

Research Article

# Mathematical Modeling of the Response of a Buck Converter to Disturbances

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**Abstract:** This study discusses the issues of mathematical modeling of the response of a step-down voltage converter to disturbing influences, which provides a two-stage transfer of a dynamic system from an arbitrary state to a state determined by the required output voltage value. Using the method of analytical construction of aggregated controllers (ACAC) with integral adaptation, a regulator of a buck converter in DC mode (CCM) and a constant frequency of the power switch is synthesized.

**Keywords:** Buck converter, Continuous current mode (CCM), Control system, Integral adaptation method, Pulse Width Modulation

## Düşürücü Voltaj Dönüştürücüsünün Bozucu Etkilere Tepkisinin Matematiksel Modellenmesi

**Öz.** Bu çalışmada, dinamik bir sistemin keyfi bir durumdan, ihtiyaç duyulan çıkış voltajı değeri tarafından belirlenen bir duruma iki aşamalı bir geçişini sağlayan, düşürücü bir voltaj dönüştürücüsünün bozucu etkilere tepkisinin matematiksel modellenmesi analiz edilmektedir. Entegre uyarlamalı birleştirilmiş denetleyicilerin analitik yapı yöntemi (ACAC) kullanılarak, DC sürekli akım modunda ve güç anahtarının sabit frekans değeri için bir düşürücü voltaj dönüştürücüsü sentezlenmiştir.

**Anahtar kelimeler:** Buck dönüştürücü, Darbe Genişlik Modülasyonu, Integral adaptasyon yöntemi, Kontrol sistemi, Sürekli akım modu

### 1.Introduction

Switching power supplies are widely used in electronics. By topology, voltage converters are classified as step-down (Buck), step-up (Boost) and inverting (Buck-Boost). Simulation of switching converters reduces the time and cost of developing the final device [1-6].

Today, many leading manufacturers of PWM controllers offer free software (software) that allows for the most accurate analysis of the reactions of an electrical circuit under the control of their products. The main disadvantage of the proposed software is the limited ability to analyze the pulse converter as a nonlinear dynamic object [7,8].

Mathematical analysis makes it possible to describe a pulse-width converter by various differential equations. Due to the

periodic switching of power switches (transistors, diodes), pulse converters are modern circuits [9,10]. Obviously, the state changes when the power switch is switched. For this reason, the mathematical analysis of such circuits is reduced to solving systems of differential equations at different time intervals and "stitching" the resulting solutions at the boundaries of the intervals according to the switching laws. The calculation is simplified when passing from a sequential solution of systems of equations to an averaged system of differential equations.

The use of a linear controller in nonlinear systems leads to a limitation of the stability region and the inability to suppress external uncertain disturbances. In order to expand the stability area, PID controllers with auto-tuning of coefficients are used. In various operating modes of the object, it can

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switch between PI, PD and PID controller. Extending the stability domain is entirely up to the developer. If the deviation of the system parameters from the calculated ones increases, or the system switches to an unforeseen mode of operation, the linear regulator will inevitably destabilize the system, since the developer did not foresee this. A breakthrough in these problems was the use of a fuzzy controller. But fuzzy logic is programmed by the developer and still does not know the structure of the controlled system. A distinctive characteristic of a nonlinear controller is the presence of all parameters of the controlled system. This allows him to bring the managed object to the specified parameters despite changes in the state of the system.

In this study, to further study the response of a buck converter to disturbances, we consider the construction of a basic mathematical model and simulation in the Matlab Simulink software package.

**2.The Basic Concepts of the Analytical Construction of Aggregate Controllers(ACAC) Method with Integral Adaptation**

The pulse converter consists of non-linear dynamic objects. By the type of topology and type of feedback, nonlinear dynamic objects have complex behavior, including instability, bifurcations, chaos, etc.

Therefore, the use of linear control laws is not capable of providing the required stability region for such nonlinear systems as a pulse-width converter. Mathematical analysis makes it possible to introduce disturbing influences and, by solving a new system of equations, obtain a nonlinear control law [11-14]. To this purpose, in this study analyzes mathematical modeling of the response of a buck converter to disturbing influences using the method of analytical construction of aggregated controllers (ACAC) with integral adaptation.

The method of analytical construction of aggregated controllers (ACAC) is based on the principle of "expansion-compression" of the phase space [11,12]. The main purpose of this controller is to move from the unpredictable behavior of the system according to the algorithm of the dissipative structure to directed motion along the required invariant manifolds - attractors. From the point of view of the theory of dynamical systems, this means the transfer of the corresponding system to a certain final state, regardless of its previous state.

Let us consider the synthesis of a buck converter controller in the continuous current mode (CCM) and a constant frequency of the power switch. The circuit diagram of the buck converter, shown in Figure 1, contains a capacitor C, an inductor L, a resistive load R, and two switches, represented respectively by a field effect transistor and a diode [15-17].

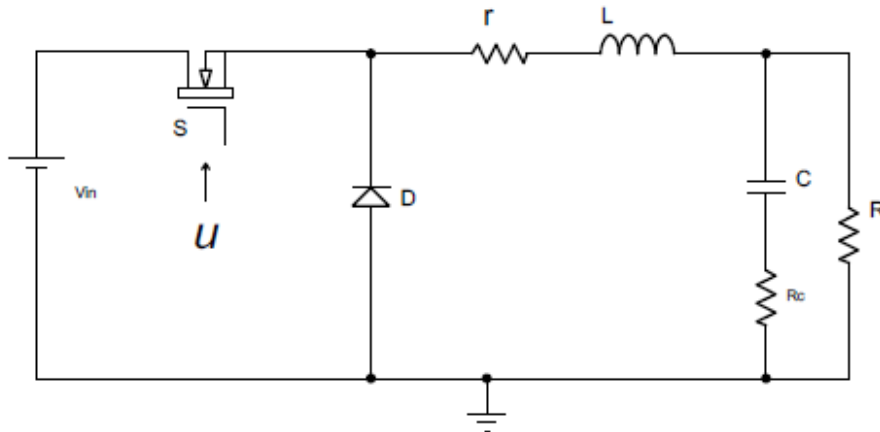


Fig. 1. Schematic diagram of the buck converter

Based on the electrical equivalent circuit of the buck converter, it is easy to notice that it is characterized by the following three working phases respectively: I) state Mosfet - on and state Diode – off; ii)state Mosfet -off and state Diode-on and iii) state Mosfet and Diode- off.

In continuous current mode, the buck converter is a switching device that has two operating states. These two states are taken into account using a logical variable  $f$  which can take two values 0 and 1.

$$\begin{cases} i_L'(t) = \frac{V_{in}(t) - (r + R_{on}) \cdot i_L(t) - V_{out}(t)}{L} \\ V_C'(t) = \frac{i_L(t) - i_{out}(t)}{C} \\ V_{out}(t) = V_C(t) + R_c(i_L(t) - i_{out}(t)) \\ V_L(t) = (V_{in}(t) - V_{out}(t)) \cdot f - V_{out}(t) \cdot \bar{f} \cdot sgn(i_L) \end{cases} \quad (1)$$

Where  $V_{in}(t)$  - is the input voltage;  $i_L(t)$ - is the inductance current;  $V_{out}(t)$  - output voltage;  $i_{out}$  - output current;  $r$  - is the resistance of the inductance winding;  $C$ - is the output capacitance;  $L$ - is the inductance,  $R$  is the load resistance and  $R_c$  is the equivalent series resistance (ESR) of the output capacitor  $C$ . As well as  $f$ - is a logical variable ;  $sgn(i_L) = 1$  if  $i_L > 0$  and  $sgn(i_L) = 0$  if  $i_L < 0$ .

In DC-DC converters, one of the most common methods for controlling the power transmitted to the load is the use of pulse-width modulation (PWM) [7, 8, 18].

Obviously, the system of equations (1) is designed for a specific circuitry. Synthesizing the control law for all circuit solutions is very difficult. The solution should be a single control action that allows you to change power keys, power key drivers. Let's simplify the mathematical model by

assuming that the frequency of the PWM signal is constant, the transistor is an ideal switch, and the equivalent series resistance of the output capacitor can also be neglected.

As a result of simplifying the system of differential equations compiled for an equivalent electrical circuit, elements remain

in the system, which, regardless of the circuit design adopted by the developer, will always be in a step-down converter [5,15]. These elements can be called the main ones, due to their influence on the dynamics and principle of operation of the converter. With these assumptions in mind, a simplified circuit diagram of the buck converter is shown in Figure2

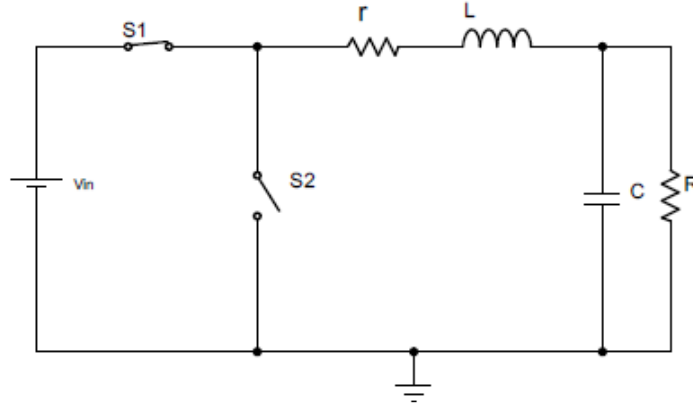


Fig. 2. Simplified diagram of a buck converter

From the point of view of synthesis by the ACAR method, this is not an adequate solution, since the more complete the model is written, the more accurate the control action. From a practical point of view, we will get a control action that allows you to change the electrical circuit for various tasks without re-synthesising the controller

Taking into account the control action parameter  $u$ , for the simplified electrical circuit of the buck converter, the averaged mathematical model can be written as

$$\begin{cases} \frac{di_L(t)}{dt} = \frac{1}{L}(V_{in}(t) \cdot u - r \cdot i_L(t) - V_{out}(t)) \\ \frac{dV_{out}(t)}{dt} = \frac{1}{C}(i_L(t) - i_{out}(t)) \end{cases} \quad (2)$$

Synthesis by the ACAC method is carried out by entering the desired variable, to the value of which our system is translated. In our case, the desired variable is the voltage we set at the output  $V_c(t)$ . The ACAC method should transfer our system from any state to  $V_c(t) = V_{out}(t)$  by means of calculating the control action. The input of the desired variable is carried out in various technique and methods. We will apply the integral adaptation method. Integral adaptation involves expanding the phase space by introducing additional integrators into the system of differential equations. These integrators actually form the feedback of the system, due to which the result of the suppression of external disturbing factors is achieved.

$$\begin{cases} \frac{di_L}{dt} = \frac{1}{L}(V_{in} \cdot u - i_L \cdot R - V_{out}) \\ \frac{dV_{out}}{dt} = \frac{1}{C}(i_L - i_{out}) - M \\ \frac{dM}{dt} = \mu(V_c - V_{out}) \end{cases} \quad (3)$$

The principle of operation of the integral adaptation method is to find the error value between the given and output voltage. By changing the values of the variable  $\mu$ , the dynamics of calculating the error value changes. Since the output voltage is determined in the second equation of system (3), it is necessary to subtract  $M(t)$  from the entire right side of the equation. This will increase the system mismatch until the error  $(V_c(t) - V_{out}(t))$  becomes zero.

We proceed to the synthesis of the control action, since we need to stabilize  $V_{out}(t)$  located in the second equation, we enter the first macro variable:

$$\Psi_1(t) = i_L(t) - \phi(V_{out}, M) \quad (4)$$

Where  $\phi(V_o, M)$  denotes internal control. Applying the main functional equation, we have

$$CR \cdot \Psi_1'(t) + \alpha_1 \Psi_1(t) = 0 \quad (5)$$

Next, substituting expression (4) into (5), we find the internal control

$$u = \frac{1}{V_{in}} \cdot (a_1 \cdot i_L + b_1 \cdot V_{out} + c_1) \quad (6)$$

In expression (6), the following notations are accepted

$$\begin{aligned} a_1 &= \left(\frac{L}{C} \cdot \phi'(V_{out}, M) - r\right); \\ b_1 &= \left[L - \left(\mu \cdot L + \frac{L}{C \cdot R}\right) \cdot \phi(V_{out}, M)\right]; \\ c_1 &= L \cdot [(\mu \cdot V_c - M) \cdot \phi'(V_{out}, M) + \phi(V_{out}, M) - \alpha_1] \end{aligned}$$

It should be noted that as a result of the system entering the neighborhood of the manifold  $\Psi_1(t) = 0$ , a dynamic decomposition occurs in the system and the behavior of the

system will be described by the following system of differential equations

$$\begin{cases} V'_{out}(t) = \frac{1}{C} \left( \phi(V_{out}, M) - \frac{V_o}{R} \right) - M \\ M'(t) = \mu \cdot (V_c - V_{out}) \end{cases} \quad (7)$$

We introduce the second macro variable for system (7)

$$\Psi_2(t) = V_{out}(t) + \beta \cdot M(t) \quad (8)$$

Applying the main functional equation, we have

$$CR \cdot \Psi_2'(t) + \alpha_2 \Psi_2(t) = 0 \quad (9)$$

Thus, from expression (6), taking into account expression (9), we obtain

$$u = \frac{1}{V_{in}} \cdot [(a_2 - b_2) \cdot V_{out} - a_2 V_c - c_2 i_L - d_2 \cdot M] \quad (10)$$

In expression (10), the following notations are accepted

$$a_2 = \mu \cdot [1 - \beta(\alpha_1 + \alpha_2)] \cdot LC;$$

$$b_2 = \frac{L(\alpha_1 + \alpha_2 + \beta\mu - \alpha_1\alpha_2RC)}{R} - \frac{L}{R^2C} + 1;$$

$$c_2 = r + L(\alpha_1 + \alpha_2 - \beta\mu);$$

$$d_2 = \mu L \cdot \left[ \frac{RC(\beta\mu + \alpha_1\alpha_2\beta - \alpha_1 - \alpha_2) + 1}{R} \right]$$

The synthesized control action ensures the transfer of the dynamic system (1) from an arbitrary state first to the neighborhood of the manifold  $\Psi_1(t) = 0$ , and then to the state determined by the required value of the output voltage  $V_c(t) = V_{out}(t)$ . The coefficients  $\alpha_1, \alpha_2, \beta$  and  $\mu$  are tuned in such a way as to obtain the desired transient. By changing the value of the coefficients, the time to reach the specified voltage  $V_c(t)$  is regulated.

### 3.Simulation Results

The physical model of a step-down converter with a control action obtained from an averaged mathematical model of an electrical circuit in the Matlab Simulink environment is shown in Fig.3. The output current and voltage waveforms obtained from the physical model are shown in fig. 4 and 5.

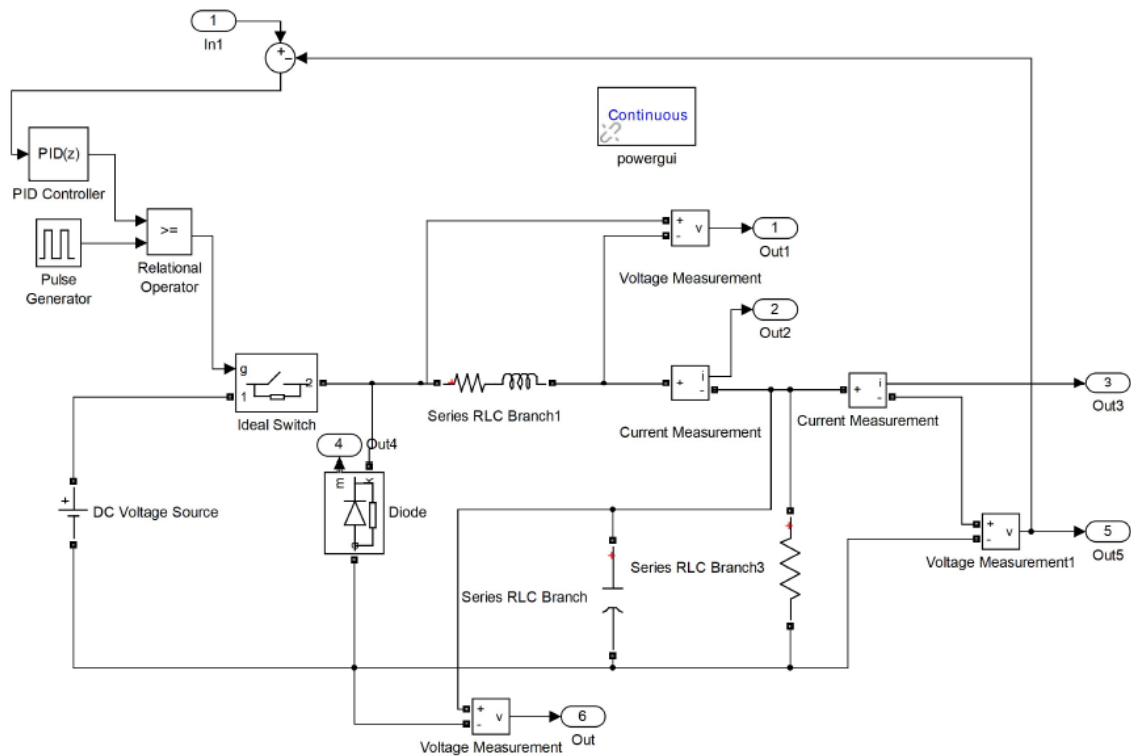


Fig. 3: Physical model of a buck converter

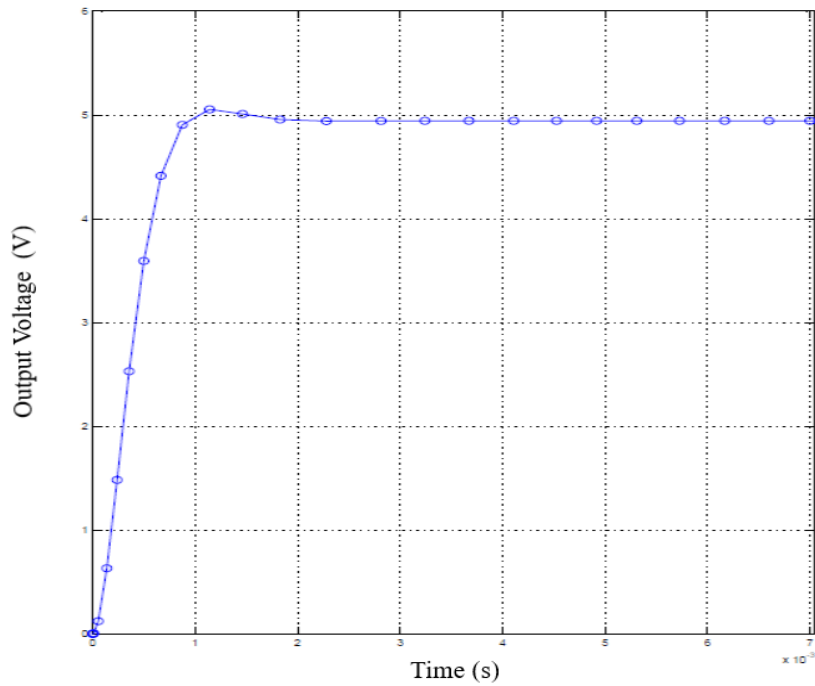


Fig. 4: Output voltage waveform obtained from the physical model

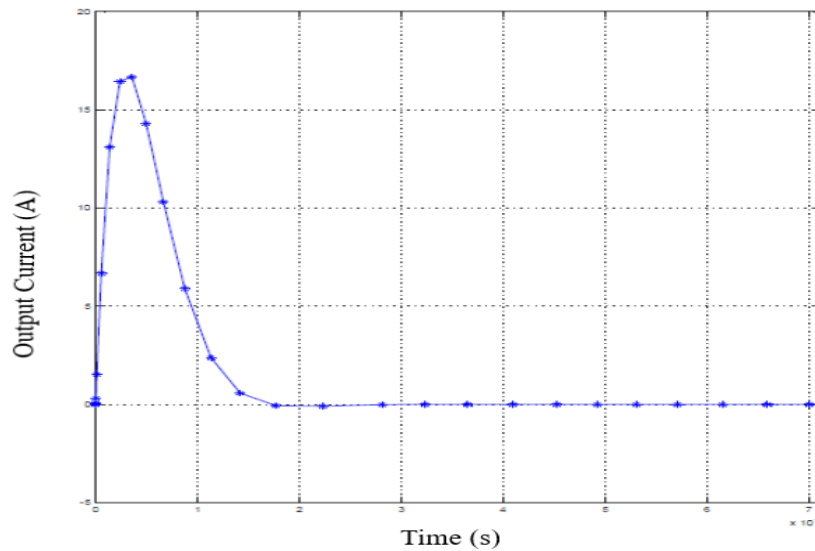


Fig. 5: Output current waveform obtained from the physical model

**4. Conclusion**

In the study “Mathematical modeling of the response of a step-down converter to disturbing influences”, we considered the issues of mathematical modeling of the reaction of a step-down voltage converter to disturbing actions, which provides a two-stage transfer of a dynamic system from an arbitrary state to a state determined by the required value of the output voltage.

The suitability of using the method of analytical synthesis of aggregated controllers with integrated adaptation for a simplified mathematical model of a buck converter in DC mode (CCM) and a constant frequency of the power switch is shown.

The results of the study can be relevant in control systems for poorly formalized dynamic objects and in decision support systems for managing nonlinear objects with chaotic modes.

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