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A Proposed New Approach for The Single Machine Scheduling Problem: Dynamic Programming

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Abstract

A traditional scheduling problem is one of the optimization problems that assign tasks to humans and machines in an optimal order. In real applications, many jobs do not have a fixed processing time. During production, some machines need to be cooled due to overheating. This activity, which takes place outside periodic maintenance, is called rate-modifying. During this time, the job times increase with each passing second as Jobs wait to be assigned. The rate of deterioration due to this increase is called deteriorating jobs. This paper considers scheduling a set of deteriorating jobs with rate-modifying activity with a single processor. During the speed change activity, the production process is halted, resulting in increased completion times of jobs. The problem arose from the problem of a machine and an automatic production line. This problem is classified as an NP-Hard problem. The problem addressed by the study has been tried to obtain optimal results with different methods by considering different factors by many authors. When a detailed literature review is made, it has been seen that no author has developed a dynamic programming model until today. The most significant advantage of dynamic programming models is that they provide solutions with the closest optimal result faster, especially in solving problems classified as Np-Hard. Therefore, a dynamic programming algorithm was developed for the first time for large job numbers of the focused problem. Therefore, this study presents a dynamic programming algorithm to calculate the optimal solution. The algorithm's efficiency is proven on an extensive randomly generated sample data set. The results prove that the proposed algorithm provides the optimal solution with much less effort than the mathematical model.

Keywords Dynamic programming, Deteriorating jobs, Rate-modifying activities, Scheduling

Jel Codes C61, C72, G11

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1. Introduction

Scheduling problems are optimization problems that assign tasks, people, and machines in an optimal sequence. In general, scheduling problems are analyzed and solved by numerous different techniques. MacCarthy & Liu (1993) classify into three categories to solve scheduling problems. Efficient Optimal Methods include easy problems, also known as polynomially solvable problems. Some scheduling problems cannot be formulated as linear programs, and no appropriate algorithms to obtain optimal solutions in a reasonable time. Therefore, heuristics methods should be applied to these kinds of problems. Lastly, optimal enumerative methods can also be used for NP-hard scheduling problems (Ozturkoglu, 2020). Dynamic Programming is another solution method for this class of scheduling problems. Dynamic Programming is one technique that can be used in optimization problems when solving a problem requires making a sequence of decisions.

This paper considers scheduling a group of deteriorating tasks on a single processor. A deteriorating job refers to tasks that are longer when processed later than when processed prior. Gupta & Gupta (1988) first introduced scheduling deteriorating jobs, and afterward, Browne & Yechiali (1990) supported this. In this paper, machine deterioration causes a rise in processing time. The machine needs maintenance or repair activity to prevent the machine's deterioration over time. Rate-modifying activity (RMA) was first introduced by Lee & Leon (2001), and it refers to the activity which improves the machine's condition. The processing times of the jobs differ based on whether a job is scheduled beforehand or afterward the RMA since the RMA provides a machine recovery.

The mentioned problem has been tried to be solved by using both mathematical and different heuristic methods. For the first time, the mathematical model that handles the deteriorated works simultaneously in a single machine belongs to Ozturkoglu & Bulfin (2011). Later, several attempts have proposed different mathematical models (Ji et al., 2013; Cheng & Cheng, 2014; Renna, 2014; Zhang et al., 2018; Kim & Kim, 2022) that schedule deteriorating jobs with RMA in s single machine.

Ji et al. (2013) proposed a general model to consider single-machine scheduling problems with focusing resource allocation, the aging effect, and the RMA factor. They completed the cases of the start and finish time of the due window. Some polynomial algorithms are proposed for each model. However, Cheng & Cheng (2014) claimed that Ji et al. (2013)'s model is only valid when the modifying rate is unitary. Therefore, they also proposed some corrections to the special-case algorithm of Ji et al. (2013). One year later, Renna (2014) integrated Ozturkoglu & Bulfin (2011)'s problem in a multi-agent architecture supporting scheduling activities in a job-shop manufacturing system. A simulation environment is developed to test the proposed approach to investigate several performance measures. The proposed approach allows to reduce drastically the number of rate-modifying activities keeping a high level of performance. Therefore, the proposed approach does not increase the completion time compared to Ozturkoglu & Bulfin (2011)'s model. Zhang et al. (2018) convert the same problem to a parallel machine problem. They developed Polynomial-time algorithms to solve the problem in less time. Kim & Kim (2022) studied scheduling problems with deteriorating jobs and rate-modifying activity. Six extended variants of the Ozturkoglu & Bulfin (2011) model are introduced. Two of the six variants involve job-dependent deterioration rates. Optimal policies are developed for the six variants.

Table 1. Comparison of the Literature Review

Performance Measure	Methods	Factors	Authors
makespan	polynomial algorithms	resource allocation, aging effect, and rate-modifying-activities	Ji et al. (2013)
makespan	special-case algorithm	resource allocation, aging effect, and rate-modifying-activities	Cheng & Cheng (2014)
total completion time	simulation	rate-modifying-activities, deteriorating jobs	Renna (2014)
Makespan, total completion time, total tardiness	polynomial algorithms	rate-modifying-activities, deteriorating jobs	Zhang et al. (2018)
Makespan, total completion time	mathematical model & heuristics	rate-modifying-activities, deteriorating jobs	Kim & Kim (2022))

The problem that the study focuses on, as shown in Table 1, has been solved by many authors with different methods. However, the increase in the number of jobs in the model slows down the solution speed. As the number of jobs increases, the problem becomes more difficult. Therefore, mathematical models cannot solve the problem after a certain number of jobs.

Consequently, researchers have tried solving the same problem quickly using different heuristic methods. A genetic algorithm reduces solution time with more significant size problems (Kim & Ozturkoglu, 2013; Chung & Kim, 2016; Chung et al., 2019).

Although it is a problem frequently encountered in actual industrial production applications, very few studies have focused on the solution methods of the problem. Studies have generally focused on single- or parallel-machine solutions to the same problem. Therefore, there are still gaps in the literature assigning fast-deteriorating jobs to a single machine. This paper addresses the single machine-scheduling problem with deteriorating jobs and RMA. We deal with a proposed problem: Ozturkoglu & Bulfin (2011). Numerous preventive maintenance activities and position-based deterioration (PBD) of jobs are combined in that study. PBD controls the actual processing time of a job depending on the position of the job sequence. The automatic-production line problem with a single machine is the motivation for this problem. During the RMA, the process of production pauses, which increases the completion of the subsequent tasks. Dynamic programming has not been used to obtain an optimal solution for the problem classified as NP-Hard for higher job numbers. In order to solve this NP-Hard problem, a dynamic programming algorithm to compute the optimal solution is presented in this study. So, this is the first study, the develop and propose a dynamic programming model to solve a single machine scheduling problem by combining rate-modifying activities and deteriorating jobs. Furthermore, a large set of randomly generated examples proves the approach's effectiveness.

This paper includes four parts. In the following section, the statement of the problem and the variables are explained. Section 3 includes the dynamic algorithm and numerical examples. The final section covers the conclusion part.

2. Problem Statement

The problem's motivation is an automatic production line with a single machine. The production process is paused through the RMA and results in an increase in completion times for the subsequent jobs. The decision related to scheduling the RMA should be made. The problem is to schedule a set of n independent jobs, defined as a set of $J = \{J_1, J_2, ..., J_n\}$, and J^c is the complement of the J. In addition, it is to decide the number of required RMAs and where they need to be given to one worker. The job can be processed at any time. One job can be done at a time by the worker. Each job has a deterioration rate, represented by α, and shows overstress of a machine from doing jobs during the time. It is accepted that the deterioration rate has an equal impact on the different jobs' time of the process. Each job has a nominal processing time p_j before deterioration. In addition, p_{ji} is the processing time of the job if done in positions after an RMA or the primary position. It is calculated as Eq. 1:

$$p_{ji} = (1 + \alpha_i)^{i-1} p_j \tag{1}$$

To formulate this problem using Dynamic Programming, we use forward dynamic programming formulation for scheduling problems. In the model, giving a RMA in previous stages affects the deterioration of jobs in the further steps.

Model Parameters

- Let J denote a subset of the n jobs and assume the set J is processed first.
- k specifies the position.
- C_k is the cost-to-go function, which identifies the processing time of the last job at the end of stage k (makespan at stage k).
- m_k a state variable that specifies the position of the last given RMA initial position.
- w_k a decision variable that is $\{0, 1\}$ and indicates whether an RMA is given at stage k or not.
- d_k is a random variable that specifies how many times a job deteriorates.
- P_J is the nominal processing time of the j^{th} job.
- r the RMA time of the processor.
- α the deterioration rate, which is between 0 and 1.

3. Forward Dynamic Programming Formulation and Numerical Example

The problem should be divided into stages for a dynamic programming algorithm, with a decision required at each stage. These states, which are defined as possible, condition the problem. For each state algorithm need to answer two critical questions;

- Which job should be scheduled?
- Where is the RMA in the schedule?

The algorithm is designed to find the optimal solution for the overall problem. The complete algorithm is shown as follows:

Algorithm

Step 0. Initialization — Set $J_0 = \varphi, w_1 = 0, m_1 = 1, d_1 = 0, C_0 = 0$

$$k = 1 \Rightarrow J_1 = \{j\}, C_1(J_1, w_1, m_1, d_1) = P_j \; j = 1, ..., n$$

Step 1. Recursive — $k = 2, ..., n$

Step 2. State equation — $J_1 = \{j\}, J_k = J_{k-1} + \{j\}, j \in J_{k-1}^c$

$$m_k = \begin{Bmatrix} m_{k-1} & \text{if } w_{k-1}=0 \\ k-1 & \text{if } w_{k-1}=1 \end{Bmatrix}$$

Step 3. Constraint

$$d_k = \begin{Bmatrix} k-m_k & \text{if } w_k=0 \\ 0 & \text{if } w_k=1 \end{Bmatrix}$$

Step 4. Cost-to-go equations

$$C_k(J_k, w_k, m_k, d_k) = \min_{\substack{j\in J \\ w_k\in\{0,1\}}} \Big\{C_{k-1}(J_k - \{j\}, w_{k-1}, m_{k-1}, d_{k-1}) + p_j(1+\alpha)^{d_k} + w_k r\Big\}$$

Step 5. Optimal value — $J_N = \{1, 2, ..., n\}$ Terminal

$$C_{\max} = \min\{(C_N(\{1,2,...,n\}, w_N, m_N)\}$$

Step 6. Objective function — $\min C_{\max}$

Until non-dominated states have been extended, the algorithm proceeds.

The proposed dynamic algorithm model is tested with the data that Ozturkoglu & Bulfin (2011) tested in their mathematical models. In their model, there are four jobs with different processing times; the rate-modifying time is assigned as five minutes. The parameters of the example are given below:

$n = 4$

$J = \{1, 2, 3, 4\}$

$\alpha = 0.20$

$r = 5$

$p_1 = 5$

$p_2 = 10$

$p_3 = 20$

$p_4 = 30$

In the proposed dynamic model, AMPL is used for coding and CPLEX 9.1 on a computer with a Pentium IV 2.8-GHz processor and 1 GB of RAM are used for the solution. When we compared the result of the dynamic programming algorithm with Ozturkoglu & Bulfin (2011)'s mathematical model with the same parameters, the optimal makespan is 73, and the optimal schedule is 3-2-RMA-4-1.

To determine the accuracy of the developed model, a comparison was made with the data set consisting of 20 jobs used by Ozturkoglu & Bulfin (2011). According to the results shown in Table 2, the developed dynamic model was faster than the mathematical model, and only a 0.01 percent gap was observed on average when compared to the Optimal results.

Table 2. Comparison results of the dynamic and mathematical model

Jobs No	Mathematical Model	Proposed Dynamic Model	Gap%
1	459.51	459.45	0.013
2	474.48	475.01	0.112
3	466.66	466.75	0.019
4	943.8	943.71	0.01
5	248.11	247,45	0.266
6	0	0	0
7	1156.6	1156.5	0.009
8	857.6	857.5	0.012
9	633.6	633.2	0.063
10	885.3	885.048	0.028
11	536.8	536.646	0.029
12	340.9	341.016	0.034
13	306.9	307.473	0.186
14	394.8	394.367	0.11
15	437	436.865	0.031
16	475.3	474.63	0.141
17	758.4	757.74	0.087
18	360.2	360.081	0.033
19	315.6	314.705	0.284
20	455.6	455.955	0.078

Our results indicate that the dynamic programming model reaches the optimal result with as much less time. Therefore, in this proposed dynamic algorithm, we extended the problem that Ozturkoglu & Bulfin (2011) presented for many jobs. We show that the objective function problem can be solved in $O(n^3)$ time.

As it is known, the problem is an NP-hard problem. Therefore, the proposed dynamic model can be applied to the problem. However, since we must decide two things at every stage, the assigned job and the RMA, it must be modeled carefully. When we look at the computational complexity of the proposed dynamic problem, the total number of evaluations at every stage is $\frac{k!}{k!(k-l)!}$ for l number of subsets. Also, in every evaluation at stage k, there are $2k$ comparisons.

4. Conclusions

In real industrial applications, we often face the problem of increasing the jobs' processing times due to machines' overstress during the production interval. Ozturkoglu & Bulfin (2011) simultaneously developed a mathematical model for the scheduling problem with deteriorating jobs and RMA. As a well-known fact that these problems are NP-hard and the size of the real problem is very large, optimum solutions can be easily obtained by their mathematical model with the small size of problems. For the mathematical model, the computational time rises, and the number of jobs rises.

The problem addressed by the study has been tried to obtain optimal results with different methods by considering different factors by many authors. When a detailed literature review is made, it has been seen that no author has developed a dynamic programming model until today. The most significant advantage of dynamic programming models is that they provide solutions with the closest optimal result faster, especially in solving problems classified as NP-Hard. Therefore, a dynamic programming algorithm was developed for the first time for large job numbers of the focused problem. This study focuses on scheduling a set of deteriorating jobs with RMA with a single processor, where increased completion times of jobs occur due to halted production process during the speed change activity. In order to measure the performance of the developed dynamic model, it was compared with the previously presented mathematical model, which obtained optimal results. The numerical example shows that the proposed algorithm provides the optimal solution with much less effort than the mathematical model. When the results are examined, the proposed dynamic model gave much faster results for larger tasks than the mathematical model developed in 2011. Compared to the optimal results, a gap of only 0.01 percent was observed on average. These results reveal the reliability of the developed dynamic model. Thus, for future studies, the model developed for larger job numbers with different performance criteria and multiple break times is suitable for use.

Declarations

Conflict of interest The authors have no competing interests to declare that are relevant to the content of this article.

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