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Dark Energy from the Scalar Field and Gauss-Bonnet Interactions

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Abstract
In this work, we study a homogeneous and isotropic cosmological model of the universe filled with the perfect fluid and scalar field. Using the framework of Brans-Dicke (BD) and Gauss-Bonnet (GB) theories of gravity, we obtain the field equations and find the expansion parameter and dynamical field functions. We suppose that an energy interaction occurs between the BD and GB components, then adopting this argument, we speculate that the BD scalar field may arise from the intrinsic properties of the GB medium. Also, we get the positive energy density and negative pressure for the baryonic part of matter, so we confirm that this property coincides with the dark energy behavior of the late time universe. We conclude that since some sort of interaction between the scalar field and GB sector provides the accelerating expansion of the universe, to recover the dark energy effect, we may no longer need a cosmological constant.
Keywords: Brans-Dicke theory, Gauss-Bonnet theory, cosmology, scalar field theory

Öz

Bu çalışmada, mükemmel akışkan ve skaler alan ile dolu homojen ve izotropik kozmolojik evren modeli incenmiştir. Brans-Dicke (BD) ve Gauss-Bonnet (GB) kütleçekim teorileri birlikte kullanılarak, alan denklemleri çözülmüş, evrenin genişleme parametresi ve dinamik alan fonksiyonları elde edilmiştir. BD ve GB bileşenleri arasında bir tür enerji etkileşimi olabileceği varsayılarak, BD skaler alanının, GB ortamının içsel özelliklerinden kaynaklanabileceği düşünülmüştür. Baryonik madde yapısının, pozitif enerji yoğunluğu ve negatif basınca sahip olduğu elde edilmiş ve böylece bu sonucun geç evren dönemi için karanlık enerji davranışıyla uyumlu olduğu vurgulanmıştır. Skaler alan ve GB sektörü arasındaki bu enerji etkileşiminin evrenin hızlanarak genişlemesini sağladığı ve böylece karanlık enerji etkisi için artık kozmolojik sabite ihtiyacımız olmayabileceği sonucu elde edilmiştir.

Anahtar Kelimeler: Brans-Dicke teorisi, Gauss-Bonnet teorisi, kozmoloji, skaler alan teorisi

I. INTRODUCTION

From the observations, we know that the universe is expanding with acceleration [1,2]. On the other hand, we don't have a sufficient theoretical and observational explanation for the origin of this expansion yet. The hypothetical reasoning for this acceleration is named dark energy and is considered as a new kind of dynamical fluid or field and has the following characteristics: it spreads across all of space, emits the strong negative pressure, and pushes opposite that of the matter and normal energy. We need to search the dark energy candidates, understand the property of space, obtain a new dynamic fluid, or a new theory of gravity. Einstein's theory of gravity says that all the forms of matter and energy affect the fabric of space-time and may govern how it evolves in time. Although Einstein's general theory of relativity is highly successful, there are some experimental and observational deviations of gravity from Einstein's theory in cosmology [3]. Searching for an alternative model, especially to explain the late-time acceleration of the universe and dark energy, becomes a fundamental problem for theoretical physicists. Brans-Dicke theory is one of the modified gravity model alternatives to Einstein's General Relativity in which Newton's Gravitation constant is not a constant but is characterized by a scalar field [4]. Moreover, the Gauss-Bonnet theory of gravity is a string motivated model [5,6,7] and used to describe the universe during the primordial period and known to explain the inflationary era [8,9,10,11]. The Gauss-Bonnet term includes the higher order derivative curvature corrections and is usually used in higher dimensional string theories since the topologically invariant GB term has no contribution to the Einstein field equations in four dimensions. In accordance with this information, the works

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[12,13,14,15,16,17,18] study the modified Brans-Dicke scalar tensor theory in the presence of Gauss-Bonnet curvature corrections and explore the present cosmological features of a four dimensional homogeneous and isotropic universe.

II. MATERIAL AND METHOD

In this work, we show that modified Brans-Dicke-Gauss-Bonnet (BD-GB) gravity may play the role of a gravitational alternative for dark energy (DE). We think that, the unknown forms of matter and their behaviors could be modeled both by BD scalar field and GB stringy correction terms, hence, this assumption may predict a late-time accelerated expansion and may clarify the evolution of the universe. Therefore, we consider a minimally coupled BD-GB theory and the Brans-Dicke action with a scalar field coupled Gauss-Bonnet term can be described by the following gravitational action,

$$S_{BD} = \int d^4x\sqrt{-g}\left[\frac{\phi R}{2} - \frac{\omega}{2\phi} g^{ab}\nabla_a\phi\nabla_b\phi - f(\phi)GB + L_m\right], \quad (1)$$

where R is the Ricci scalar, ϕ is the Brans-Dicke scalar field, ω is the Brans-Dicke coupling constant, GB is Gauss-Bonnet invariant term and $f(\phi)$ is the Gauss-Bonnet coupling scalar function which may be responsible for the of dark energy, L_m describes the ordinary matter Lagrangian [19]. The Gauss-Bonnet invariant is defined as,

$$GB = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}, \quad (2)$$

with R_{ab} and R_{abcd} represent the Ricci and Riemann tensor.
Applying the variation principle with respect to the metric tensor and the scalar field, we get the field equations of gravity and the continuity equation of the scalar field. The variation with respect to g^{ab} gives,

$$-\frac{\phi G_{ab}}{2} + \frac{\omega}{2\phi}\left(\nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla_c\phi\nabla^c\phi\right)$$
$$+\frac{1}{2}(\nabla_a\nabla_b\phi - g_{ab}\Box\phi) - 2(\nabla_a\nabla_b f)R$$
$$+2g_{ab}(\Box f)R + 4(\nabla^c\nabla_a f)R_{bc} + 4(\nabla^c\nabla_b f)R_{ac}$$
$$-4(\Box f)R_{ab} - 4g_{ab}(\nabla^c\nabla^d f)R_{cd} + 4(\nabla^c\nabla^d f)R_{acbd}$$
$$+8\pi T^{(m)}_{ab} = 0, \quad (3)$$

where $T^{(m)}_{ab}$ is the energy-momentum tensor of ordinary matter and $G_{ab} = R_{ab} - \frac{1}{2}g_{ab}R$ is the Einstein tensor. The variation with respect to ϕ becomes,

$$\frac{R}{2} - \frac{\omega}{2\phi^2}g^{ab}\nabla_a\phi\nabla_b\phi + \frac{\omega}{\phi}\nabla_c\nabla^c\phi - \frac{df}{d\phi}G = 0, \quad (4)$$

where ∇_a is the covariant derivative operator and $\Box \equiv g^{ab}\nabla_a\nabla_b$ is the d'Alembertian operator. The usual BD scalar field equations together with GB correction can be obtained by taking trace of (3) and incorporating into (4), but we prefer using (3) in this work.
Since equation (3) contains different contents of matter and energy, we can express each ingredient separately. Hence, the contributions from the BD scalar field can be written as (the first line of the (3))

$$\mathcal{G}^{(\phi)}_{ab} = -\frac{\phi G_{ab}}{2} + \frac{\omega}{2\phi}\left(\nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla_c\phi\nabla^c\phi\right)$$
$$+\frac{1}{2}(\nabla_a\nabla_b\phi - g_{ab}\Box\phi). \quad (5)$$

Also, the contributions from the GB term which contains the field $f(\phi)$ in (3) can be addressed as,

$$\mathcal{G}^{(GB)}_{ab} = -2(\nabla_a\nabla_b f)R + 2g_{ab}(\Box f)R + 4(\nabla^c\nabla_a f)R_{bc}$$
$$+4(\nabla^c\nabla_b f)R_{ac} - 4(\Box f)R_{ab} - 4g_{ab}(\nabla^c\nabla^d f)R_{cd}$$
$$+4(\nabla^c\nabla^d f)R_{acbd}. \quad (6)$$

Hence, the equation (3) can be expressed as,

$$\mathcal{G}^{(\phi)}_{ab} + \mathcal{G}^{(GB)}_{ab} + 8\pi T^{(m)}_{ab} = 0 \quad (7)$$

We consider the flat, spatially homogeneous and isotropic cosmological geometry which represented by the Friedmann-Lemaître-Robertson-Walker (FLRW) line element as,

$$ds^2 = -dt^2 + a^2(t)\left(dr^2 + r^2 d\Omega^2_{(2)}\right), \quad (8)$$

in comoving coordinates, where $d\Omega^2_{(2)} = d\theta^2 + \sin^2\theta d\varphi^2$ is the line element on the unit 2-sphere.
The Gauss-Bonnet invariant for the metric tensor (8) becomes $GB = 24^2(\dot{H} + H^2)$ and Ricci tensor reads the $R = 6\dot{H} + 12H^2$ where H denotes Hubble's parameter defined as $H = \frac{\dot{a}}{a}$ and the "dot" represent the derivative with respect to the cosmic time t.
Our main purpose is to look for accelerating cosmological solutions of the space-time filled with the matter of perfect fluid with stress-energy tensor,

$$T^{(m)}_{ab} = \left(P^{(m)} + \rho^{(m)}\right)u_a u_b + P^{(m)}g_{ab}, \quad (9)$$

where $u_a u^a = -1$ is the fluid 4-velocity. From the field equations (3) and (4), the Friedmann, acceleration, and scalar field equations for the geometry (8) become,

$$3H^2 = \frac{16\pi\rho^{(m)}}{\phi} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 3H\frac{\dot{\phi}}{\phi} + 24H^3\frac{\dot{f}}{\phi}, \quad (10)$$

$$-2\dot{H} = \frac{16\pi}{\phi}\left(\rho^{(m)} + P^{(m)}\right) + \omega\frac{\dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} - H\frac{\dot{\phi}}{\phi}$$

$$-16H\dot{H}\frac{\dot{f}}{\phi}-8H^2\left(\frac{\ddot{f}}{\phi}-H\frac{\dot{f}}{\phi}\right), \quad (11)$$

$$\omega\left(\frac{\ddot{\phi}}{\phi}+3H\frac{\dot{\phi}}{\phi}-\frac{1}{2}\frac{\dot{\phi}^2}{\phi^2}\right)-\frac{R}{2}+24\frac{df}{d\phi}H^2(\dot{H}+H^2)=0. \quad (12)$$

We consider a barotropic fluid which relates the energy density $\rho^{(m)}$ and pressure as $P^{(m)}=(\gamma-1)\rho^{(m)}$ (for constant γ). The energy conservation equation $\nabla_a T^{ab}_{(m)}=0$ is satisfied for the energy density of $\rho^{(m)}(a)=\frac{\rho_0}{a^{3\gamma}}$, where ρ_0 is the non-negative integration constant which is obtained from the nonzero component of energy conservation equation, $\dot{\rho}^{(m)}+3H\left(P^{(m)}+\rho^{(m)}\right)=0$. Here, as we have said, the matter part of the universe is modeled with perfect fluid and assumed that there is no energy transfer with the other BD scalar field and GB components. Besides, it is clear that, scalar field will affect the dynamics of the universe via the field equations. Moreover, there could be the energy transfer between the other ingredients of space-time, hence, we suppose,

$$\nabla_a \mathcal{G}^{a0}_{(\phi)}=-\Theta(t),$$

$$\nabla_a \mathcal{G}^{a0}_{(GB)}=\Theta(t), \quad (13)$$

for the nonzero component of energy conservation equation. Where $\Theta(t)$ is the interaction term between the BD scalar field and GB contents. The nature of $\Theta(t)$ is not known, and it may arise from the microscopic properties of space-time or it depends on the interaction between the scalar and GB fields. Note that if interaction term is $\Theta(t)>0$, the energy transfers from medium $\mathcal{G}^{(\phi)}_{ab}$ to the medium $\mathcal{G}^{(GB)}_{ab}$ or vice versa [20]. It means that we can consider a cosmological model containing a scalar field that interacts with the stringy properties of space-time which might originated from the GB term. In this case, the exchange of energy between the scalar field part and GB part may offer a reasonable explanation for the observed late time acceleration of the universe.

In the next section of this paper, using the above setup, we derive the power law expressions and in the third section, we obtain the solutions for the exponential dynamical functions of the field equations. Finally, in the last section, we present a brief conclusion.

1.1. Power Law Solutions

In this part, we begin by assuming that the scale factor $a(t)$, scalar field $\phi(t)$ and the GB coupling function $f(\phi)$ are simply given by,

$$a(t)=a_0 t^q,$$

$$\phi(t)=\phi_0 t^2,$$

$$f(t)=f_0\phi^2, \quad (14)$$

where q, a_0, ϕ_0 and f_0 are constants to be determined from the field equations (10) - (12). The measure of cosmic acceleration of the expansion of the universe in the power law is given by the deceleration parameter q_0. It satisfies,

$$q_0=-\frac{\ddot{a}a}{\dot{a}^2}=-\frac{q-1}{q}, \quad (15)$$

and produces the accelerated expansion simply for $q>1$. Using (19), if we admit the values of f_0 and ϕ_0 are bigger than zero, the transfer of energy always occurs from BD scalar field to GB sector.

Also, substituting the field functions (14) into the Friedmann equation (10) and into the acceleration equation (11), we obtain the constant $\gamma=0$. Then, energy density and pressure for the baryonic matter become as following,

$$\rho^{(m)}=-P^{(m)}=\frac{\phi_0[3q^2(q-3)+2\omega(5q+1)]}{16\pi(q-1)}=constant, \quad (16)$$

and the BD scalar field and GB coupling field satisfy that,

$$\phi(t)=\phi_0 t^2 \quad \text{and} \quad f(t)=\frac{\phi_0(2q-1-2\omega)}{16q^2(q-1)}t^4, \quad (17)$$

where $f_0=\frac{(2q-1-2\omega)}{16\phi_0 q^2(q-1)}$. Here, note that, we require the value $2q-1-2\omega>0$ should always satisfied for the positive value of $f(t)$.

Moreover, substituting the field functions (14) into (13), the interaction term is obtained as,

$$\nabla_a \mathcal{G}^{a0}_{(\phi)} = \frac{3\phi}{2}\left[-\dot{H}\frac{\dot{\phi}}{\phi}+\omega\frac{\dot{\phi}}{\phi}\left(H\frac{\dot{\phi}}{\phi}-\frac{1}{6}\frac{\dot{\phi}^2}{\phi^2}+\frac{1}{3}\frac{\ddot{\phi}}{\phi}\right)\right]=-\Theta_0 t^{-1},$$

$$\nabla_a \mathcal{G}^{a0}_{(GB)}=12\dot{f}H^2(\dot{H}+H^2)=\Theta_0 t^{-1}, \quad (18)$$

where the constant Θ_0 satisfies

$$\Theta_0=3q\phi_0(2q-1-2\omega), \quad (19)$$

here, the value of positive $2q-1-2\omega>0$ also required for the appropriate direction of energy transfer in (18), otherwise, the interaction will be in the opposite direction and hence f_0 reads negative value. It can easily seen that two equations in (18) have the opposite sign, hence the total energy is conserved in this theory. Also, the interaction term is not a constant but changes in time by the factor t^{-1}.

Here, the transfer of energy from the BD scalar field to the GB stringy sector yields the universe a dark energy dominated era, therefore, this scenario may describe the late-time dynamics of the accelerated expansion. If we consider the stress-energy tensor of GB contribution as a perfect fluid type of matter, using (6), energy density and pressure of this component satisfy the following relations,

$$\rho^{(GB)} = 12H^3\dot{f}(t) = \frac{3q\phi_0(2q-1-2\omega)}{(q-1)} = \textit{constant},$$

$$P^{(GB)} = -4H\left[2\dot{f}(\dot{H}+H^2)+H\ddot{f}\right] = -\frac{(2q+1)}{3q}\rho^{(GB)}, \quad (20)$$

here, it is worthwhile to express that the GB dark sector also satisfies the positive energy condition since the numerator has the positive value, and both energy density and pressure remain constant while the universe is expanding. In this type of interaction, we choose the scalar field as $\phi(t) = \phi_0 t^2$ and the equation of state (EoS) (16) for baryonic fluid becomes $P^{(m)} = -\rho^{(m)}$ that implies the vacuum energy dominance or so-called dark energy behavior according to Λ Cold Dark Matter model (ΛCDM) [21]. On the other hand, EoS (20) for the GB content, gives $P^{(GB)} = -\frac{(2q+1)}{3q}\rho^{(GB)}$. This rate corresponds to the value $-1 < -\frac{(2q+1)}{3q} < -\frac{1}{3}$ and behaves as an effective quintessence era since the value of q is always bigger than one as we mentioned before [6]. Unknown properties of scalar field or its interaction with the GB sector somehow provide this expansion. Or we can simply speculate that, some intrinsic features of the GB sector might be the source of the scalar field. Nevertheless, understanding the origin and true nature of the scalar field is undoubtedly crucial for a valuable theory, however, lack of knowledge both about the GB sector and its practical applications constrains us to analyze the dynamics of matter contents and to obtain a more precise theory.

1.2. Exponential Solutions

The solution of (11) and (12) for the exponential expansion satisfies,

$$a(t) = a_0 e^{\lambda_a t},$$
$$\phi(t) = \phi_0 e^{\lambda_\phi t},$$
$$f(\phi) = f_0\phi(t, \quad (21)$$

where λ_a and λ_ϕ are the expansion parameters and $f_0 = \frac{12\lambda_a^2 - \lambda_\phi(6\lambda_a+\lambda_\phi)\omega}{48\lambda_a^4}$.

The matter part of ingredients behaves as perfect fluid as we mentioned before and the energy density and pressure for the baryonic matters have the following form,

$$\rho(t)^{(m)} = \frac{\phi_0\left(6\lambda_a^2(\lambda_a-\lambda_\phi)+\lambda_\phi^2(5\lambda_a+\lambda_\phi)\omega\right)}{32\pi\lambda_a} e^{\lambda_\phi t},$$

$$P(t)^{(m)} = -\frac{3\lambda_a+\lambda_\phi}{3\lambda_a}\rho(t)^{(m)}, \quad (22)$$

where the positive energy density and negative pressure is easily satisfied if the numerator of $\rho(t)$ has the positive values. However, since the value of λ_ϕ should always be nonzero, the rate $\frac{3\lambda_a+\lambda_\phi}{3\lambda_a}$ is never become the unity, hence, this solution doesn't recover the pure cosmological constant behavior. On the other hand, depending the value of λ_ϕ, the field may satisfy the so called quintessence or phantom-like dark energy in which the value $-1 < -\frac{3\lambda_a+\lambda_\phi}{3\lambda_a} < -\frac{1}{3}$ for the quintessence field [6] and $-\frac{3\lambda_a+\lambda_\phi}{3\lambda_a} < -1$ for the phantom field [22]. Nevertheless, our model always satisfies the accelerated expansion for the values $\lambda_a > 1$.

The interaction term (13) between the BD and GB sectors becomes,

$$\Theta(t) = \frac{\lambda_\phi\theta_0}{4}\left[12\lambda_a^2 - \lambda_\phi(6\lambda_a+\lambda_\phi)\omega\right]e^{\lambda_\phi t}. \quad (23)$$

Here, if we require the condition $f(t) > 0$ on the GB scalar field in (21), the interaction energy is therefore transferred from the scalar field to the GB medium and vice versa. In any case, however, we can speculate that the scalar field is generated from intrinsic properties of the geometry which is contributed from the GB term.

For the perfect fluid type of GB content, we obtain the energy density and pressure as following,

$$\rho^{(GB)} = \frac{\lambda_\phi\theta_0}{4\lambda_a}\left[12\lambda_a^2 - \lambda_\phi(6\lambda_a+\lambda_\phi)\omega\right]e^{\lambda_\phi t},$$

$$P^{(GB)} = -\frac{2\lambda_a+\lambda_\phi}{3\lambda_a}\rho^{(GB)}, \quad (24)$$

here, we see that, the GB dark sector also satisfies the positive energy condition and negative pressure if the numerator of (21) has the positive value.

Now, if we criticize the rates for equations of state in (22) and (24), for example, for $\lambda_\phi = \lambda_a$ the baryonic matter satisfies the phantom-like dark energy while the GB component becomes the pure cosmological constant nature of dark energy [21]. Moreover, for example, for $\lambda_\phi = -\lambda_a$, both the baryonic matter and the GB content may satisfy the quintessence type of dark energy.

III. CONCLUSION

In this work, we have studied the late-time dynamics of a four-dimensional homogeneous and isotropic FLRW universe based on the Brans-Dicke scalar-tensor theory in the presence of the Gauss-Bonnet invariant. Our model contains the modification of source term that includes the scalar field and the modifications of curvature term as a scalar field non-minimally couples to the curvature R and to the other string inspired higher-order curvature squared corrections. In the model, we have considered the cosmological perfect fluid matter and assumed that the energy of the perfect fluid content is conserved, but there is an energy exchange between the scalar field and the GB components. We proposed that this interaction can conveniently explain the origin of dark energy.

We have shown that the power law evolution of the scale factor and scalar field yields the power law form of the GB coupling function. We introduced the $\phi_0 t^2$ type of the scalar field and derived the corresponding analytic solution of the field equations. We obtained that if there is no energy transfer between perfect fluid matter and the other (BD and GB) sectors, spacetime expands with acceleration, negative pressure is exerted on the barotropic content of matter, and hence all visible objects move away from each other with acceleration. We have also investigated the exponential solution and find similar results. In each scenario, we concluded that dark energy can arise both from the energy density of the scalar field and from the generalized nonminimal coupling to the GB invariant. This result shows that considering GB and BD theories in the same model provide us the additional dynamical degrees of freedom and this model might be the relevant explanation of the nature of dark energy. Even ΛCDM model describes the cosmology in the very simplest form, the flatness of the universe at a large scale and also a highly fine-tuned explanation for the accelerated expansion of the universe (or so-called vacuum energy) leaves the model further unsolved problems. On the other hand, in our work, higher-order curvature terms provide a relevant explanation of the late time acceleration then we explain the dark energy without introducing a vacuum energy or a cosmological constant. Moreover, the scalar field offers a viable description of the inflationary era and then solves the flatness problem. In our work, a scalar field is naturally included in the BD theory and also combined with the string-motivated GB invariant. As a result, considering a scalar field and the higher-order curvature terms in an action might be a promising model to explain both the early universe and late-time acceleration.

Our scenario seems to be consistent with the observed late-time dynamics of the universe but, we need further works to clarify how this scalar field is generated from the GB interaction and hence leads the dark energy behavior and also how these theoretical results are compared and supported by recent observational data.

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