



## EFFECTIVE SOLUTIONS FOR COMMON PROBLEMS OF ARTIFICIAL POTENTIAL FIELD BASED PATH PLANNING ALGORITHMS FOR MOBILE ROBOTS

### MOBİL ROBOTLAR İÇİN YAPAY POTANSİYEL ALAN TABANLI YOL PLANLAMA ALGORİTMALARININ ORTAK SORUNLARINA ETKİLİ ÇÖZÜMLER

<https://doi.org/10.20854/bujse.1214752>

Muhammet Emre Akarsu<sup>1,\*</sup>, Ömer Çetin<sup>2</sup>

#### Abstract

Autonomous Path Planning (APP) capability is one of the main factors determining the autonomous level of a mobile robot. Although different methods are used for APP in the literature, the path planning approach based on Artificial Potential Fields (APF) has a very common usage area with its modeling ease and computational performance. APF-based APP, which is a grid-based path planning approach, is usually performed by combining a repulsive and attractive component that models many basic motions with a certain equation and calculating the gradient of this potential field to obtain the vector field. In this study, the basic models used for APF-based APP are examined, and how they are realized and how the resultant potential field is produced are mentioned. Although APF-based APP approaches have advantages, they also have problems such as local minimum, obstacles positioned too close, oscillation, and targets positioned too close to obstacles. Within the scope of the study, these problems were defined one by one and the approaches suggested in the literature for the solution of these problems were mentioned in detail. As a result, it has been seen that to obtain an effective APF-based APP solution, it is necessary to generate a convolutional vector field, limit the fundamental potential fields with exponential functions, use virtual potential fields and perform models with harmonic functions.

#### Özet

Otonom Yol Planlaması (OYP) yeteneği, bir mobil robotun otonom seviyesini belirleyen başlıca faktörlerden birisidir. Literatürde her ne kadar farklı yöntemler otonom yol planlaması için kullanılıyor olsa dahi, Yapay Potansiyel Alanlara (YPA) dayalı yol planlaması yaklaşımı modelleme kolaylığı ve hesaplama performansı ile oldukça yaygın bir kullanım alanına sahiptir. Grid tabanlı bir yol planlaması yaklaşımı olan YPA tabanlı OYP, genellikle çok sayıda temel hareketi modelleyen itici ve çekici yönde bileşenin belirli bir denklem ile bir araya getirilmesi ve bu potansiyel alanın gradientinin hesaplanarak vektör alanın elde edilmesi ile gerçekleştirilir. Bu çalışma kapsamında, YPA tabanlı OYP amacıyla kullanılan temel modeller incelenmiş, nasıl gerçekleştirildiğine değinilmiş ve bileşke potansiyel alanın nasıl üretildiğinden bahsedilmiştir. Her ne kadar YPA tabanlı OYP yaklaşımlarının avantajları olsa dahi, yerel minimum, çok yakın konumlandırılmış engeller, osilasyon ve engellere çok yakın konumlandırılmış hedef gibi problemleri de vardır. Çalışma kapsamında bu problemlerin teker teker tanımlamaları yapılmış ve literatürde bu problemlerin çözümü için önerilen yaklaşımlara detaylı olarak değinilmiştir. Sonuç olarak etkin bir YPA tabanlı OYP çözümü elde etmek için kıvrımsız vektör alanı üretilmesi, temel potansiyel alanların üssel fonksiyonlar ile sınırlandırılması, sanal potansiyel alanların kullanılması ve harmonik fonksiyonlar ile modellemelerin gerçekleştirilmesi gerektiği görülmüştür.

**Keywords:** Aeronautical Engineering, National Defense, Space

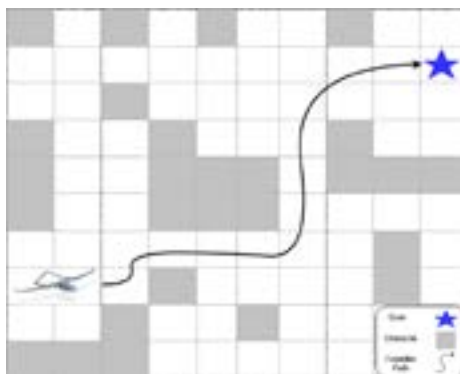
**Anahtar Kelimeler:** Otonom Sistemler, UAV, Yapay Potansiyel Alanlar, Yol Planlaması

<sup>1,\*</sup> Corresponding Author: National Defense University, Istanbul, Turkey, m.emreakarsu@gmail.com, <https://orcid.org/0000-0003-2051-6330>

<sup>2</sup> National Defense University, Izmir, Turkey, ocerin@msu.edu.tr, <https://orcid.org/0000-0001-5176-6338>

## 1. INTRODUCTION

Autonomous Path Planning (APP) ability is one of the major problems for autonomous mobile robots (Zhang et al., 2022; Rybus et al., 2022). Different kinds of robots can apply different levels of autonomy. APP capabilities are seen as an important component to determine the autonomous level of a mobile robot. APP can be defined as finding the optimal or feasible path for reaching to target point while avoiding static or dynamic obstacles in the robot environment. The basic concept of APP and obstacle avoidance strategy is as given in Figure 1.



**Figure 1:** Problem definition scheme of Autonomous Path Planning.

As seen the Figure 1, in the 2D space APP has two major components which are the target point to achieve and obstacles to avoid. However, the figure is 2D, it is also very similar to 3D space and even though the given obstacles in the figure are static, they can be dynamic mobile in the same manner. These options are not changing the definition of APP for a mobile robot, but it affects the complexity of the solution. While there are many different approaches in the literature to implement the APP on a mobile robot, the most common ones are listed below:

- Geometric Algorithms Approach
- Evolutionary Algorithms Approach
- Grid-Based/Heuristic Approach (Klančar et al., 2022)
- Mixed Integer Linear Programming Approach
- Artificial Potential Fields
- Vector Field Histogram

Navigation characteristics and dynamic environment changes are obtained extensions of the approaches and their improvement versions as seen as an example of Rapidly Exploring Random Tree Star (RRT\*) algorithm (Wang et al., 2022). Grid-based algorithms (Zammit & Kampen, 2022; Zhang & Wu et al., 2022) and their combinations with different APP approaches are common to find feasible path (Xiang et al., 2022). Especially in static environment, genetic algorithms (Lamini et al., 2018; Yuan et al., 2022) can be used to solve path planning problems like path optimization etc. Reinforcement learning method is also using to APP by mobile robots to find target point in both previously known and unknown environments (Xi et al., 2022; Wang et al., 2022; Zheng et al. 2022) Evolutionary algorithms and linear programming approaches are also other commonly using APP for autonomous systems (Chen et al., 2022; Yang et al., 2012). Evolutionary algorithms are one of the heuristic-

based approach. This approach has powerful characteristics of robustness for problem solving of complex optimization problem. Evolutionary algorithms have the ability for escaping from local minima. Creating the fitness function can be difficult. Also, it is important to find a good function that accurately represents the data. For this reason, using hybrid methods give better performance result.

Artificial Potential Field (APF) approach is one of the most common techniques in APP for mobile robots (Sabudin et al., 2021) because of collision and obstacle avoidance features are inherently a part of APP. With the collision and obstacle avoidance features, APP can be integrated in 2D environment easily for Unmanned Ground Vehicles (UGV) (Feng et al., 2021) and it is also suitable for the 3D environment, especially for Unmanned Aerial Vehicles (UAV) (Tang et al., 2019). Furthermore, these features revealed some advantages like fast response and low computational time, easy to implement in a real-time application (Szczepanski et al., 2021; Yan et al., 2021). In the APF-based path planning approach, different basic modeling approaches can be seen to model the target to be achieved and the static/dynamic obstacles to be avoided, and the production of straight, curved, spiral roads to reach the target. Combining multiple base models with a meaningful mathematical approach is enough to produce APP while avoiding obstacles and collisions.

Although APF-based path planning approaches are one of the widely used methods to obtain a high level of autonomy, especially with APP with obstacle/collision avoidance and easily adaptable for different kinds of environments, it is seen that many problems are encountered in the literature like the local minimum problem (Koren & Borenstein, 1991), the closely spaced obstacles problem (GE & Cui, 2000), and the oscillation problem (Duhé et al., 2021). The APP problems show similarities for wheeled, walking, and even flying robots, it should be considered as difficult to meet the need for 3D modeling within the framework of robots' mobility. UAV systems, the use of which has become widespread with increasing momentum in recent years, need the development of autonomous path planning approaches that can produce the optimum path that avoids static or moving obstacles and reaches the target, with the mobility capabilities of the six degrees of freedom (6DOF) in 3D space. In fact, within the path planning approach to be developed, it is necessary to find solutions that can prevent collision within a formation. By nature, 3D modeled YPA-based APP solutions are capable of meeting all these demands.

In this study, the definition of the basic models that are used for APF-based APP, the demonstration of how path planning is carried out by combining these basic APF models, the introduction of the different methods used in the literature to produce APF-Based APP, and the typical APF-based APP problems encountered during the application of common methods, in a comparative way. Also, it is aimed to explain which method overcomes these problems better. Although there are APF-based APP studies in the literature, the aim of this study is to contribute to the literature by presenting the complete solution of the basic APF problems to the reader.

The remainder of the article is organized as follows, in section 2, the APF-based path planning approach and its basic model representation is defined. Besides, potential field based major motion models for 2D and 3D environments that predict attractive, repulsive, and circular motion as examples are discussed in this section. It is mentioned how models realized using harmonic functions can offer a solution to APF-based APP probes. Section 3 describes commonly encountered problems of APF-based APP as types of defects like, the local minimum problem, the closely spaced of obstacles problem, the oscillation problem are

examined one by one. Chapter 4 explains which methods can be used to solve the traditional problems of artificial potential area-based path planning approaches. In the last part, the approaches required for the effective implementation of the YPA-based YPP problem will be evaluated together and the effectiveness of the methods will be revealed.

The motivation of this study is to explain to the reader how the artificial potential field-based path planning approach can be developed by examining the classical problems of artificial potential fields-based path planning approaches and the solution methods of these problems. In addition, it is to propose models and supports the production of algorithms in order to develop the most efficient APF-based autonomous path planning, which is abstracted from traditional problems.

## 2. ARTIFICIAL POTENTIAL FIELDS BASED PATH PLANNING

The APF was developed by Khatib (Khatib, 1985). During the evaluation of the basic algorithm, recently, the APF approach is commonly using for APP besides obstacle/collision avoidance for mobile robots (Li et al., 2010; Rezaee & Abdollahi, 2012)

The APF approach has two different forces and their potential fields which names are attractive force, repulsive force as defined in the basic definition of APP in Figure 1. Attractive potential field and repulsive potential fields represent the target point and obstacle in an orderly. Forces within the potential area are usually calculated based on the distance between the mobile robot position and the target/obstacle points. This distance can be converted to unit measures using grid-based spaces. The attractive force aims to move the mobile robot to the target. It is possible to define moving target points as well as dynamically define obstacles. This will cause the pulling forces to be recalculated only at certain intervals. The repulsive force is exerted by the obstacles to keep the mobile robot out of the obstacle position.

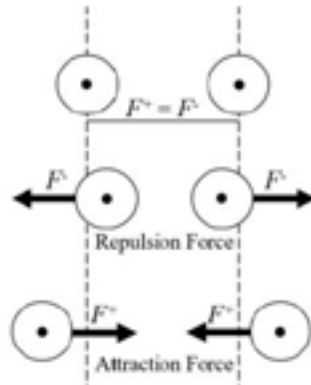


Figure 2: Basic structure of APF.

The Figure 2 shows these repulsive and attractive forces respectively. Attractive force represented by  $F^+$  and repulsive force represented by  $F^-$  in the Figure 2. The artificial potential fields generated by  $U$  and total potential field formula is represented as in Equation (1).

$$U(p) = \sum_i U_{a_i}(p_i) + \sum_j U_{r_j}(p_j) \quad \text{Equation (1)}$$

The total potential field is represented by  $U(p)$ , attractive potential field is represented by  $U_a(p)$  and repulsive potential field is represented by  $U_r(p)$ . The number of attractive forces

( $i$ ) and repulsive ( $j$ ) can be increased due to the number of points in the total field. The potential fields obtained from the above equation and the basic repulsive and attractive models of the APF approach can be calculated as the sum of each component one by one for each cell of the grid-based total area where the movement will be planned, and a total potential field that can be applied for the entire area can be obtained. If we consider the moving robot as a point-charged particle in this area, the gradient of the potential field value of the cell in which the particle is located is accepted as the vector force acting on the particle instantaneously, thus representing the robot's ability to reach the target point in the most optimal way by avoiding obstacles. The directing force that is affecting mobile robot movement must be calculated again when one of the actors like the robot, target and obstacle change its position. However, the problem can be considered a bit more complex than it is in this basic definition. In the remainder of this section, sample modelling approaches are described on how to model the obstacles in the field for the creation of attractive vector and repulsive vector fields.

### 2.1. Attractive Force

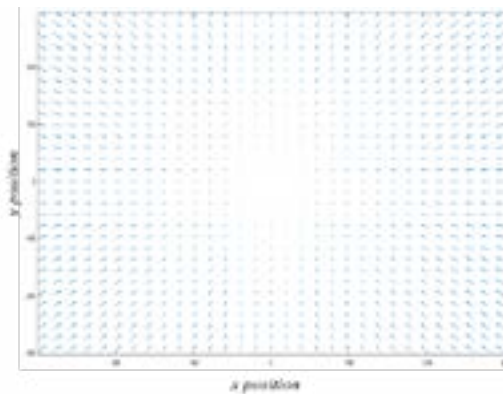
The attractive potential field force generated by the  $U_a$  target can be described as in Equation (2).

$$U_{a_i}(p) = \frac{1}{2}k_a d(p, p_g)^2 \quad \text{Equation (2)}$$

In the equation,  $p$  point represents robot's current position,  $p_g$  point represent goal position.  $d(p, p_g)^2$  is the calculation of the Euclidean distance of the robot and goal positions.  $k_a$  is a scaling factor. The attractive potential field is derived by computing the negative gradient of Equation (2) as seen from the Equation (3) as follows:

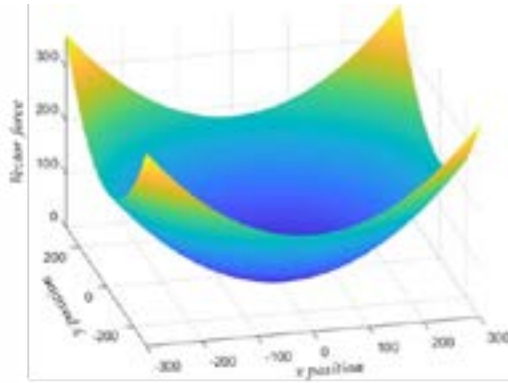
$$F_a = -\nabla U_a(p) \quad \text{Equation (3)}$$

The attractive vector force is generated by  $F_a$  in Equation (3) can be seen from the Figure 3 as an example.



**Figure 3:** Basic attractive vector forces of APF.

Vector forces of the attractive field can be seen from Figure 4 for the attractive point in the APF. The attractive force strength is decreasing when robot getting closer to target.



**Figure 4:** Basic attractive forces of APF.

## 2.2. Repulsive Force

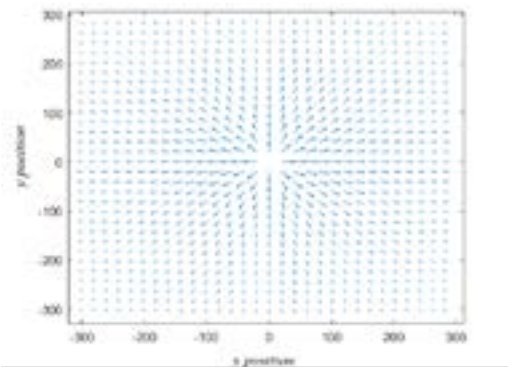
The repulsive force keeps the robot away from obstacles. The repulsive potential field and force is result of total obstacle effects. So, it can be represented as in Equation (4) and each repulsive potential field can be represented as in Equation (5).

$$U_{rj}(p) = \begin{cases} \frac{1}{2}k_{rj} \left( \frac{1}{d(p,p_{obs})} - \frac{1}{d_0} \right) & \text{if } d(p,p_{obs}) \leq d_0 \\ 0 & \text{if } d(p,p_{obs}) > d_0 \end{cases} \quad \text{Equation (4)}$$

The repulsive potential field  $U_r$  is generated from obstacle locations  $p_{obs}$ . in Equation (4).  $k_{rj}$  is the repulsive field constant,  $p$  shows robot's current position and  $d_0$  is the obstacle influence threshold. The gradient of the repulsive field is given by the Equation (5).

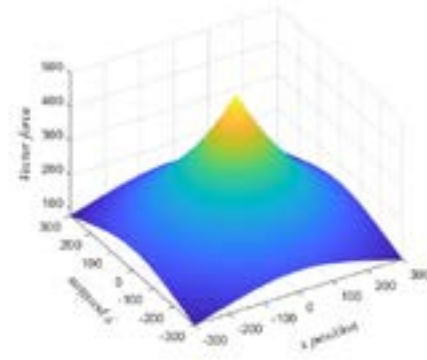
$$F_r = \nabla U_r(p) \quad \text{Equation (5)}$$

The repulsive force is represented by  $F_r$  in Equation (5) can be seen from the Figure 5 as an example.



**Figure 5:** Basic repulsive vector forces of APF.

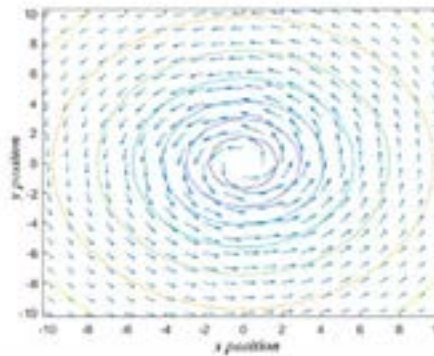
Vector forces of the repulsive field can be seen from Figure 6 for the attractive point in the APF. The repulsive force strength is increasing when robot getting closer to obstacle.



**Figure 6:** Basic repulsive forces of APF.

### 2.3. Circular Model

However repulsive and attractive models are basic structures for the APF-based APP, they are not enough to model possible attributes of the mobile robot. Because of that additional models must be defined, for example, circular, linear, etc. In this paper circular model is selected as an example model, due to the similarity of the modelling of others. The repulsive force can be represented with different geometric model like point repulsion and circular repulsion model. For circular repulsion model, every vector field can turn clockwise (CW) or counter-clockwise (CCW). With these model, different environment situation can be modelled in APF for APP.



**Figure 7:** Basic repulsive vector forces of APF.

As seen from the Figure 7, CW/CCW attractive APF model can be obtained by using Equation (6), Equation (7) and Equation (8).

$$[g_x, g_y] = -\nabla F_{ri} \quad \text{Equation (6)}$$

$$g_{x\_circ} = g_x * \cos\alpha - g_y * \sin\alpha \quad \text{Equation (7)}$$

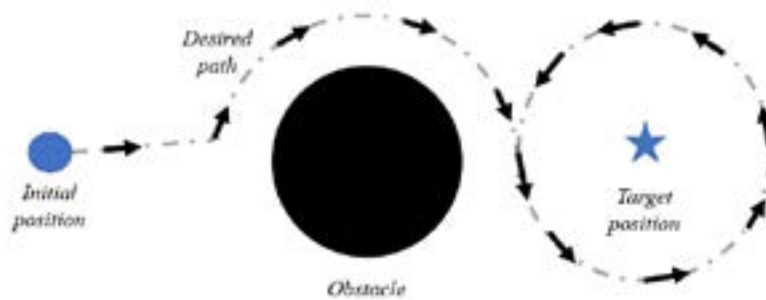
$$g_{y\_circ} = g_x * \sin\alpha + g_y * \cos\alpha \quad \text{Equation (8)}$$

Parameters  $g_x$  and  $g_y$  represents the gradient component of  $F_{r_i}$  and  $\alpha$  is angle that is using for defining direction of vector. By changing sign of  $\alpha$  CW and CCW model can be generated.  $g_{x\_circ}$  and  $g_{y\_circ}$  represents the new components of CW and CCW vector.



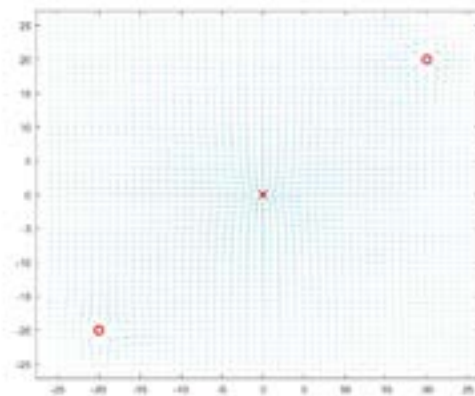
## 2.4. Combination of Basic Models

To produce the path in the field autonomously, different type of vector fields must be combined. However, all type of basic potential fields doesn't have to be exist in the field, usually a few types of them must be used together to obtain desired path autonomously. In this paper an attractive, repulsive, and circular basic models are used together as an example scenery.



**Figure 8:** Desired APP result.

It can be described as a mobile robot will make a circle pattern around the target point while avoiding obstacle in the field as seen from Figure 8. It is possible to increase variety and number of basic APF models in a field.



**Figure 9:** Combination of basic vector fields.

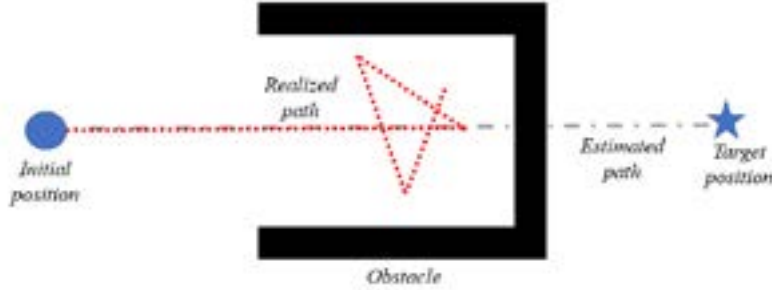
To achieve the path that is defined in the Figure 8, single attractive, repulsive, and circular basic APF models are generated and then combined by using Equation (1). As a result, the vector field that is shown in Figure 9 is obtained.

## 3. COMMON PROBLEMS OF APF BASED APP

However, APF based APP has lots of advantages and easy to implement as seen from the previous chapter it also has different defects. It is clear that grid resolution of the field, directly affect the optimum path that is generated by APF. Also, the number of obstacles, the number of models to define mobile robot behaviors and the definition of target/obstacle as static or dynamic directly affects the computation performance of the approach. But all the previously

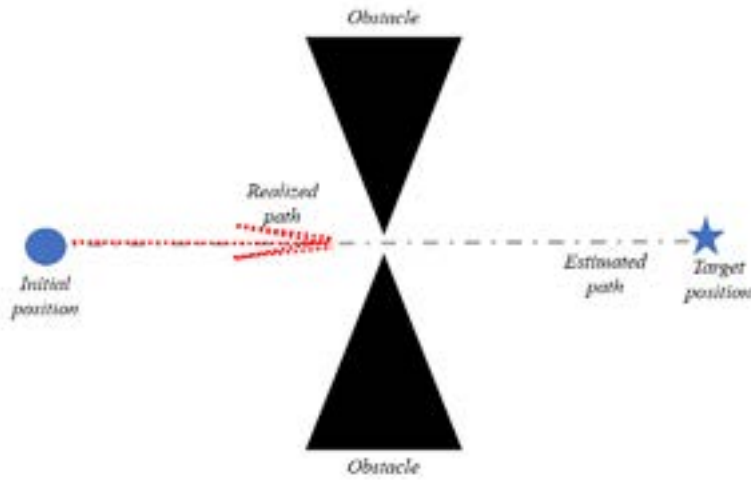


mentioned issues are problem specific defects for the APF based APP. Furthermore, common defects can be listed as local minima, closely space of obstacle, oscillation problem and Goal Not Reachable when Obstacle are Nearby (GNRON) in the literature and they are handled in this chapter of work one by one.



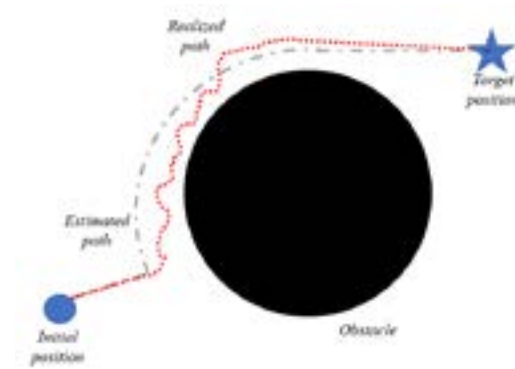
**Figure 10:** Local minimum problem of APF based APP.

Obstacles can be located such a point that their potential fields lead the moving robot into a dead end as seen from the Figure 10. This problem is called as local minimum problem for APF (Matoui et al., 2015). The local minimum problem usually occurs when the target point is located near and behind obstacles that are likely to create a local minimum. It occurs when the mobile robot under the attracting field effect is caught in a trap at the minimum point that occurs unintentionally due to the location of the obstacles.



**Figure 11:** Closely space of obstacle problem of APF based APP.

When two or more obstacles are located close to each other, and the path that goes to target passed from this point as seen from the Figure 11, closely space of obstacle problem can be occurred (Tang et al., 2010). To reach the target point behind the obstacles, mobile robot must pass from this gab, but it is not possible under the affect of total repulsive force when the robot becomes to close the obstacles.



**Figure 12:** Oscillation problem of APF based APP.

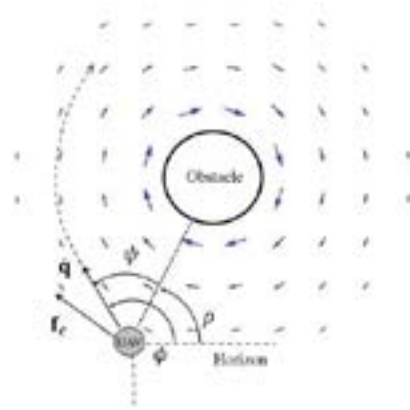
Because of obstacle and target point position, robot makes same maneuvers in the same position as seen from the Figure 12. This problem is defined as oscillation problem in the literature (Li et al., 2021; Zheyi & Bing, 2021). This problem occurs mostly in areas defined as static. If target position and obstacle positions are dynamic, oscillation problem is not occurred. Oscillation problem can be solved by adding security margin. If robot reach the security margin, it will change its direction.

In addition, there is another problem in APF approach that is GNRON. When the goal position is near to obstacle, robot will not reach to the goal point because of attractive and repulsive force have same value.

#### **4. A LITERATURE REVIEW FOR COMMON APF BASED APP PROBLEMS**

Several methods have been suggested to solve typical common problems of APF based APP in the literature (Wang et al., 2022; Weerakoon et al., 2015; Heidari & Saska, 2021). In fact, most of the suggested methods have same manner that modification of APF or combination with different approach like fuzzy logic (Liu et al., 2021), artificial intelligent methods (Li et al., 2021, Matoui et al., 2015,) etc. In this work, we will discuss commonly used approaches to solve local minimum problem, the closely spaced obstacles problem, the oscillation problem and GNRON in traditional APF based APP approach. In fact, there are major points to achieve successful APP by using APF; first point is modelling basic components, second one can be listed as distance between components, and the last one is how to combine basic components.

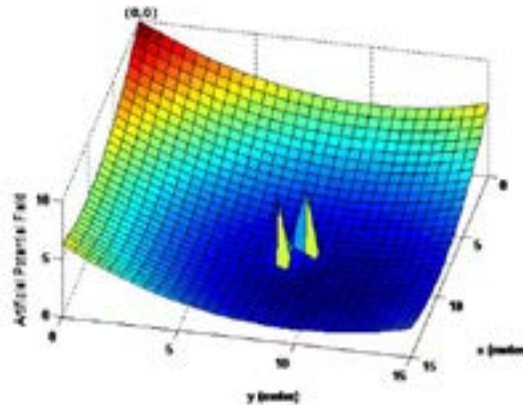
To solve the local minima problem, Choi et al. (2020) developed the curl-free vector field to obstacle avoidance. They suggested to use curved based basic models instead of linear ones. The curl-free vector field is produced around an obstacle. The direction of the vector field is applied by angle differences between mobile robot position and relative position vector from the robot to the obstacle. As a result, it can prevent local minima problem but in some cases, solution can be different from optimum path.



**Figure 13:** Concept of curl-free vector field (Choi et al., 2020).

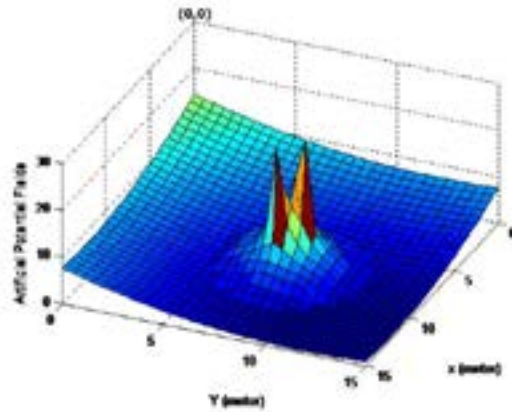
In Figure 13, basic concept of curl-free vector field has given. The method is generated around an obstacle.  $\rho$  is the angle of relative position vector between robot and obstacle. The path angle of robot is represented by  $\phi$ . The defined angle difference is represented by  $\psi = \phi - \rho$ . By using rotation matrix, vectors direction can be defined as CW or CCW. If there are many obstacles in the environment and their position is near to each other, the solution may not produce results. Furthermore, for the robots like UAVs in the 3D space, it may encounter many different types of obstacles. So, this method may not be compatible for 3D environment.

Iswanto, and colleagues were focused general problem of APF algorithm that is called local minima (Iswanto et al., 2019). The proposed a modification of the APF with adding virtual potentials to in front of the obstacle. With this approach local minima points will be eliminated by the modifying equation of the repulsive force of the APF. The Figure 14 and Figure 15 is shown effect of virtual potential field. With this effect robot avoids from obstacles. With virtual potential field approach, GNRON problem has solved.



**Figure 14:** Khatib's potential in local minima environment (Iswanto et al., 2019).

As seen from the Figure 14, two obstacles are very closed to each other, and it is possible to reach local minimum point. Furthermore, if the goal is located closely and behind the obstacles, it is impossible to reach it by using natural form of APF based APP.



**Figure 15:** The proposed APF potential in local minima environment (Iswanto et al., 2019).

As seen from the Figure 15, additional virtual potential fields are added to the same field which is shown with Figure 15. By using virtual potential fields, it is impossible to reach local minimum point. Even this approach solves local minima and GNRON problem, it will increase complexity and computational time of algorithm. If robot has a unit which can be used for parallel computation like General Purpose Computing on Graphical Processing Units (GPGPU), it may use in real-time robotic applications.

Harmonic functions can be used for solution of local minima problem in APF approach. Faria, and his colleagues used harmonic functions to generate the APF and avoid obstacle without local minima problem (Faria et al., 2004). Harmonic functions to define basic components of potential field can be used as seen from Equation (9).

$$f(x, y) = -\log(\alpha(x - x_a)^2 + \gamma(y - y_a)^2) \quad \text{Equation (9)}$$

By using Equation (9) spherical symmetry form can be produced with changing  $\gamma$  variable's value.  $\alpha$  is the magnitude factor of potential field.  $x_a, y_a$  parameters are definition of the center of the field. Thanks to low computational time and complexity of harmonic function based APF, it can be used for formation flight (Cetin & Yilmaz, 2014; Cetin & Yilmaz, 2016), dynamic/static obstacle environment, 2D and 3D space.



**Figure 16:** Limitation functions for basic APF components (Cetin, 2015).

Another solution for common APF based APP problems is using limitation functions, instead of combining basic components affecting all over the field. By limiting affect of the basic APF components, especially for obstacles, local minimum and GNRON problems can be solved. Different types of limit functions like rigid, linear and exponential that can be seen from Figure 16, are able to limit affect of potential field. By considering maneuver capabilities of mobile robot, usually exponential limit functions are suitable. Using linear limit functions may cause oscillation problem also in some cases.

## 5. CONCLUSION

In this paper, APF based APP approaches are defined in detail. As seen from the definition of APF based APP, it is easy to implement and modelling. However, besides its benefits, there are common problems like local minima, the closely spaced obstacles problem, the oscillation problem and GNRON as explained. Major drawbacks are solving the common problems together. Their limitations, chronic issues and solutions, usage areas and combination with other APP algorithms are discussed.

To achieve a real-time, in other words, it provides fast calculation opportunity, and effective path planning algorithm by using artificial potential fields, basic component of the APF must be modelling by using harmonic functions and they must be limited by using exponential functions. Besides, overcome problems such as local minima and simulation, curl-free vector fields and virtually supported potential fields must be calculated. By modeling APF with using harmonic functions while limiting basic models with exponential functions also using virtual potentials basic problems of the APF can be solved effectively.

## REFERENCES

- Cetin, O. (2015). Parallel programming based path planning for multi autonomous unmanned vehicles. [Doctoral dissertation, Turkish Air Force Academy]. Dissertation ID: 397130. <https://tez.yok.gov.tr/UlusalTezMerkezi/tezSorguSonucYeni.jsp>
- Cetin, O., & Yilmaz, G. (2014). GPGPU accelerated real-time potential field based formation control for Unmanned Aerial Vehicles. 2014 International Conference on Unmanned Aircraft Systems (ICUAS), 2014, 103-114. <https://doi.org/10.1109/ICUAS.2014.6842245>
- Cetin, O., & Yilmaz, G. (2016). Real-time Autonomous UAV Formation Flight with Collision and Obstacle Avoidance in Unknown Environment. *Journal of Intelligent & Robotic Systems*, 84, 415–433. <https://doi.org/10.1007/s10846-015-0318-8>
- Chen, J., Ling, F., Zhang, Y., You, T., Liu, Y., & Du, X. (2022). Coverage path planning of heterogeneous unmanned aerial vehicles based on ant colony system. *Swarm and Evolutionary Computation*, 69. <https://doi.org/10.1016/j.swevo.2021.101005>
- Choi, D., Lee, K., & Kim, D. (2020). Enhanced Potential Field-Based Collision Avoidance for Unmanned Aerial Vehicles in a Dynamic Environment. *AIAA Scitech 2020 Forum, Detect and Avoid Technologies for UAS*. <https://doi.org/10.2514/6.2020-0487>
- Dai, J., Qiu, J., Yu, H., Zhang, C., Wu, Z., & Gao, Q. (2022). Robot Static Path Planning Method Based on Deterministic Annealing. *Machines* 2022, 10 (8), 600. <https://doi.org/10.3390/machines10080600>
- Duhé, JF., Victor, S., & Melchior, P. (2021). Contributions on Artificial Potential Field Method for Effective Obstacle Avoidance. *Fractional Calculus and Applied Analysis*, 24, 421–446. <https://doi.org/10.1515/fca-2021-0019>
- Faria, G., Romero, R. A. F., Prestes, E., & Idiart, M. A. P. (2004). Comparing harmonic functions and potential fields in the trajectory control of mobile robots. *IEEE Conference on Robotics, Automation and Mechatronics*, 2004, 2, 762-767. <https://doi.org/10.1109/RAMECH.2004.1438014>

- Feng, S., Qian, Y., & Wang, Y. (2021). Collision avoidance method of autonomous vehicle based on improved artificial potential field algorithm. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 235 (14), 3416-3430. <https://doi.org/10.1177/09544070211014319>
- GE, S. S., & Cui, Y. J. (2000). New potential functions for mobile robot path planning. *IEEE Transactions on Robotics and Automation*, 16 (5), 615-620. <https://doi.org/10.1109/70.880813>
- Heidari, H., & Saska, M. (2021). Collision-free trajectory planning of multi-rotor UAVs in a wind condition based on modified potential field. *Mechanism and Machine Theory*, 156, 104140 <https://doi.org/10.1109/STA.2015.7505223>
- Iswanto, I., Ma'arif, A., Wahyunggoro, O., & Cahyadi, A. I. (2019). Artificial Potential Field Algorithm Implementation for Quadrotor Path Planning. *International Journal of Advanced Computer Science and Applications (IJACSA)*, 2018, 10 (8). <https://doi.org/10.14569/IJACSA.2019.0100876>
- Khatib, O. (1985). Real-time obstacle avoidance for manipulators and mobile robots. *Proceedings. 1985 IEEE International Conference on Robotics and Automation*, 500-505, <https://doi.org/10.1109/ROBOT.1985.1087247>.
- Klančar, G., Zdešar, A., & Krishnan, M. (2022). Robot Navigation Based on Potential Field and Gradient Obtained by Bilinear Interpolation and a Grid-Based Search. *Sensors* 2022, 22 (9), 3295. <https://doi.org/10.3390/s22093295>
- Koren, Y., & Borenstein, J. (1991). Potential field methods and their inherent limitations for mobile robot navigation. *Proceedings. 1991 IEEE International Conference on Robotics and Automation*, 2, 1398-1404. <https://doi.org/10.1109/ROBOT.1991.131810>
- Lamini, C., Benhlila, S., & Elbekri, Ali. (2018). Genetic Algorithm Based Approach for Autonomous Mobile Robot Path Planning. *Procedia Computer Science*, 127, 180-189. <https://doi.org/10.1016/j.procs.2018.01.113>
- Li, C., Cui, G., & Lu, H. (2010). The design of an obstacle avoiding trajectory in unknown environment using potential fields. *The 2010 IEEE International Conference on Information and Automation*, 2010, 2050-2054, <https://doi.org/10.1109/ICINFA.2010.5512513>.
- Li, G., Yamashita, A., Asama, H., & Tamura, Y. (2012). An efficient improved artificial potential field based regression search method for robot path planning. *2012 IEEE International Conference on Mechatronics and Automation*, 2012, 1227-1232 <https://doi.org/10.1109/ICMA.2012.6283526>.
- Li, H., Gong, D., & Yu, J. (2021). An obstacles avoidance method for serial manipulator based on reinforcement learning and Artificial Potential Field. *Int J Intell Robot Appl*, 5, 186-202. <https://doi.org/10.1007/s41315-021-00172-5>
- Liu, G., Du, Y., Li, X., & Dou, S. (2021). Research on Path Planning of Logistics Storage Robot Based on Fuzzy Improved Artificial Potential Field Method. In: Liu, S., Bohács, G., Shi, X., Shang, X., Huang, A. (eds) *LISS 2020*, 265. [https://doi.org/10.1007/978-981-33-4359-7\\_19](https://doi.org/10.1007/978-981-33-4359-7_19)
- Matoui, F., Boussaid, B., & Abdelkrim, M. N. (2015). Local minimum solution for the potential field method in multiple robot motion planning task. *2015 16th International*

- Conference on Sciences and Techniques of Automatic Control and Computer Engineering (STA), 452-457. <https://doi.org/10.1109/STA.2015.7505223>
- Rezaee, H., & Abdollahi, F. (2012). Adaptive artificial potential field approach for obstacle avoidance of unmanned aircrafts. 2012 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM), 2012, 1-6. <https://doi.org/10.1109/AIM.2012.6305268>.
- Rybus, T., Wojtunik, M., & Basmadji, F. L. (2022). Optimal collision-free path planning of a free-floating space robot using spline-based trajectories. *Acta Astronautica*, 190, 395-408. <https://doi.org/10.1016/j.actaastro.2021.10.012>
- Sabudin, E. N., Omar, R., Debnath, S. K., & Sulong, M. S. (2021). Efficient robotic path planning algorithm based on artificial potential field. *International Journal of Electrical and Computer Engineering (IJECE)*, 11 (6), 4840-4849. <https://doi.org/10.11591/ijece.v11i6.pp4840-4849>
- Szczepanski, R., Bereit, A., & Tarczewski, T. (2021). Efficient Local Path Planning Algorithm Using Artificial Potential Field Supported by Augmented Reality. *Energies* 2021, 14 (20), 6642. <https://doi.org/10.3390/en14206642>
- Tang, J., Sun, J., Lu, C., & Lao, S. (2019). Optimized artificial potential field algorithm to multi-unmanned aerial vehicle coordinated trajectory planning and collision avoidance in three-dimensional environment. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, 233 (16), 6032-6043. <https://doi.org/10.1177/095441001984443>
- Tang, L., Dian, S., Gu, G., Zhou, K., Wang, S., & Feng, X. (2010). A novel potential field method for obstacle avoidance and path planning of mobile robot. 2010 3rd International Conference on Computer Science and Information Technology, 2010, 633-637. <https://doi.org/10.1109/ICCSIT.2010.5565069>.
- Wang, H., Li, G., Hou, J., Chen, L., & Hu, N. (2022). A Path Planning Method for Underground Intelligent Vehicles Based on an Improved RRT\* Algorithm. *Electronics* 2022, 11 (3), 294. <https://doi.org/10.3390/electronics11030294>
- Wang, S., Lin, F., Wang, T., Zhao, Y., Zang, L., & Deng, Y. (2022). Autonomous Vehicle Path Planning Based on Driver Characteristics Identification and Improved Artificial Potential Field. *Actuators* 2022, 11 (2), 52. <https://doi.org/10.3390/act11020052>
- Wang, W., Wu, Z., Lou, H., & Zhang, B. (2022). Path Planning Method of Mobile Robot Using Improved Deep Reinforcement Learning. *Journal of Electrical and Computer Engineering*, 2022, Article ID: 5433988. <https://doi.org/10.1155/2022/5433988>
- Weerakoon, T., Ishii, K., & Nassiraei, A. A. F. (2015). An Artificial Potential Field Based Mobile Robot Navigation Method to Prevent from Deadlock. *JAISCR*, 2015, 5 (3), 189-203. <https://doi.org/10.1515/jaiscr-2015-0028>
- Xi, M., Yang, J., Wen, J., Liu, H., Li, Y., & Song, H. H. (2022). Comprehensive Ocean Information-Enabled AUV Path Planning Via Reinforcement Learning. *IEEE Internet of Things Journal*, 9, 18, 17440-17451. <https://doi.org/10.1109/JIOT.2022.3155697>



- Xiang, D., Lin, H., Ouyang, J., & Huang, D. (2022). Combined improved A\* and greedy algorithm for path planning of multi-objective mobile robot. *Scientific Reports*, 12 (1), 13273. <https://doi.org/10.1038/s41598-022-17684-0>
- Yan, X., Jiang, D., Miao, R., & Li, Y. (2021). Formation Control and Obstacle Avoidance Algorithm of a Multi-USV System Based on Virtual Structure and Artificial Potential Field. *J. Mar. Sci. Eng* 2021, 9 (2), 161. <https://doi.org/10.3390/jmse9020161>
- Yang, L., Qi, J., & Han, J. (2012). "Path planning methods for mobile robots with linear programming" in *Proceedings of 2012 International Conference on Modelling, Identification and Control, ICMIC 2012* (pp. 641–646).
- Yuan, J., Liu, Z., Lian, Y., Chen, L., An, Q., Wang, L., & Ma, B. (2022). Global Optimization of UAV Area Coverage Path Planning Based on Good Print Set and Genetic Algorithm. *Aerospace* 2022, 9 (2), 86. <https://doi.org/10.3390/aerospace9020086>
- Zammit, C., & Kampen, E. J. V. (2022). Comparison Between A\* and RRT Algorithms for 3D UAV Path Planning. *Unmanned Systems*, 10 (2), 129-146. <https://doi.org/10.1142/S2301385022500078>
- Zhang, T.W., Xu, G.H., Zhan, X. S., & Han, T. (2022). A new hybrid algorithm for path planning of mobile robot. *The Journal of Supercomputing*, 78, 4158-4181. <https://doi.org/10.1007/s11227-021-04031-9>
- Zhang, Z., Wu, J., Dai, J., & He, C. (2022). Optimal path planning with modified A-Star algorithm for stealth unmanned aerial vehicles in 3D network radar environment. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, 236 (1), 72-81. <https://doi.org/10.1177/0954410021100738>
- Zheng, J., Mao, S., Wu, Z., Kong, P., & Qiang, H. (2022). Improved Path Planning for Indoor Patrol Robot Based on Deep Reinforcement Learning. *Symmetry* 2022, 14 (1), 132. <https://doi.org/10.3390/sym14010132>
- Zheyi, C. & Bing, X. (2021). AGV Path Planning Based on Improved Artificial Potential Field Method, *IEEE International Conference on Power Electronics, Computer Application (ICPECA 2021)*, 32-37, <https://10.1109/ICPECA51329.2021.9362519>