problem, we convert the ordinary differential equation in time to algebraic equation and the most commonly used method is the α family of interpolation. In doing this, a weighted average of the time derivative of the dependent variable is approximated to two consecutive time steps by linear interpolation of the values of the variables of the two steps.
For a given time step s, eq. (8) becomes

$$\left[K_{ij}^e\right]\left\{P_D\right\}_s+\left[M_{ij}^e\right]\left\{\dot{P}_{Dj}\right\}_s=\left\{Q_i^e\right\}_s \tag{14}$$

For the next time step s+1, eq. (8) becomes

$$\left[K_{ij}^{e}\right]\left\{P_{D}\right\}_{s+1}+\left[M_{ij}^{e}\right]\left\{\dot{P}_{Dj}\right\}_{s+1}=\left\{Q_{i}^{e}\right\}_{s+1} \tag{15}$$

Multiply eq. (14) by $(1-\alpha)$ and eq. (15) by α , then we add the two resulting equations, we have

$$\left[M_{ij}^{e}\right]\left[(1-\alpha)\left\{\dot{P}_{Dj}\right\}_{s}+\alpha\left\{\dot{P}_{Dj}\right\}_{s+1}\right]+\left[K_{ij}^{e}\right]\left[(1-\alpha)\left\{P_{Dj}\right\}_{s}+\alpha\left\{P_{Dj}\right\}_{s+1}\right]=(1-\alpha)\left\{Q_{i}^{e}\right\}_{s}+\alpha\left\{Q_{i}^{e}\right\}_{s+1} \tag{16}$$

The α family of interpolation for time consideration is given as:

$$(1-\alpha)\left\{\dot{P}_{Dj}\right\}_{s}+\alpha\left\{\dot{P}_{Dj}\right\}_{s+1}=\frac{\left\{P_{Dj}\right\}_{s+1}-\left\{P_{Dj}\right\}_{s}}{\Delta t_{s+1}} \tag{17}$$

Substitute eq. (17) into eq. (16) and using the Crank-Nicholson Scheme where $\alpha=1/2$,

$$\left[\left[M_{ij}^{e}\right]+\frac{\Delta t_{s+1}}{2}\left[K_{ij}^{e}\right]\right]\left\{P_{Dj}\right\}_{s+1}=\left[\left[M_{ij}^{e}\right]-\frac{\Delta t_{s+1}}{2}\left[K_{ij}^{e}\right]\right]\left\{P_{Dj}\right\}_{s}+\frac{\Delta t_{s+1}}{2}\left[\left\{Q_{i}^{e}\right\}_{s}+\left\{Q_{i}^{e}\right\}_{s+1}\right] \tag{18}$$

Substituting the initial condition, we have:

$$\left\{\bar{Q}_{i}^{e}\right\}=\left[\frac{1}{\Delta t_{1}}\left[M_{ij}^{e}\right]+\frac{1}{2}\left[K_{ij}^{e}\right]\right]\left\{P_{Dj}\right\}_{1}-\left[\frac{1}{\Delta t_{1}}\left[M_{ij}^{e}\right]-\frac{1}{2}\left[K_{ij}^{e}\right]\right]\left\{P_{Dj}\right\}_{0} \tag{19}$$

3. Results and Discussion

The flow regime is identified as a steady-state flow if the pressure at every location in the reservoir remains constant, i.e., does not change with time . Mathematically, this condition is expressed as:

$$P=P_{e}=\text{Constant at } r=r_{e}\text{, i.e., } P_{D}\left(r_{D}\rightarrow r_{eD},t_{D}\right)=0 \text{ and } \frac{\partial P}{\partial t}=0\ \forall \text{ r and t} \tag{20}$$

This equation states that the rate of change of pressure *p* with respect to time *t* at any location *i* is zero. In reservoirs, the steady-state flow condition can only occur when the reservoir is completely recharged and supported by strong aquifer or pressure maintenance operations, i.e. this condition is appropriate when pressure is being maintained in the reservoir due to either natural water influx or artificially by the injection of some displacing fluid. The pressure can also be maintained as a result of gas cap expansion support.

The semi-steady state flow equations are frequently applied when the rate, and consequently the position of the closed boundary surrounding a well, is slowly varying functions of time. If the production rate of an individual well is changed, for instance, due to closure for repair or increasing the rate to obtain a more even fluid withdrawal pattern throughout the reservoir, there will be a brief period when transient flow conditions prevail followed by stabilized flow for the new distribution of individual well rates .

Thus, this solution of the diffusivity equation models radial flow of slightly compressible liquid in a homogeneous reservoir of uniform thickness; reservoir at uniform pressure before production; unchanging pressure at the outer boundary; and production at constant rate from a single well (centred in the reservoir) with wellbore radius.

The results obtained from this analysis were shown in the form of graphs of dimensionless production rate. These were shown in Figs. 2 to 7.These were shown for different dimensionless radii ranging between 100 and 1000000. It was seen from the graph that the dimensionless production rate history of the reservoir was not captured at the initial stage between the dimensionless time of zero and the respective dimensionless times in Figs. 2 and 7. This was due to the fact that, within these regions, the reservoir was still at the infinite acting region. After the infinite acting period, it was observed that the dimensionless production rate decreases significantly and later becomes uniform because the withdrawn fluid has been completely replaced. This condition remains throughout the entire life of the reservoir presumably.

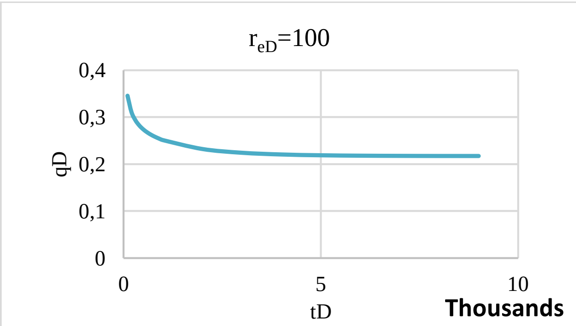

Figure 2. A graph of q_D against t_D at r_{eD}=100

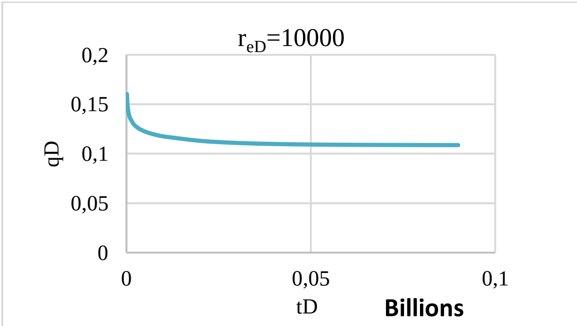

Figure 5. A graph of q_D against t_D at r_{eD}=10000

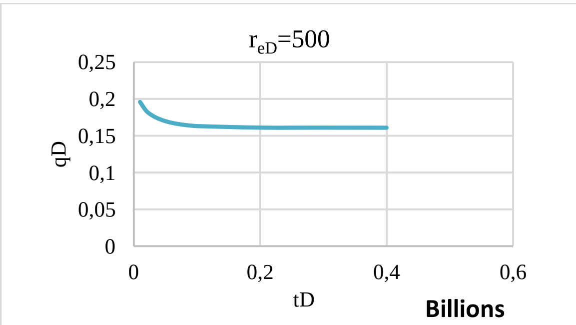

Figure 3. A graph of q_D against t_D at r_{eD}=500

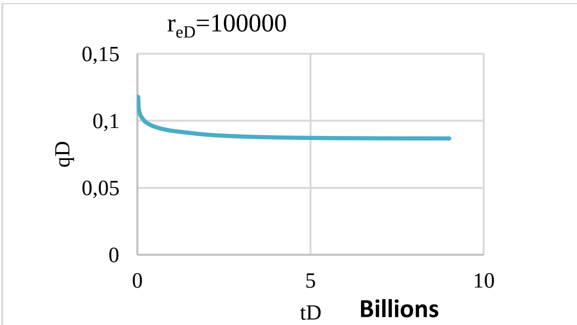

Figure 6. A graph of q_D against t_D at r_{eD}=100000

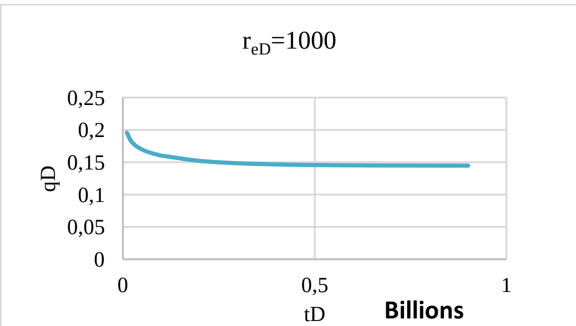

Figure 4. A graph of q_D against t_D at r_{eD}=1000

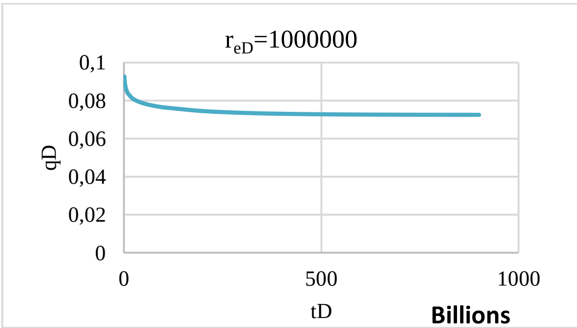

Figure 7. A graph of q_D against t_D at r_{eD}=1000000

Also in this study, the results obtained from this analysis were shown in the form of graphs of dimensionless cumulative production against dimensionless time. These were shown in Figures 8 to 13. These were shown for different dimensionless radii ranging between 20 and 1000000. It was seen from the graph that the dimensionless cumulative production history of the reservoir was not captured at the initial stage between the dimensionless time of zero and the respective dimensionless times in Figures 8 and 13. This was due to the fact that, within these regions, the reservoir was still at the infinite acting region. After the infinite acting period, it was observed that the dimensionless production rate increases uniformly throughout the entire life of the reservoir.

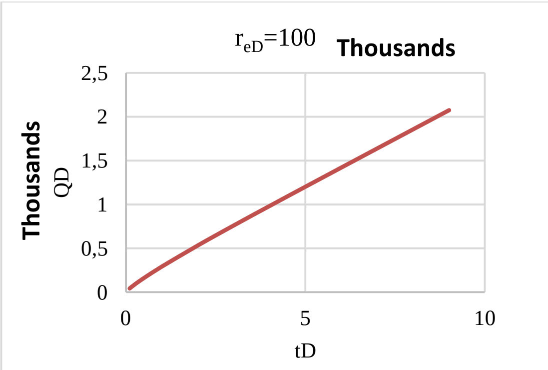

Figure 8. A graph of Q_D against t_D at r_{eD}=100

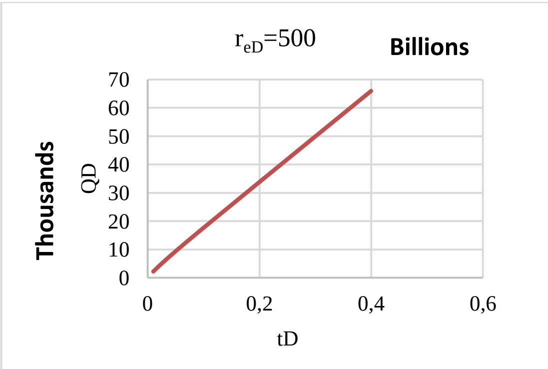

Figure 9. A graph of Q_D against t_D at r_{eD}=500

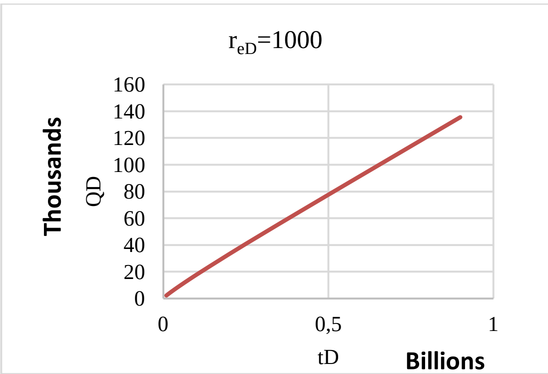

Figure 10. A graph of Q_D against t_D at r_{eD}=1000

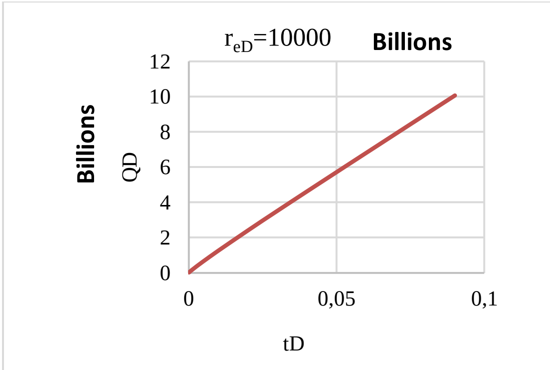

Figure 11. A graph of Q_D against t_D at r_{eD}=10000

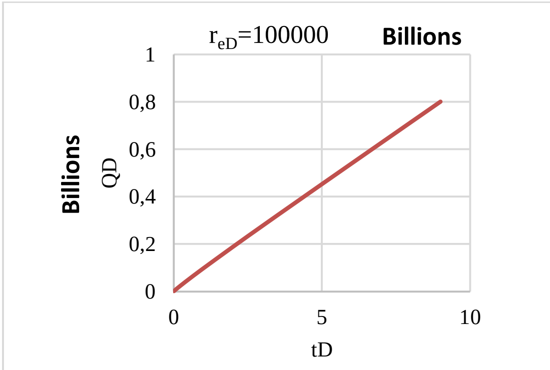

Figure 12. A graph of Q_D against t_D at r_{eD}=100000

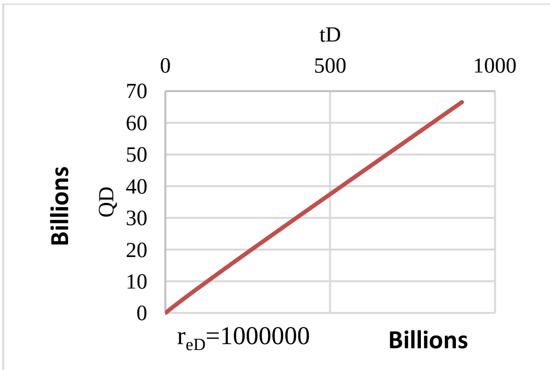

Figure 13. A graph of Q_D against t_D at r_{eD}=1000000

The accuracy of the results was also obtained by computing a table of percentage errors. Table 1 shows the percentage error between the finite element method solutions and Christine [14]. The results show the level of discrepancies between the two results. It was shown that there was a strong agreement between the two results.

Table 1. Percentage error between this work and Christine [14] result at various dimensionless time and radii (100-10000)

r_{eD}=100			r_{eD}=500			r_{eD}=1000			r_{eD}=10000		
t_D	Q_D% **error**	q_D% **error**	t_D	Q_D% **error**	q_D% **error**	t_D	Q_D% **error**	q_D% **error**	t_D	Q_D% **error**	q_D% **error**
100	0.0033	0.0413	10000	0.0650	0.0729	10000	0.0204	0.0729	100000	0.0813	0.0891
200	0.0019	0.0460	20000	0.0349	0.0778	20000	0.0204	0.0778	200000	0.0431	0.0940
300	0.0014	0.0487	30000	0.0242	0.0806	30000	0.0204	0.0806	300000	0.0297	0.0968
400	0.0011	0.0507	40000	0.0187	0.0826	40000	0.0204	0.0826	400000	0.0228	0.0989
500	0.0009	0.0522	50000	0.0153	0.0841	50000	0.0204	0.0842	500000	0.0185	0.1004
600	0.0008	0.0534	60000	0.0129	0.0852	60000	0.0204	0.0855	600000	0.0157	0.1017
700	0.0007	0.0545	70000	0.0112	0.0861	70000	0.0204	0.0866	700000	0.0136	0.1028
800	0.0006	0.0554	80000	0.0099	0.0867	80000	0.0204	0.0875	800000	0.0120	0.1038
900	0.0005	0.0562	90000	0.0089	0.0872	90000	0.0204	0.0883	900000	0.0108	0.1046
1000	0.0005	0.0569	100000	0.0081	0.0876	100000	0.0204	0.0891	1000000	0.0098	0.1053
2000	0.0003	0.0616	200000	0.0042	0.0887	200000	0.0204	0.0939	2000000	0.0051	0.1103
3000	0.0002	0.0638	300000	0.0029	0.0887	300000	0.0204	0.0962	3000000	0.0035	0.1131
4000	0.0001	0.0648	400000	0.0022	0.0888	400000	0.0204	0.0974	4000000	0.0027	0.1152

Table 2. Percentage error between this work and Christine [14] result at various dimensionless time and radii (50000-1000000)

r_{eD}=100000			r_{eD}=1000000			r_{eD}=50000		
t_D	Q_D% **error**	q_D% **error**	t_D	Q_D% **error**	q_D% **error**	t_D	Q_D% **error**	q_D% **error**
10000000	0.1711	0.0122	1000000000	0.0022	0.1544	10000000	0.1711	0.1217
20000000	0.0892	0.0121	2000000000	0.0011	0.1593	20000000	0.0892	0.1275
30000000	0.0609	0.0129	3000000000	0.0008	0.1622	30000000	0.0609	0.1295
40000000	0.0465	0.0132	4000000000	0.0006	0.1642	40000000	0.0465	0.1315
50000000	0.0377	0.0133	5000000000	0.0005	0.1658	50000000	0.0377	0.1331
60000000	0.0317	0.0134	6000000000	0.0004	0.1671	60000000	0.0317	0.1344
70000000	0.0274	0.0135	7000000000	0.0003	0.1682	70000000	0.0274	0.1355
80000000	0.0242	0.0136	8000000000	0.0003	0.1692	80000000	0.0242	0.1364
90000000	0.0216	0.0137	9000000000	0.0003	0.1700	90000000	0.0216	0.1373
100000000	0.0196	0.0138	10000000000	0.0002	0.1708	100000000	0.0196	0.1380
200000000	0.0102	0.0143	20000000000	0.0013	0.1757	200000000	0.0102	0.1429
300000000	0.0069	0.0146	30000000000	0.0009	0.1786	300000000	0.0069	0.1458
400000000	0.0053	0.0148	40000000000	0.0006	0.1806	400000000	0.0053	0.1479
500000000	0.0043	0.0149	50000000000	0.0005	0.1822	500000000	0.0043	0.1494
600000000	0.0036	0.0151	60000000000	0.0004	0.1835	600000000	0.0036	0.1506
700000000	0.0031	0.0152	70000000000	0.0004	0.1846	700000000	0.0031	0.1514
800000000	0.0027	0.0153	80000000000	0.0003	0.1856	800000000	0.0027	0.1521
900000000	0.0024	0.0154	90000000000	0.0003	0.1864	900000000	0.0024	0.1527

4. Conclusion

In this research, we have formulated the finite element base models for the diffusivity equation under the steady state flow regime. The result obtained where used to analyse the production rate and cumulative production for different radii and time. In the constant pressure outer boundary condition where the pressure at the external boundary of the reservoir was held constant because the reservoir was been recharged by a very strong aquifer.

The accuracy of the results has been validated by comparing the results with existing results in literature. The result from this research shows a strong positive correlation between the results with existing results in literature.
Finally, the accuracy of the results obtained was admissible and it therefore shows that the Finite element method can be used in approximating the dimensionless production rate and cumulative production values of fluid in circular reservoirs.

5. Acknowledgements

We want to acknowledge the Department of Production Engineering, University of Benin, Nigeria for serving as a platform to embark on this research.

6. Author Contribution Statement

All the authors in this research made on or two contributions towards the success of this research. Author 1, provided the finite element solution of the diffusivity equation while Author 2 used the result from the first author to analyze the reservoir production rate and cumulative production. Author 3 supervised the research and help in the validation of the result using relevant literature.

7. Ethics Committee Approval and Conflict of Interest

There is no need for an ethics committee approval in the prepared article and there is no conflict of interest with any person/institution in the prepared article.

8. References

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