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Reaction of Quasielastic Knock-Out of Nucleons From Nuclei by Protons

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Abstract. On the basis of non-relativistic theory in the distorted-wave approximation in three dimensions, a theory of from nuclei by nucleons of intermediate energies has been developed. On the baizing on this theory the differential cross sections of quasielastic knock-out reactions of protons with shells $1p$, $1S$ in the nucleus ^{16}O and ^{12}C have been calculated, which allows to determine the orbital momentum of the nucleons in the nucleus before scattering by the angular distribution of emitted protons.

Keywords: Reaction A (p, 2p)B, distorted-wave approximation, ^{16}O and ^{12}C nuclei.

1. INTRODUCTION

Investigation of nucleon knock-out reactions at intermediate initial energies and the experiments, in which they were studied, confirmed the correctness of the model of direct knock-out particles from nucleus. Despite some successes in describing the mechanism of nuclear reactions, many questions still remain open. To some extent this is due to some unresolved issues from nuclear theory: the lack of reliable data on the intrans nuclear wave functions, two-body potentials and others. Serious problem is the parameterization of the distorted wave scattered nucleons [1].

The further investigation of the particles ejected by fast nucleons may give additional information about the wave functions of the ground state of nuclei and the distribution of nucleon moments of the in this state [2].

Based on the theory of direct interactions of nucleons with the surface protons and neutrons of the nucleus [3], using previously developed theory of the scattering of protons of intermediate energies in distorted-wave approximation [4,5], and getting the analytical form of expression of the differential cross section, we can receive more accurate information about the knocked-out nucleons before the reaction in the nucleus.

Now consider the derivation of the formula determining the angular distribution of protons in the reaction of surface interaction of the incident neutrons on the nucleus of the target.

We write the differential cross section for the reaction A (n, np) in the following form [6]:

$$d\sigma_{n,np} = (2\pi)^4 \frac{m}{k} d\mathbf{p}_f d\mathbf{p}_p \delta(E_i - E_f - E_p - E_N - E_R) \ \frac{1}{2J_i + 1} \sum_{\sigma_i \sigma_f} \sum_{M_i M_f} \left| T_{if} \right|^2 \qquad (1)$$

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Here, $\mathbf{p}_f$, $\mathbf{p}_p$ are three-dimensional moments of the scattered neutrons and ejected protons, E_i, E_f are the kinetic energies of the incident and scattered neutrons. The energy of the ejected protons E_p and the separation energy of the least bound proton is E_N. Finally, the recoil energy of the daughter nucleus is $E_R = \frac{P_R^2}{2M_{A-1}}$ which is determined by the momentum of the recoil nucleus ($\mathbf{P}_R$) and in its turn, is connected with the missing mass (M_R) from the reaction, based on the law of conservation of energy:

$$M_R = [(M_A - m_p + E_i - E_f - E_p)^2 - P_R^2]^{1/2} \tag{2}$$

For the missing mass at that the relationship between the separation energy of the nucleons (E_N) and mass of daughter nucleus (M_{A-1}), known from the experiment is used:

$$E_N = M_R - M_{A-1} \tag{3}$$

Consider the derivation of the formula determining the angular distribution of knocked out protons in the reaction of surface interaction of the incident neutron on the nucleus - A. We write for this case the wave function of the initial state $n + A$ of the system as follows:

$$\psi_{A-1}(\mathbf{r})\psi_{nl}(r_p)\mathrm{Y}_{lm}(\theta_p, \varphi_p)\psi_{\mathbf{k_i}}(\mathbf{r}') \tag{4}$$

where r_p, θ_p, φ_p are the polar coordinates of the proton in the nucleus A, $\mathbf{r}$ are the coordinates of all other nucleons in the nucleus, $\mathbf{r}'$ and $\mathbf{k}_i$, respectively, the coordinate and wave vector of the incident neutron. Final state after the emission of a proton by the nucleus, corresponds to the core $A-1$, which wave function at large distances nucleons from the nucleus has the form

$$\psi_{A-1}(\mathbf{r})\psi_{\mathbf{k}_f}(\mathbf{r}')\psi_{\mathbf{k}_p}(\mathbf{r}_p), \tag{5}$$

where $\mathbf{k}_f$ and $\mathbf{r}'$ are respectively, the wave vector and the coordinate of the scattered neutron and wave vector of the ejected proton. Potential $\upsilon(|\mathbf{r}' - \mathbf{r}_p|)$ responsible for the direct interaction depicts the interaction of the incident neutron with a proton of the nucleus A, $U(|\mathbf{r}_p - \mathbf{r}|)$ is the potential of interaction of emitted proton with the nucleus of the balance, which is different from zero only on the "surface" of the nucleus, i.e. $r_n = r_p = R$.

The matrix element of the nucleus transition is represented as

$$T_{if} =< f\left|\int d\mathbf{r}' d\mathbf{r}_p \psi_{\mathbf{k}_f}^{(-)*}(\mathbf{r}')\psi_{\mathbf{k}_p}^{*}(\mathbf{r}_p)\upsilon(|\mathbf{r}' - \mathbf{r}_p|)\psi_{\mathbf{k}_i}^{(+)}(\mathbf{r}')\psi_{nl}(\mathbf{r}_p)\mathrm{Y}_{lm}(\theta_p.\varphi_p)W(\mathbf{r}_p)\delta(|\mathbf{r}_p - R|)\right|i> \tag{6}$$

where

$$W(\mathbf{r}_p) = \int \psi_{A-1}^{*}(\mathbf{r})U(|\mathbf{r}_p - \mathbf{r}|)\psi_{A-1}(\mathbf{r})d\mathbf{r} \tag{7}$$

The wave functions of the scattered neutrons, obtained in [2] from the solution of the nonrelativistic Schrodinger equation is written in the following form

$$\psi^{(-)*}_{\mathbf{k}_f}(\mathbf{r}')\psi^{(+)}_{\mathbf{k}_i}(\mathbf{r}') = \ell^{i[\mathbf{qr}'+\phi(\mathbf{r}',\mathbf{q})]} \quad (8)$$

where the distorting function depending on the distribution of nucleons in nuclei, is given in [7].

Expressing the interaction potential $\upsilon(|\mathbf{r}'-\mathbf{r}_p|)$ through the amplitude of nucleon-nucleon scattering f_{NN} $(\mathbf{q}')$ presented in [7] for the transition matrix element of the nucleus we obtain:

$$T_{if} = \frac{-\hbar^2}{2\pi^2 m}\langle f|\iint \ell^{i[\mathbf{qr}'+\phi]}\ell^{-i\mathbf{k}_p\mathbf{r}_p}\ell^{-i\mathbf{q}(\mathbf{r}'-\mathbf{r}_p)} f_{NN}(\mathbf{q}')\psi_{nl}(\mathbf{r_p})\mathrm{Y}_{lm}(\theta_\mathbf{p},\varphi_\mathbf{p})W(\mathbf{r_p})\delta(|\mathbf{r_p}-\mathbf{R}|)d\mathbf{r}'d\mathbf{q}'d\mathbf{r_p}|i\rangle$$
(9)

In order to simplify the calculation the wave function of the knocked-out proton is taken as a plane wave.

To calculate the matrix element (9), we replacing the phase variables $\mathbf{u}=\mathbf{r}'-\mathbf{r}_p$ after integration obtain:

$$T_{if} = -\frac{4\pi\hbar^2}{m}\langle f|\int f_{NN}(\mathbf{q}_{eff})\ell^{i[\mathbf{qr}_p+\phi(\mathbf{r}_p)]}\ell^{-i\mathbf{k}_p\mathbf{r}_p}\psi_{nl}(\mathbf{r}_p)Y_{lm}(\theta_p,\varphi_p)W(\mathbf{r}_p)\delta(|\mathbf{r}_p-\mathbf{R}|)d\mathbf{r}_p|i\rangle \quad (10)$$

Now, as shown in Fig. 1, we choose a coordinate system in which $Oz \uparrow\uparrow \mathbf{q}$, denoting $\cos(\hat{\mathbf{q}}\hat{\mathbf{r}}) = \mu$, $\mathbf{r} = \{r\mu\varphi\}$. This will allow taking into account the loss of energy, to write momentum transferred by the incident nucleon to the nucleus of the target, as:

$$|\mathbf{q}| = |\mathbf{k}_i - \mathbf{k}_f| = \sqrt{k_i^2+k_f^2-2k_ik_f\cos\vartheta} = (\frac{2m}{\hbar^2})^{1/2}\sqrt{E_i+E_f-2E_i^{1/2}E_f^{1/2}\cos\vartheta} \quad (11)$$

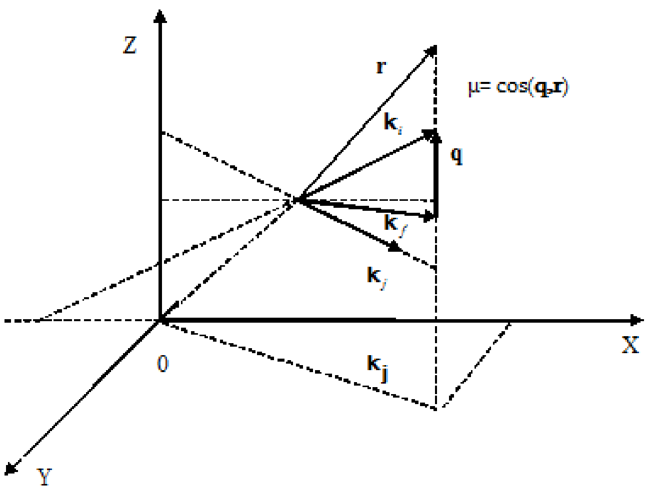

Fig.1. Momentum of incident (k_i), trace (k_f) and ejected particles (k_j) in the three-dimensional coordinate system with momentum transfer $\mathbf{q}=\mathbf{k}_i-\mathbf{k}_f$.

Scattering angle $\vartheta = \vartheta_1 + \vartheta_2$ and deflection angles of the incident (ϑ_1) and the scattered particles (ϑ_2) respectively x-axis and the scattering angle of the ejected particles in three-dimensional coordinate system $\vartheta_3 = (\hat{\mathbf{k}}_i \hat{\mathbf{k}}_j)$ are related as follows:

$$\begin{aligned} \cos(\hat{\mathbf{r}}\hat{\mathbf{k}}_i) &= \mu \sin \vartheta_1 + \cos\vartheta_1 \sqrt{1-\mu^2}\cos\varphi \\ \cos(\hat{\mathbf{r}}\hat{\mathbf{k}}_f) &= -\mu \sin(\vartheta - \vartheta_1) + \cos(\vartheta - \vartheta_1)\sqrt{1-\mu^2}\cos\varphi \\ \cos(\hat{\mathbf{r}}\hat{\mathbf{k}}_j) &= -\mu \sin(\vartheta_3 - \vartheta_1) + \cos(\vartheta_3 - \vartheta_1)\sqrt{1-\mu^2}\cos\varphi \end{aligned} \qquad (12)$$

$$tg\vartheta_1 = \frac{E_i^{1/2}}{E_f^{1/2}} \frac{1}{\cos\vartheta} - tg\vartheta \qquad (13)$$

We note at once that angles ϑ and ϑ_3 are determined from the experiment.

Applying the recursion formula derived in [7], simplifying the exponential factor in the amplitude of the process, we obtain:

$$T_{if} = -\frac{\hbar^2 k_i \sigma_{NN}}{2m}(i+\varepsilon_0)\ell^{\frac{-\beta_0^2 \mathbf{q}^2}{2}} W(R)\sum_{n=0}^{4}(a_{n1} + ia_{n2})\frac{\partial^n I(q)}{\partial q^n}, \qquad (14)$$

where

$$I(q) = \int \ell^{i(\mathbf{q}-\mathbf{k}_p)\mathbf{R}} \psi_{nl}(R) Y_{lm}(\theta_p, \varphi_p) d\Omega \quad , \qquad (15)$$

and

$$W(R) = \left\langle f \left| \int \psi^*_{A-1}(\mathbf{r}) \; U(|\mathbf{R}-\mathbf{r}|)\psi_{A-1}(\mathbf{r}) d\mathbf{r} \right| i \right\rangle , \qquad (16)$$

Explicit expressions are given in functional a_{n1} and a_{n2} are presented in [7].

An explicit expression for U_0 potential in the center of the nucleus as well as a, b_0 options, depending on the distribution of nucleon density in nuclei, as well as β_0^2 parameter (the slope of the diffraction cones), which is a part of the amplitude of free NN – interactions, are presented in [7].

To calculate the integral of spherical functions included in (15) we use the plane wave expansion of spherical functions:

$$\ell^{i(\mathbf{q}-\mathbf{k}_p)\mathbf{R}} = \sum_{L=0}^{\infty} i^L \sqrt{4\pi(2L+1)} \; j_L(|\mathbf{q}-\mathbf{k}_p| R) \mathrm{Y}^*_{L0}(\theta), \qquad (17)$$

Considering the orthogonally of which we obtain

$$I(q)=\sum_{L=0}^{\infty} i^L \sqrt{4\pi(2L+1)}\ \ j_L(|\mathbf{q}-\mathbf{k}_p|R)\psi_{nL}(R) \quad , \tag{18}$$

here $j_L(|\mathbf{q}-\mathbf{k}_p|R)$ - Bessel function and

$$|\mathbf{q}\text{-}\mathbf{k}_p|=\sqrt{q^2-2|\mathbf{q}||\mathbf{k}_p|\cos(\tfrac{\pi}{2}-\vartheta_1+\vartheta_3)+k_p^2}\ . \tag{19}$$

It is known that the experiments on the angular distribution of the reaction products of direct interaction provides insights about the properties of the energy levels of nuclei.

Therefore, the expression (18) allows determining L for the angular distribution of emitted protons.

Finally, we pass to the calculation of integral (16) on the potential of interaction of the ejected proton with the nucleons of the residual nucleus, the potential of the two partially interaction is taken into account in the asymptotic form due to one-pion exchange, which allows us to write

$$W=-\frac{4\pi\hbar^2}{m}\gamma\int\frac{\ell^{-k_0|\mathbf{R}-\mathbf{r}|}}{|\mathbf{R}-\mathbf{r}|}\rho(\mathbf{r})d\mathbf{r} \tag{20}$$

here $\gamma=g^2/\hbar c=0.081\pm0.002$ is the coupling constant, determining the value of the potential, obtained by analyzing the scattering π - meson on nucleons, reverse value of which $k_0=\frac{m_\pi c}{\hbar}$, corresponds the radius of action of nuclear forces.

To calculate the integral (20) the distribution of nucleons density in the ground state of the residual nucleus is chosen in the form of the Fermi - functions.

$$\rho_N(r)=\rho_0(1+\ell^{\frac{r-R}{b}})^{-1} \tag{21}$$

Applying the method of pole calculating of integral [7], we get:

$$W=W_0(b)\ell^{-k_0R}(\cos bk_0-i\sin bk_0) \tag{22}$$

Where

$$W_0(b)=i(2\pi)^3\frac{8\gamma\hbar^2}{m}bR\rho_0\{1+\frac{3i}{2}\frac{\pi b}{R}-\frac{1}{2}(\frac{\pi b}{R})^2\} \tag{23}$$

We examined the knock-out of protons from the nuclei by nucleons scattering the, interaction of (n, np)-type. However the obtained results can be directly applied to the reactions (p, nn), (p, 2p) and (n, 2n) as well, because the Coulomb interaction is ignored.

As direct interaction is essential at middle and high energies of nucleons, the influence of Coulomb interaction on the angular distribution is of little importance. Thus, the final expression of the differential cross section of quasielastic scattering of nucleons by nuclei is written as:

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_f} = \aleph_0(q)\sum_{L=0}^{\infty} i^{2L}(2L+1)\left|\psi_{n'L}(R)\sum_{n=0}^{3} a_n(q)\frac{\partial^n j_L(|\mathbf{q}-\mathbf{k}_p|)}{\partial q^n}\right|^2 \tag{24}$$

where

$$\aleph_0(q) = (2\pi)^5\hbar^4 m^2\sigma_{NN}^2(1+\varepsilon_0^2)\ell^{-\beta_0^2 q^2}|W(R)|^2 E_f^{1/2}(E_i - E_f - E_N - E_R)^{1/2} \tag{25}$$

As in the future these reactions will be studied on light nuclei, it is desirable to give explicit expressions of radial wave functions of nucleons $\psi_{n'L}(R)$ for 1s and 1p states in nuclei obtained from the solution of nonrelativistic Schrodinger equation for a spherically symmetric potential:

$$\psi_{1s}(R) = 2\frac{R}{a_0}\pi^{-\frac{1}{4}}exp(-\frac{R^2}{2a_0^2}) \tag{26}$$

$$\psi_{1p}(R) = \left(\frac{8}{3}\right)^{\frac{1}{2}}\frac{R^2}{a_0^2}\pi^{-\frac{1}{4}}exp(-\frac{R^2}{2a_0^2}) \tag{27}$$

The proposed approach allows calculating the differential cross-section of nucleons knockout by protons with energy $T_p = 1GeV$, using the variation of the parameter b, characterizing the thickness of the surface of nucleus. Results of specific calculations of the reactions A (p, 2p) in nucleus ^{16}O compared with the experimental data are shown in Fig. 2.

The calculations are mainly carried out for different angles of emission of slow protons ($\vartheta_3 = 61^\circ;64^\circ;67^\circ;73^\circ$) at a fixed angle of scattering of fast protons $\vartheta = 13{,}4^\circ$. The figure shows the results only for the angle of departure of slow protons $\vartheta_3 = 61^\circ$. Analysis of the results shows that the scattering cross section of protons depends on the angles weakly.

Nuclei of ^{16}O and ^{12}C can emit protons from levels 1p and 1s so the differential cross section was calculated for every of these cases. In the experiment slow protons were recorded at energies E = (60-105) MeV (The agreement between theoretical and experimental cross sections at recording of fast protons take place namely at these energies of slow protons.)

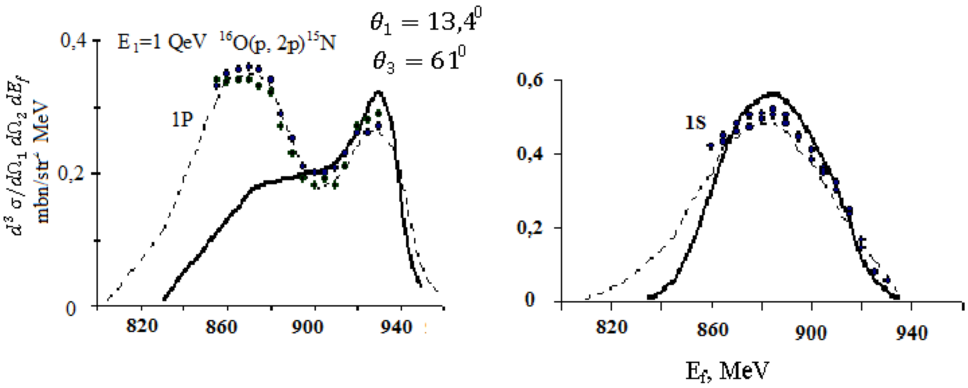

Fig.2. Experimental (dots) and theoretical (solid line) differential cross sections of the reaction of quasielastic knockout of protons from sub shells 1p and 1s ^{16}O nucleus at angles $\vartheta_1 = 13{,}4^\circ$, $\vartheta_3 = 61^\circ$. Dashed lines-the results of investigation [8], calculated with Hartri-Fock wave functions considering excitation of nucleus.

As it seen from fig.2 and 3, theoretical graphs obtained in presented article on knockout protons from level 1*p*, unlike the experimental, have only one maximum. Apparently, this is consequence of examination the absolute quasielastic proton knockout, so residual nucleus does not excite. However, the displacement of the maximum towards high energies is connected with not taking into account the distortion in the wave function of knocked out protons.

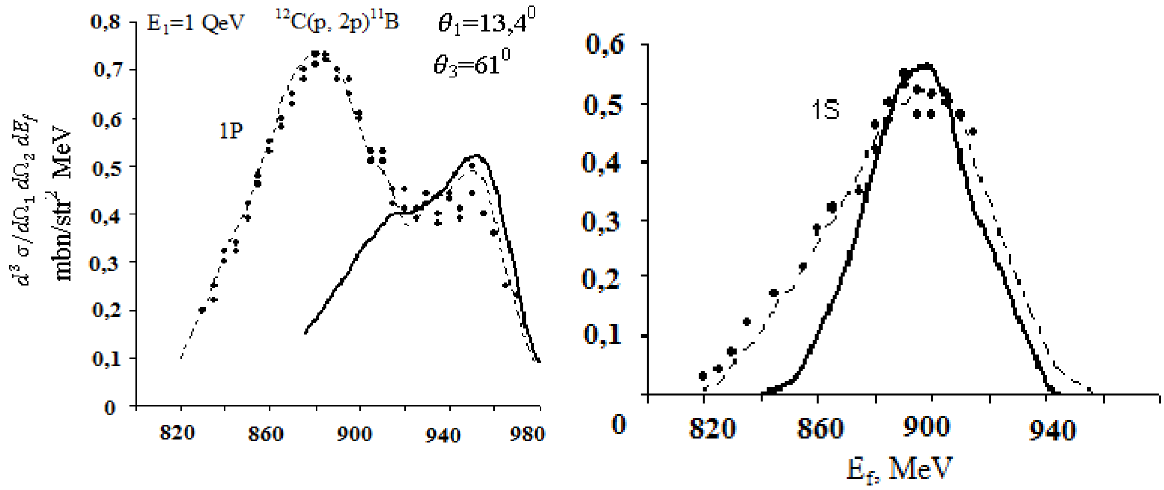

Fig. 3. Experimental (dots) and theoretical (solid line) differential cross sections of the reaction of quasielastic knockout of protons from sub shells 1p and 1s ^{12}C nucleus at angles $\vartheta_1 = 13{,}4^\circ$, $\vartheta_3 = 61^\circ$. Dashed lines-the results of investigation [8], calculated with Hartri-Fock wave functions considering excitation of nucleus.

The theoretical curves, calculated in distorted-wave impulse approach, where for the nucleus nucleons Hartri-Fock wave functions were used, are presented for comparison on fig. The authors of reference [8] considered the excitation of the residual nucleus, as well.

On the of abovementioned theory the calculated differential cross sections of the reaction on quasielastic proton knock out from sub shells 1p and 1s in nuclei ^{16}O and ^{12}C allow to define orbital momentums of these nucleons in nuclei before scattering by angular distribution of emitted protons.

So, concluding, we may confirm that similar derivations are convenient for the practical use with analytical wave functions of nucleus.

From the analysis of the obtained data it follows that the reaction of the knock out requires more accurate revision inter nucleus wave functions, in particular, more exact accounting of excitation the residual nucleus and distortions in the wave functions of knocked out from nuclei nucleons.

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