
Araştırma Makalesi / Research Article

Investigation of Numerical Methods SPH, ALE, Coupled MM-ALE with LBE and CONWEP Empirical method for Simulation of the Spherical Free Air Blast Loading with Using LS Dyna

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ABSTRACT: Different simulation methods can be used to model the response of structures to the effects of blast loading. While some simulation methods are based on empirical blast loading principles, other methods use a fluid structure interaction algorithm to predict post-explosion shock waves and their physical effects. In this study, CONWEP, SPH, ALE, and coupled MM-ALE with LBE numerical methods were compared against each other according to results from an experimental study. Each method was compared with the test results in terms of solution time, convergence, and the use for different explosive types and environments. According to this comparison, it was concluded that empirical methods can be used for more limited environmental conditions and blast types, ALE numerical methods can give very sensitive results even in different solution sets but the solution time is long. Meanwhile in SPH method, the interaction of the air and blast shock cannot be fully modelled. According to the results of the study, the hybrid method is consistent with the test results in terms of peak pressure with a deviation of 7.44% at P1 and 2.29% at P2 under spherical free air blast loading conditions. However, since the effects of reflected pressure cannot be modelled exactly in the hybrid method, the ALE method should be preferred in cases with more complex geometries.

Keywords: Free Air Blast, Numerical Simulation, Explosion, Spherical Blast Load, High Explosive.

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Table 1. Nomenclature and abbreviations

Term	Definition	Units
CONWEP	Conventional Weapons Program	
SPH	Smooth Particle Hydrodynamics	
ALE	Arbitrary Lagrangian Eulerian	
MM-ALE	Multimaterial Ale	
LBE	Load Blast Enhanced	
TNT	Trinitrotoluene	
UFC-3-340-02	Structures To Resist The Effects Of Accidental Explosions	
p_0	Ambient Pressure	
$p_{so,max}$	Peak Overpressure	<i>kPa</i>
td	Positive Phase Duration	s
so	Incoming Pressure Values	subscript
$p_{so,min}$	A Negative Shock Front With A Peak Amplitude	<i>kPa</i>
$t\bar{d}$	Negative Phase Duration	
Z	Scaled Distance	$m/kg^{1/3}$.
$p_r(t)$	Static pressure at any time for Friedlander Equation	<i>kPa</i>
q_0	Dynamic Pressure	<i>kPa</i>
$C_1, C_2, C_3, C_4, C_5, C_6$	Constants for ideal gases	
ρ and ρ_0	Current and reference state densities for air	kg/m^3
p	Function of, the internal energy of air	<i>J</i>
γ	Specific heat ratio	
E	Specific internal energy	kJ/kg
D	Detonation velocity	<i>m/s</i>
PCJ	Chapman-Jouguet pressure	<i>kPa</i>
A, B	Linear coefficients	
R1, R2 and ω	Nonlinear coefficients	
$V = v/v_0$	The volume of detonation products / volumes of undetonated High Explosive	
P	Pressure	<i>kPa</i>
E	Detonation energy per unit volume of the high explosive	<i>J</i>

1. INTRODUCTION

Air blast loadings should be taken into account at the design stage to minimize the damage to structures, people, or vehicles in case of terrorist attacks, defence needs of countries, or explosion-related accidents. Briefly; an explosion is an event that usually occurs with a sudden increase in temperature and gas release, causing rapid volume expand and the release of extremely high energy, accompanied by a very loud sound. From the centre of the explosive, the air creates a pressure increase. This pressure change causes a pressure distribution called shock wave. In order to investigate and model the effects of the blast loads on the structures, it is necessary to estimate the shock wave parameters according to the explosive type, amount and the environment in which the explosive is located. Experimental estimation of the blast load is costly and requires long processes. Numerical calculation methods are promising and frequently used calculation methods in modelling blast loads and structures exposed to blast loads.

Numerous studies have been conducted on the numerical calculation of the free air blast and the comparison of various methodologies. In a study by Zakrisson et al., numerical and experimental studies were carried out on two different scenarios. The first of these scenarios is the research in

which a cylindrical NSP 71 explosive is detonated in a steel pot and the other is the study in which a cylindrical NSP 71 explosive is detonated in free air. Zakrisson et al. modelled the test systems with the empirical explosion loading method, also known as CONWEP, and the ALE Multimaterial group. Apart from the two-dimensional modelling, they were also solved in 3D using the mapping method with Cartesian mesh. Simulation and experimental results were compared in terms of impulse and deflection on the target plate. According to the results of the research, the plate displacement was estimated with a deviation of 9.4-11.1%, and the deviation in the total impulse was calculated as only 1.0-1.6% in the solution calculated using the ALE method (Zakrisson et al., 2011). Another research conducted by Tabatabaei and Volz compared the LBE, MM-ALE, coupled MM-ALE with LBE methods, in which a 36 kg TNT explosive was placed at a standoff distance of 168 cm from the target concrete panel (Tabatabaei and Volz, 2012). The Cartesian mesh was applied in all numerical models, and no mesh sensitivity analysis was performed in this study. The results of the two sensors measuring the reflected pressure and the free air pressure were compared in terms of peak pressure values and solution times. It was observed that in coupled method results at the sensor, for which the free air pressure was calculated, largely overlapped with the test results, while the results of the reflected pressure calculation were found to be below the test results in all methods. Fichera et al. worked on the numerical modelling of explosions caused by landmines in the sand at different depths (Fichera, and Scapin, 2013). The pressure peak values formed at different altitudes in the air as a result of the explosion of 100 gr C4 explosive in a steel tube and sand environment simulated by the ALE method were compared with the experimental data. The other test system is the comparison of the pressures on the target plate in the air environment as a result of the explosion of 250 gr pentolite explosive underground in the sand. All cases in this research by Fichera et al. were simulations of blasts at different depths in a sand environment (Fichera and Scapin, 2013).

Trajkovski et al. performed a series of free air blast numerical simulations to investigate the effects of mesh size and bias parameters on the incident and reflected pressure values using the MM-ALE method (Trajkovskiet al., 2014). They also examined the effects of varying the standoff distance of the target from the explosive on the reflected pressure. Accordingly, reflected pressure parameters are negatively affected at distances less than 4 times the explosive diameter. Besides, they concluded that at least 10 elements are required across the explosive radius. Han and Liu, unlike the others, used the coupled MM-ALE method and compared the changes of incident pressures according to the scaled distance values in UFC 3-340-02 guideline with numerical results. It was concluded that the coupled MM-ALE method is useful in cases where the scaled distance is above 0.4, the problems that may arise from large air elements can be compensated by increasing the air density up to a certain rate, and larger air elements can be used in simulations where the scaled distance is relatively large (Han and Liu, 2015). Flis and Dobrociński, simulated hemispherical detonation waves on 1 kg of TNT detonated on a 1m x 1m steel plate on the ground plane with the numerical methods CONWEP, MM-ALE and SPH and concluded that the MMALE method is the only method that gives approximate results in contact explosions (Flis and Dobrociński, 2015). They observed that the CONWEP method could not generate detonation waves in the correct form in explosions that were closer than three times the charge radius and when the scaled distance was about $1 \text{ m/kg}^{\frac{1}{3}}$. Rebelo and Cismasiu modeled the free air blast on a 2.17m x 2.6 m steel plate from a standoff distance of 3 m with 8 kg TNT, using the numerical methods CONWEP, MM-ALE and combined MM-ALE by modelling in 2D and 3D (Rebelo and Cismasiu, 2017). The pressures of 10 different points, two on the lower surface of the part, two on the upper surface, and six in the explosive direction at different distances, were compared with the test results. They concluded that the pressures on the bottom surface of the

plate could not be obtained due to the lack of shading and focusing properties of the CONWEP method. In addition, it follows that the use of the 2D to 3D mapping method shortened the solution time considerably and the resolution precision was not impaired.

Different studies such as Erdik and Uçar's study also compared blast loading techniques by referencing Tabatatei and Volz's (2012) test design and findings (Erdik and Uçar, 2018). In terms of incident pressure and reflected peak pressure, the pressure waves created by the explosion of 34 kg of TNT over 1675 mm of a 1830x1830x165 mm steel plate were measured and estimated. In Erdik and Uçar's study, the hybrid technique was found to be effective when measured in terms of total solution time and accuracy when compared to CONWEP, ALE, and combined MM-ALE methods (Erdik and Uçar, 2018). Brief information regarding recent studies using numerical approaches to evaluate and calculate blast loads was provided. This study differs from previous studies in that it compares four alternative methodologies on the same experimental system and identifies crucial parameters for spherical free-air blast modelling. When the amount of explosive material detonates, it creates dense, high-pressure gases. The air surrounding the explosive is forced out from the centre of the explosion and compressed as a result of the explosion at supersonic speed. Because, the air is a compressible fluid, the pressure and density increase with a shock front up to a certain distance from the centre of the explosion and then return to the atmospheric conditions (Baker, 1974).

The blast wave is described by a rapid increase in air pressure from the ambient (p_0) to the point of peak overpressure ($p_{so,max}$) at a certain distance from the explosive. The pressure returns to ambient pressure in time (td) known as the positive phase duration. The subscript "so" indicates incoming pressure values, that is, the pressure value measured by a pressure transducer just behind the shock front or parallel to the propagation of the shock front. Overpressure term is the pressure increased above normal atmospheric pressure which is inflicted by the blast wave (Rigby, 2014). After the positive phase, the negative phase period occurs due to the excessive expansion of the air. In the negative phase, a negative shock front with a peak amplitude $p_{so,min}$ occurs during $t\bar{d}$ below atmospheric pressure. At the end of the $t\bar{d}$ period, the pressure returns to ambient pressure. An ideal free air blast pressure-time graph can be found in previous literature (Rigby, 2014), and is obtained by the time-dependent integration of the pressure change, that is, the area under the pulse pressure time curve. When a blast wave strikes a rigid surface, mass, momentum, and energy are conserved at the interface. As a result, the pressure, temperature, and density of the blast wave rise above their initial values. The reflected pressure is the overpressure at a rigid surface, and its values are shown by the subscript "r". It is assumed that the detonation effect varies within a certain rule with the mass of the explosive and the distance of the detonator from the target. The concept of scaled distance or cube root similarity rule was proposed at separate times and independently by Cranz and Hopkinson (Cranz, 1926), (Hopkinson, 1915); It reveals that there is a similarity between pressure, duration and impacts for explosives of different masses and explosives at certain distances from the target as described in a previous study (Rigby, 2014). For example, an explosion with mass W and distance R has similar explosion pressures at a distance KR with mass K^3W . The unit of scaled distance expressed as Z will be $m / kg^{1/3}$.

$$Z = \frac{R}{W^{1/3}} \quad (1)$$

When the explosive diameter and the distance of the explosive to the target are K times; the explosive mass would be K^3 times. In this case, the scaled distance Z does not change. However, ta ,

which is the time to reach the maximum pressure, and td , which is the time to fall from the maximum pressure to the ambient pressure, increase with K times. Therefore, the impulse also changes at the same rate linearly. The relationship between the coefficient K , ta , i and td is linear. According to UFC-3-340-02 guideline published by The United States Department of Defense “Structures to Resist the Effects of Accidental Explosions, 2008” (formerly known as TM-5 1300) for empirical estimation of blast load; where the scaled distance is the same, the maximum pressure and the reflected pressure are considered the same.

With regards to previous literature studies, the current study aims to determine the incident and reflected pressure values on the target as a result of blast waves in spherical waveforms of explosive charge. Comparisons of experimental data with the outcomes of numerical calculations were also taken into consideration in several investigations. This study's major goal is to compare the experimental findings from the same scenario with all of the numerical approaches that have been discussed in the literature for spherical free-air blast calculations. Numerical calculations and modelling carried out with using LS-DYNA commercial software with version of V.4.5.24.

2. MATERIALS AND METHODS

An explosion in free air can simply be expressed as a shock wave propagating outward from the centre of the explosive in a spherical form. Shock wave parameters can be obtained analytically by solving the conservation equations of momentum, mass, and energy on the explosive side and the free air side. Although the empirical method gives very fast and predictive results, numerical methods have been developed for situations involving complex structures and explosives. Euler elements are used for modelling compressible fluids such as air, and Lagrange-type elements in which the material moves with the nodes are used for target structures. Although the explosion event is modelled very precisely with these numerical methods, many parameters are needed. All these parameters are explained below. On the other hand, detailed information about test setup is given.

2.1 CONWEP (LBE) Method

The CONWEP method implemented in LS-DYNA is based on the empirical blast loading function based on TNT data from Kingery and Bulmash and the study of Randers-Pehrson and Bannister (Randers et al., 1997), (Zakrisson et al., 2011), (Kingery and Bulmash, 1984). This blast load, modelled with the *LOAD_BLAZT_ENHANCED (LBE) calculation model in LS-DYNA, serves to simulate an explosion in a Lagrange structure without the need to model any Euler domain. Spherical or hemispherical blast loads can be simulated in the free-air blast environment. The load acts on a surface consisting of a set of predefined segments, such as solid elements or shell elements. The CONWEP method uses the Friedlander Equation (Eq.2) (Friedlander, 1946).

$$p_r(t) = p_{r,maks} \left(1 - \frac{t}{t_d}\right) e^{-b \frac{t}{t_d}} \quad (2)$$

Because high-order polynomial curve fitting is quite cumbersome, the estimation of shock wave parameters for spherical and TNT explosives by scaled distance is expressed by a series of curves which can be found in a previous study in the open literature (UFC 3-340-02, 2008). These curves also form the basis of the CONWEP computer code, which calculates the explosion parameters using the empirical method. It is also based on this work in the US Department of Defence (2008) Design Guide UFC-3-340-02 (Hyde, 1991).

For spherical TNT explosives, in cases where the explosive mass and the distance of the explosive to the target are known, the maximum pressure reflected pressure, impulse and ta and td graph on the graph in (UFC 3-340-02, 2008) correspond to the y value where the scaled distance value on the x -axis cuts the relevant curve. But the impulses, td and ta also depend on the cube root of the explosive mass. On the other hand, the pressure acting on the front side of the target could be found by using the data on variation of peak dynamic pressure versus peak incident pressure (UFC 3-340-02, 2008). When correlating the P_{so} value attained from to the x -axis of variation of peak dynamic pressure versus peak incident pressure, the dynamic pressure q_0 will be the point that corresponds to the y -axis. Thus, the total pressure acting according to the drag coefficient of the target surface can be calculated. In LS-DYNA application of this method, the crucial card is LOAD BLAST ENHANCED which applies the LOS CONWEP pressure prediction to the surfaces in the model, taking into account the explosive mass and standoff distance.

2.2 Smooth Particle Hydrodynamics (SPH) Method

The Smooth Particle Hydrodynamics (SPH) method is a technique that examines the change of the particle over time, or in other words, follows the particle and uses Lagrangian equations in the solution. In shock wave problems particle methods are not only in the approximate solution of continuous fluid equations; states that it is a more fundamental approach than the continuity equations, which includes molecular dynamics systems in its infrastructure, revealing the difficult particle system equations. The SPH method can be used in solving hydrodynamic problems of field variables, which are generally in the form of partial differential equations. For this, firstly, the discretization of the domain of the problem, and then the field functions and the derivatives of these functions, these partial differential equations are converted into ordinary differential equations that depend only on time. Finally, these ordinary differential equations can be solved by any integration method.

Particles are randomly dispersed throughout the field without interconnection. An integral definition method (kernel approximation) is used to approximate the function valid in the domain. After the kernel approach, another approach using particles is made and this is called "particle approximation". In this step, the function and derivative values of the particles distributed in the area to be used in the definition of the integral are calculated by averaging the neighbourhoods of each particle at a certain distance. The particle approach is iterated at each time step, and the function values are updated at each step. The field variables functions, which are in the form of partial differential equations, turn into ordinary differential equations after the particle approximation. After this step, the solution of ordinary differential equations is done. One of the most important advantages here is that the solution can be made with the "explicit" integration algorithm. That is, in each step, the values obtained in the previous step are used to calculate the time variation of the field functions. In SPH method, the explosive is defined with the material model HIGH EXPLOSIVE BURN, the steel plate with the JOHNSON COOK material model, and the EOS of the explosive is defined with the JWLL (JONES WILKINGS LEE) card. The AUTOMATIC NODES TO SURFACE card is used in modelling the contact between the particles and the steel plate surface. While the number of particles used in the SPH method directly affects the convergence of the result, there are also some important parameters. These;

SOFSCCL; Scale factor for constraint forces of soft constraint option

TSSFAC; Scale factor for the computed time step.

CSLH; Constant applied to the smoothing length of the particles.

2.3 Arbitrary Lagrangian Eulerian (ALE) Method

Explosions involve high-pressure shock waves as well as liquid and gas flows. A Lagrangian finite element mesh is not always possible in the explosive charge region. The time step per iteration will be very small since the liquid-form elements around the explosive are severely deformed when the Lagrangian mesh structure is applied, which significantly increases the computation time. Also, there may be numerical approximation errors due to mesh distortions.

Frequently, fluid flow problems are determined using an Euler structure, but the need to accurately monitor material mixing and material interfaces can become huge and computationally takes a long time (Rigby, 2014). Also, Eulerian analysis usually requires a large number of elements (Hallquist, 2006). The Lagrangian method is the approach in which solids are represented. With the Lagrangian method, the nodes of the elements are connected to the material and do not separate from each other, they move together. When the material is deformed, the mesh also deforms with the material, but there is no mass transfer between the elements. Although the Lagrangian method is efficient in terms of calculation time and the ease of application of boundary conditions, the most common problem is the excessive deterioration of quality and the inability of analysis to converge at high deformations. In ALE method, an arbitrary reference coordinate system is created in addition to the Lagrange and Euler coordinate systems. With the following equation, the material derivative can be arranged according to the reference coordinates (Messahel and Soulie, 2013).

$$\frac{\partial f(X_i)}{\partial t} = \frac{\partial f(x_i)}{\partial t} + w_i \frac{\partial f(x_i)}{\partial x_i} \quad (3)$$

In equation 3, X_i is the Lagrangian coordinate, x_i is the Euler coordinate, w_i is the relative velocity. When we denote the velocity of the material by v and the velocity of the mesh by u , the relative velocity is $w = v - u$

Thus, the basic equations for the ALE formulation are given by the following conservation equations:

Conservation of Mass;

$$\frac{\partial \rho}{\partial t} = -\rho \frac{\partial v_i}{\partial x_i} + -w_i \frac{\partial \rho}{\partial x_i} \quad (4)$$

Conservation of Momentum;

$$\rho \frac{\partial v_i}{\partial t} = \sigma_{ij,j} + \rho b_i - \rho w_i \frac{\partial v_i}{\partial x_j} \quad (5)$$

In equations 4 and 5, $\sigma_{ij,j}$ is the stress tensor and is defined by, $\sigma = -P.Id + \tau$. Here τ is the shear stress from the constitutive model and P is the pressure. For fluid and explosive gas, the pressure is calculated by an equation of state.

Conservation of Energy;

$$\rho \frac{\partial E}{\partial t} = \sigma_{ij} v_{i,j} + \rho b_i v_i - \rho w_j \frac{\partial E}{\partial x_j} \quad (6)$$

In the Euler method, $u = 0$ is taken since the net does not move. Therefore, the relative velocity w is equal to v , that is, the material velocity. In equation 6, $\rho b_i v_i$ and $\rho w_j \frac{\partial E}{\partial x_j}$ are expressed as advection terms. And it allows the material to move in the mesh (Messahel and Soulie, 2013). The ALE method allows the element nodes to move independently and randomly, allowing the elements

to deform without being warped even under high deformation so that the analysis converges and reaches the solution while maintaining the high-quality mesh during the analysis. The ALE method connects the mesh to the material as in the Lagrangian method but allows the mass transfer (the material to flow through the mesh) as in the Euler method. The computational steps are divided into the Lagrangian phase and the advection phase. The mesh structure can be advected to the original shape or a more advantageous shape. Or it may not be advected at all.

The *MAT_NULL material card with a combination of *LINEAR_POLYNOMIAL Equation of State card is used for modelling the ambient air. The pressure p is expressed as a function of, the internal energy of air, and $\mu = \rho / \rho_0 - 1$, where ρ and ρ_0 are the current and reference state densities, relatively (Rebelo and Cismasiu, 2017).

$$p = C_0 + C_1\mu + C_2\mu^2 + C_3\mu^3 + (C_4 + C_5\mu + C_6\mu^2)E \quad (7)$$

In equation 7, $C_1, C_2, C_3, C_4, C_5, C_6$ are constants and for ideal gases, $C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = 0$ and $C_4 = C_5 = \gamma - 1$ and the equation is reduced to the ideal gas equation of state.

$$p = (\gamma - 1)E\rho/\rho_0 \quad (8)$$

In Equation 8, γ is the specific heat ratio and E is the specific internal energy. The specific heat ratio is the ratio of the heat capacity at a constant temperature to the heat capacity at a constant volume. For air, $\gamma = 1,4$. The specific internal energy of $E = 253,4 \text{ kJ/kg}$ gives an atmospheric pressure of $101,36 \text{ kPa}$ (Rigby, 2014). While modeling the detonation process, the material model of the explosive and also the EOS should be determined. LS-DYNA typically uses MAT HIGH EXPLOSIVE BURN card, which requires the density of the explosive to be defined, ρ , detonation velocity, D , and Chapman-Jouguet pressure (PCJ). In the EOS_JWL (Jones-Wilkins-Lee Equation of state), which is empirical, used for high explosives, the volume, pressure, and energy relationship of the explosive is defined as follows (Lee et al., 1968).

$$p = A \left(1 - \frac{\omega}{R_1 V}\right) e^{-R_1 V} + B \left(1 - \frac{\omega}{R_2 V}\right) e^{-R_2 V} + \frac{\omega E}{V} \quad (9)$$

In equation 9, A, B, R_1, R_2 and ω are constants, V is volume and E is internal energy. The parameters for air and Pentolite are shown in Table 2 with the JWL parameters.

Different from the SPH method, some characteristic cards are used in the ALE method to simulate an explosion in the LS-DYNA software. These;

ALE MULTI MATERIAL GROUPS; The keyword used to model the interaction between explosive and air elements also allows individual elements to contain more than one material (Luo et al., 2004).

CONSTRAINED LAGRANGE IN SOLID; defines fluid structure interactions.

CONTROL ALE; allows the addition of reference pressure to the free surfaces of the ALE, this keyword also allows the adjustment of various global parameters such as the advection method (meth), the number of cycles between advection (nadv) and the smoothing controls.

HOURLASS; Hourglass modes are non-physical, zero-energy modes that do not create any stress or strain but can affect solution accuracy. It is used to reduce the unreal deformation of elements in zero-energy modes.

MAT NULL; defines the material characteristics of the surrounding air.

LINEAR POLYNOMIAL; defines the equation of state parameters of the surrounding air.
 EOS GRUNEISEN; defines the equation of state parameters of steel plate.

Table 2. ALE Parameters were used in this study (Schwer, 2016)

		MAT_NULL								
Air	ρ_0	1.225								
		EOS_LINEAR_POLYNOMIAL								
	C_0	C_1	C_2	C_3	C_4	C_5	C_6	E_0	V_0	
	0.0	0.0	0.0	0.0	0.4	0.4	0.0	2.55e-6	1	
		MAT_HIGH_EXPLOSIVE_BURN								
Pentolite	ρ_0	D	PCJ							
	1650	7360	23.5							
		EOS_JWL								
	A	B	R_1	R_2	w	E_0	V_0			
	531.77	8.93	4.6	1.05	0.33	8	1			
		MAT_JOHNSON_COOK								
Mild Steel Plates (AS3678-250)	ρ_0	G	E	PR	A	B	N	C	M	
	7.850e-06	77	203	0.3	0.35	0.275	0.36	0.0022	1	
	TM	TR	$EPSO$	CP	PC	$SPALL$	IT	FMIN		
	1673	293	1	4400	-1000	1	1	1e-6		
		EOS_GRUNEISEN								
	C	$S1$	$S2$	$S3$	$GAMA0$	A	E_0	VO		
	4596	1.49	0	0	2.17	0.46	0	0		

2.4 Coupled MM-ALE with LBE Method

As described in the preceding section, empirical blast loads are applied to the air domain simulated with the MM-ALE formulation in this linked approach known as Coupled MM-ALE with LBE, which has significant benefits over using either methodology for air blast simulations (Slavik, 2009). The empirical method although computationally efficient is impractical if there is an obstacle to the shock wave between the explosive and the target, which may cause any shock wave reflection, or shock wave merging. In the empirical method, the blast energy focus resulting from shock wave merging or reflection is not considered. A relatively large air domain and a relatively fine mesh structure are required in the MM-ALE formulation. It causes high computational complexity than the CONWEP method (Rebelo and Cismasiu, 2017).

In this method, which is a combination of ALE and CONWEP methods, the air domain is modelled with the ALE formulation, while the pressure values caused by the explosion are applied to the ALE layer known as the ambient layer with the empirical formulation in the LOAD_BLAST_ENHANCED card. The blast wave propagates along the shock wave in the air domain modelled as ALE. Due to the usage of strong features of each model without modelling extensive air domains, Coupled MM-ALE with the LBE method can help to simulate shadowing and focusing. (Slavik, 2009).

The Coupled MM-ALE with the LBE solver's primary drawback is the extensive air domain that must often be included in the air blast model to prevent boundary effects. The necessity to use very fine mesh resolution to appropriately resolve the air shock adds to the huge domain's computing complexity (Slavik, 2009). Some special cards define this method as well.

INITIAL VOLUME FRACTION GEOMETRY; lets you fill a volume with ALE multi-material groups.

ALE REFERENCE SYSTEM GROUP; helps to adjust the air domain and related parameters exposed to mesh expansion.

2.5 Test Descriptions

Explosion tests take place in a very short period, so the sensors and necessary hardware to be used need to be very sensitive. On the other hand, due to the difficulty of procuring the necessary explosive, academic studies in this field are also limited. For this reason, the experiments and results described below in the literature belongs to Boyd (2000) were used in this study. Experimental test report includes the test methodology, device specifications and the engineering parameters, that's why in this study the numerical results had been compared according to Boyd's (2000) test report's results.

A square target mild steel plate (AS3678-250) with one an edge of 1200 mm and a thickness of 5 mm is fixed to a more rigid heavy steel frame with 24 high tensile bolts with a pre-tensioned with 11.06 N.m. moment by Boyd shown in Figure 1a (Boyd, 2000). One of these experiments was carried out at a 500 mm standoff distance and 250 gr pentolite charge (E14). 100 mm of each side is fixed on the concrete block, and 1000 mm of the plate is left to move freely with the explosion load. Necessary sensors have been placed as can be seen in Figure 1b to record pressure, acceleration and displacement data. Boyd's (Boyd, 2000) experiments were performed with the help of the LS-Dyna program for numerical calculation with four different methods. SPH, ALE and Coupled MM-ALE with LBE methods were solved separately with different mesh sizes, and their results were compared in terms of mesh size and total computation time.

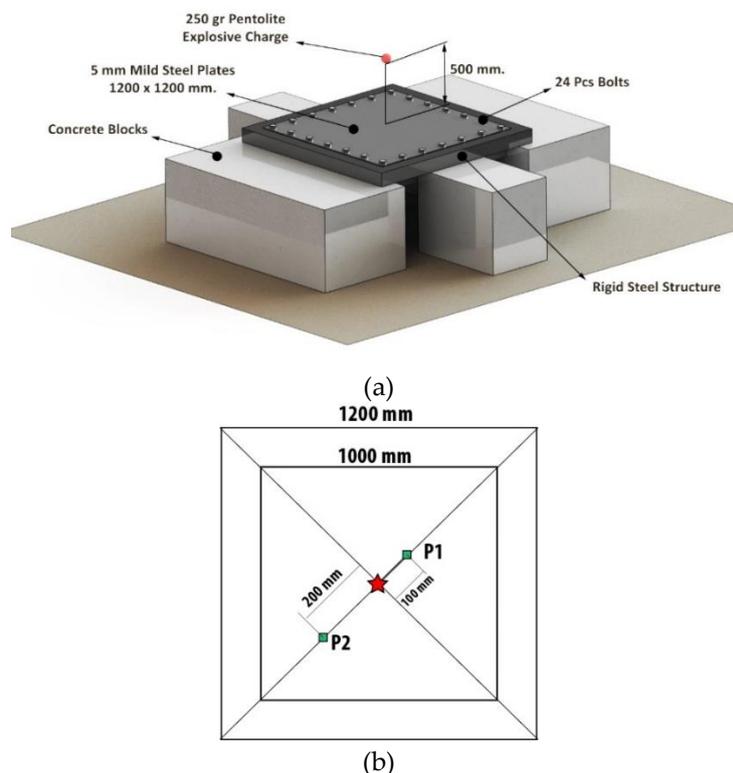


Figure 1. Experimental work showing (a) setup and instrumentation of Boyd's study and (b) scheme of points that are located on the plate for related sensors

According to Boyd's (2000) test number E14 the peak overpressure was measured 9.4 MPa at point P1 and 8.7 MPa at point P2.

2.6 Numerical Analysis

Boyd's experimental study with 250 gr PETN explosive was solved with LS-DYNA finite element software, using CONWEP, which solved the problem empirically, Coupled MM-ALE with LBE, which solved the problem semi-empirically, and ALE, which solved it completely numerically. SPH method uses hydrodynamic approaches which are generally in the form of partial differential equations (Boyd, 2000).

In SPH method, four different simulations were carried out with a different number of particles. The number of particles directly affects the convergence of the solution and also extends the solution time. The simulation was modelled with 2176, 8176, 137376 and 1114121 particles, respectively, and SOFSCCL=0.25, TSSFAC=0.001, and CSLH=1.2 were taken in these models. Figure 2 shows the schematic representation of the numerical model using the SPH method.

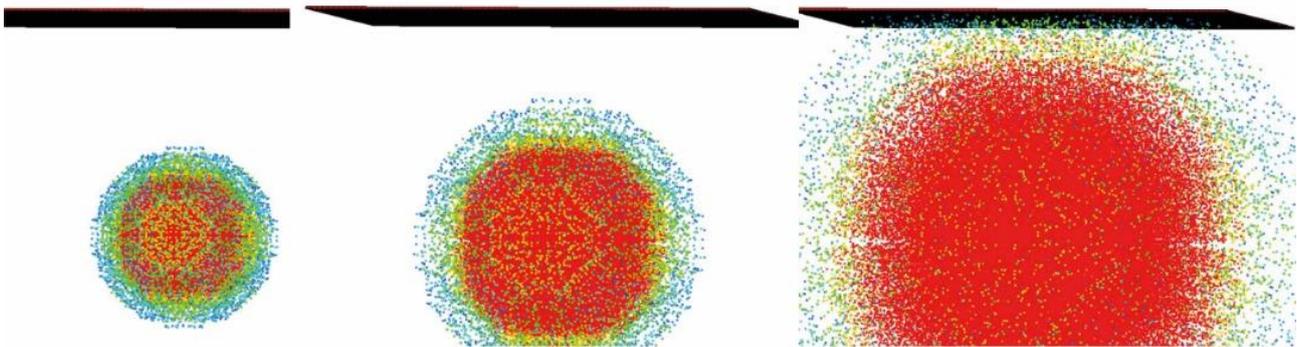


Figure 2. Scheme of SPH (Smooth Particle Method)

In the finite element method, since the physical quantities are transferred over the elements and nodes in the mesh structure, it is important to construct and select the mesh structure in order to model the shock wave propagation in spherical form. Therefore, it is important that the number of elements from the explosive core is so high that it can spread spherically and that it is arranged radially. For this reason, a preliminary study was carried out to create the mesh structure and according to the results of this first study, refinements were made in the regions under high pressure. Elements were enlarged from the explosive core to the air environment boundary using the bias method. Afterwards, different calculations were made by reducing the number of elements and the results were expected to converge. Considering the convergence of the results and the solution time, the optimum number of elements was determined for the convergent result.

To determine the mesh sensitivity for each method, different mesh structures were used, and the results were compared in Table 3. For coarse meshes in both types of calculation methods (ALE and Coupled MM-ALE with LBE) the approximate element size is chosen 25 mm. Especially in the ALE method, the mesh size must be chosen very small relative to the air domain. For instance, in this study, the minimum element size is 2.5 mm (in the condition of fine mesh) where the explosive diameter is 63 mm. If the same element size is chosen in the whole model the computation time will be extremely high likewise the total number of the element. Hence, the bias function was used in ALE and Coupled MM-ALE with LBE models to increase element size from the core of the explosion to the boundary of the air domain in direction of shock propagation. The bias function is used to adjust the spacing ratio of nodes along the edge. This function is very useful for FEM problems where the nodes need to be clustered intensively in a specific volume or area of the mesh structure.

All simulations were modelled in a 3D radial mesh structure in the Coupled MM-ALE with LBE and ALE methods, where explosives and fluids are defined by the Euler formulation. The shock

front formed by spherical explosives in the free air domain proceeds radially from the centre outward due to its nature. Since the shock waveform is distorted in Cartesian mesh structures, a structure in which the mesh size increases as a rule from the core to the outward with the radial mesh structure by using a bias feature is used. The air domain and the spherical explosive mesh are shown in Figure 3. The form of the shock wave in the analysis with radial and Cartesian mesh is given in Figure 4.

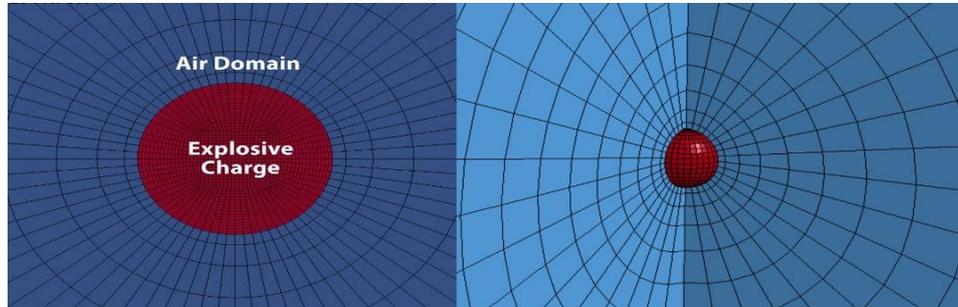


Figure 3. 3D radial mesh structure of explosive and air domain for ALE

As mentioned in the previous section, the CONWEP method works according to a mathematical expression developed as a result of controlled experiments. Since these controlled experiments are performed with TNT, they make a convergence with TNT equivalents in different types of explosives. However, it is not very useful for complex analyses because the velocity of detonation and the amount of energy released by each explosive is different. In addition, the CONWEP method offers an option only for spherical or hemispherical explosions. TNT equivalent for pentolite explosive was taken as 1.12 and calculations were made accordingly (Kingery and Coulter, 1983).

In the ALE method, the steel plate is modelled as Lagrange, and explosives and fluids as Euler-type elements. In this method, a large number of parameters are needed to estimate the blast physics. The parameters required for this study were obtained from the literature. The advantage of the ALE method is that it can be applied to all explosive types of different types and different geometries. The time interval between outputs is defined as 0.001 whereas the scale factor for the computed time steps is defined as 0.3.

In the solution with the Coupled MM-ALE with LBE method, the explosive is defined by the determination of the initial detonation point in the air domain by using a VOLUME FRACTION GEOMETRY card. The biggest advantage of this method is that it can produce solutions in a shorter time compared to the ALE method and can be applied to different types of explosives and geometries.

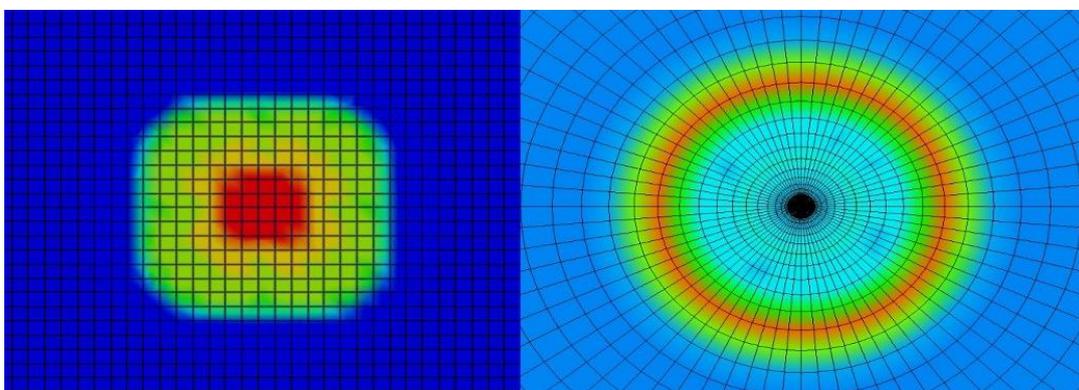


Figure 4. The difference in shock propagation between the cartesian and radial mesh.

On the other hand, the pressures acting on the monitoring points P1 and P2, on the target plate were estimated according to the UFC-3-340-02 guideline and are shown in Table 5. The scaled distance was calculated at $0.78 \text{ m} / \text{kg}^{1/3}$ for points P1 and $0.82 \text{ m} / \text{kg}^{1/3}$ for point P2. In the calculations for the front wall of the target, the c_d the coefficient was taken as 1 and the pressure acting on the relevant points was estimated (Karlos and Solomos, 2013).

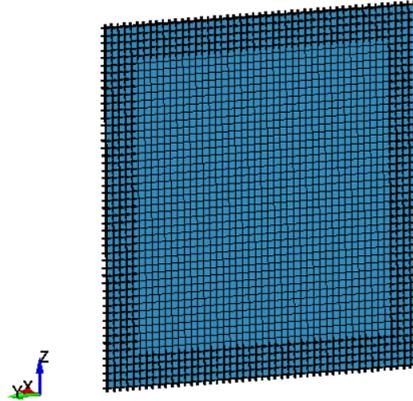


Figure 5. Defining Boundary Conditions of the Plate

The steel plate had been stabled under steel flange that was bolted with 24 pieces high tensile bolts with 11.06 [N.m.] in the test set up. According to report it is emphasized that the target plate's 1000 mm square central area were free to move. Since there is no any information about the bolts types and locations, related region elements were assumed to be fixed elements in any direction. The movements of the elements in the 200 mm wide part surrounding the plate in the x, y, z, rx, ry and rz directions were fixed and the boundary conditions are determined (Figure 5).

2.7 Mesh Convergence Study

Mesh convergence was addressed in a series of calculations in which the element size was progressively reduced with a refinement ratio of 1.5 in the ALE method, 1.25 in the hybrid method, and 8 in the SPH method. In this study, the size of the square elements of the plate is taken as 12.5 mm. In addition, since the CONWEP algorithm doesn't use air media, there are no relations between mesh specifications and results. All the cases and methods to be solved are given in Table 3. At the same time, the results obtained with different mesh qualities were compared with the experimental results, and error rates were given. Relative error rates were calculated as below;

$$\varepsilon = \frac{\text{Calculated Value} - \text{Measured Value}}{\text{Measured Value}} \times 100 \quad (10)$$

Roache calculated the convergence rate based on three results with a certain refinement rate between mesh sizes as follows (Roache, 1998).

$$p = \log \left[\frac{F_1 - F_3}{F_1 - F_2} \right] / \log(r) \quad (11)$$

Here F_i is the result of interest in different simulations and r is the mesh refinement ratio. When the results of the three different methods are compared, there is an asymptotic convergence. The following equation is used to obtain the convergence point (Roache, 1998).

$$F_0 = F_1 + (F_1 - F_2)/(r^p - 1) \quad (12)$$

Here, F_0 indicates the point at which the analysis will converge. The mesh convergence analysis obtained for the three methods is given in Table 4. Mesh convergence analysis was performed only for the P1 point results.

Table 3. Comparative results of ALE coupled MM-ALE with LBE methods with a different number of elements.

Method	Mesh Number	Number of Elements / Particles	Approximate Mesh Sizes	CPU time	Calculated Average Peak Pressure at P1	ε -Relative Errors for P1 (%)
ALE	1	158256	2.5 mm to 250 mm (bias)	21467 seconds	~13.4 MPa	43%
ALE	2	106651	3.7 mm to 250 mm (bias)	19245 seconds	~14.5 MPa	54%
ALE	3	92340	4 mm to 250 mm (bias)	18831 seconds	~14.9 MPa	59%
ALE	4	16301	5,5 mm to 250 mm (bias)	12305 seconds	~17.2 MPa	83%
COUPLED MM-ALE	1	4098304	12.5 mm	19112 seconds	~8.7 MPa	-7%
COUPLED MM-ALE	2	3235912	16 mm	13456 seconds	~8.1 MPa	-14%
COUPLED MM-ALE	3	2983506	20 mm	11304 seconds	~6.9 MPa	-27%
COUPLED MM-ALE	4	514304	25 mm	8306 seconds	~5.1 MPa	-46%
SPH	1	1114121	12.5 mm (Target Plate)	82379 seconds	~19.1 MPa	103%
SPH	2	137376	12.5 mm (Target Plate)	34521 seconds	~22.5 MPa	139%
SPH	3	8176	12.5 mm (Target Plate)	7884 seconds	~29.4 MPa	213%
SPH	4	2176	12.5 mm (Target Plate)	2905 seconds	~35.2 MPa	274%

Table 4. Mesh convergence study for three methods

Method	Refinement Ratio	Converged Number of Elements / Particles	Converged Mesh Sizes	Converged Peak Pressure at P1	Convergence Errors for P1 (%)
ALE	1.5	-	1.6 mm	~12.95 MPa	7.64%
COUPLED MM-ALE	1.25	-	10.5 mm	~9.1 MPa	4.39%
SPH	8	379820	-	~17.42 MPa	9.64%

Element sizes and numbers that can be used in the numerical modelling to be done in the next stage are given in Table 3. Since the convergence error rates are less than 10% and the increase in the number of elements will increase the CPU time, the experimental comparison has been made with the simulations with the smallest element sizes.

3. RESULTS AND DISCUSSION

The results of four different methods with finer mesh sizes and the empirically calculated results with the help of the UFC-3-340-02 guideline are compared with the results of Boyd's (Boyd, 2000) experimental study in Table 5.

Although the decrease in mesh size in the ALE and Coupled MM-ALE with LBE methods increases the calculation time, it is seen that the calculated pressures converge to the experimental results. The plate was used as low-carbon steel (AS3678-250) in the test system. The plate is exposed to vibration as a result of the deflection of the plate with the blast pressure. Therefore, in the data read by a pressure transducer fixed on the part, the pressure is in a fluctuating form as a result of the acceleration acting on the part. For this reason, cases where the part is considered rigid give more clean results in terms of peak pressures acting on the relevant points. Relevant diagrams for cases where the plate is considered rigid are given in Figure 6 and 8. In cases where the plate is considered

elastic, the pressure acting on the part causes accelerations and deflections and are given in Figure 7. This causes fluctuations in the values of the pressure sensor fixed on the plate. The peak pressure at P1 and P2 points and time graphs are given in Figure 7 for comparison with test data.

Table 5. Comparative results of CONWEP, ALE, Coupled MM-ALE and SPH methods and experimental study.

Method	Explosive Charge	Calculated Average Peak Pressure at P1	Error Rates According to Experimental Results at P1	Calculated Average Peak Pressure at P2	Error Rates According to Experimental Results at P2
E14 Experiment	250 gr Pentolite	9.4 MPa	-	8.7 MPa	-
UFC-3-340-02 (UFC 3-340-02, 2008)	280 gr TNT	6.4 MPa	31.91%	5.8 MPa	33.34%
CONWEP	280 gr TNT	~9.6 MPa	2.12%	~7.8 MPa	10.34%
ALE	250 gr Pentolite	~13.4 MPa	42.55%	~9.1 MPa	4.59%
COUPLED MM-ALE	250 gr Pentolite	~8.7 MPa	7.44%	~8.5 MPa	2.29%
SPH	250 gr Pentolite	~19.1 MPa	103.19%	~16.3 MPa	87.35%

When the results were examined, the CONWEP method gave the most approximate results considering the calculation time and the calculated pressures. However, as the CONWEP method uses TNT data, there is a time shift with respect to the test data as can be seen in Figure 6a and Figure 6b. This is because the TNT detonation velocity is higher than the pentolite detonation velocity.

With the decrease in element size, the peak pressure values in the ALE and SPH methods decrease and converge to the required value, while there is an inverse relationship in the hybrid method. The reason for this is that the pressure wave that occurs as a result of the explosion in the hybrid method is modelled as a rapid volume expansion in the fluid media. The pressure increase as a result of this expansion is carried over the Euler elements by the advection method.

When all the results are considered, it is observed that the hybrid method gives the most reliable results in free air blast loading due to the solution time and more overlap with the experimental results. In the ALE method, considering the mesh convergence analysis, it is seen that the peak pressure at the P1 point will converge at 12.95 MPa. According to this result, an error of 37.7% is calculated. Element size needs to be reduced for both air elements and explosive elements for more sensitive solutions. In the solutions made by the SPH method, it is observed that the solution will converge at 17.42 MPa, thus creating an 85% error rate. Although the pressures calculated at both P1 and P2 points in the SPH method are considerably higher than the test values, the accuracy increases with the increase in the number of particles. On the other hand, the time of the pressure wave reaching the relevant points coincides with the test data.

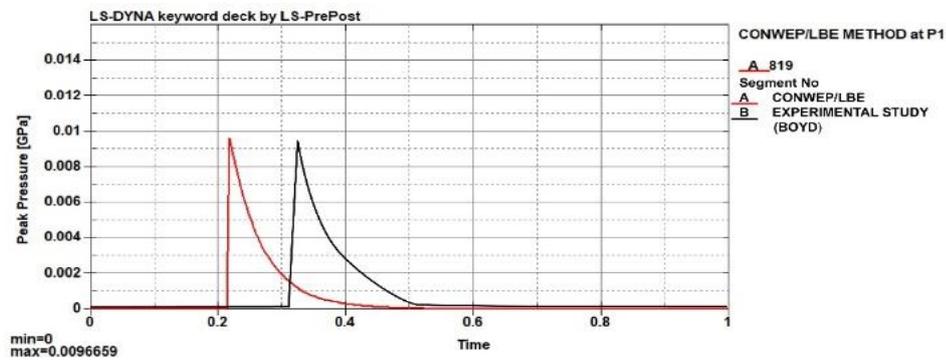
It is observed that the slope of the pressure from the beginning to the peak is less than the test data in ALE and SPH methods. This is due to the dimensions of the air elements at the point of contact of the pressure with the plate. To shorten the total solution time, the element dimensions are increased with the bias function starting from the center of the explosive. The size of the air elements at the point of contact is relatively larger.

According to the results, the hybrid method has advantages over other methods in terms of both solution time and convergence in completely spherical free air burst loading cases. However, in complex geometries, the reflected pressure has a significant effect on the pressure acting on the target. The effect of reflected pressure cannot be modelled in the hybrid method. For this reason, as can be seen in Table 3, the peak pressure decreases as the element size decreases. In ALE and SPH methods, there is an inverse relationship between element size and peak pressure. Compared to other methods,

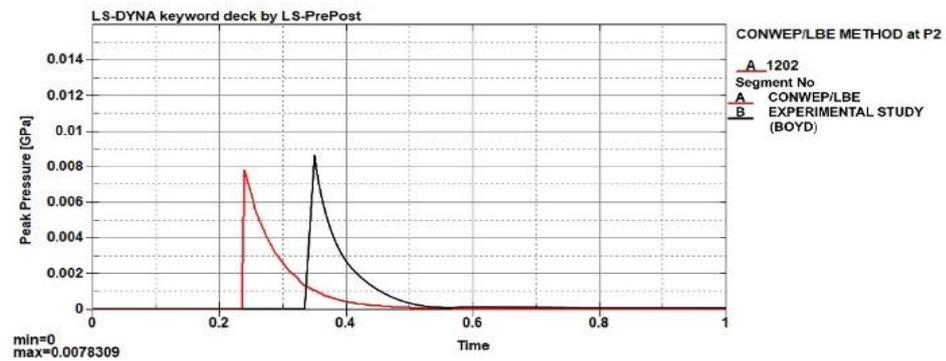
more fluctuations are observed in the SPH method. The reason for this is that the air domain is not modelled. The appropriate calculation method selection criteria for different scenarios are given in Table 6 below. Accordingly, criteria such as explosive geometry, fluid environment, solution time, explosive type, and modelling of shock wave parameters are the most important parameters for numerical design. In pressure transducers placed on the target plate, the vibration of the target causes fluctuations in the pressure values.

Table 6. Summary of selection criteria of CONWEP, ALE, Coupled MM-ALE and SPH methods and experimental study.

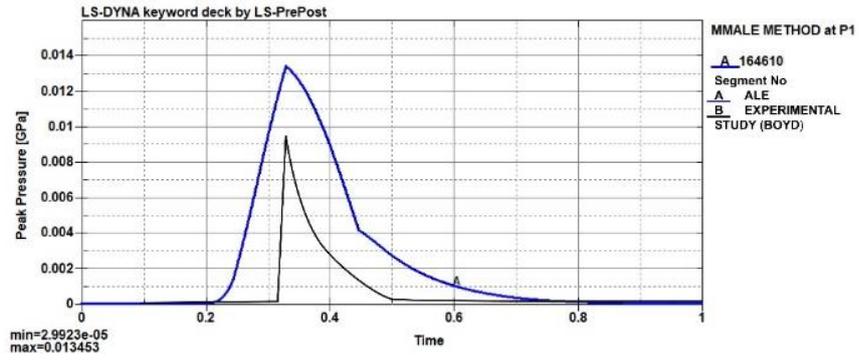
Method	Explosive Geometry	Type of Explosive	CPU Time	Modelling of Blast Shock Wave	Modelling of Blast Media
CONWEP	Spherical	TNT	Short	Not Possible	Not Possible
ALE	Any	Any	Extremely Long	Possible	Possible
COUPLED MM-ALE	Spherical	Any	Moderate	Possible	Possible
SPH	Any	Any	Moderate	Not Possible	Not Possible



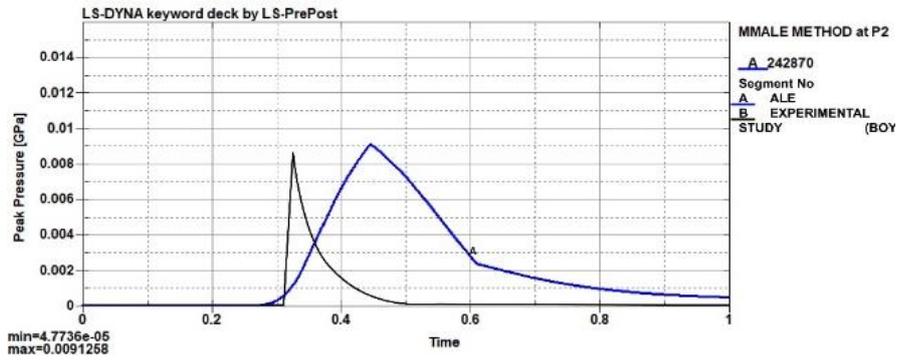
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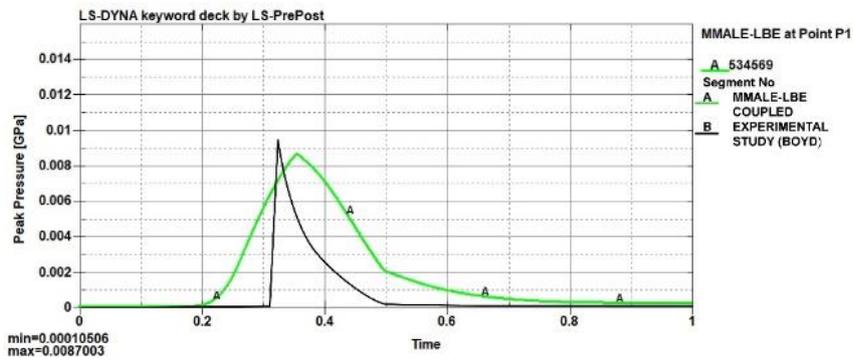
(b)



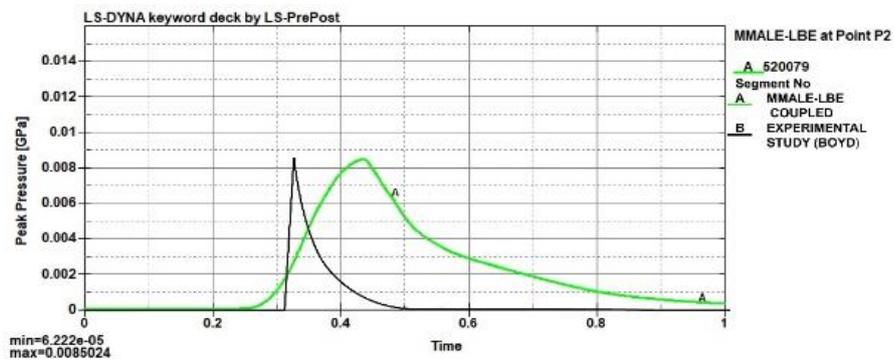
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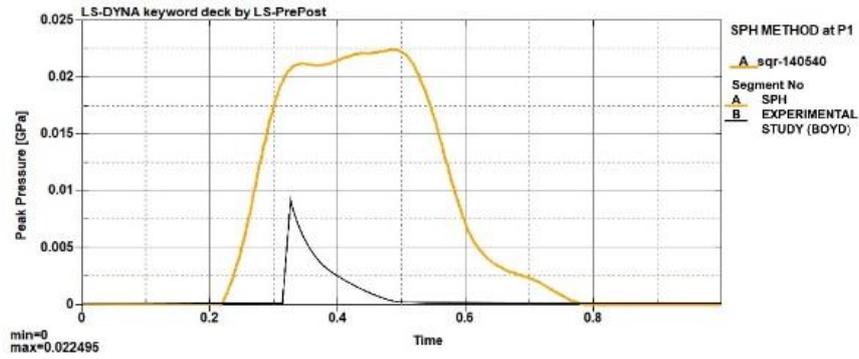
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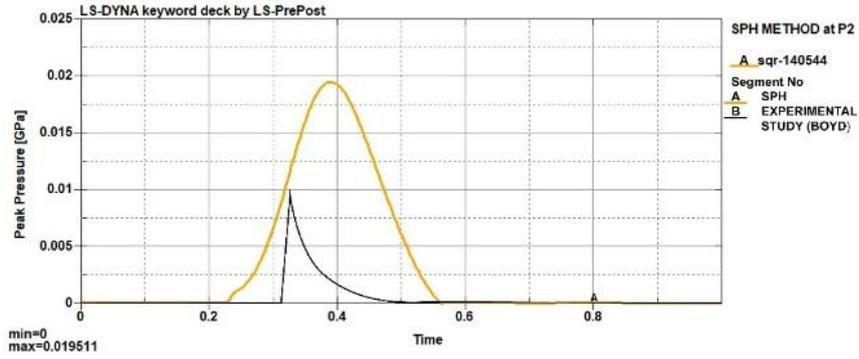
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(f)

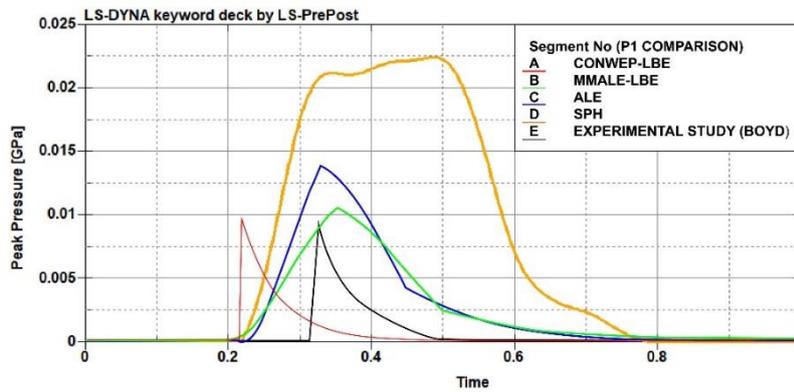


(g)

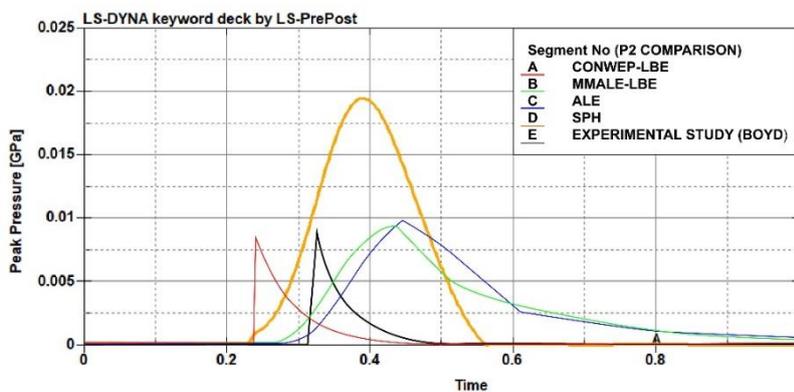


(h)

Figure 6. Peak pressure-time diagram at point P1 and P2 when the plate is considered rigid, obtained by the (a-b) CONWEP method (c-d) ALE method (e-f) coupled MM-ALE with LBE method (g-h) SPH method

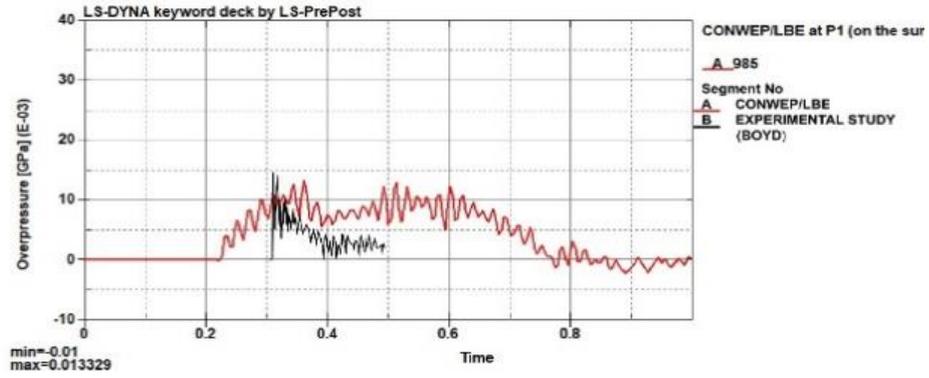


(a)

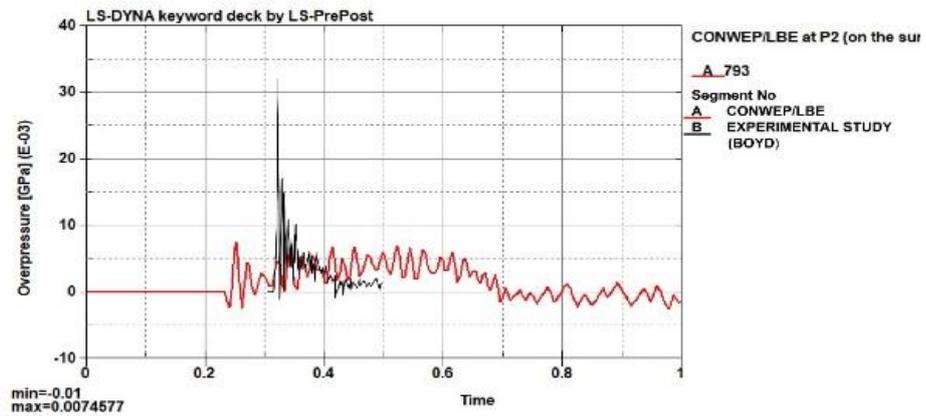


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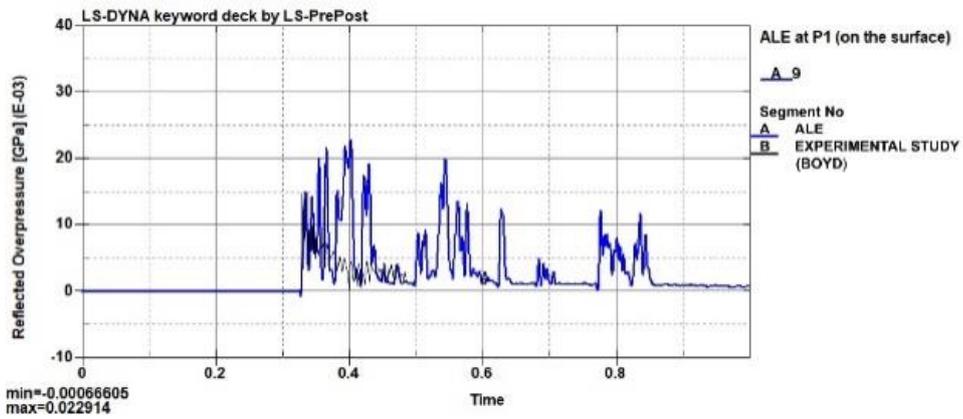
Figure 7. The peak pressure-time diagram at the (a) P1 and (b) P2 points obtained by different methods



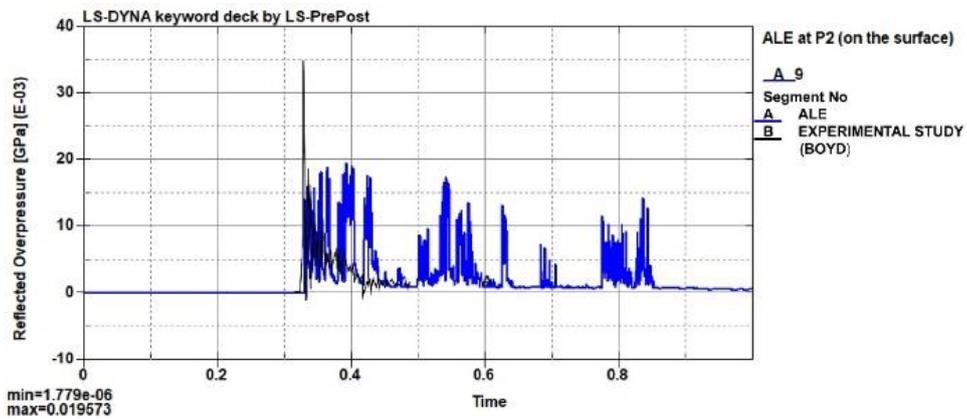
(a)



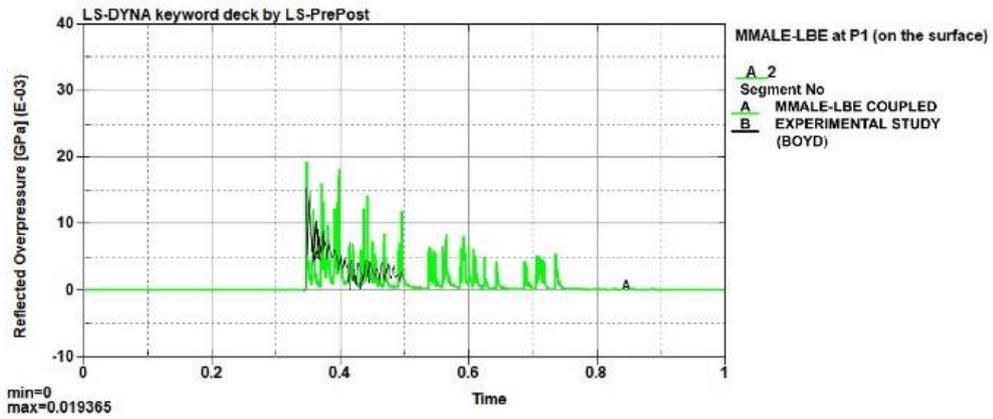
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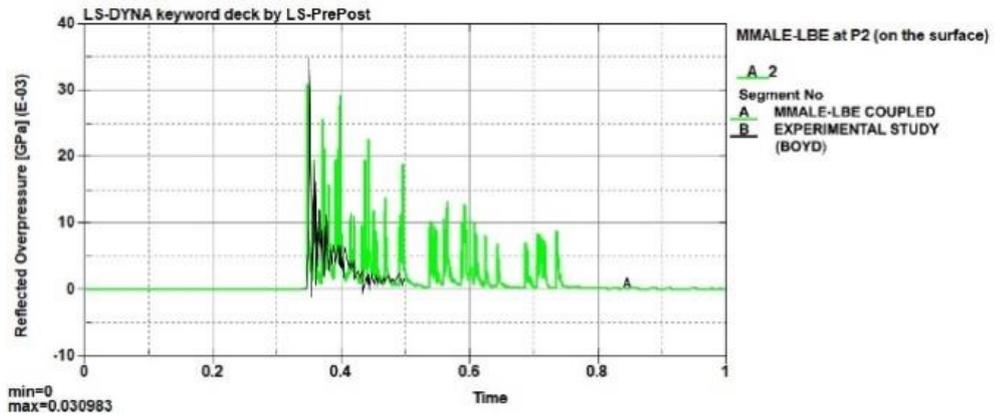
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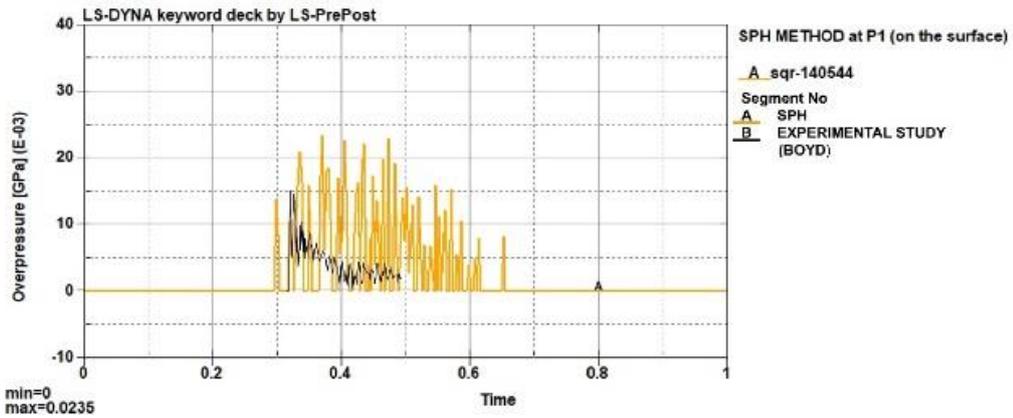
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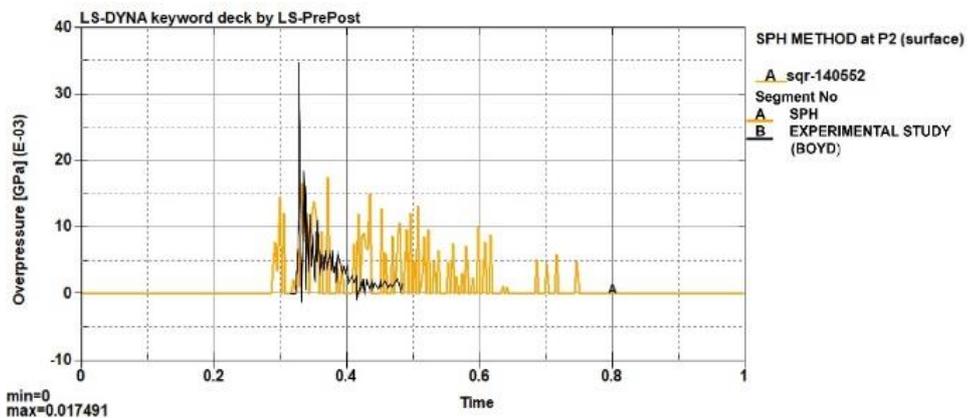
(e)



(f)



(g)



(h)

Figure 8. Peak pressure-time diagram at points P1 and P2, when the plate is considered elastic obtained by the (a-b) CONWEP method (c-d) ALE method (e-f) coupled MM-ALE with LBE method (g-h) SPH method

4. CONCLUSION

In this study, Boyd's (Boyd, 2000) experimental work on the target steel plate at a certain distance under a spherical explosion load was tried to be verified by using different numerical methods. As a result of different calculations, the following conclusions were reached.

- Explosive type, geometry and ambient conditions may differ in real situations subject to blast load. Therefore, it is not correct to call a certain calculation method more reliable. Each method can be evaluated for analysis in different situations.
- Although the ALE method and hybrid methods are using same approach, spherical propagation of the waveform is mostly related with the both explosive and air domains mesh structure. In ALE method it is obtained that the excessive error rate caused by the mesh structure. Despite using the same parameters, the error rate in the hybrid method is quite low.
- In the literature, experimental studies have been exposed to structures exposed to spherical or hemispherical explosion loads. More experiments are needed for explosives of different geometries and different types, especially to accurately determine the necessary parameters in the ALE and COUPLED MM-ALE with LBE methods.
- Numerical methods present promising results, especially for mid-range blast charges where the scaled distance is between 0.4 and 1.
- In nonlinear analysis like blast loading, the scale factor for the computed time step must be defined below 0,67. Reducing this value increases the solution time and can be solved with different values to control the convergence of the solution.
- As the element size gets smaller, numerical methods give more accurate results. As the scaled distance decreases, the element size must also decrease.
- Compared to other methods, the SPH method can be used in preliminary design processes in terms of solution time and results. However, convergence studies should be done regarding the particle number and SOFSCL, TSSFAC, and CSLH parameters.

5. CONFLICT OF INTEREST

Author(s) approve that to the best of their knowledge, there is not any conflict of interest or common interest with an institution/organization or a person that may affect the review process of the paper.

6. AUTHOR CONTRIBUTION

Formal analysis, Ibrahim Savrukoğlu; Investigation, İbrahim Savrukoğlu and Kubilay Aslantaş; Data curation, Ibrahim Savrukoğlu; Writing-original draft, İbrahim Savrukoğlu and Kubilay Aslantaş.

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