



Beta decay rates for $^{50,52}\text{Fe}$ isotopes by Pyatov-Salamov method

Şule DUYGU, Necla ÇAKMAK

Department of Physics, Faculty of Science, Karabuk University, Karabuk, Turkey

Received: 30.09.2016; Accepted: 15.11.2016

Abstract. The Pyatov-Salamov Method (PM) has been used to study the Gamow-Teller (GT) transitions for chosen iron isotopes. The eigenvalues and eigenfunctions of the restored Hamiltonian have been solved within the framework of proton-neutron quasiparticle random phase approximation (pn-QRPA). The allowed GT beta decay half-life values have been calculated for the nuclei under consideration. The results have been compared with the schematic model (SM) calculations and corresponding experimental data.

Keywords: Pyatov-Salamov method, pn-QRPA approximation, Gamow-Teller

1. INTRODUCTION

The charge-exchange excitations in nuclei play an active role in the research of the nuclear structure. Reliable evidence on the Gamow-Teller (GT) strength distribution at relatively low energies is of great importance in understanding the basic astrophysical processes such as nucleosynthesis, stellar collapse, r process, $2\nu\beta\beta$ decay and supernova formation [1,2]. Theoretical descriptions of the GT strength distributions in medium and heavy mass nuclei are given in many studies [3-9]. Kuzmin and Soloviev calculated the fragmentation of the GTR in heavy nuclei within the framework of the quasiparticle phonon model [3]. Their results showed that the GTR spread only up to the excitation energies of ~ 30 MeV. Unlu studied GT 1^+ states using the method developed by Pyatov and Salamov [7].

In the present study, the broken super symmetry property of the pairing interaction between nucleons was restored and the effect of restoration on the GTR for $^{46-52}\text{Fe}$ isotopes was studied within the framework of the pnQRPA method with the separable residual GT effective interactions in the particle-hole (ph) and particle-particle (pp) channels. The same calculations were performed in the phenomenological shell model basis also (schematic model (SM)). The obtained results in both methods were compared with each other and with the corresponding experimental data.

2. THEORETICAL FORMALISM

The schematic model (SM) Hamiltonian (the schematic model Hamiltonian does not include h_0 term) for GT excitations in the quasiparticle representation is usually accepted in the following form:

$$H_{SM} = H_{sqp} + h_{ph} + h_{pp}, \quad (1)$$

where H_{sqp} is the single quasiparticle (sqp) Hamiltonian, h_{ph} and h_{pp} are respectively. The GT effective interactions in the ph and pp channels. As known, the effective interaction constants in the ph and pp channels are fixed from the experimental value of the GTR energy and the β decay $\log ft$ values between the low energy states of the parent and daughter nuclei. The super symmetry property of the pairing part

* Corresponding author. *Email address:* ssuleduyguu@gmail.com

in total Hamiltonian was restored according to the Pyatov's method. In this respect, certain terms which naturally do not commute with the GT operator were excluded from total Hamiltonian and the broken commutativity of the remaining part due to the shell model mean-field approximation was restored by adding an effective interaction term h_0 as follows [51]:

$$[H_{SM} - (h_{ph} + h_{pp}) - (V_1 + V_C + V_{ls} + h_0, G_{1\mu}^\pm)] = 0 \quad (2)$$

or

$$[H_{sqp} - V_1 - V_C - V_{ls} + h_0, G_{1\mu}^\pm] = 0 \quad (3)$$

where V_1 , V_C and V_{ls} are the isovector, Coulomb and spin orbital term of the shell model potential, respectively. The restoration term h_0 in eq. (3) is included in a separable form:

$$h_0 = \sum_{\rho=\pm} \frac{1}{4\gamma_\rho} \sum_{\mu=0,\pm 1} [(H_{sqp} - V_1 - V_C - V_{ls}), G_{1\mu}^\rho]^\dagger [H_{sqp} - V_1 - V_C - V_{ls}, G_{1\mu}^\rho]. \quad (4)$$

The strength parameter γ_ρ of the h_0 effective interaction is found from the commutation condition in eq. (3) and the following expression is obtained for this constant.

$$\gamma_\rho = \frac{\rho(-1)^\mu}{2} \left\langle 0 \left| \left[[H_{sqp} - V_1 - V_C - V_{ls}, G_{1\mu}^\rho], G_{1\mu}^\rho \right] \right| 0 \right\rangle$$

Let us consider a system of nucleons in a spherical symmetric average field with pairing forces. In this case, the corresponding quasiparticle Hamiltonian of the system is given by

$$H_{sqp} = \sum_{\tau, j_m} \varepsilon_{j_\tau} \alpha_{j_\tau m_\tau}^\dagger \alpha_{j_\tau m_\tau}, \quad \tau = n, p \quad (5)$$

where ε_{j_τ} is the sqp energy of the nucleons with angular momentum j_τ , and $\alpha_{j_\tau m_\tau}^\dagger$ ($\alpha_{j_\tau m_\tau}$) is the quasiparticle creation (annihilation) operator. The GT operator in the quasiparticle space according to quasiboson approximation is given as follows:

$$G_{1\mu}^- = \sum_{np} [\bar{b}_{np} C_{np}^\dagger(\mu) + -1^{1+\mu} b_{np} C_{np}(-\mu)], \quad G_{1\mu}^- = [G_{1\mu}^-]^\dagger,$$

where $C_{np}^\dagger(\mu)$ and $C_{np}(\mu)$ are the quasiboson creation and annihilation operators.

$$C_{np}^\dagger(\mu) = \sqrt{\frac{3}{2j_n + 1}} \sum_{m_n, m_p} (-1)^{j_p - m_p} \langle j_p m_p 1\mu | j_n m_n \rangle \alpha_{j_n m_n}^\dagger \alpha_{j_p - m_p}^\dagger$$

and $C_{np}(\mu) = [C_{np}^\dagger(\mu)]^\dagger$.

These operators satisfy the following commutation rules in the quasiboson approximation:

$$[C_{np}(\mu), C_{n'p'}^\dagger(\mu')] = \delta_{nn'} \delta_{pp'} \delta_{\mu\mu'}, \quad [C_{np}(\mu), C_{n'p'}(\mu')] = 0. \quad (6)$$

The h_0 , h_{ph} and h_{pp} effective interactions in quasiboson approximation are described in the following forms:

$$h_0 = \sum_{npn'p'} \mu \rho \frac{1}{2\gamma_\rho} E_{np}^\rho E_{n'p'}^\rho [C_{np}(\mu) + \rho(-1)^{1+\mu} C_{np}^\dagger(-\mu)]$$

$$\cdot \left[C_{n'p'}^\dagger(\mu') + \rho'(-1)^{1+\mu'} C_{n'p'}(-\mu') \right], \quad (7)$$

$$h_{ph} = 2\chi_{ph} \sum_{n p n' p' \mu \rho} \left[\bar{b}_{np} C_{np}^\dagger(\mu) + (-1)^{1+\mu} b_{np} C_{np}(-\mu) \right] \cdot \left[\bar{b}_{n'p'} C_{n'p'}(\mu') + (-1)^{1+\mu'} b_{n'p'} C_{n'p'}^\dagger(-\mu') \right] \quad (8)$$

$$h_{pp} = -2\chi_{pp} \sum_{n p n' p' \mu \rho} \left[d_{np} C_{np}^\dagger(\mu) - (-1)^{1+\mu} \bar{d}_{np} C_{np}(-\mu) \right] \cdot \left[d_{n'p'} C_{n'p'}(\mu') - (-1)^{1+\mu'} \bar{d}_{n'p'} C_{n'p'}^\dagger(-\mu') \right], \quad (9)$$

where $E^\rho, \bar{b}, b, \bar{d}, d$ are the reduced matrix elements. The complete expressions of these matrix elements are given in refs [10]. Thus, the total Hamiltonian of the system according to PM is given as

$$H_{PM} = H_{sqp} + h_{ph} + h_{pp} + h_0. \quad (10)$$

The eigenvalues and eigenfunctions of the Hamiltonian solved within the framework of the pnQRPA method (see detailed in refs [10]) and the $0^+ \rightarrow 1^+ \beta^\pm$ transition matrix elements are calculated using the following expressions:

$$M_{\beta^-}^i(0^+ \rightarrow 1_i^+) = \langle 1_i^+, \mu | G_{1\mu}^- | 0^+ \rangle = \langle 0 | [Q_i(\mu), G_{1\mu}^-] | 0 \rangle = -\sum_{np} (\psi_{np}^i b_{np} + \varphi_{np}^i \bar{b}_{np}) \quad (11)$$

$$M_{\beta^+}^i(0^+ \rightarrow 1_i^+) = \langle 1_i^+, \mu | G_{1\mu}^+ | 0^+ \rangle = \langle 0 | [Q_i(\mu), G_{1\mu}^+] | 0 \rangle = \sum_{np} (\psi_{np}^i \bar{b}_{np} + \varphi_{np}^i b_{np}) \quad (12)$$

The β^\pm recuded matrix elements are given by

$$B_{GT}^\pm(\omega_i) = \sum_{\mu} \left| M_{\beta^\pm}^i(0^+ \rightarrow 1_i^+) \right|^2. \quad (13)$$

The β^\pm transition strengths (S^\pm) must fulfill the Ikeda sum rule (ISR).

$$S^\pm = \sum_i B_{GT}^\pm(\omega_i), \quad (14)$$

$$ISR = S^{(-)} - S^{(+)} \cong 3(N - Z). \quad (15)$$

The ft values for the allowed GT β transitions are finally calculated using

$$ft = \frac{D}{\left(\frac{g_A}{g_V}\right)^2 4\pi B_{GT}^\pm(I_i \rightarrow I_f, \beta^\pm)} \quad (16)$$

3. RESULTS AND CONCLUSIONS

The allowed GT β -decay half-lives have been investigated for chosen iron isotopes. The fixed values of the strength parameters of the ph and the pp interactions are given in Table 1 and the same values are used both PM and SM calculations. It should be noted that the agreement between theory and experiment depends on the choice of the strength parameters of the ph and pp interaction. In numerical calculations, the Woods-Saxon potential with Chepurnov parameterization has been used and the pair correlation constants have been chosen as $C_n = C_p = 12/A^{1/2}$ for open shell nuclei [12]. The basis used in our

calculation contained all the neutron-proton transitions which change the radial quantum number n by $\Delta n = 0, 1, 2, 3$. The reliability of our basis was tested by calculating the Ikeda Sum Rule. The calculations has been obtained without any quenching factor. The calculated values of the half-lives are given in Table 2. The results have been compared with the experimental data [11]. The calculated half-lives in PM are closer to the corresponding experimental values. This shows that the restoration term is important in understanding the GT 1^+ states. The fixed χ_{ph} value in PM is greater than that in SM and the fixed χ_{pp} value in PM is smaller than that in SM. Both PM and SM give a good agreement with the corresponding experimental data depending on the choice of the χ_{ph} and χ_{pp} parameters. The restoration of supersymmetry property of the pairing interaction is physically important.

Table 1. The fixed parameter values of the PM and SM for iron-50 and iron-52 isotopes. The χ_{ph} and χ_{pp} values are in units of $5.2A^{-0.7}$ MeV and $0.52A^{-0.7}$ MeV, respectively.

A	χ_{ph}	χ_{pp}
50	1.0	0.1
52	1.0	0.1

Table 2. Allowed GT β -decay half-lives for $^{50,52}\text{Fe}$ isotopes calculated using the pn-QRPA (SM) and pn-QRPA (PM) models.

A	Exp [11]	pn-QRPA (PM)	pn-QRPA (SM)
50	0.155 s	0.245 s	1.548 s
52	8.275 h	10.32 h	0.145 h

Acknowledgements

The authors would like to thank Serdar Unlu for his contributions to this study.

REFERENCES

1. Pyatov, N.I. and Salamov, D.I, 1977, *Nucleonica* 22, 127
2. Alford, W.P. and Spicer B.M., 1998, *Advances in nuclear physics* (Plenium, New York) Vol. 24
3. Kuzmin, V.A. and Soloviev, V.D., 1984, *J. Phys. G : Nucl. Part. Phys.* 10, 1507
4. Colo, G. et al, 1994, *Phys. Rev. C*50, 1496
5. Dang, N.D. et al, 1997, *Phys. Rev. Lett.* 79, 1638
6. Babacan, T. et al., 2005, *Phys. Rev. C*71, 037303
7. Unlu, S., 2012, *Physics of Atomic Nuclei*, 75, 8
8. Unlu, S. et al., 2008, *Pramana J. Phys.* 71, 521
9. Cakmak, N. et al, 2010, *Pramana J. Phys.* 75, 649
10. Cakmak, N., 2008, *The study of charge-exchange collective excitations in odd mass nuclei by Pyatov method*, Ph.D. Thesis (Anadolu University, Turkey)
11. Audi, G. et al., 2012, *Chin. Phys. C* 36, 1287
12. Soloviev, V.G., 1976, *Theory of Complex Nuclei*, (Pergamon, New York)