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Stability of the program manifold of automatic indirect control systems taking into account the external load

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Abstract

The problems of stability system that arises in the construction of different automatic systems of indirect control are considered. It is known that a given program is not always exactly performed, since there are always initial, constantly acting perturbations. Therefore, it is also reasonable to require the program manifold's stability itself relatively to some quality indicator. In the first part, the stability being investigated of automatic indirect control systems with rigid and tachometric feedback. Necessary and sufficient conditions of the program manifold's absolute stability are established separately. In the second part, automatic systems of indirect control in the presence of an external load are considered. The equations of the hydraulic actuator, taking into account the action of an external load, are presented in a convenient form for research. Then it reduces to studying the stability of the system of equations with respect to a given program manifold. By constructing Lyapunov's functions for the system in canonical form, sufficient conditions of the program manifold's absolute stability are obtained. The results obtained can be used in the formation of stable indirect systems automatic control.

Keywords: indirect control system external load rigid and tachometric feedbacks Lyapunov function .
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1. Introduction and Motivation

Inverse problems of ordinary differential equations dates back to the fifties of the last century. The first work in this domain was published by N.P. Erugin in [1]. Where he set and solved the problem of formation a system of differential equations along the specified curve. Futher, this problem was developed in the following works [2, 3, 4, 5, 6, 7, 8] as a problem formation of differential equations' systems, to construct automatic systems control for a given manifold, for dynamics inverse problems, for the construction program motion's systems. These problems have aroused great interest among mathematicians and mechanics because of their viability. Due to the fact that the program manifold is exposed to various influences when solving different problems, it became necessary to study the stability of the manifold itself [4], [5], [9], [10], [11], [12]. A detailed review of these studies can be found in the following papers [2], [8], [9], [12]. Nowadays, the study of the stability of program manifold in relation to some indicators has become a separate theory.

Definition 1.1. *A set $\Omega(t)$ is called a program manifold of ordinary differential equation if, from that $x_0 \in \Omega(t_0)$ follows $x(t, t_0, x_0)) \in \Omega(t)$ for all $t \geq t_0$.*

For a given smooth program manifold $\Omega(t)$ we consider the problem of constructing the following system of differential equations

$$\dot{x} = f\left(t, x\right), \tag{1}$$

where f, x are n dimensional vectors, continuous in all variables $f \in R^n$ is satisfied to existence conditions of the solution $x(t) = 0$, and $\Omega(t)$ is described as next equations

$$\Omega\left(t\right) \equiv \omega(t, x) = 0, \tag{2}$$

where an $s$dimensional vector ω $(s \leq n)$ is continuous together with its partial derivatives, in the following domain:

$$G\left(\rho\right) = \left\{(t, x) : t \in I \wedge \left\|\omega\left(t, x\right)\right\| \leq \rho < \infty, I = [0, \infty)\right\},$$

including the manifold $\Omega(t)$, and the Jacobian rank $H = \dfrac{\partial \omega}{\partial x}$ is equal to $rankH = s$ at all points of $\Omega(t)$.

This program manifold (2) is performed exactly only if it satisfies the conditions of the initial values $\omega(t_0, x_0) = 0$ of the system state vector. However, these conditions are not always satisfied due to the presence of other perturbing forces. Therefore, when constructing program motion systems, advisable to take into consideration of the program manifold's stability.

We find the derivative of the manifold $\Omega(t)$ with respect to time t due to the system (1), we obtain

$$\dot{\omega} = \frac{\partial \omega}{\partial t} + Hf\left(t, x\right) = F(t, x, \omega), \tag{3}$$

where $H = \dfrac{\partial \omega}{\partial x}$ is the Jacobi matrix and $F(t, x, \omega)$ is a certain s dimensional Erugin vector function, satisfying conditions $F(t, x, 0) \equiv 0$ [1].

Together with equation (3), we consider the system with rigid and tachometric feedback of the next type

$$\begin{array}{l} \dot{x} = f(t, x) - B\xi - D\dot{\xi}, \quad t \in I = [0, \infty), \\ \dot{\xi} = \varphi(\sigma), \quad \sigma = P^T\omega - Q\xi - N\dot{\xi}, \end{array} \tag{4}$$

where $x \in R^n$ is a state vector of the object, continuous in all variables $f \in R^n$ is satisfied to existence conditions of the solution $x(t) = 0$, $B \in R^{n\times r}$, $D \in R^{n\times r}$, $P \in R^{s\times r}$, $Q \in R^{r\times r}$, $N = diag\|N_1, ..., N_r\|$ are matrices, Q and N are matrices, respectively, of rigid and tachometric feedback, $\xi, \varphi, \sigma \in R^r$ are vectors, $\omega \in R^s (s \leq n)$ is a vector, $\varphi(\sigma) \in R^r$ is a continuous and differentiable to σ control vector function on deviation from the specified program manifold, satisfying to conditions:

$$\varphi(0) = 0 \quad 0 < \varphi_i(\sigma_i)\sigma_i \leq k_i\sigma_i^2 \quad \forall \sigma_i \neq 0, \tag{5}$$

$$-k_i^{(1)} \leq \frac{d\varphi_i(\sigma_i)}{d\sigma_i} \leq k_i^{(2)} \quad \forall i_1^r, \tag{6}$$

where $k_i, k_1^{(1)}, \dots, k_r^{(1)}$ are non-negative finite numbers, $k_1^{(2)}, \dots, k_r^{(2)}$ are real positive numbers or infinities, and all integrals

$$\lim_{|\sigma_\nu|\to\infty} \Phi = \lim_{|\sigma_\nu|\to\infty} \int_0^{|\sigma_\nu|} \kappa_i(\sigma_i)\varphi_i(\sigma_i)d\sigma_i \to \infty \tag{7}$$

diverge at $|\sigma_\nu| \to \infty$,where

$$\kappa_i(\sigma_i) = 1 + N\frac{d\varphi_i(\sigma_i)}{d\sigma_i}. \tag{8}$$

In order to manifold (2) to be integral also for the system (4) at $\omega = 0$ condition $\xi = 0$ must be satisfied. The latter takes place if and only if $Q > 0$.

Definition 1.2. *A program manifold $\Omega(t)$ of the system control with rigid and tachometric feedbacks is called absolutely stable in relation to a vector function ω, if it is asymptotically stable on the whole at all functions $\omega(t_0, x_0)$ and $\varphi(\sigma)$ satisfying to the conditions (5)-(7).*

Formulation of the problem. *To get the condition of absolute stability of a program manifold $\Omega(t)$ of the control systems with rigid and tachometric feedbacks with regard to the specified vector function ω.*

2. Necessary conditions for the absolute stability of the program manifold of feedback control systems

Consider a feedback control system (4) with a given linear integral manifold with regard to the vector function ω:

$$\omega \equiv Ux + g(t) = 0, \tag{9}$$

where $U \in R^{s\times n}$ is given constant matrix, $g(t) \in R^s$ is given continuous vector function.

We find the derivative of the manifold (9) with respect to time t due to the system (4), and in expression (3) assuming that

$$F(t, x, \omega) = -A\omega, \tag{10}$$

where $-A \in R^{s\times s}$ is stable matrix, we obtain

$$\begin{aligned} &\dot\omega = -A\omega - UB\xi - UD\dot\xi, \quad t \in I = [0, \infty),\\ &\dot\xi = \varphi(\sigma), \quad \sigma = P^T\omega - Q\xi - N\dot\xi, \end{aligned} \tag{11}$$

here nonlinearity $\varphi(\sigma)$ satisfies conditions (5)-(7).

We suppose that

$$\det \begin{Vmatrix} A & UB \\ -P^T & Q \end{Vmatrix} \neq 0. \tag{12}$$

Then in the system (11) we can go to the new coordinate system (η, σ) using a non-singular transformation

$$\eta = -A\omega - UB\xi; \quad y = P^T\omega - Q\xi, \tag{13}$$

where

$$y = \sigma + N\varphi(\sigma). \tag{14}$$

Differentiating (13) with respect to time t due to the system (11) and taking into account (14) we obtain

$$\begin{aligned} &\dot\eta = -A\eta - G\varphi(\sigma),\\ &\kappa(\sigma)\dot\sigma = P^T\eta - M\varphi(\sigma), \end{aligned} \tag{15}$$

here

$$G = -AUD + UB, \quad M = P^T UD + q,$$
$$\kappa(\sigma) = diag\|\kappa_1(\sigma_1), \dots, \kappa_r(\sigma_r)\|,$$

$\kappa_i(\sigma_i)$ is defined by formula (8).

Theorem 2.1. *Let the Erugin function $F(t, x, \omega)$ have the form (10) and the nonlinearity $\varphi(\sigma)$ satisfies the conditions (5)-(7). Then for absolute stability of program manifold of the control system with feedback (4), it is necessary that matrix $(-A)$ there be Hurwitz and the following inequality is valid*

$$|\kappa^{-1}UQ + P^T A^{-1} UB| > 0. \tag{16}$$

Proof. For absolute stability of program manifold of the feedback control system (4) it is necessary that the following system

$$\begin{aligned} \dot{\eta} &= -A\eta - Gh\sigma, \\ \dot{\sigma} &= \kappa^{-1} P^T \eta - \kappa^{-1} M h\sigma, \end{aligned} \tag{17}$$

was asymptotically stable. Here $\varphi(\sigma) = h\sigma, \quad h = diag\|h_1, \dots, h_r\|, \quad \kappa = E + h \cdot N.$

For systems (17) to be asymptotically stable, the following inequality must be satisfied

$$\det \begin{Vmatrix} A & Gh \\ -\kappa^{-1}P^T & \kappa^{-1}Mh \end{Vmatrix} 0 \vee |\kappa^{-1}| \cdot |A| \cdot |M + P^T A^{-1} G| \cdot |h| > 0$$

or taking into account the values of the matrices G, M :

$$|\kappa^{-1}| \cdot |h| \cdot \Delta > 0. \tag{18}$$

Due to the fact that

$$\Delta = |A| \cdot |Q + P^T A^{-1} UB| \wedge |h| > 0$$

from (18), taking into account the Hurwitz property of the matrix $(-A)$, we obtain inequality (16). □

3. Sufficient conditions for absolute stability of program manifold of feedback control systems

We express the vector σ in terms of η and ξ. From (13) we define

$$\omega = -A^{-1}\eta - A^{-1}UB\xi$$

and substituting it into (11) we obtain

$$\sigma = C^T \eta - \Gamma\xi - N\varphi(\sigma), \tag{19}$$

where

$$C = -A^{-T}P, \quad \Gamma = P^T A^{-1} B + Q.$$

We compose for system (15) the Lyapunov function of the type

$$V = \eta^T L \eta + \zeta^T \tau \zeta + 2 \int_0^\sigma \varphi^T(\sigma) \beta \kappa(\sigma) d\sigma,$$

where L, τ, β are constant symmetric matrices

$$L = L^T > 0 \wedge \tau = \tau^T > 0, \quad \beta = diag\|\beta_1, \dots, \beta_r\|, \tag{20}$$
$$sign\beta_\nu = sign\kappa_\nu(\sigma_\nu),$$

and ζ is a variable vector

$$\zeta = P^T A^{-1}\eta + N\varphi(\sigma) = -\Gamma\xi.$$

Differentiating the function V with respect to t, by virtue of (15) and on the basis of properties (20), we obtain

$$-\dot{V} = \eta^T\Lambda\eta + 2\eta^T\Lambda_1\varphi(\sigma) + \varphi^T(\sigma)\Lambda_2\varphi(\sigma) + 2\sigma^T\tau\Gamma\varphi(\sigma),$$

where

$$\Lambda = A^L + LA, \quad \Lambda_1 = LG - P\beta - A^{-T}P\tau\Gamma,$$
$$\Lambda_2 = N\tau\Gamma + \Gamma^T\tau N + \beta M + M^T\beta.$$

Let it be satisfied

$$\tau\Gamma = E. \tag{21}$$

Then, based on properties (5), for any $-\dot{V} > 0$, it is sufficient to satisfy Sylvester's condition

$$\det\left\| \begin{array}{cc} \Lambda & \Lambda_1 \\ \Lambda_1^T & \Lambda_2 \end{array} \right\| > 0. \tag{22}$$

Theorem 3.1. *Let the Erugin function $F(t,x,\omega)$ have the form (10), the matrix $(-A)$ is Hurwitz and the conditions (12), (16), (20) be satisfied. Then, for absolute stability of program manifold of feedback control systems (4), it suffices to satisfy equality (21) and inequality (22).*

4. Stability of the program manifold with rigid and tachometric feedbacks control systems, with external load

Together with equation (1), now we consider the following system of indirect automatic control with rigid and tachometric feedbacks, taking into account the external load

$$\begin{array}{l} \dot{x} = f(t,x) - b_1\xi, \quad t \in I = [0, \infty), \\ \dot{\xi} = \varphi(\sigma)\psi(\nu), \quad \sigma = p^T\omega - q\xi - N\dot{\xi}, \end{array} \tag{23}$$

where $b_1 \in R^n, p \in R^s$ are constant values, q, N are rigid and tachometric feedbacks constant coefficients, σ is the impulse-signal total control, a differentiable with regard to σ function ξ satisfies the following conditions

$$\begin{array}{c} \varphi(0) = 0 \wedge \varphi(\sigma)\sigma > 0 \quad \forall\sigma \neq 0, \\ \left.\dfrac{\partial\varphi}{\partial\sigma}\right|_{\sigma=0} < \chi > 0, \end{array} \tag{24}$$

and the function $\psi(\nu)$ points to the actions of an external load. In order to manifold (2) to be integral also for the system (23), (24) at $\omega = 0$ condition $\xi = 0$ must be satisfied. This condition is satisfied for $q \neq 0$.

When the coordinates ξ, σ change multiplier $\psi(\nu)$ deforms the function $\varphi(\sigma)$. Here, ν is a complex discontinuous function of the automatic control system. In the general case, ν has the following form [13]:

$$\nu = 1 - (a\ddot{\xi} + b\dot{\xi} + c\xi)\ sign\sigma,$$

in the simplest case, it looks like this:

$$\nu = 1 - c\xi\ sign\sigma, \tag{25}$$

here a, b, c are real numbers, and $sign\sigma$ is the Kroneker function:

$$sign\sigma = \begin{cases} +1 & \text{at} \quad \sigma > 0, \\ 0 & \text{at} \quad \sigma = 0, \\ -1 & \text{at} \quad \sigma < 0. \end{cases}$$

The functions $\varphi(\sigma), \psi(\nu)$ were described by A.M. Letov in [14]:

$$\dot{\xi} = \varphi(\sigma) \cdot \psi(\nu),$$

where the function $\varphi(\sigma)$ is continuous in σ and satisfies condition (5).

He led to a convenient form for the study of the equation of the hydraulic actuator, taking into account the external load, obtained by V.A. Khokhlov [13]:

$$\dot{x} = \mu\sqrt{\frac{g}{\gamma}} \cdot \frac{l}{F}\sqrt{p_0 - \triangle p \ sign\sigma} \cdot \sigma.$$

Here $\mu\sqrt{\frac{g}{\gamma}} \cdot \frac{l}{F}$ is the constructive constant, μ is the flow coefficient, σ is the spool displacement, $p_0 = p_k - p_\sigma, p_k$ is pressure in the supply line, p_σ is pressure at the drain, $\triangle p$ is pressure difference in the cameras of the actuator, determined by the load.

The function $\varphi(\sigma)$ denotes the expression $\sqrt{\frac{gp_0}{\gamma}} \cdot \frac{l}{F}$, which defines the speed of an unloaded executor, and the multiplier $\psi(\nu) = \sqrt{1 - \frac{\triangle p}{p_0} sign\sigma}$ points to the actions of an external load.

From a physical point of view, the throttle control of a hydraulic actuator is always $p_0 > \triangle p \ sign\sigma$ is satisfied in the presence of a positional load.

The multiplier $\psi(\nu)$, when ν depends on the deflection of the control element ξ, its speed $\dot{\xi}$ and its acceleration $\ddot{\xi}$ is determined as follows:

$$\psi(\nu) = \begin{cases} 1 & \text{at} \quad \nu \geq 1, \\ \sqrt{\nu} & \text{at} \quad 0 < \nu < 1, \\ 0 & \text{at} \quad \nu \leq 0. \end{cases} \tag{26}$$

Considering that the manifold (2) is also an integral for system (18), (19) and selecting the Erugin function of the type (11), we arrive at the next system with relation to ω:

$$\begin{aligned} &\dot{\omega} = -A\omega - \bar{b}\xi, \quad t \in I = [0, \ \infty), \\ &\dot{\xi} = \varphi(\sigma)\,\psi(\nu), \\ &\sigma = p^T\omega - q\xi - N\dot{\xi}, \end{aligned} \tag{27}$$

where $\bar{b} = Hb_1, H = \dfrac{\partial\omega}{\partial x}$, and $-A(s \times s)$ is a constant Hurwitz matrix, the nonlinearity $\varphi(\sigma)$ satisfies conditions (24), and the multiplier $\psi(\nu)$ is determined by formula (26).

Statement of the problem. To get the condition of absolute stability of a program manifold $\Omega(t)$ of the control systems (27) with rigid and tachometric feedbacks in the presence of an external load with relation to ω under conditions (24), (25).

In the area of operation of the hydraulic actuator ($\dot{\xi} \neq 0$) by introducing the following notation

$$\dot{\xi} = z \tag{28}$$

due to (24), (25), from the third equation of system (27) we obtain

$$z = \varphi(\sigma)\sqrt{1 - ((a\dot{z} + bz + c\xi) \ sign\sigma)}. \tag{29}$$

Solving equation (29) with respect to $\dot{z}$, under the condition $\sigma \neq 0$, also taking into account notation (28), instead of the second equation of system (27) we obtain the following system

$$\begin{aligned} &\dot{\xi} = z, \\ &\dot{z} = \lambda_1\xi + \lambda_2 z + \phi(\sigma, z) \ sign\sigma, \end{aligned} \tag{30}$$

where

$$\lambda_1 = -\frac{c}{a}, \lambda_2 = -\frac{b}{a},$$
$$\phi(\sigma, z) = \frac{\varphi^2(\sigma) - z^2}{a\varphi^2(\sigma)}.$$

Here the constants λ_1 and λ_2 are negative numbers, and the function $\psi(\sigma, z)$ in the work area for all $\sigma \neq 0$, due to the second equation in (27) and (24), (25) satisfies the condition

$$\sigma \leq \phi(\sigma, z) \leq \frac{1}{\sigma},$$

and in the stooping region ($\dot{\xi} = 0$) we have $\xi = const$.

In the domain of hydraulic actuator operation due to (30) system of equations (27) can be written as:

$$\begin{aligned} &\dot{\omega} = -A\omega - b\xi, \quad t \in I = [0, \infty), \\ &\dot{\xi} = z, \\ &\dot{z} = \lambda_1\xi + \lambda_2 z + \phi(\sigma, z)\ sign\sigma, \\ &\sigma = p^T\omega - q\xi - N\dot{\xi}, \end{aligned} \tag{31}$$

In order to investigate the stability of the system (31), it is necessary to reduce it to the canonical form.

Assume that $q \neq 0$ and $N \neq 0$. Taking into account the second equation of (31), we write the last equation as follows

$$\sigma = p^T\omega - q\xi - Nz. \tag{32}$$

Introducing the notation

$$\omega_{s+1} = \xi,\ a_{k,s+1} = b,\ a_{s+1,j} = \frac{p_j}{N},$$
$$p_{s+1} = -q,\ m_k = \varepsilon_k = 0,\ m_{s+1} = 1,$$
$$\varepsilon_{s+1} = \frac{c}{b}N,$$
$$g = p^T A_1\omega + \frac{b}{a}(p_j + \varepsilon_j), \quad M = \frac{b}{a} + \frac{1}{N}p^T m,$$

and excluding the variable z by virtue of (32) and differentiating σ from the second and third equations of (31), we obtain a system of equations

$$\begin{aligned} &\dot{\omega} = -A\omega - \frac{m_k}{N}\sigma, \quad t \in I = [0, \infty), \\ &\dot{\sigma} = g^T\omega - M\sigma - N\phi(\sigma, z)\ sign\sigma, \end{aligned} \tag{33}$$

where $m^T = (0, \ldots, 0, 1)^T$, and g, M are expressed in terms of the coefficients of the initial equation.

System (33) is reduced to the canonical form [14]

$$\begin{aligned} &\dot{\eta} = -\rho\eta + \sigma, \\ &\dot{\sigma} = \beta^T\eta - M\xi - N\phi(\sigma, z) sign\sigma, \end{aligned} \tag{34}$$

where $\rho = diag(\rho_1, \ldots, \rho_s), \beta$ is a constant vector.

We will compose the Lyapunov function for system (34) of the following type

$$V = \sum_{i=1}^{s+1}\sum_{k=1}^{s+1} \frac{l_i l_k}{\rho_1 + \rho_k}\eta_1\eta_k + \frac{1}{2}\sum_{k=1}^{n} L_k\eta_K^2 + \sum_{i=1}^{s-m} C_i\eta_{m+i}\eta_{m+i+1} + \frac{1}{2}l_{s+2}\sigma^2. \tag{35}$$

Here $l_1, \ldots, l_m$ are real and $l_{m+1}, \ldots, l_{m+s+1}$ are complex pairwise conjugate numbers.

Differentiating (35) with respect to time t, we find the derivative $\dot{V}$ by virtue of the system (34). In order for $-\dot{V} > 0$ to be, it is enough to satisfy the next equalities

$$L_k + l_{s+2}\beta_k + 2l_k \sum_{i=1}^{n+1} \frac{l_i}{\rho_i + \rho_k} = 0 \quad \forall\ k = \overline{1, m}, \tag{36}$$

$$C_j + l_{s+2}\beta_{m+j} + 2l_{m+j} \sum_{i=1}^{s+1} \frac{l_i}{\rho_i + \rho_k} = 0 \quad \forall\ m = \overline{1, s-m+1}. \tag{37}$$

Theorem 4.1. *If the Erugin function is linear with respect to ω and there are L_k, C_i positive real numbers, l_{s+2}, in addition, nonlinearity $\varphi(\sigma)$ satisfies the conditions (24) and the function $\psi(\nu)$ defined by (25), (26). Then in order that, the program manifold of feedbacks system control in the presence of an external load was absolute stability with relation to ω it is enough fulfillment of the equalities (36) and (37).*

5. Conclusion

The necessary and sufficient conditions of absolute stability for the program manifold of automatic indirect control systems are established separately relative to the given function. Also the suffusion conditions are obtained of automatic systems rigid and tachometric feedbacks in the presence of the external load. The results can be used in the formation of stable automatic systems indirect control.

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7. Conflicts of interest

This work does not have any conflicts of interest.

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