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Parameter Estimation for a Class of Fractional Stochastic SIRD Models with Random Perturbations

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Abstract

The classical SIRD model is extended to the conformable fractional stochastic SIRD model. The differences between the fractional stochastic SIRD model and the integer stochastic SIRD model are analyzed and compared using COVID-19 data from India. The results show that when the order of the fractional stochastic SIRD model is between $[0.93, 0.99]$, the root mean square error between the simulated value and the real value of the number of infections is smaller than that of the integer stochastic SIRD model. Then, the maximum likelihood estimation of the parameters of the conformable fractional stochastic SIRD model is carried out, and compared with the maximum likelihood estimation results of the parameters of the integer stochastic SIRD model, It can be seen that the root mean square error of the fractional stochastic SIRD model is smaller when the fractional order is between $[0.93, 0.99]$.

1. Introduction

In recent years, with the wide spread of COVID-19, the loss of individuals and society has gradually increased. And the study of mathematical models of infectious diseases has become an increasingly important topic, which can be divided into deterministic models and stochastic models [1]. Stochastic models have been studied mainly for models corresponding to transitions of individuals into different epidemic regimes over time.

The SIR model was first proposed by Kermack and Mckendrick in 1927 [2], which laid the foundation for the study of the dynamics of infectious diseases. Becker used the least square method and maximum likelihood method to estimate the parameters of the model and defined the initial infection rate [3]. Timmer discussed the parameter estimation problem of nonlinear stochastic differential equations based on the sampled time series [4]. Buckingham-Jeffery et al. considered the Gaussian process approximation based on the approximation of random moment closure and the approximation based on the approximation of linear time non-uniform SDE to infer the parameter characteristics of the random SEIR model [5]. Senel proposed a single-parameter estimation method to avoid potential problems such as limited and noisy data when using SIR Model to estimate COVID-19 [6]. Morato et al. formulated a nonlinear model predictive control scheme based on the SIRD model with time-varying parameters, and they proposed an identification method consisting of analytical regression, least squares optimization, and autoregressive model fitting, which can fully predict the infection curve in a large range [7].

In recent years, fractional infectious disease models have begun to attract a surge of research. Farman et al. applied the Laplace Adomian decomposition method to give the approximate solution of the nonlinear system of the Caputo SEIR model [8]. Rajagopal et al. compared the predictive ability of the fractional SEIRD model and the classical SEIRD model by using Italian COVID-19 data [9]. Basti et al. proposed an improved mathematical model for fractional SIRD in the sense

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of Caputo-Katugampola fractional derivatives. The existence and uniqueness of the solution of the improved SIRD model are studied by applying the properties of Schauder and Banach's fixed point theorems [10]. Mohammadi et al. proposed a SIRD model in the sense of Caputo fractional order, discussed the stability of the model and the existence and uniqueness of nonnegative solutions, and obtained approximate responses by implementing the fractional Euler method [11]. Fouladi et al. analyzed the identifiability and sensitivity of the integer and Caputo fractional SEIRD models and proved that the fractional infectious disease model was more suitable for predicting the real situation by comparing the quality of fitting [12].
In 2014, Khalil et al. proposed Conformable fractional derivatives, which is a natural expansion of integer derivatives [13]. Several scholars have applied conformable fractional derivatives to models in mathematics, physics, transportation, and other fields. In 2021, Akinyemi et al. established a time-varying nonlinear differential equation in the sense of conformable fractional order and obtained the exact solution of the equation by using the sub-equation method [14]. In 2022, Ashraf et al. studied the $(2+1)$ dimensional Conformable fractional transmission line equation in a nonlinear system. By combining exponential, polynomial, trigon, and hyperbolic functions, Multi-wave, M-shaped rational, and interaction solutions are obtained [15]. In 2022, Yuxiao Kang et al. established a conformable fractional time-varying gray Riccati traffic flow model based on viscoelastic fluid and applied it to modeling traffic flow and traffic congestion degree in multiple scenarios [16]. The relevant definitions and properties are introduced below.

Definition 1.1 ([13]). *Given the function $f(t):[0,\infty)\to R$, for all $t>0$, $\alpha\in(0,1]$, the α-order conformable fractional derivative of $f(t)$ is defined as*

$$T_\alpha f(t)=\lim_{\varepsilon\to 0}\frac{f\left(t+\varepsilon t^{1-\alpha}\right)-f(t)}{\varepsilon},$$

when $t=0$, $T_\alpha f(0)=\lim_{t\to 0^+}T_\alpha f(t)$.

Lemma 1.2 ([13]). *The relationship between the α-order conformable fractional derivative of $f(t)$ and the first derivative of $f(t)$ can be represented as*

$$T_\alpha f(t)=t^{1-\alpha}\frac{\mathrm{d}f(t)}{\mathrm{d}t},$$

specifically, $T_1 f(t)=\frac{\mathrm{d}f(t)}{\mathrm{d}t}$.

In this paper, we propose a stochastic SIRD model in the sense of conformable fractional order and obtain maximum likelihood estimates for the model parameters, taking into account the impact of various uncertainties on infectious diseases in real-world situations. Finally, the COVID-19 data from April 1, 2020, to July 31, 2020, in India is used for example analysis to compare the effect of fitting the raw data with a fractional stochastic SIRD model at different fractional orders. The parameter estimates for the fractional stochastic SIRD model are computed using maximum likelihood estimation. The results show that when the order of the fractional stochastic SIRD model is between $[0.93,0.99]$, the root mean square error between the simulated value and the real value of the number of infections is smaller than that of the integer stochastic SIRD model. Moreover, compared with the maximum likelihood estimation of the parameters of the integer stochastic SIRD model, it can be seen that when the fractional order is between $[0.93,0.99]$, the root mean square error of the fractional stochastic SIRD model is smaller.

2. Model Introduction

The SIRD model divides the tested population into Susceptible(S), Infected(I), Recovered(R), and Dead(D) populations. A susceptible person is a person who has not been infected, lacks immunity and is susceptible to infection after contact with an infected person; An infected population is already infected; Recovered populations are those that have been cured of their disease; Dead populations refers to a person who has died as a result of illness and is no longer involved in the process of infection and contagion. The classical SIRD model can be expressed as the following system of differential equations [17]

$$\begin{cases}\dfrac{\mathrm{d}S(t)}{\mathrm{d}t}=-\dfrac{\beta S(t)I(t)}{N},\\ \dfrac{\mathrm{d}I(t)}{\mathrm{d}t}=\dfrac{\beta S(t)I(t)}{N}-\gamma I(t)-\mu I(t),\\ \dfrac{\mathrm{d}R(t)}{\mathrm{d}t}=\gamma I(t),\\ \dfrac{\mathrm{d}D(t)}{\mathrm{d}t}=\mu I(t),\end{cases}\tag{2.1}$$

where $t\geq 0$, $S(0)\geq 0$, $I(0)\geq 0$, $R(0)\geq 0$, $D(0)\geq 0$. Let the total population of the area be N, satisfying $S(t)+I(t)+R(t)+D(t)=N$. β is the infection rate, and it represents the probability of a susceptible person being infected; γ is the recovery rate, and it indicates the likelihood of recovery of the infected person; μ is the mortality rate, all of which are positive numbers.
The classical SIRD model has a stable and interference-resistant performance, and considering the advantage that the fractional SIRD model can better fit the data by adjusting the order of the fractional derivatives in the model, in this paper we extend

the classical SIRD model to the conformable fractional SIRD model by using the definition and properties of conformable fractional derivatives.

$$\begin{cases} T_\alpha S(t) = t^{1-\alpha}\dfrac{\mathrm{d}S(t)}{\mathrm{d}t} = -\dfrac{\beta S(t)I(t)}{N}, \\ T_\alpha I(t) = t^{1-\alpha}\dfrac{\mathrm{d}I(t)}{\mathrm{d}t} = \dfrac{\beta S(t)I(t)}{N} - \gamma I(t) - \mu I(t), \\ T_\alpha R(t) = t^{1-\alpha}\dfrac{\mathrm{d}R(t)}{\mathrm{d}t} = \gamma I(t), \\ T_\alpha D(t) = t^{1-\alpha}\dfrac{\mathrm{d}D(t)}{\mathrm{d}t} = \mu I(t). \end{cases} \tag{2.2}$$

α is the order of the fractional order derivative of conformable, and $\alpha \in (0,1]$. Since the parameters β, γ, and μ of the real-life SIRD model are easily affected by the variable environment, and there is some randomness in the number of susceptibilities, infections, recoveries, and deaths at the next moment, we give a conformable fractional stochastic SIRD model.

Let $B_1(t),\ B_2(t),\ B_3(t),\ B_4(t)$ be a set of mutually independent standard Brownian motions, $\sigma_i\,(i=1,2,3,4)$ be a non-negative white noise intensity, and build the following Conformable fractional stochastic SIRD model

$$\begin{cases} \mathrm{d}S(t) = -t^{\alpha-1}\dfrac{\beta S(t)I(t)}{N}\mathrm{d}t + \sigma_1 S(t)\mathrm{d}B_1(t), \\ \mathrm{d}I(t) = t^{\alpha-1}\left(\dfrac{\beta S(t)I(t)}{N} - \gamma I(t) - \mu I(t)\right)\mathrm{d}t + \sigma_2 I(t)\mathrm{d}B_2(t), \\ \mathrm{d}R(t) = t^{\alpha-1}\gamma I(t)\mathrm{d}t + \sigma_3 R(t)\mathrm{d}B_3(t), \\ \mathrm{d}D(t) = t^{\alpha-1}\mu I(t)\mathrm{d}t + \sigma_4 D(t)\mathrm{d}B_4(t). \end{cases} \tag{2.3}$$

Parameter estimation is an extremely important ingredient in the study of infectious disease models. Considering that the maximum likelihood estimator is widely used and converges well, the following discussion deals with the maximum likelihood estimator for the parameters β, γ, and μ of the conformable fractional stochastic SIRD model.

3. Parameter Estimation

The discretization of equation (2.3) using Euler's method gives

$$\begin{cases} S(t+1) = S(t) - t^{\alpha-1}\dfrac{\beta S(t)I(t)}{N}\Delta t + \sigma_1 S(t)\Delta B_1(t), \\ I(t+1) = I(t) + t^{\alpha-1}\left(\dfrac{\beta S(t)I(t)}{N} - \gamma I(t) - \mu I(t)\right)\Delta t + \sigma_2 I(t)\Delta B_2(t), \\ R(t+1) = R(t) + t^{\alpha-1}\gamma I(t)\Delta t + \sigma_3 R(t)\Delta B_3(t), \\ D(t+1) = D(t) + t^{\alpha-1}\mu I(t)\Delta t + \sigma_4 D(t)\Delta B_4(t), \end{cases} \tag{3.1}$$

where $\Delta B_i(t) = B_i(t+\Delta t) - B_i(t)$, $i = 1,2,3,4$. Taking into account the reasonableness of the division of time, let $\Delta t = 1$. Since $B_1(t), B_2(t), B_3(t), B_4(t)$ are independent of each other. Based on the properties of the multidimensional normal distribution [18], we can get the probability density function of the fractional stochastic SIRD model as

$$f(t) = \frac{1}{A}\exp\left\{-\frac{1}{2}\left[\left(\frac{S(t+1)-S(t)+t^{\alpha-1}\frac{\beta S(t)I(t)}{N}}{\sigma_1 S(t)}\right)^2 + \left(\frac{I(t+1)-I(t)-t^{\alpha-1}(\frac{\beta S(t)I(t)}{N}-\gamma I(t)-\mu I(t))}{\sigma_2 I(t)}\right)^2 + \left(\frac{R(t+1)-R(t)-t^{\alpha-1}\gamma I(t)}{\sigma_3 R(t)}\right)^2 + \left(\frac{D(t+1)-D(t)-t^{\alpha-1}\mu I(t)}{\sigma_4 D(t)}\right)^2\right]\right\} \tag{3.2}$$

where $A = 4\pi^2\sigma_1 S(t)\sigma_2 I(t)\sigma_3 R(t)\sigma_4 D(t)$. The joint probability density function, the maximum likelihood function $L(\theta)$, is computed from the above equations as follows.

$$L(\theta) = \prod_{t=1}^{n-1}\left\{\frac{1}{A}\exp\left[-\frac{1}{2}\left(\left(\frac{S(t+1)-S(t)+t^{\alpha-1}\frac{\beta S(t)I(t)}{N}}{\sigma_1 S(t)}\right)^2 + \left(\frac{D(t+1)-D(t)-t^{\alpha-1}\mu I(t)}{\sigma_4 D(t)}\right)^2 + \left(\frac{I(t+1)-I(t)-t^{\alpha-1}\left(\frac{\beta S(t)I(t)}{N}-\gamma I(t)-\mu I(t)\right)}{\sigma_2 I(t)}\right)^2 + \left(\frac{R(t+1)-R(t)-t^{\alpha-1}\gamma I(t)}{\sigma_3 R(t)}\right)^2\right)\right]\right\}, \tag{3.3}$$

Taking the logarithm of $L(\theta)$, this gives the log-likelihood function of the fractional stochastic SIRD model as

$$\ln L(\theta) = -\sum_{t=1}^{n-1}\left\{\ln A - \frac{1}{2}\left[\begin{array}{l}\left(\frac{S(t+1)-S(t)+t^{\alpha-1}\frac{\beta S(t)I(t)}{N}}{\sigma_1 S(t)}\right)^2 + \left(\frac{I(t+1)-I(t)-t^{\alpha-1}\left(\frac{\beta S(t)I(t)}{N}-\gamma I(t)-\mu I(t)\right)}{\sigma_2 I(t)}\right)^2 \\ +\left(\frac{R(t+1)-R(t)-t^{\alpha-1}\gamma I(t)}{\sigma_3 R(t)}\right)^2 + \left(\frac{D(t+1)-D(t)-t^{\alpha-1}\mu I(t)}{\sigma_4 D(t)}\right)^2\end{array}\right]\right\}. \tag{3.4}$$

The maximum likelihood estimates of the parameters β, γ and μ in the fractional stochastic SIRD model are thus obtained as

$$\begin{pmatrix}\hat{\beta}\\ \hat{\gamma}\\ \hat{\mu}\end{pmatrix} = \begin{pmatrix}\sigma_2^2C_2+\sigma_1^2C_1 & -\sigma_1^2NC_3 & -\sigma_1^2NC_3\\ -\sigma_3^2C_3 & \sigma_3^2NC_4+\sigma_2^2NC_5 & \sigma_3^2NC_4\\ -\sigma_4^2C_3 & \sigma_4^2NC_4 & \sigma_4^2NC_4+\sigma_2^2NC_6\end{pmatrix}^{-1}\begin{pmatrix}\sigma_1^2NA_2-\sigma_2^2NA_1\\ \sigma_2^2NA_4-\sigma_3^2NA_3\\ \sigma_2^2NA_5-\sigma_4^2NA_3\end{pmatrix}, \tag{3.5}$$

where

$$A_1=\sum_{t=1}^{n-1}\frac{t^{\alpha-1}(S(t+1)-S(t))I(t)}{S(t)},\quad A_2=\sum_{t=1}^{n-1}\frac{t^{\alpha-1}(I(t+1)-I(t))S(t)}{I(t)},\quad A_3=\sum_{t=1}^{n-1}\frac{t^{\alpha-1}(I(t+1)-I(t))}{I(t)},$$
$$A_4=\sum_{t=1}^{n-1}\frac{t^{\alpha-1}(R(t+1)-R(t))I(t)}{R^2(t)},\quad A_5=\sum_{t=1}^{n-1}\frac{t^{\alpha-1}(D(t+1)-D(t))I(t)}{D^2(t)},$$
$$C_1=\sum_{t=1}^{n-1}t^{2\alpha-2}S^2(t),\quad C_2=\sum_{t=1}^{n-1}t^{2\alpha-2}I^2(t),\quad C_3=\sum_{t=1}^{n-1}t^{2\alpha-2}S(t),\quad C_4=\sum_{t=1}^{n-1}t^{2\alpha-2},\quad C_5=\sum_{t=1}^{n-1}\frac{t^{2\alpha-2}I^2(t)}{R^2(t)},\quad C_6=\sum_{t=1}^{n-1}\frac{t^{2\alpha-2}I^2(t)}{D^2(t)}.$$

Maximum likelihood estimates of infection rate β, recovery rate γ, and mortality rate μ can are obtained by calculating equation (3.5) using MATLAB software. The fractional stochastic SIRD model can have the lowest possible estimation error by finding a suitable value between $(0,1]$.

4. Example Analysis

Data for the COVID-19 epidemic in India from April 1, 2020, to July 31, 2020, are summarized from Worldometers and the WHO website, and the raw data for the number of infections are plotted in MATLAB as shown in Figure 4.1.

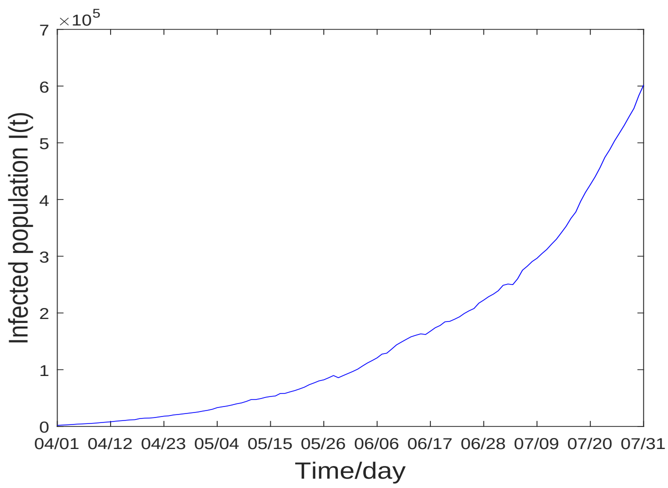

Figure 4.1: Graph of raw data on the number of infections

According to the parameter values of the SIRD model in reference [19], the initial parameters $\beta = 0.0559$, $\gamma = 7.3594\times10^{-4}$ and $\mu = 1.3030\times10^{-5}$ of the fractional SIRD model are obtained by using the lsqcurvefit function in MATLAB software. Using the second-order Adams-Bashforth method [20], numerical solutions of the number of infected people when α is $1, 0.99$, and 0.98 are obtained, as shown in Figure 4.2. The three curves in Figure 4.2 are in good agreement with the true data curves in Figure 4.1, but there is a gap between the simulated and true values for the number of infected people at the intermediate epoch. The fractional SIRD model can make the simulated values of the number of infections as close as possible to the true values by a reasonable choice of the fractional order. The fractional order was continuously adjusted to observe the effect of fitting the fractional SIRD model to the raw data, and the fit was found to be better when α was equal to 0.99. Consider next the stochastic SIRD model. Here $\alpha = 1$, the fractional SIRD model, is the integer SIRD model.

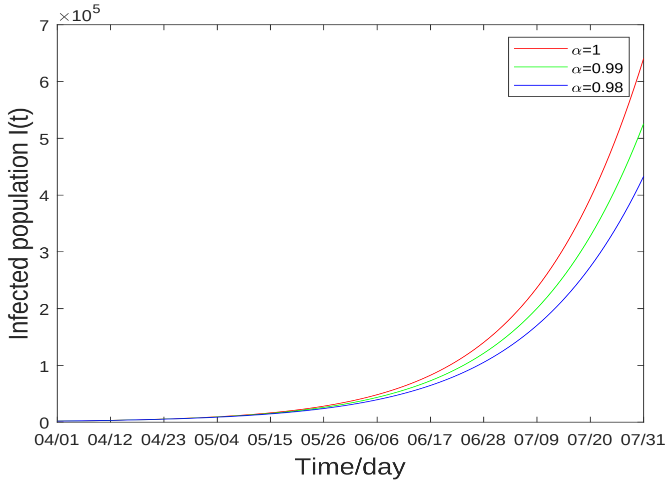

Figure 4.2: Simulation of the number of infected people in the fractional SIRD model

A series of simulation values for the number of infected individuals can be obtained by adjusting the values of the fractional order α, and the noise strength σ_2, of the stochastic SIRD model of the infected population. It can be seen from Table 4.1 that under different noise intensities, when $\alpha = 0.98$, the root mean square error reaches the minimum. And α is between $[0.93, 0.99]$, the root mean square error of the fractional stochastic SIRD model is smaller than that of the integer stochastic SIRD model. The smaller the value of the root mean square error, the better the fit, so the fractional stochastic SIRD model has a better fit than the integer stochastic SIRD model when $\alpha \in [0.93, 0.99]$, and the best result is when $\alpha = 0.98$, respectively.

	$\sigma_2 = 0.0001$	$\sigma_2 = 0.0002$	$\sigma_2 = 0.0003$	$\sigma_2 = 0.0004$
$\alpha = 1$	1.36×10^5	1.38×10^5	1.39×10^5	1.40×10^5
$\alpha = 0.99$	8.04×10^4	8.10×10^4	8.16×10^4	8.19×10^4
$\alpha = 0.98$	5.25×10^4	5.28×10^4	5.28×10^4	5.30×10^4
$\alpha = 0.97$	5.91×10^4	5.88×10^4	5.85×10^4	5.81×10^4
$\alpha = 0.93$	1.34×10^5	1.33×10^5	1.33×10^5	1.33×10^5
$\alpha = 0.92$	1.46×10^5	1.46×10^5	1.46×10^5	1.45×10^5

Table 4.1: RMSE of the number of infected people in the stochastic SIRD model

The maximum likelihood estimates of the parameters β, γ, and μ of the integer and fractional stochastic SIRD models are carried out respectively, the root mean square error between the simulation value and the true value of the number of infected people is calculated at the same time. The parameter results and root mean square errors are shown in Table 4.2. It can be seen from Table 4.2 that when $\alpha = 0.98$, the root mean square error reaches the minimum 5.2452×10^4, and when $\alpha \in [0.93, 0.99]$, the root mean square error of the fractional stochastic SIRD model is smaller than that of the integer stochastic SIRD model. Thus, when $\alpha \in [0.93, 0.99]$, the maximum likelihood estimator for the fractional stochastic SIRD model yields better results than that for the integer stochastic SIRD model.

	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\mu}$	RMSE
$\alpha = 1$	0.0559	7.3646×10^{-4}	1.3046×10^{-5}	1.3589×10^5
$\alpha = 0.99$	0.0559	7.3647×10^{-4}	1.3047×10^{-5}	7.9728×10^4
$\alpha = 0.98$	0.0559	7.3652×10^{-4}	1.3048×10^{-5}	5.2452×10^4
$\alpha = 0.97$	0.0559	7.3660×10^{-4}	1.3057×10^{-5}	5.9574×10^4
$\alpha = 0.93$	0.0559	7.3669×10^{-4}	1.3069×10^{-5}	1.3407×10^5
$\alpha = 0.92$	0.0559	7.3678×10^{-4}	1.3077×10^{-5}	1.4668×10^5

Table 4.2: Maximum likelihood estimation results and RMSE of stochastic SIRD model parameters

This section analyzes the goodness-of-fit of the fractional stochastic SIRD model and the integer stochastic SIRD model to the raw data and compares and analyzes the maximum likelihood estimation results for the parameters of the integer stochastic SIRD model and the fractional stochastic SIRD model. The results show that when the value of α is between $[0.97, 0.99]$, the root mean square error estimated by the fraction stochastic SIRD model is smaller than that of the integer stochastic SIRD model, indicating that the fraction stochastic SIRD model has better estimation effect than the integer stochastic SIRD model, which also indicates that the fraction stochastic SIRD model is more suitable for the actual situation.

5. Conclusion

In this paper, we propose a conformable fractional stochastic SIRD model and perform maximum likelihood estimation of the parameters in the model. Moreover, data from the COVID-19 outbreak in India is used for example analysis. By comparing the simulation graphs of the number of infected persons between the integer stochastic SIRD model and the fractional stochastic SIRD model, the results show that the fitting ability of the original data can be greatly improved by adjusting the order of the fractional stochastic SIRD model reasonably. Also, maximum likelihood estimation results for the parameters of the integer and fractional stochastic SIRD models are compared. It is found that the root mean square error of the fractional stochastic SIRD model is smaller than that of the integer stochastic SIRD model when the fractional order number is between $[0.97, 0.99]$, and when the fractional order is 0.98, the fractional stochastic SIRD model has the best parameter estimation effect. It has been shown that a reasonable choice of the fractional order has a certain effect on the correct understanding and estimation of the model parameters of infectious diseases, and a positive effect on the prediction and prevention of infectious diseases. In recent years, infectious diseases have been ravaging the globe. The study of infectious diseases has become a hot topic. With the deepening of infectious disease models, the estimation of model parameters is becoming more and more important. In future studies, parameter estimation of more complex fractional infectious disease models, such as the SEIRDV model, or models that take into account parameters such as vaccine coverage and level of government intervention, should be considered. It can better adapt to the more complex situation in the present era and optimize parameter estimation methods to reduce estimation errors, leading to more accurate predictions of infectious disease trends and reasonable measures to control the epidemic situation promptly.

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