A Clustering Algorithm For The Capacitated Vehicle Routing Problems With Stochastic Demands

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ABSTRACT

In today's world, logistics problems are crucial for the supply chain management. The vehicle routing problems are one of the most studied combinatorial optimization problems in the logistics literature. In real life applications, all of the parameters of the problem may not be known. In this paper, we considered the capacitated vehicle routing problem with stochastic demands. Uniform and normal distributions were analyzed on customer demands to observe the stochastic nature of the problem. These methods were compared by using GAMS with different test problems which were taken from the literature. The clustering analysis including the K-means algorithm also applied on large-sized test problems. All of the obtained results were presented in detail.

Keywords: Clustering Analysis, K-Means, Modelling, Stochastic Demand, VRP

Stokastik Talepli Kapasite Kısıtlı Araç Rotalama Problemleri için Bir Kümeleme Algoritması

ÖZ

Günümüz dünyasında, tedarik zinciri yönetimi için lojistik problemler çok kritiktir. Araç rotalama problemi, lojistik literatüründe en çok çalışılan kombinatoryal optimizasyon problemlerinden biridir. Gerçek hayat uygulamalarında problemin tüm parametreleri bilinmeyebilir. Bu çalışmada, stokastik talepli kapasite kısıtlı araç rotalama problemi ele alınmıştır. Problemin stokastik yapısını gözlemlemek için müşteri talepleri düzgün ve normal dağılımlar ile analiz edilmiştir. Bu yöntemler, literatürden alınan farklı test problemleri ile GAMS kullanılarak karşılaştırılmıştır. Büyük boyutlu test problemleri için k-ortalamalar algoritmasını içeren kümeleme analizi uygulanmıştır. Elde edilen tüm sonuçlar ayrıntılı olarak gösterilmiştir.

Anahtar Kelimeler: Kümeleme Analizi, K-Ortalamalar, Modelleme, Stokastik Talep, ARP

INTRODUCTION

Today, transportation is an important problem to find the best routes with minimum costs for many different firms. There are different types of transportation problems. Vehicle routing problems (VRP) are mainly considered in this area. VRPs are first studied by Dantzig and Ramser [1]. After this, there have been many studies on this topic. The most studied variants of VRPs are capacitated VRP, open VRP, VRP with time windows, split delivery VRP, VRP with backhauls etc.
In the nature of the problem, parameters are generally known in advance. But in real life, all parameters may not have to be known. Therefore, several of the problem parameters such as demand, time, distance and others may be stochastic [2,3,4,5]. Therefore, the stochastic VRP is an important variant of the VRP. Stochastic VRP considers one or more parameters of the problem as stochastic or unknown during the planning horizon. These parameters may be the presence of the customers, the nature of the customer demand, service time, time windows etc.
The most common variants of stochastic VRP are VRP with stochastic customers (Dror and Trudeau [2]; Laporte et al. [3]). Smith et al. [4] considered stochastic demand dynamic VRP with multiple vehicles priority. Tripathi et al. [5] analyzed stochastic customer demand using ant based simulation. Erera et al. considered the same problem with duration constraints [6]. Moghaddam et al. [7] considered stochastic demand by using advanced particle swarm optimization. Goodson [8] studied stochastic customer demand. They analyzed the problem with the cyclic order simulated annealing procedure to solve the multi-compartment VRP. Niu et al. [9] studied multi-objective algorithm for green vehicle routing problem with stochastic demands. Florio et al. [10] analyzed the branch-and-price algorithm for the vehicle routing problem with stochastic demands. Gaur et al. [11]

improved approximation algorithms for cumulative VRP with stochastic demands. Xia et al. [12] presented a discrete algorithm for the regarding problem. Salavati-Khoshghalb et al. [13] proposed an exact algorithm for the VRP with stochastic demand. Islam et al. [14] studied the mixed fleet based green clustered logistics problem under CO2 emission cap.

There are also different papers focusing on clustering. Kamisli and Durak [15] studied capacitated, open and heterogeneous fleet vehicle routing problems by applying the cluster first-route second method to solve these problems. In the research report of Bruwer [16], the k-medoid clustering method is modeled and evaluated for the Capacitated VRP. Comert et al. [17] studied capacitated VRP. All customers were clustered using K-means, K-medoids and random clustering with considering vehicle capacity.

The objective of the problem is minimizing total routing cost. In real life, there are not always deterministic parameters for the regarding problem to solve (Alpaslan Takan and Agayeva [18]). Therefore, the stochastic data must be included in the model to better describe the real world. In this paper, we studied the capacitated vehicle routing problems with stochastic demands. To analyze the problem, two different distributions are used which are uniform and normal. After handling the problem as stochastic, we used clustering analysis to solve large-sized test problems easily. All computational results are explained in detail.

The remainder of this paper is organized as follows. The details of the mathematical model, methods used for stochastic demands and the clustering analysis including the k-means algorithm are given in the materials and method part. The computational results are demonstrated. Finally, concluding remarks are provided.

MATERIALS AND METHODS

Mathematical Model of the Problem

The three-index formulation of the capacitated vehicle routing problem which was introduced by Toth and Vigo [19] is given below:

Notation

n: total number of customers
K: total number of vehicles
i,j: customer indices $i,j = \{1,\dots,n\}$. 1 is the depot location.
k: set of vehicles $k = \{1,\dots,K\}$
d_i: demand of customer i
Q_k: capacity of vehicle k
c_{ij}: cost of traveling from customer i to customer j

Decision variables

$$x_{ijk}: \begin{cases} 1, & \text{if vehicle k travels from customer i to customer j} \\ 0, & \text{otherwise} \end{cases}$$

Positive variables

u_i, u_j

The objective function:

$$Min\ z = \sum_{i=1}^{n}\sum_{j\neq i}^{n}\sum_{k=1}^{K} c_{ij}x_{ijk} \quad (1)$$

Constraints:

$$\sum_{i=1}^{n}\sum_{j\neq i}^{n} d_i\, x_{ijk} \leq Q_k \qquad \forall k \quad (2)$$

$$\sum_{i=1}^{n} x_{imk} - \sum_{j=1}^{n} x_{mjk} = 0 \quad m = 2,..,n;\ \forall\, k \quad (3)$$

$$\sum_{i=1}^{n}\sum_{k=1}^{K} x_{ijk} = 1 \qquad j = 2,..,n \quad (4)$$

$$u_i - u_j + n\sum_{k=1}^{K} x_{ijk} \leq n-1 \quad i,j = \overline{2,n}\,, i \neq j \quad (5)$$

$$x_{ijk} \in \{0,1\} \qquad \forall\, i,j,k \quad (6)$$

The objective function (1) is the minimization of the total traveling cost. Constraint (2) describes the capacity constraint. The total capacity of the route can not exceed the capacity of that vehicle. Constraint (3) states the vehicle flow. If a vehicle travels from city i to city m, this vehicle also travels from city m to city k (or depot). Constraint (4) states that a vehicle must travel to each city exactly once. Constraint (5) is the subtour elimination constraint which is described by Miller et al. [20]. Constraint (6) describes the binary variable.

Methods Used For Stochastic Demands

The parameters Q_k, d_i and c_{ij} in the model of the capacitated vehicle routing problem examined above are deterministic. For the stochastic part of this model which we are considering, demand values are taken with random variables (Agayeva and Alpaslan Takan [21]). To take into account various uncertain situations, demands are analyzed with two different approaches as uniform and normal distribution. Below explains the application details of these distributions.

a)Uniform distribution: It refers to situations that correspond to the homogeneous division of claims. In this case, we simulate the demands as a uniformly distributed random variable defined in the interval between the smallest demand and the largest demand, in other words on the interval : $[d_{min}, d_{max}]$:$D \sim U[d_{min}, d_{max}]$

b) Normal distribution: It is a type of continuous probability distribution for a real-valued random variable. The second type for simulation of stochastic demands is Normal distribution $N(\mu,\sigma^2)$; where $\mu = \overline{D}_a$ and $\sigma^2 = s_D^2$.

Clustering Analysis

Clustering analysis is a common technique for statistical data analysis. It is used in many fields such as pattern recognition, image analysis, bioinformatics, data compression, computer graphics, machine learning etc. In this paper, for clustering the customers, we have considered K means algorithm. This method has the advantage to solve large-sized vehicle routing problems easily. The main steps of clustering algorithms are given below.

Step 1. Cluster the customers regarding their coordinates. Weka is used for K-means algorithm.

Step 2. Solve VRP with stochastic demands for each cluster by using GAMS. For each cluster, total costs are calculated.
In the following, the k-means algorithm is explained in detail.

K-Means Algorithm

The k-means algorithm is an unsupervised learning and clustering algorithm. The K-means clustering method is to partition a dataset consisting of N data objects into K clusters given as input parameters. The aim is to ensure that the clusters obtained at the end of the partitioning process have maximum similarities within clusters and minimum similarities between clusters. K means algorithm assumes Euclidean space. It picks one point at random, then k-1 other points, each as far away as possible from the previous points.

i. Initialize the center of clusters
ii. Attribute the closest cluster to each data point
iii. Set the position of each cluster to the mean of all data points belonging to that cluster.
iv. Repeat steps 2-3 until convergence.

RESULTS AND DISCUSSION

To analyze the effect of the inclusion of uncertainty in the model, Augerat [22] and Christofides et al.'s [23] test problems were studied (data was taken from http://vrp.atd-lab.inf.puc-rio.br/index.php/en/). First, VRP with stochastic demand problem was solved by GAMS. All test problems could not be solved by GAMS. For this reason, clustering algorithms were applied and then all costs for each cluster was calculated for the total routing costs. Table 1 shows the detailed computational results obtained by GAMS/Cplex solver. To solve the problems, the time limit of GAMS was given 1 hour for each problem. First column of Table 1 indicates the name of the studied test problem (For example: A-n32-k5 means 32 customers and 5 vehicles), the second column shows the vehicle capacity, the third column is the optimal solution of the problems. Column 4 presents the solution found when the customer demands have a uniform distribution. The last column shows the results of problems when the customer demands have a normal distribution.

Table 1. Solution of the problem with stochastic demand

Test problem	Vehicle capacity	Optimal solution	Uniform distribution	Normal distribution
A-n32-k5	100	784	1002.3	988.76
A-n44-k6	100	937	1125.6	1114.1
A-n54-k7	100	1167	...	...
B-n31-k5	100	672	959	956.38
B-n38-k6	100	805	936.5	924.6
B-n45-k5	100	751	1012.2	1003.8
B-n57-k9	100	1598	...	...
M-n101-k10	200	820	...	...
M-n121-k7	200	1034	...	....
M-n151-k12	200	1053	...	....

The results show that when the number of customers are greater than 45, GAMS/Cplex could not find the solution in the given time limit(for the problems A-n54-k7, B-n57-k9, M-n101-k10, M-n121-k7, M-n151-k12). For the other instances, normal distribution gave better results than uniform distribution.
In Table 2, solutions of the clustered problem with stochastic demand were given for both distributions.

Table 2. Solution of the clustered problem with stochastic demand

Test problem	Vehicle capacity	Optimal solution	Uniform distribution	Normal distribution
A-n32-k5	100	784	1102	996.4
A-n44-k6	100	937	1229	1365.1
A-n54-k7	100	1167	1332.5	1342.2
B-n31-k5	100	672	865	993.5
B-n38-k6	100	805	984.4	966.3
B-n45-k5	100	751	1004	991.1
B-n57-k9	100	1598	1755.6	1886.3
M-n101-k10	200	820	1222.5	1116.9
M-n121-k7	200	1034	1365	1456.2
M-n151-k12	200	1053	1544.4	1226.8

From Table 2, It is observed that all large-sized test problems could be solved by clustering k-means method. First all problems were clustered, and then total costs for each cluster were calculated. For instance, for the problem M-n151-k12, normal distribution gave better result but for the problem M-n121-k7, uniform distribution found better solution than normal.
Figure 1 shows an example of the WEKA clustering of the M-n101-k10 test problem by using the k-means algorithm. There are 10 clusters for the regarding problem. Each cluster is shown by different colors (The blue point in the middle of the graph indicates the depot location).

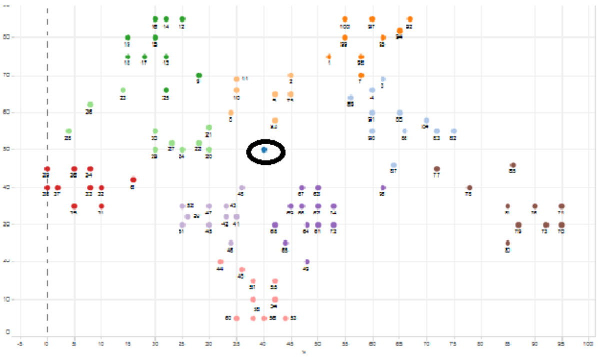

Figure 1. An example of the WEKA clustering by using the k-means algorithm (test problem:M-n101-k10)

CONCLUSIONS

Within the scope of this study, the stochastic demand vehicle routing problem, which is frequently encountered in the literature is discussed. Today, stochastic models are of great importance for expressing these situations, as sudden and unexpected changes are encountered a lot in real life problems. These models emphasize the consideration of previously unnoticed or unexpected effects. Therefore, stochastic models reflect real life better than deterministic models. The problem under consideration has different application areas in real life. In [24], the authors focus on the optimal control problem for stochastic switching systems with the quadratic cost function. In [18], the authors also analyzed the stochastic demand VRP for the small-sized test problems and different test problems which consider a maximum number of 23 customers were studied by uniform, exponential and poisson distributions. The papers [6,7,10] consider the vehicle routing problem with stochastic demand. Unlike these studies, in this paper, stochastic demand VRP is considered for the large sized problems by using clustering methodology (maximum number of customers were taken 151). Two different distributions which are uniform and normal also applied to the studied test problems. By working on 10 different test problems, the results obtained by GAMS/Cplex solver were compared in detail. Generally, better results were observed when customer demands fit the normal distribution. Since GAMS could not find a solution when the problem size increases, the clustering method k-means was used for obtaining the solution for the regarding problems. In this way, large-scale problems could also be solved easily. This is the motivation and the difference of this paper.
Different clustering approaches may be used in future studies. The advantages of the methods can be discussed by using different clustering approaches. The applied methods may also be analyzed in more detail by considering different test problems from the literature.

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