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GROWTH PROPERTIES OF GENERALIZED ITERATED ENTIRE AND COMPOSITE ENTIRE AND MEROMORPHIC FUNCTIONS

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Abstract. In this paper we consider generalized iteration of entire functions and prove some growth properties of generalized iterated entire functions and composition of entire and meromorphic functions under certain restrictions on (p,q) orders of functions.

1. INTRODUCTION AND DEFINITIONS

It is well known that for any two transcendental entire functions $f(z)$ and $g(z)$, $\lim_{r\to\infty} \frac{M(r,f\circ g)}{M(r,f)} = \infty$. In a paper [10] Clunie proved that the same is also true when maximum modulus functions are replaced by their characteristic functions. Singh [15] proved some results dealing with the ratios of $\log T(r, f\circ g)$ and $T(r,f)$ under some restrictions on the order of f and g. After this several authors {see [9], [13] } made close investigation on comparative growth of $\log T(r, f\circ g)$ and $T(r,g)$ by imposing certain restrictions on order of f and g.

If $f(z)$ and $g(z)$ be entire functions then following the iteration process of Lahiri and Banerjee [12], we write

$f_1(z) = f(z)$
$f_2(z) = f(g(z)) = f(g_1(z))$
$f_3(z) = f(g(f(z))) = f(g_2(z))$
$f_4(z) = f(g(f(g(z)))) = f(g_3(z))$
. . . .
$f_n(z) = f(g(f(g(....(f(z)$ or $g(z))$ according as n is odd or even)))
and so are
$g_1(z) = g(z)$
$g_2(z) = g(f(z)) = g(f_1(z))$
$g_3(z) = g(f(g(z))) = g(f_2(z))$
$g_4(z) = g(f(g(f(z)))) = g(f_3(z))$

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$\ldots \quad \ldots \quad \ldots$

$g_n(z) = g(f(g(f(....(g(z)$ or $f(z))$ according as n is odd or even))).

Clearly all $f_n(z)$ and $g_n(z)$ are entire functions.

Following this iteration process several papers {see [1], [2], [3], [4] } on growth properties of entire functions have appeared in the literature where growing interest of researchers on this topic has been noticed.

Recently Banerjee and Mondal [5] introduced another type of iteration called generalised iteration to study {see [5], [6] } some growth properties of entire functions.

Let f and g be two non-constant entire functions and α be any real number satisfying $0 < \alpha \leq 1$. Then the generalized iteration of f with respect to g is defined as follows:

$$f_{1,g}(z) = (1-\alpha)z + \alpha f(z)$$
$$f_{2,g}(z) = (1-\alpha)g_{1,f}(z) + \alpha f(g_{1,f}(z))$$
$$f_{3,g}(z) = (1-\alpha)g_{2,f}(z) + \alpha f(g_{2,f}(z))$$
$$\ldots. \quad \ldots. \quad \ldots.$$
$$f_{n,g}(z) = (1-\alpha)g_{n-1,f}(z) + \alpha f(g_{n-1,f}(z))$$

and so are

$$g_{1,f}(z) = (1-\alpha)z + \alpha g(z)$$
$$g_{2,f}(z) = (1-\alpha)f_{1,g}(z) + \alpha g(f_{1,g}(z))$$
$$g_{3,f}(z) = (1-\alpha)f_{2,g}(z) + \alpha g(f_{2,g}(z))$$
$$\ldots. \quad \ldots. \quad \ldots.$$
$$g_{n,f}(z) = (1-\alpha)f_{n-1,g}(z) + \alpha g(f_{n-1,g}(z)).$$

Following Sato [14], we write $\log^{[0]} x = x$, $\exp^{[0]} x = x$ and for positive integer m, $\log^{[m]} x = \log(\log^{[m-1]} x)$, $\exp^{[m]} x = \exp(\exp^{[m-1]} x)$.

Let $f(z) = \sum\limits_{n=0}^{\infty} a_n z^n$ be an entire function.Then the (p,q)-order and lower (p,q)-order of $f(z)$ are denoted by $\rho_{(p,q)}(f)$ and $\lambda_{(p,q)}(f)$ respectively and defined by[7]

$$\rho_{(p,q)}(f) = \lim_{r\to\infty} \sup \frac{\log^{[p]} T(r,f)}{\log^{[q]} r} \text{ and } \lambda_{(p,q)}(f) = \lim_{r\to\infty} \inf \frac{\log^{[p]} T(r,f)}{\log^{[q]} r},\ p \geq q \geq 1.$$

Definition 1.1. A real valued function $\varphi(r)$ is said to have the property P_1 if

i) $\varphi(r)$ is non negative ;

ii) $\varphi(r)$ is strictly increasing and $\varphi(r) \to \infty$ as $r \to \infty$;

iii) $\log \varphi(r) \leq \delta \varphi(\frac{r}{4})$ holds for every $\delta > 0$ and for all sufficiently large values of r.

Remark 1.1. If $\varphi(r)$ satisfies the property P_1 then it is clear that $log^{[p]}\varphi(r) \leq \delta\varphi(\frac{r}{4})$ holds for every $p \geq 1$.

The purpose of this paper is to compare the growth of generalized iterated entire functions with composition of a meromorphic function and an entire function imposing certain restrictions on (p,q)-order and lower (p,q)-order. Throughout the paper we assume that f is a meromorphic function and g, h and k are non-constant entire functions such that the maximum modulus functions of h, k and all of their generalized functions satisfy property P_1. We do not explain the standard notations and definitions of the theory of meromorphic functions as those are available in [11].

2. LEMMAS

In this section we state some known results in the form of lemma which will be needed in the sequel.

Lemma 2.1. *([11]) If $f(z)$ be regular in $|z| \leq R$, then for $0 \leq r < R$*
$T(r,f) \leq \log^+ M(r,f) \leq \frac{R+r}{R-r} T(R,f)$.
In particular if f be entire, then for all large values of r
$T(r,f) \leq \log^+ M(r,f) \leq 3T(2r,f)$.

Lemma 2.2. *([8]) If f is meromorphic and g is entire then for all large values of r*
$T(r, f\circ g) \leq (1+o(1)) \frac{T(r,g)}{\log M(r,g)} T(M(r,g),f)$.
Since g is entire so using Lemma 2.1, we have
$T(r, f\circ g) \leq (1+o(1)) T(M(r,g),f)$.

Lemma 2.3. *([10]) Let $f(z)$ and $g(z)$ be entire functions with $g(0) = 0$. Let β satisfy $0 < \beta < 1$ and let $C(\beta) = \frac{(1-\beta)^2}{4\beta}$. Then for $r > 0$*
$M(r, f\circ g) \geq M(C(\beta)M(\beta r, g), f)$.
Further if $g(z)$ is any entire function, then with $\beta = 1/2$, for sufficiently large values of r
$M(r, f\circ g) \geq M(\frac{1}{8}M(\frac{r}{2}, g) - |g(0)|, f)$.
Clearly $M(r, f\circ g) \geq M(\frac{1}{16}M(\frac{r}{2}, g), f)$.

3. MAIN THEOREMS

In this section, we present the main results of this paper.

Theorem 3.1. *Let g, h and k be three entire functions and f be a meromorphic function with $\rho_{(p,q)}(f) < \infty, \rho_{(p,q)}(g) < \infty$ and $\lambda_{(p,q)}(g) < \min\{\lambda_{(p,q)}(h), \lambda_{(p,q)}(k)\}$. Then*
$\limsup\limits_{r\to\infty} \frac{\log^{[p+(n-2)(p+1-q)]} T(r,h_{n,k})}{\log^{[p]} T(r,f_{2,g})} = \infty$, *where $f_{2,g}(z)$ is the generalized composition of f with respect to g.*

Proof. Since $\lambda_{(p,q)}(g) < \min\{\lambda_{(p,q)}(h), \lambda_{(p,q)}(k)\}$ we can choose $\epsilon > 0$ in such a way that $\lambda_{(p,q)}(g) + \epsilon < \min\{\lambda_{(p,q)}(h) - \epsilon, \lambda_{(p,q)}(k) - \epsilon\}$. Using Lemma 2.1 and 2.3, we have for all large values of r

$$\begin{aligned}
T(r,h_{n,k}) &\geq \tfrac{1}{3}\log M(\tfrac{r}{2}, h_{n,k}) \\
&\geq \tfrac{1}{3}\log\{M(\tfrac{r}{2}, \alpha h(k_{n-1,h})) - M(\tfrac{r}{2}, (1-\alpha)k_{n-1,h})\} \\
&\geq \tfrac{1}{3}\log\{\alpha M(\tfrac{1}{16}M(\tfrac{r}{4}, k_{n-1,h}), h) - (1-\alpha)M(\tfrac{r}{2}, k_{n-1,h})\} + O(1) \text{ [for } \alpha \neq 1] \\
&= \tfrac{1}{3}[\log M(\tfrac{1}{16}M(\tfrac{r}{4}, k_{n-1,h}), h) - \log M(\tfrac{r}{2}, k_{n-1,h})] + O(1).
\end{aligned}$$

So for all sufficiently large values of r, we get

$$\begin{aligned}
\log^{[p]} T(r,h_{n,k}) &\geq \log^{[p+1]} M(\tfrac{1}{16}M(\tfrac{r}{4}, k_{n-1,h}), h) - \log^{[p+1]} M(\tfrac{r}{2}, k_{n-1,h}) + O(1) \\
&> (\lambda_{(p,q)}(h) - \epsilon)\log^{[q]}(\tfrac{1}{16}M(\tfrac{r}{4}, k_{n-1,h})) - \log^{[p+1]} M(\tfrac{r}{2}, k_{n-1,h}) + O(1) \\
&> (\lambda_{(p,q)}(h) - \epsilon)\log^{[q]} M(\tfrac{r}{4}, k_{n-1,h}) - \\
&\qquad \tfrac{1}{2}(\lambda_{(p,q)}(h) - \epsilon)\log^{[q]} M(\tfrac{r}{4}, k_{n-1,h}) + O(1), \text{using property } P_1 \\
&= \tfrac{1}{2}(\lambda_{(p,q)}(h) - \epsilon)\log^{[q]} M(\tfrac{r}{4}, k_{n-1,h}) + O(1) \qquad (3.1)
\end{aligned}$$

$$\begin{aligned}
\text{or, } \log^{[p+(p+1-q)]} T(r,h_{n,k}) &> \log^{[p]}\{\log M(\tfrac{r}{4},k_{n-1,h})\}+O(1)\\
&\geq \log^{[p]} T(\tfrac{r}{4},k_{n-1,h})+O(1)\\
&> \tfrac{1}{2}(\lambda_{(p,q)}(k)-\epsilon)\log^{[q]} M(\tfrac{r}{4^2},h_{n-2,k})+O(1), \text{using}(3.1)\\
\text{or, } \log^{[p+2(p+1-q)]} T(r,h_{n,k}) &> \log^{[p]} T(\tfrac{r}{4^2},h_{n-2,k})+O(1)\\
&> \tfrac{1}{2}(\lambda_{(p,q)}(h)-\epsilon)\log^{[q]} M(\tfrac{r}{4^3},k_{n-3,h})+O(1).
\end{aligned}$$

Proceeding similarly after some steps we get for even n

$$\begin{aligned}
\log^{[p+(n-2)(p+1-q)]} T(r,h_{n,k}) &> \tfrac{1}{2}(\lambda_{(p,q)}(h)-\epsilon)\log^{[q]} M(\tfrac{r}{4^{n-1}},k_{1,h})+O(1)\\
&= \tfrac{1}{2}(\lambda_{(p,q)}(h)-\epsilon)\log^{[q]} M\{\tfrac{r}{4^{n-1}},(1-\alpha)z+\alpha k\}+O(1)\\
&\geq \tfrac{1}{2}(\lambda_{(p,q)}(h)-\epsilon)\{\log^{[q]} M(\tfrac{r}{4^{n-1}},k)-\log^{[q]} M(\tfrac{r}{4^{n-1}},z)\}+O(1)
\end{aligned}$$

$$\geq \tfrac{1}{2}(\lambda_{(p,q)}(h)-\epsilon)[\exp^{[p-q]}\{\log^{[q-1]}(\tfrac{r}{4^{n-1}})\}^{\lambda_{(p,q)}(k)-\epsilon}-\log^{[q]}(\tfrac{r}{4^{n-1}})]+O(1). \tag{3.2}$$

On the other hand using Lemma 2.2 for a sequence of values of r tending to infinity

$$\begin{aligned}
T(r,f_{2,g}) &= T\{r,(1-\alpha)g_{1,f}+\alpha f(g_{1,f})\}\\
&\leq T(r,g_{1,f})+T(r,f(g_{1,f}))+O(1)\\
&\leq T(r,g_{1,f})+(1+o(1))T(M(r,g_{1,f}),f)+O(1)\\
\text{or, } \log^{[p]} T(r,f_{2,g}) &\leq \log^{[p]} T(r,g_{1,f})+\log^{[p]} T(M(r,g_{1,f}),f)+O(1)\\
&< \log^{[p]} T(r,g_{1,f})+(\rho_{(p,q)}(f)+\epsilon)\log^{[q]} M(r,g_{1,f})+O(1)\\
&\leq \log^{[p]} T(r,z)+\log^{[p]} T(r,g)+(\rho_{(p,q)}(f)+\epsilon)[\log^{[q]} M(r,z)\\
&\qquad +\log^{[q]} M(r,g)]+O(1) \qquad (3.3)\\
&< \log^{[p+1]} r+(\rho_{(p,q)}(g)+\rho_{(p,q)}(f)+2\epsilon)\log^{[q]} r+\\
&\qquad \exp^{[p-q]}(\log^{[q-1]} r)^{\lambda_{(p,q)}(g)+\epsilon}+O(1). \qquad (3.4)
\end{aligned}$$

From (3.2) and (3.4) we get for a sequence of values of r tending to infinity

$$\frac{\log^{[p+(n-2)(p+1-q)]} T(r,h_{n,k})}{\log^{[p]} T(r,f_{2,g})} > \frac{\frac{1}{2}(\lambda_{(p,q)}(h)-\epsilon)[\exp^{[p-q]}\{\log^{[q-1]}(\frac{r}{4^{n-1}})\}^{\lambda_{(p,q)}(k)-\epsilon}-\log^{[q]}(\frac{r}{4^{n-1}})]+O(1)}{\log^{[p+1]} r+(\rho_{(p,q)}(g)+\rho_{(p,q)}(f)+2\epsilon)\log^{[q]} r+\exp^{[p-q]}(\log^{[q-1]} r)^{\lambda_{(p,q)}(g)+\epsilon}+O(1)}. \tag{3.5}$$

When n is odd as in (3.2) we get

$$\log^{[p+(n-2)(p+1-q)]} T(r,h_{n,k}) > \tfrac{1}{2}(\lambda_{(p,q)}(k)-\epsilon)[\exp^{[p-q]}\{\log^{[q-1]}(\tfrac{r}{4^{n-1}})\}^{\lambda_{(p,q)}(h)-\epsilon} -\log^{[q]}(\tfrac{r}{4^{n-1}})]+O(1). \tag{3.6}$$

From (3.4) and (3.6) we get for a sequence of values of r tending to infinity

$$\frac{\log^{[p+(n-2)(p+1-q)]} T(r,h_{n,k})}{\log^{[p]} T(r,f_{2,g})} > \frac{\frac{1}{2}(\lambda_{(p,q)}(k)-\epsilon)[\exp^{[p-q]}\{\log^{[q-1]}(\frac{r}{4^{n-1}})\}^{\lambda_{(p,q)}(h)-\epsilon}-\log^{[q]}(\frac{r}{4^{n-1}})]+O(1)}{\log^{[p+1]} r+(\rho_{(p,q)}(g)+\rho_{(p,q)}(f)+2\epsilon)\log^{[q]} r+\exp^{[p-q]}(\log^{[q-1]} r)^{\lambda_{(p,q)}(g)+\epsilon}+O(1)} \tag{3.7}$$

From (3.5) and (3.7) we get

$$\limsup_{r\to\infty}\frac{\log^{[p+(n-2)(p+1-q)]} T(r,h_{n,k})}{\log^{[p]} T(r,f_{2,g})} = \infty.$$

Remark 3.1. In the above theorem if we take $\rho_{(p,q)}(g) < \min\{\lambda_{(p,q)}(h),\lambda_{(p,q)}(k)\}$ instead of $\lambda_{(p,q)}(g) < \min\{\lambda_{(p,q)}(h),\lambda_{(p,q)}(k)\}$ then limit superior is replaced by limit inferior.

Theorem 3.2. *Let g, h and k be three entire functions and f be a meromorphic function such that $\rho_{(p,q)}(f) < \infty$, $\rho_{(p,q)}(g) < \infty, \lambda_{(p,q)}(h) > 0$ and $\lambda_{(p,q)}(k) > 0$. Then*

$$\begin{aligned}
\limsup_{r\to\infty}\frac{\log^{[2p+1-q]} T(r,f_{2,g})}{\log^{[p+(n-1)(p+1-q)]} T(r,h_{n,k})} &\leq \frac{\rho_{(p,q)}(g)}{\lambda_{(p,q)}(k)}, \qquad \textit{if } n \textit{ is even.}\\
&\leq \frac{\rho_{(p,q)}(g)}{\lambda_{(p,q)}(h)}, \qquad \textit{if } n \textit{ is odd.}
\end{aligned}$$

Proof**.** We may suppose that $\rho_{(p,q)}(g)$ is finite. Otherwise the result is obvious. First suppose that n is even. Then by (3.3) we get for large values of r

$$\log^{[p]} T(r, f_{2,g}) < \log^{[p]} T(r, z) + \log^{[p]} T(r, g) + (\rho_{(p,q)}(f) + \epsilon)[\log^{[q]} M(r, z) + \log^{[q]} M(r, g)] + O(1)$$
$$< \log^{[p+1]} r + (\rho_{(p,q)}(g) + \rho_{(p,q)}(f) + 2\epsilon) \log^{[q]} r + \exp^{[p-q]}(\log^{[q-1]} r)^{\rho_{(p,q)}(g)+\epsilon} + O(1).$$

So,

(3.8) $$\log^{[2p+1-q]} T(r, f_{2,g}) < \log^{[2p+2-q]} r + \log^{[p+1]} r + (\rho_{(p,q)}(g) + \epsilon) \log^{[q]} r + O(1).$$

From (3.2) we get for large values of r

$$\log^{[p+(n-2)(p+1-q)]} T(r, h_{n,k}) > \tfrac{1}{2}(\lambda_{(p,q)}(h) - \epsilon)[\exp^{[p-q]}\{\log^{[q-1]}(\tfrac{r}{4^{n-1}})\}^{\lambda_{(p,q)}(k)-\epsilon} - \log^{[q]}(\tfrac{r}{4^{n-1}})] + O(1).$$

So,

(3.9) $$\log^{[p+(n-1)(p+1-q)]} T(r, h_{n,k}) > (\lambda_{(p,q)}(k) - \epsilon) \log^{[q]} r - \log^{[p+1]}(\tfrac{r}{4^{n-1}}) + O(1).$$

Now from (3.8) and (3.9) we get for all large values of r

$$\frac{\log^{[2p+1-q]} T(r,f_{2,g})}{\log^{[p+(n-1)(p+1-q)]} T(r,h_{n,k})} < \frac{\log^{[2p+2-q]} r + \log^{[p+1]} r + (\rho_{(p,q)}(g)+\epsilon) \log^{[q]} r + O(1)}{(\lambda_{(p,q)}(k)-\epsilon) \log^{[q]} r - \log^{p+1}(\frac{r}{4^{n-1}}) + O(1)}$$
$$= \frac{(\rho_{(p,q)}(g)+\epsilon) \log^{[q]} r[1 + \frac{\log^{[2p+2-q]} r + \log^{[p+1]} r + O(1)}{(\rho_{(p,q)}(g)+\epsilon) \log^{[q]} r}]}{(\lambda_{(p,q)}(k)-\epsilon) \log^{[q]} r[1 - \frac{\log^{[p+1]}(\frac{r}{4^{n-1}}) + O(1)}{(\lambda_{(p,q)}(k)-\epsilon) \log^{[q]} r}]}.$$

Therefore, $\limsup\limits_{r\to\infty} \frac{\log^{[2p+1-q]} T(r,f_{2,g})}{\log^{[p+(n-1)(p+1-q)]} T(r,h_{n,k})} \leq \frac{\rho_{(p,q)}(g)}{\lambda_{(p,q)}(k)}$.

When n is odd we get as in (3.9)

(3.10) $$\log^{[p+(n-1)(p+1-q)]} T(r, h_{n,k}) > (\lambda_{(p,q)}(h) - \epsilon) \log^{[q]} r - \log^{[p+1]}(\tfrac{r}{4^{n-1}}) + O(1).$$

Now from (3.8) and (3.10) the remaining part of the theorem easily follows.

Theorem 3.3. *Let f be a meromorphic function and g, h and k be three entire functions such that $\rho_{(p,q)}(f) < \infty$, $\rho_{(p,q)}(g) < \infty, \lambda_{(p,q)}(h) > 0$ and $\lambda_{(p,q)}(k) > 0$. Then*

$$\limsup_{r\to\infty} \frac{\log^{[2p+2-q]} T(r,f_{2,g})}{\log^{[p+1+(n-1)(p+1-q)]} T(r,h_{n,k})} \leq 1.$$

Proof**.** From (3.8) we get for all large values of r

$$\log^{[2p+1-q]} T(r, f_{2,g}) < \log^{[2p+2-q]} r + \log^{[p+1]} r + (\rho_{(p,q)}(g) + \epsilon) \log^{[q]} r + O(1)$$
$$= (\rho_{(p,q)}(g) + \epsilon) \log^{[q]} r[1 + \frac{\log^{[2p+2-q]} r + \log^{[p+1]} r + O(1)}{(\rho_{(p,q)}(g)+\epsilon) \log^{[q]} r}].$$

Therefore,

(3.11) $$\log^{[2p+2-q]} T(r, f_{2,g}) < \log^{[q+1]} r + O(1).$$

Again from (3.9) we get for all large values of r and for even n

$$\log^{[p+(n-1)(p+1-q)]} T(r, h_{n,k}) > (\lambda_{(p,q)}(k) - \epsilon) \log^{[q]} r - \log^{[p+1]}(\tfrac{r}{4^{n-1}}) + O(1)$$
$$= (\lambda_{(p,q)}(k) - \epsilon) \log^{[q]} r[1 - \frac{\log^{[p+1]}(\frac{r}{4^{n-1}}) + O(1)}{(\lambda_{(p,q)}(k)-\epsilon) \log^{[q]} r}].$$

Therefore,

(3.12) $$\log^{[p+1+(n-1)(p+1-q)]} T(r, h_{n,k}) > \log^{[q+1]} r + O(1).$$

When n is odd we get from (3.10)

$$\log^{[p+(n-1)(p+1-q)]} T(r, h_{n,k}) > (\lambda_{(p,q)}(h) - \epsilon) \log^{[q]} r - \log^{[p+1]}(\tfrac{r}{4^{n-1}}) + O(1)$$

$$= (\lambda_{(p,q)}(h) - \epsilon) \log^{[q]} r[1 - \frac{\log^{[p+1]}(\frac{r}{4^{n-1}}) + O(1)}{(\lambda_{(p,q)}(h) - \epsilon) \log^{[q]} r}].$$

So,

$$\log^{[p+1+(n-1)(p+1-q)]} T(r, h_{n,k}) > \log^{[q+1]} r + O(1). \tag{3.13}$$

Therefore from (3.11), (3.12) and (3.13) we get

$$\limsup_{r\to\infty} \frac{\log^{[2p+2-q]} T(r, f_{2,g})}{\log^{[p+1+(n-1)(p+1-q)]} T(r, h_{n,k})} \leq 1.$$

Remark 3.2. In the above theorems if we take relative iteration instead of generalized iteration and take $q = 1$ then the results coincide with the results of Banerjee and Jana [3].

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