A Lorenz-like Chaotic OTA-C Circuit and Memristive Synchronization

Şüle Zeynep Aydın, Gökçe Nur Beken, and Zehra Güler Çam Taşkıran

1Department of Electrical Electronics Engineering, Marmara University, Istanbul, Türkiye, 2Department of Electrical Electronics Engineering, Yıldız Technical University, Istanbul, Türkiye.

ABSTRACT In this paper, a new set of lorenz-like hyper-chaotic equation set is obtained using the anti-control procedure. The chaoticity of the system is verified by MATLAB simulations using mathematical analysis methods. A new OTA-C circuit is designed for the new equation set. In the difference term addition technique, synchronizing the OTA-C circuit with a memristor rather than a resistor is proposed. Circuit design and synchronization are performed in PSpice simulation. The fact that the transconductance of the OTA element can be easily adjusted with a bias current provides the parameters that will make the proposed dynamic circuit a chaotic oscillator. The advantage of the proposed synchronization method is that the memristor automatically reaches the value that will provide the required weight of the differential term required for synchronization, rather than the computational methods used to determine the weight.

INTRODUCTION

Chaotic systems are nonlinear systems highly sensitive to initial conditions. It is important to create new chaotic systems due to their widespread use in secure communication, cryptography, chemical reactions, etc. In 1963, the first chaotic attractor was found by Lorenz (1963). Following that, Rössler (1976), Fabrikant (1979), and Chua et al. (1993b) generated new chaotic equations. Many different methods have been used while producing new chaotic systems. Generating a new chaotic equation set with the control parameter method is a widely used method Deng et al. (2014); Zhou et al. (2008); Lü et al. (2002).

Chua’s chaotic circuit design with memristor pioneered the work of chaotic circuit design. Later, in most studies, chaotic circuit design was made using the operational amplifier (OPAMP) component Fan et al. (2019); Sundarapandian and Pehlivan (2012); Pappu et al. (2017); Pehlivan and Uyaroğlu (2010); Lai et al. (2017); Akgul et al. (2016); Cao and Zhao (2021). Only a few studies on circuit implementation of the chaotic system are based on OTA Karawanich and Prommee (2022); Yildirim (2022). The advantage of OTA over OPAMP component is its high output impedance, wide band gap, and transconductance gain which can be changed with bias current. This provides an important advantage in chaotic circuits. The chaotic circuit design with OTA presented in Karawanich and Prommee (2022); Yildirim (2022) has been designed, but there is no study on its synchronization. In this study, a simpler structure is proposed by using only OTA, capacitor, and analog multiplier.

According to Carroll and Pecora (1995), Pecora and Carrol proposed the concept of first chaos synchronization, which is the foundation of chaotic secure communication. Following that, passive components such as resistors, inductors, and capacitors were used Chua et al. (1993a); Yao et al. (2020); Zhang et al. (2020); Xu et al. (2019a); Yao et al. (2019). Synchronization studies are available by using active components such as Deniz et al. (2018); Uyaroğlu and Pehlivan (2010). Considering the important effect of the memristor in chaotic circuits, synchronization studies with memristor have become widespread in recent years. The memristor has less power consumption than other components because it is a passive component. In addition, although the memristor is nonlinear, it provides linear behavior in a certain frequency range. In this study, because of the memristor’s properties, the OTA-C chaotic circuit is synchronized with the memristor.

In the literature, there is a method of synchronizing memristors by connecting them in anti-parallel. With this method, it is possible to change the receiver and transmitter, but since the structure draws current from both the receiver and transmitter sub-circuits, the original ordinary differential equation set could not be preserved on the transmitter side Gambuzza et al. (2015). Whereas,
In most other methods, the original equations are preserved on the transmitting side, while only different terms are involved on the receiving side. There are articles that synchronize with different connection types besides anti-parallel connection, but the same mathematical deformation is also present in them Zhang et al. (2020b); Escudero et al. (2020); Wang et al. (2021); Xu et al. (2019b).

The method proposed in this study is based on the method of adding the difference term Cuomo et al. (1993) which is already found in the literature, to obtain this term over the memristor rather than the resistor. Instead of finding this coefficient with an optimization algorithm and producing this term with a suitable resistor, the memristor element connected instead of the resistor, both creates this coefficient and changes its value as long as there is a synchronization error due to the error expression passing over it, and reaches the value where error-free synchronization is provided by itself. In this way, the coefficient is self-adjusted by the value change of the memristor. The researcher eliminates the time cost by itself. In this way, the coefficient is self-adjusted by the value change of the memristor. The researcher eliminates the time cost.

Thus, parameters are chosen as $l_2 = 1_3 = 0$. In this case, the control parameter is $u = 1_1 x$. The new Lorenz-like chaotic equation is obtained in the Equation 4.

$$
\begin{align*}
\dot{x} &= \sigma(y - x) + l_1 x \\
\dot{y} &= x(p - z) - y \\
\dot{z} &= xy - \beta z \\
\end{align*}
$$

The new system is chaotic when parameter values $\sigma = 10, p = 28, \beta = 8/3, l_1 = 1$. At initial conditions $x(0) = 0.9, y(0) = 0.5, z(0) = 0.1$, the attractors of the system are in Figure 1. As time passes, the orbits around this created attractor scan the entire space, never passing a point they passed. The chaotic state of the new system is investigated by time series, frequency analysis, Jacobian matrix, Lyapunov exponents, and bifurcation diagram analysis.

The method proposed in this study is based on the method of adding the difference term Cuomo et al. (1993) which is already found in the literature, to obtain this term over the memristor.

$\Delta V = -\sigma + l_1 - 1 - \beta = -38/3$. Since it is $\Delta V < 0$, the behavior of the system is chaotic at the right initial conditions. Lyapunov exponents are expressions of interactions and differences between trajectories of phase space characteristics formed under close initial conditions. If the largest exponent is negative, the system converges to a value over time and becomes independent of initial conditions Özer and Akin (2005). If the largest exponent is positive, the distance between the orbits increases and the system is sensitive to initial conditions, that is, chaotic. If there are multiple positive Lyapunov exponents, the system is hyperchaotic Wolf et al. (1985). The new system’s Lyapunov exponents are shown in Figure 4. The Lyapunov exponents obtained with the parameters of the system selected as $\sigma = 10, p = 28, \beta = 8/3, l_1 = 1$ are $L_1 = 8.38852, L_2 = 0.632274, L_3 = -21.6813$. Since there are two positive Lyapunov exponents, the new set of equations is hyperchaotic.

By using Lyapunov Exponents, the Lyapunov dimension or Kaplan-Yorke dimension can be calculated as in Equation 6, Grassberger and Procaccia (1983).

$$
D_ky = j + \frac{1}{|L_{j+1}|} \sum_{i=1}^{j} L_i
$$

$j$ is the largest integer for which $0 \leq L_1 + \ldots + L_j$. For the proposed circuit $j = 2$ and the Kaplan-Yorke dimension $D_{ky}$ can be calculated as 7.

$$
D_ky = 2 + \frac{L_1 + L_2}{|L_3|} = 2.41597
$$
(a) \( V_x - V_y - V_z \) chaotic attractor.

(b) \( V_y - V_z \) chaotic attractor.

(c) \( V_x - V_z \) chaotic attractor.

(d) \( V_x - V_y \) chaotic attractor.

Figure 1 Phase portraits of the system.

Figure 2 Time series of the system.

Figure 3 Frequency spectrum of the system.

Figure 4 Lyapunov exponents of the system.
For original Lorenz system with the well-known coefficients
\( \sigma = 10, p = 28, \beta = 8/3 \), and \( L_1 = 0.054129, L_2 = 0.727225, L_3 = -14.448021, j \) is also equal to 2 and the Kaplan-Yorke dimension,

\[
D_{ky} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.05408
\]  

(8)

Thus, the new chaotic system has a larger Kaplan-Yorke dimension than the original Lorenz system.

The bifurcation diagram is the points at which the variables \( x \) and \( y \) intersect the equation’s solution curve on the plane formed by the two variables for each value of the parameter \( p \). The bifurcation diagram obtained for \( 1 < p < 350 \) and initial conditions \((0.9, 0.5, 0.1)\) in the new set of equations is shown in Figure 5.

![Figure 5 Bifurcation diagram of the system.](image)

**OTA-C CHAOTIC CIRCUIT DESIGN**

The new chaotic equation set circuit design is created using OTA, analog multiplier (AM), and capacitor components. While generating the OTA-C circuit, each chaotic state variable is represented by voltage state variables corresponding to a capacitor voltage. The expressions of the derivatives of these state variables are tried to be formed as the sum of the terms of the current magnitudes divided by the capacitor values, according to

\[
\frac{dV_x}{dt} = \frac{g_{m_1}}{C_x} (V_y - V_x) + \frac{g_{m_2}}{C_x} V_x
\]

\[
\frac{dV_y}{dt} = \frac{g_{m_1}}{C_y} V_x - \frac{k g_{m_2}}{C_y} V_x V_x - \frac{g_{m_1}}{C_y} V_y
\]

\[
\frac{dV_z}{dt} = \frac{k g_{m_5}}{C_z} V_x V_y - \frac{g_{m_1}}{C_z} V_z
\]

(9)

Taken as \( C_x = C_y = C_z = 10nF, g_{m_1} = 100\mu S, g_{m_2} = 27\mu S, g_{m_3} = 1nS, g_{m_4} = 280\mu S, g_{m_5} = 10\mu S, g_{m_6} = 1nS \).

While performing PSpice simulations of the circuit in Figure 6, the ideal OTA model realized with discrete elements and the AD633 integrated circuit macro model as analog multiplier were used. The multiplier constant of the AD633 IC is \( k = 0.1 \) V\(^{-1}\). The simulation results of the voltage values of the state variables of the circuit according to time are given in Figure 7. Chaotic attractors are also shown in Figure 8.

![Figure 6 OTA-C chaotic circuit of the system.](image)

**Figure 7 Time series of the simulated system.**

**SYNCHRONIZATION OF OTA-C CIRCUIT WITH MEMRISTOR**

The synchronization of two chaotic circuits with different initial conditions is provided by a memristor and a circuit with OTA by adding the difference term attached to it (Sambas et al. 2013).
According to this method, the equation of the receiver is as in Equation 10.

\[
\begin{align*}
V_x^r &= \sigma(V_y - V_z) + V_x^r \\
V_y^r &= V_x^r(p - V_z) - V_y^r \\
V_z^r &= V_x^r V_y^r - \beta V_z - \xi (V_z - V_z^r)
\end{align*}
\]

To ensure that the circuits are both chaotic and synchronized, the value of \(\xi\) should be either optimized or observed by drawing a bifurcation diagram of the error as shown below. According to the bifurcation diagram in the Figure 9, synchronization is provided in the proposed circuit for \(\xi > 1.8\).

**Figure 8** Phase portraits of the simulated system.

**Figure 9** Bifurcation diagram of the error.

In this study, it is suggested that the necessity of optimizing the resistance value is eliminated by replacing the fixed resistor with the memristor element. The proposed method is to start from any state of the memristor and wait for the desired coefficient to occur spontaneously due to the nature of the memristor. In this way, when the coefficient needs to be updated due to a change in the circuit due to time or environmental factors, it will automatically reach the needed value and be synchronized again.

The weight of the difference term addition circuit is self-adjusted by the value change of the memristor. The synchronization circuit is given in Figure 10. Accordingly, for Equation 10, it will be \(\xi = \frac{1}{C_r(M|R)}\). Due to the nature of the memristor, as long as there is an error, the memristance value will change in the direction of reducing the error, since \(V_z - V_z^r = 0\) after synchronization is achieved, no current will flow from this part of the circuit and the circuits will operate synchronously.

The parameter values of the receiver and transmitter circuits are the same. The initial conditions of the receiver circuit are \(v_x(0) = 0.05V, v_y(0) = 0.01V, v_z(0) = 0.05V\), the initial conditions of the transmitter circuit are \(v_x(0) = 0.09V, v_y(0) = 0.05V, v_z(0) = 0.01V\). The value of the resistor connected in parallel with the memristor is \(R = 30\,\text{k}\Omega\). Synchronization is realized over the \(z\) state variable of the receiver and transmitter circuit. The simulation results are shown in Figure 11. Circuits synchronized at 75ms. It is shown that this is the contribution of the OTA-C design of the chaotic circuit and the memristor circuit model used in synchronization.

In the memristor simulations, the PSpice code of the memristor model proposed by Joglekar was used (Haron et al. 2014). This model has been proposed for titanium dioxide memristor nanostructures (Joglekar and Wolf 2009). The window function associated with the \(p\) exponent is used to provide the necessary nonlinearity. The \(p\) parameter is usually between 1 and 100. It is defined by Equations 11 and 12, where the memristor model represents the Joglekar window (Joglekar and Wolf 2009):

\[
f_j(x) = 1 - (2x - 1)^2p
\]
\[
\begin{align*}
\frac{dx}{dt} &= k f(x) \\
v &= i[R_{\text{ON}} x + R_{\text{OFF}} (1 - x)] \\
k &= \frac{\mu R_{\text{ON}}}{D^2}
\end{align*}
\] (12)

where \(x\) is the memristor state variable, \(f(x)\) is the window function, \(p = 10\) is a parameter of the Window Function, \(k = 1000\) is a constant dependent on memristor physical parameters, \(\mu = 10^{-14} m^2/(Vs)\) is the ionic drift mobility, \(D = 10\text{nm}\) is the memristor length, \(i\) is the memristor current, \(v\) is the applied voltage, \(R_{\text{ON}} = 100\Omega\) and \(R_{\text{OFF}} = 16k\Omega\) are the ON and OFF resistances of the memristor.

\[\text{Figure 10} \quad \text{Synchronization of chaotic OTA-C circuits.}\]

\[\text{Figure 11} \quad \text{OTA-C chaotic synchronization charts.}\]

**CONCLUSION**

In this study, a new chaotic equation set is obtained from the Lorenz equation using the anti-control procedure. Then, the circuit of this equation set is designed. Ideal OTA, capacitor, and analog multiplier are used in the designed circuit. This provides it less costly in case of physical implementation. The synchronization of the circuit was realized in a short time of 75ms using the memristor and differential receiver circuit with OTA. At the same time, the use of a memristor component provided low power consumption and time-saving.

**Conflicts of interest**

The authors declare that there is no conflict of interest regarding the publication of this paper.

**Availability of data and material**

Not applicable.
LITERATURE CITED


Deniz, H. I., Z. G. C. Taskiran, and H. Sedef, 2018 Chaotic lorenz synchronization circuit design for secure communication. In 2018 6th International conference on control engineering & information technology (CEIT), pp. 1–6, IEEE.


