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Research Article / Araştırma Makalesi

Performance Evaluation of Magnitude-Based Fuzzy Analytic Hierarchy Process (MFAHP) Method

Magnitüde Bağlı Bulanık Analitik Hiyerarşi Süreci (MBAHS) Yöntemi Performans Değerlendirmesi

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ABSTRACT

In Analytic Hierarchy Process (AHP), which is a very common method in Multi-Criteria Decision Making (MCDM) problems, the use of fuzzy set theory, which allows human judgment to be expressed more realistically, has gained popularity in recent years. However, this situation causes more computational complexity due to the way fuzzy numbers are expressed and the operators used. In this study, the results of real various problems in a hierarchical structure with the Magnitude Based Fuzzy Analytic Hierarchy Process (MFAHP) were compared with the results of the Modified Fuzzy Logarithmic Least Squares method (MFLLSM) and Buckley's Geometric Means method (GM), which are two known methods to obtain accurate weight values. The results show that there is no statistically significant difference between MFAHP and the results of these two methods. In the performance comparison, although it is known that it produces incorrect results, unfortunately, the results of Chang's Extent Analysis method on fuzzy AHP (FEA) are also included because it is a widely used method. As another important finding of this study, it can be said that MFAHP is faster than both methods when the running times are compared. Finally, software for the calculations of these methods mentioned in the study has been developed and link shared.

Keywords: Fuzzy multi-criteria decision making, fuzzy analytic hierarchy process, performance evaluation

ÖZ

Çok Kriterli Karar Verme (ÇKKV) problemlerinde oldukça yaygın bir yöntem olan Analitik Hiyerarşi Sürecinde (AHS), insan yargısının daha gerçekçi bir şekilde ifade edilmesini sağlayan bulanık küme teorisinin kullanımı son yıllarda önem kazanmıştır. Ancak bu durum, bulanık sayıların ifade edilme şekli ve kullanılan operatörler nedeniyle daha fazla hesaplama karmaşıklığına neden olmaktadır. Bu çalışmada, hiyerarşik yapıdaki çeşitli gerçek hayat problemlerinin Magnitüde Bağlı Bulanık Analitik Hiyerarşi Süreci (MBAHS) ile elde edilen sonuçların doğruluğu, doğru ağırlık değerleri elde etmekte kullanılan iki yöntem olan Modifiye Bulanık Logaritmik En Küçük Kareler yöntemi (MFLLSM) ve Buckley'nin Geometrik Ortalamalar yöntemi (GM) sonuçları ile karşılaştırılmıştır. Sonuçlar, MBAHS ile bu iki yöntemin sonuçları arasında istatistiksel olarak anlamlı bir fark olmadığını göstermektedir. Performans karşılaştırmasında hatalı sonuçlar ürettiği bilinse de ne yazık ki yaygın olarak kullanılan bir yöntem olduğu için bulanık AHS'de Chang'in Extent Analizi (CEA) yöntemi sonuçları da yer almaktadır. Bu çalışmanın bir diğer önemli bulgusu olarak çalışma süreleri karşılaştırıldığında da MBAHS'nin her iki yöntemden daha hızlı olduğu söylenebilir. Son olarak çalışmada adı geçen bu yöntemlerin hesaplamalarının yapılabileceği bir yazılım geliştirilmiş ve bağlantısı paylaşılmıştır.

Anahtar Kelimeler: Bulanık çok kriterli karar verme, bulanık analitik hiyerarşi süreci, performans değerlendirmesi

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1. INTRODUCTION

In its most general definition, decision-making problems are described as the problem of one or more decision-makers choosing one or more of the alternatives by considering various criteria and possibly the sub-criteria of these criteria. However, as the number of decision-makers, criteria, and alternative concepts, which are the fundamental concepts of this definition, increases, the problem's solution becomes quite complex. For this reason, Multi-Criteria Decision Making (MCDM) methods are used to help decision-makers choose alternatives, especially in complex decision-making processes, by considering the criteria analytically..

The Analytic Hierarchy Process (AHP) method proposed by Saaty (1977; 1980) is widely preferred among the different MCDM methods. Prioritization, resource allocation, business process reengineering, quality management, and planning are some of the domains where the AHP method is applied (Vaidya & Kumar, 2006; Korkmaz, Gökçen & Çetinyokuş, 2008; Amiri, 2010; Dweiri, Kumar, Khan & Jain, 2016).

The reasons for the widespread use of this method are the hierarchical structure of the problems, the ease of calculation, and the calculation of both criterion weights and alternative priorities. AHP provides a hierarchical structure for a problem, starting with the goal and continuing through criteria, sub-criteria, and alternatives. This method enables decision-makers to systematically evaluate relations and see more clearly what to compare. Thus, the global priorities of the alternatives are determined by making pairwise comparisons of the necessary elements. Pairwise comparisons, on the other hand, are frequently ambiguous. Because the number of elements to be compared increases, it becomes difficult to compare each pair with exact numbers (Xu & Liao, 2013). As it is known, linguistic expressions in Saaty's relative importance scale are represented in ascending order of importance, from 1 for "equally important" to 9 for "extremely more important". However, studies on fuzzy AHP have started to increase rapidly with the use of fuzzy numbers with the thought that they can better reflect these linguistic expressions instead of the integers defined for the linguistic expressions in the scale. As a result, fuzzified techniques have gained popularity in recent decades, and various fuzzy AHP (FAHP) methods have been presented (Liu, Eckert & Earl, 2020). However, integrating the concept of fuzziness into the AHP complicates the computational process. The use of fuzzy sets in AHP, on the other hand, makes the computing process more difficult. For this reason, FAHP methods, which give accurate results similar to the classical AHP method and can be applied easily at the same time, should become widespread.

The main motivation of this study is to evaluate the results of the Magnitude Based Fuzzy Analytic Hierarchy Method (MFAHP), a new FAHP method proposed by (Kinay & Tezel, 2022), on real examples. In this evaluation, the results of the Geometric Mean Method (GM) (Buckley, 1985) and Modified Fuzzy Logarithmic Least Squares Method (MFLLSM) (Wang, Elhag & Hua, 2006) were considered. Because using these two methods, accurate results are obtained. However, both methods, especially MFLLSM, are difficult to apply, especially in social sciences, and the computational load is high. In addition, Fuzzy Extent Analysis (FEA) (Chang, 1996) results are also included in the comparisons. This method is widely used due to its easy application, but unfortunately, it produces wrong results and the necessity of not using it has been mentioned in various studies (Liu et al., 2020; Ahmed & Kiliç, 2019). Although the computational load is significantly lower than other methods, studies are still being carried out to cope with the rapidly increasing computational load depending on the problem structure and representation. In summary, a parallel computing method has been developed (Balli & Bahadır, 2013), which allows the system to run faster and increases efficiency and performance for FEA operations that require a large number of computation-intensive operations. In this study, according to the results obtained from real numerical examples, the fact that the working time of the MFAHP method is not significantly different from the FEA can be considered an important advantage of the MFAHP method over the MFLLSM and GM methods, where accurate results are obtained. Therefore, the research hypothesis of our study is that the MFAHP method will provide accurate results while requiring less computation time compared to the GM and MFLLSM methods on real examples.

The main research contributions (RC) of this study can be summarized as follows:

- **RC1:** It is statistically shown that the results obtained using the MFAHP method are as accurate as the results obtained using the GM and MFLLSM methods.
- **RC2:** It is shown that the MFAHP method is faster in terms of computation time.
- **RC3:** Software has been developed and shared for the solution of all methods used in the study.

The rest of this paper is structured as follows. Related studies are presented in Section 2. The information used in the content of the study and the application of the MFAHP method on an example and the results are described in Section 3. The performance comparisons of the MFAHP with the MFLLSM, the GM, and the FEA are presented in Section 4. And finally, conclusions will be highlighted in Section 5.

2. RELATED WORKS

In (Mardani, Jusoh & Zavadskas, 2015), the applications and methods of fuzzy Multi-Criteria Decision Making (FMCDM) methods have been reviewed and it has been said that AHP and FAHP methods are the most preferred methods in decision-making problems. In 1983, the first method of FAHP was proposed (Van Laarhoven & Pedrycz, 1983). The Fuzzy Logarithmic Least Squares Method (fuzzy LLSM) is used to derive weights in the type of triangular fuzzy numbers from pairwise comparison matrices containing triangular fuzzy numbers. Buckley (1985) used the trapezoidal numbers and the geometric mean approach to achieve the fuzzy pairwise comparison. Kwiesielewicz (1996) presented a generalized pseudo-inverse approach that used spectral decomposition to solve the fuzzy LLSM. In (Boender, 1989), a change in the normalization process is proposed to prevent deviations in weight values resulting from the normalization process used in the fuzzy LLSM method. Ruoning and Xiaoyan (1996) developed a fuzzy LLSM depending on the notion of distance in a fuzzy evaluation scale. In (Chang, 1996), the extent analysis method on fuzzy AHP (FEA) was proposed by Chang by obtaining synthetic extent values of pairwise comparisons. Büyüközkan et al. (2004) provide a survey of FAHP algorithms with their main features, advantages, and disadvantages. The fuzzy LLSM approach presented in (Van Laarhoven & Pedrycz, 1983) was developed by (Wang, Luo & Hua, 2008) and it was named modified fuzzy LLSM (MFLLSM). This method can be expressed as a constrained nonlinear optimization model proposed such that normalized triangular fuzzy weights can be obtained.

It is also mentioned in (Wang, Luo & Hua, 2008) that real weights cannot be obtained with the FEA method, and this may lead to wrong decisions. However, Kubler et al. (2016) emphasize that owing to its simplicity of use, it is still a popular method in many domains. According to Ahmed & Kiliç (2019), FEA produces the least accurate results.

The main purpose of this study is to evaluate the performance of MFAHP results on real examples in a hierarchical structure and to show that this method gives accurate results and has a low computational load. In order to derive weight values or in other words the priority vectors, in the MFAHP method, the magnitude value of each fuzzy number was considered as suggested in (Abbasbandy & Hajjari, 2009). Studies show that in fuzzy AHP methods, comparison judgments are expressed as triangular fuzzy numbers in pairwise comparison matrices at a rate of 91% (Lie et al., 2020). Therefore, for the magnitude calculation used in the MFAHP method, the method suggested by (Abbasbandy & Hajjari, 2009) was preferred, which gives sufficient results in the comparison of fuzzy triangular numbers.

In this study, all examples used in calculations were obtained from articles published in indexed journals. Therefore, we did not check again the consistencies of the fuzzy pairwise comparison matrices which are used as the preference relations in the examples mentioned in Section 4.

3. BASIC CONCEPTS AND METHODS

3.1. Fuzzy Membership Function

The concept of fuzzy sets, first proposed in (Zadeh, 1965), is used to solve problems with ambiguous descriptions. Fuzzy sets can be thought of as a general representation of crisp sets and these are the sets of objects defined by a membership function. The membership function determines the degree of belonging of the elements to the related set. The degree of belonging of the elements to the set can take all membership degrees from "does not belong to the set" to "belongs to the set". That is, the degree of belonging of the elements to the related set is defined in [0,1].

The following is the definition of the triangular fuzzy membership function as used in the examples in Section 4 of this study.

Definition 1. $A = (l,m,u)$ on U=$(-\infty, \infty)$ is expressed as a triangular fuzzy number, and its membership function $\mu_A: U \rightarrow [0,1]$ is given as:

$$\mu_A(x) = \begin{cases} \frac{(x-l)}{(m-l}, & l < x < m \\ 1, & x = m \\ \frac{(u-x)}{(u-m)}, & m < x < u \\ 0, & otherwise \end{cases} \tag{1}$$

3.2. Extent Analysis on Fuzzy AHP

The extent analysis on fuzzy AHP (FEA) was proposed by Chang (1996) and it is summarized as follows.

The following is a fuzzy pairwise comparison matrix with judgments expressed as triangular fuzzy membership functions:

$$A = (a_{ij})_{n\times n} = \begin{bmatrix} (1,1,1) & (l_{12}, m_{12}, u_{12}) & \dots & (l_{1n}, m_{1n}, u_{1n}) \\ (l_{21}, m_{21}, u_{21}) & (1,1,1) & \dots & (l_{2n}, m_{2n}, u_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (l_{n1}, m_{n1}, u_{n1}) & (l_{n2}, m_{n2}, u_{n2}) & \dots & (1,1,1) \end{bmatrix} \quad (2)$$

where $a_{ij}=(l_{ij},m_{ij},u_{ij})$, and $a_{ji}= a_{ij}^{-1}=(1/u_{ij},1/m_{ij},1/l_{ij})$ for i,j = 1,. . . n, i≠j. First, the sum values for the rows of the fuzzy pairwise comparison matrices are obtained as follows:

$$RS_i = \sum_{j=1}^{n} a_{ij} = \left(\sum_{j=1}^{n} l_{ij}, \sum_{j=1}^{n} m_{ij}, \sum_{j=1}^{n} u_{ij} \right), i = 1, ..., n. \quad (3)$$

Then in the second step, the row sums are normalized as in Equation (4).

$$S_i = \sum_{j=1}^{n} a_{ij} \otimes \left[\sum_{k=1}^{n} \sum_{j=1}^{n} a_{kj} \right]^{-1} = \frac{RS_i}{\sum_{j=1}^{n} RS_j} = \left(\frac{\sum_{j=1}^{n} l_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} u_{kj}}, \frac{\sum_{j=1}^{n} m_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} m_{kj}}, \frac{\sum_{j=1}^{n} u_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} l_{kj}} \right), i = 1, ..., n. \quad (4)$$

In the third step, Equation (5) is used to calculate each possibility value:

$$V(S_i \geq S_j) = \begin{cases} 1 \quad , & m_i \geq m_j \\ \dfrac{(u_i - l_j)}{(u_i - m_i) + (m_j - l_j)}, & l_j \leq u_i, i, j = 1, \dots, n; j \neq i \\ 0 \quad , & otherwise \end{cases} \quad (5)$$

where $S_i=(l_i,m_i,u_i)$ and $S_j=(l_j,m_j,u_j)$ and $V(S_i \geq S_j)$ value is shown in Fig.1.

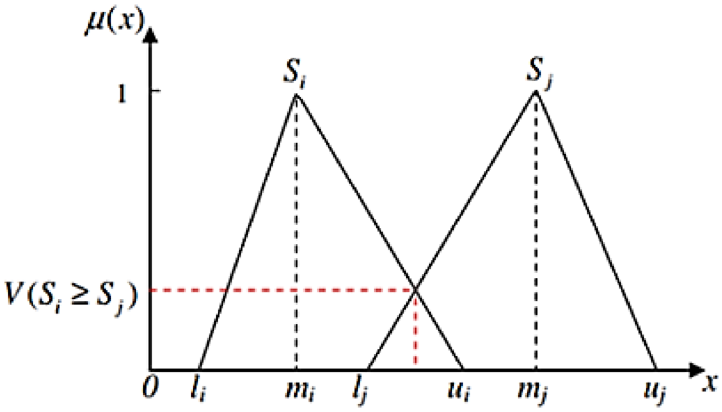

Figure 1. . Graphical representation of $V(S_i \geq S_j)$.

In the fourth step, the degree of possibility of S_i over all other (n-1) fuzzy numbers is calculated as:

$$V(S_i \geq S_j \mid j = 1, \dots, n; j \neq i) = \min_{j\in \{1,\dots,n\}, j \neq i} V(S_i \geq S_j, i = 1, \dots, n. \quad (6)$$

Finally, the weight values are obtained as follows.

$$w_i = \frac{V(S_i \geq S_j \mid j = 1, \ldots, n; j \neq i)}{\sum_{k=1}^{n} V(S_k \geq S_j \mid j = 1, \ldots, n; j \neq k)}, i = 1, \ldots, n. \tag{7}$$

where the weight values are crisp values.

3.3. Geometric Mean Method

The Geometric mean method was proposed by Buckley in 1985. In the first step, for each fuzzy pairwise comparison matrix expressed as in Equation (2), the geometric mean of each criterion is calculated as follows:

$$z_i = \left(\prod_{j=1}^{n} a_{ij}\right)^{1/n}, i = 1, \ldots, n. \tag{8}$$

Then, weight values r$_i$ of each criterion or each alternative are obtained by Equation (9) as follows:

$$r_i = z_i \otimes [z_1 \oplus z_2 \oplus \ldots \oplus z_n]^{-1} \tag{9}$$

In the third step, obtained weight values are converted into crisp values by using the Center of Area defuzzification method as in Equation (10):

$$S_i = \frac{l_i + m_i + u_i}{3}, i = 1, \ldots, n. \tag{10}$$

In the final step, these weight values are normalized by Equation (11) to obtain the normal weight values.

$$w_i = \frac{S_i}{\sum_{i=1}^{n} S_i}, i = 1, \ldots, n. \tag{11}$$

where the weight values are crisp values.

3.4. Modified Fuzzy Logarithmic Least Squares Method

The MFLLSM is improved by Wang et al. (2006) to determine the local fuzzy weights of the fuzzy pairwise comparison matrix in Equation (2). Each decision problem that was used in calculations in Section 4 has only one decision-maker. Therefore, the way the method is defined in (Wang et al., 2008) is as follows.

$$minJ = \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \left((lnw_i{}^L - lnw_j{}^U - lnl_{ij})^2 + (lnw_i{}^M - lnw_j{}^M - lnm_{ij})^2 + (lnw_i{}^U - lnw_j{}^L - lnu_{ij})^2\right) \tag{12}$$

$$s.t. \begin{cases} w_i^L + \sum_{j=1, j\neq i}^{n} w_j^U \geq 1, \\ w_i^U + \sum_{j=1, j\neq i}^{n} w_j^L \leq 1, \\ \sum_{i=1}^{n} w_i^M = 1 \qquad \text{i=...,n} \\ \sum_{i=1}^{n} (w_i^L + w_i^U) = 2, \\ w_i^U \geq w_i^M \geq w_i^L > 0. \end{cases} \tag{13}$$

After this stage, the global fuzzy weights can be obtained by solving the following two linear programming models and an equation for each decision alternative A$_i$ (i=1,. . . ,n) as follows:

$$w_{A_i}^L = \min_{W \in \Omega_W} \sum_{j=1}^{m} w_{ij}^L w_j, i = 1, ..., n, \tag{14}$$

$$w_{A_i}^U = \min_{W \in \Omega_W} \sum_{j=1}^{m} w_{ij}^U w_j, i =, 1..., n, \tag{15}$$

$$w_{A_i}^{M} = \sum_{j=1}^{m} w_{ij}^{M} w_{j}^{M}, i = 1, \ldots, n, \tag{16}$$

where Ω_W=W=(w_1,. . . ,w_m $)^T$ | $w_j^L \leq w_j \leq w_j^U$,$\sum_{j=1}^{m}$ w_j =1,j=1,. . . ,m is the set of weights, (w_j^L,w_j^M,w_j^U) is the normalized fuzzy weight of criterion j (j=1,. . . ,m), and (w_{ij}^L,w_{ij}^M,w_{ij}^U) is the normalized fuzzy weight of alternative A_i concerning the criterion j (i=1,. . . ,n; j=1,. . . ,m).

3.5. Magnitude-Based Fuzzy Analytic Hierarchy Process

In many disciplines, ranking fuzzy numbers is an important part of decision-making, and numerous scholars have created various ranking approaches (Abbasbandy & Hajjari, 2009; Wang & Kerre, 2001a; Wang & Kerre, 2001b; Chutia & Chutia, 2017; Bortolan & Degani, 1985).

In FAHP methods, preferred values in fuzzy pairwise comparison matrices are usually normal triangular fuzzy numbers. This means that the height of the triangular membership function is equal to 1. For this reason, the magnitude of a fuzzy number proposed by (Abbasbandy & Hajjari, 2009), which forms the basis of the MFAHP method proposed by (Kinay & Tezel, 2022), has been preferred because it is easy and effective for FAHP calculations. The MFAHP method is defined in (Kinay & Tezel, 2022) as follows.

In the first step, row sum values for each fuzzy pairwise comparison matrix are obtained using Equation (3).

The second step is to apply the normalization process as stated in (Wang et al., 2008; Wang & Elhag, 2006) as follows.

$$S_i = \frac{RS_i}{\sum_{j=1}^{n} RS_j} = \left(\frac{\sum_{j=1}^{n} l_{ij}}{\sum_{j=1}^{n} l_{ij} + \sum_{k=1,k\neq i}^{n} \sum_{j=1}^{n} u_{kj}}, \frac{\sum_{j=1}^{n} m_{ij}}{\sum_{k=1}^{n} + \sum_{j=1}^{n}, m_{kj}}, \frac{\sum_{j=1}^{n} u_{ij}}{\sum_{j=1}^{n} u_{ij} + \sum_{k=1,k\neq i}^{n} \sum_{j=1}^{n} l_{kj}} \right), i = 1, \ldots, n. \tag{17}$$

In the third step, the magnitude values of each S_i value are calculated as given in Equation (18).

$$Mag(S_i) = \frac{l_i + 10m_i + u_i}{12}, i = 1, \ldots, n. \tag{18}$$

In the last step, magnitude values for each S_i value are normalized by Equation (19).

$$w_i = \frac{Mag(s_i)}{\sum_{j=1}^{n} Mag(s_j)}, i = 1, \ldots, n. \tag{19}$$

where the weight values are crisp values.

3.6. An example application of the MFAHP method

In this subsection, the computations of the MFAHP method were demonstrated using the example of the shipping registry selection problem given in (Celik, Er & Ozok, 2009). This problem has three main criteria (C1, C2, and C3). The main criteria have four, three, and three sub-criteria (C11-C14, C21-C23, and C31-C33), respectively, and it is intended to determine the most preferred one among the four alternatives (A1-A4). Detailed information about this problem, its hierarchical structure, and fuzzy pairwise comparison matrices can be found in (Celik, Er & Ozok, 2009).

The MFAHP method will be illustrated step-by-step using the fuzzy pairwise comparison matrix values which are generated for the three criteria in Table 1.

Table 1. *Fuzzy pairwise comparison matrix for three criteria*

Criteria	C1	C2	C3
C1	(1, 1, 1)	(5/2, 3, 7/2)	(3/2, 2, 5/2)
C2	(2/7, 1/3, 2/5)	(1, 1, 1)	(2/3, 1, 3/2)
C3	(2/5, 1/2, 2/3)	(2/3, 1, 3/2)	(1, 1, 1)

As the first step, RS_i values are calculated for each criterion using Equation (3), and the results were obtained as follows;
RS_1=(5.0000,6.0000,7.0000)
RS_2=(1.9524,2.3333,2.9000)
RS_3=(2.0667,2.5000,3.1667)

Table 2. *Results obtained with MFAHP.*

	Local weights of alternatives with respect to C1				
	C11	C12	C13	C14	Weights
Weights	0.2007	0.2012	0.2567	0.3413	
A1	0.1235	0.1478	0.2012	0.1042	0.1417
A2	0.3779	0.2839	0.2012	0.3484	0.3035
A3	0.3550	0.4117	0.2012	0.3228	0.3159
A4	0.1436	0.1566	0.3964	0.2245	0.2387
	Local weights of alternatives with respect to C2				
	C21	C22	C23		Weights
Weights	0.2315	0.5527	0.2158		
A1	0.1725	0.3352	0.3762		0.3064
A2	0.2294	0.2332	0.2966		0.2460
A3	0.3411	0.2751	0.1636		0.2663
A4	0.2570	0.1565	0.1636		0.1813
	Local weights of alternatives with respect to C3				
	C31	C32	C33		Weights
Weights	0.2516	0.2516	0.4969		
A1	0.3324	0.2960	0.1403		0.2278
A2	0.2499	0.3788	0.4529		0.3832
A3	0.1673	0.2056	0.1786		0.1826
A4	0.2503	0.1195	0.2282		0.2064
	Global weights of alternatives				
	C1	C2	C3		Weights
Weights	0.5510	0.2167	0.2323		
A1	0.1417	0.3064	0.2278		0.1974
A2	0.3035	0.2460	0.3832		0.3096
A3	0.3159	0.2663	0.1826		0.2742
A4	0.2387	0.1813	0.2064		0.2188

In the second step, the normalization operation in Equation (17) is applied for all three rows of the matrix (the rows represent the main criteria in this matrix) as follows.

$$S_1 = RS_1 \otimes [RS_1 \oplus RS_2 \oplus RS_3]^{-1} = \left(\frac{5.0000}{(5.0000+6.0667)}, \frac{6.0000}{(10.8333)}, \frac{7.0000}{(7.0000+4.0190)}\right) = (0.4518, 0.5539, 0.6356) \tag{20}$$

$$S_2 = RS_2 \otimes [RS_1 \oplus RS_2 \oplus RS_3]^{-1} = \left(\frac{1.9524}{(1.9524+10.1667)}, \frac{2.3333}{(10.8333)}, \frac{2.9000}{(2.9000+7.0667)}\right) = (0.1611, 0.2154, 0.2910) \tag{21}$$

$$S_3 = RS_3 \otimes [RS_1 \oplus RS_2 \oplus RS_3]^{-1} = \left(\frac{2.0667}{(2.0667+9.9000)}, \frac{2.5000}{(10.8333)}, \frac{3.1667}{(3.1667+6.9524)}\right) = (0.1727, 0.2308, 0.3129) \tag{22}$$

In the third step, the magnitude values of the normalized row totals are calculated using Equation (18) as follows.

$$Mag(S_1) = \frac{0.4518 + 10 * 0.5539 + 0.6356}{12} = 0.5522 \tag{23}$$

$$Mag(S_2) = \frac{0.1611 + 10 * 0.2154 + 0.2910}{12} = 0.2171 \tag{24}$$

$$Mag(S_3) = \frac{0.1727 + 10 * 0.2308 + 0.3129}{12} = 0.2328 \quad (25)$$

In the last step, normalized weight values are obtained with Equation (19) as follows.
$W=(0.5510,0.2167,0.2323)^T$

Global weights are found for each alternative by multiplying the local weights according to the hierarchical structure, starting from the sub-criteria. As a result, local and global weight values of MFAHP were obtained as in Table 2.

4. PERFORMANCE ANALYSIS

Table 3. *The weights obtained by FEA, MFLLSM, MFAHP, and GM.*

	FEA		MFLLSM		MFAHP		GM	
	Weights	Rank	Weights	Rank	Weights	Rank	Weights	Rank
(Celik, Er & Ozok, 2009)	0.0429	4	0.1935	4	0.1974	4	0.1941	4
	0.3583	2	0.3133	1	0.3096	1	0.3125	1
	0.3878	1	0.2710	2	0.2742	2	0.2682	2
	0.2110	3	0.2232	3	0.2188	3	0.2253	3
(Kahraman, Cebeci & Ruan, 2004)	0.0418	3	0.3088	2	0.3155	2	0.3091	2
	0.6179	1	0.2845	3	0.2821	3	0.2829	3
	0.3404	2	0.4081	1	0.4024	1	0.4080	1
(Arikan & Dağdeviren, 2013)	0.1672	1	0.1613	2	0.1606	2	0.1613	2
	0.1337	4	0.1345	5	0.1340	5	0.1327	5
	0.1295	5	0.1349	4	0.1355	4	0.1342	4
	0.1235	6	0.1292	7	0.1287	6	0.1286	7
	0.1640	2	0.1642	1	0.1645	1	0.1630	1
	0.1628	3	0.1495	3	0.1487	3	0.1505	3
	0.1193	7	0.1311	6	0.1280	7	0.1297	6
(Arif et al., 2021)	0.2560	1	0.2664	1	0.2489	2	0.2693	1
	0.2560	1	0.2517	2	0.2497	1	0.2455	2
	0.1970	3	0.1471	3	0.1670	3	0.1473	3
	0.1806	4	0.1376	4	0.1489	4	0.1416	4
	0.0894	5	0.1063	5	0.0994	5	0.1062	5
	0.0210	6	0.0934	6	0.0862	6	0.0902	6
(Büyüközkan, Çifçi & Güleryüz, 2011)	0.1339	3	0.1882	4	0.1803	4	0.1841	4
	0.3147	2	0.3050	1	0.3015	1	0.3027	1
	0.4307	1	0.3046	2	0.2944	2	0.3008	2
	0.1207	4	0.2105	3	0.2238	3	0.2124	3
(Dong, Li & Zhang, 2015)	0.4099	1	0.3609	1	0.3626	1	0.3600	1
	0.2672	3	0.3100	3	0.3031	3	0.3070	3
	0.3229	2	0.3310	2	0.3342	2	0.3329	2
(Dong, Li & Zhang, 2015)	0.4234	1	0.3553	1	0.3569	1	0.3530	2
	0.2513	3	0.2953	3	0.2876	3	0.2930	3
	0.3253	2	0.3512	2	0.3555	2	0.3540	1
(Aydogan, Demirtas & Dagdeviren, 2015)	0.1617	2	0.3163	2	0.3153	2	0.3173	2
	0.6959	1	0.4020	1	0.3996	1	0.3996	1
	0.1425	3	0.2827	3	0.2852	3	0.2831	3
(Isaai et al., 2011)	0.4686	1	0.3648	1	0.3658	1	0.3662	1
	0.2796	2	0.3095	3	0.3154	3	0.3073	3
	0.2518	3	0.3284	2	0.3187	2	0.3265	2
(Jaganathan, Erinjeri & Ker, 2007)	0	4	0.0738	4	0.0719	4	0.0735	4
	0.3467	1	0.3392	2	0.3725	1	0.3549	1
	0.3328	2	0.3647	1	0.3169	2	0.3474	2
	0.3205	3	0.2257	3	0.2387	3	0.2242	3
(Praščević & Praščević, 2016)	0.6372	1	0.4206	1	0.4227	1	0.4221	1
	0.2926	2	0.3281	2	0.3297	2	0.3295	2
	0.0702	3	0.2516	3	0.2476	3	0.2484	3
(Sehra, Brar & Kaur, 2013)	0.2338	2	0.3191	2	0.2920	2	0.2972	2
	0.1952	3	0.2842	3	0.2689	3	0.2842	3
	0.5711	1	0.3967	1	0.4391	1	0.4186	1
(Yuen & Henry, 2008)	0.2214	3	0.3282	2	0.3249	2	0.3298	2
	0.4157	1	0.3600	1	0.3607	1	0.3558	1
	0.3630	2	0.3127	3	0.3143	3	0.3144	3
(Tyagi et al., 2017)	0	3	0.1703	3	0.1848	3	0.1711	3
	0	3	0.1971	2	0.2311	2	0.1955	2
	0.8475	1	0.4746	1	0.4288	1	0.4726	1
	0.1525	2	0.1615	4	0.1553	4	0.1608	4
(Aydogan, Delice & Papajorgji, 2013)	0.4270	1	0.3113	1	0.3169	1	0.3089	1
	0.0898	4	0.2125	3	0.2068	4	0.2115	3
	0.1121	3	0.2088	4	0.2100	3	0.2086	4
	0.3711	2	0.2707	2	0.2663	2	0.2710	2

The global weights of the examples used to compare these four methods are shown in Table 3. When all the examples' global weights are considered, MFLLSM, MFAHP, and GM provide remarkably similar results and ranks, with a few exceptions. FEA, on the other hand, has the same rankings in just five of fifteen examples. In the example with seven alternatives (Arikan & Dağdeviren, 2013), the top five rows in ranked alternatives are the same for MFAHP, MFLLSM, and GM results. However, for the same example, only the third-ranking value of the FEA method is the same as the other methods.

At this stage, it is important to analyze these similarities and differences observed in Table 3. Therefore, the analysis of variances method (ANOVA) was used to determine the similarity of the results obtained by FEA, MFLLSM, MFAHP, and GM. It determines statistical differences between the mean differences of absolute error values of global weights between the methods. It is important to note that while ANOVA can show that at least two of the groups are significantly different from each other, it cannot show which groups are different from each other. Therefore, a post hoc test was performed to analyze the results further. The equality of variances was first checked using Levene's test, and the normality assumption was checked using the Q-Q plot. The results show that the assumptions of variance homogeneity and normality were violated in all instances. Therefore, Kruskal-Wallis Test for independent samples, given in Table 4, was used, indicating that the methods do not produce significantly similar results based on absolute error values.

But it does not indicate which subgroups are causing this difference. As a result, Mann-Whitney post hoc tests were used to further investigate Kruskal-Wallis results, as given in Table 5. Also, graphical representations of mean differences are presented in Fig.2. Mann-Whitney tests and Fig.2 shows that FEA produces significantly different results than others. When MFLLSM, MFAHP, and GM are compared, it is seen that they have remarkably similar performance.

Table 4. *Kruskal-Wallis test result for independent samples.*

Total N	342
Test Statistic	187.401[a]
Degree of Freedom	5
Asymptotic Sig.(2-sided test)	0.000

[a] The test statistic is adjusted for ties.

Table 5. *Mann-Whitney tests for pairwise comparisons of absolute value of weight differences.*

	Test Statistic	Std. Test Statistic	Sig.	Adj. Sig.[a]
MFLLSM-GM vs MFAHP-GM	-42.430	-2.291	0.022	0.329
MFLLSM-GM vs MFLLSM-MFAHP	51.877	2.801	0.005	0.076
MFLLSM-GM vs FEA-MFAHP	171.772	9.275	0.000	0.000*
MFLLSM-GM vs FEA-MFLLSM	175.009	9.450	0.000	0.000*
MFLLSM-GM vs FEA-GM	176.018	9.504	0.000	0.000*
MFAHP-GM vs MFLLSM-MFAHP	9.447	0.510	0.610	1.000
MFAHP-GM vs FEA-MFAHP	129.342	6.984	0.000	0.000*
MFAHP-GM vs FEA-MFLLSM	132.579	7.159	0.000	0.000*
MFAHP-GM vs FEA-GM	133.588	7.213	0.000	0.000*
MFLLSM-MFAHP vs FEA-MFAHP	119.895	6.474	0.000	0.000*
MFLLSM-MFAHP vs FEA-MFLLSM	123.132	6.649	0.000	0.000*
MFLLSM-MFAHP vs FEA-GM	124.140	6.703	0.000	0.000*
FEA-MFAHP vs FEA-MFLLSM	3.237	0.175	0.861	1.000
FEA-MFAHP vs FEA-GM	-4.246	-0.229	0.819	1.000
FEA-MFLLSM vs FEA-GM	-1.009	-0.054	0.957	1.000

Each row tests the null hypothesis that the Samp-1 and Samp-2 dist.s are the same. Asymptotic significances (2-sided tests) are displayed. The significance level is .05.

[a] Significance values have been adj. by the Bonferroni correction for multiple tests.

* Absolute value of weight differences is significant at the 0.05 level.

As a result, FEA is not a suitable method to obtain priorities from the fuzzy pairwise comparison matrix in that it can assign an unreasonable zero value as weights of some essential sub-criteria and criteria. These assignments lead to inaccurate results. Because the weights obtained by FEA do not indicate the relative importance of alternatives or criteria, this method is not recommended for use.

On the other hand, we claimed that MFAHP produces results close to the results of MFLLSM, and GM while reducing the cost of computation to the level of FEA at least. MFLLSM involves complicated calculations, which cannot be performed easily without using professional optimization software packages, to obtain the local fuzzy weights by solving a constrained nonlinear optimization model for each fuzzy comparison matrix although it makes a correct decision and handles all these problems. However, MFAHP, GM, and FEA have a much lower processing load. This situation easily can be seen in Table 6 and in Fig.3.

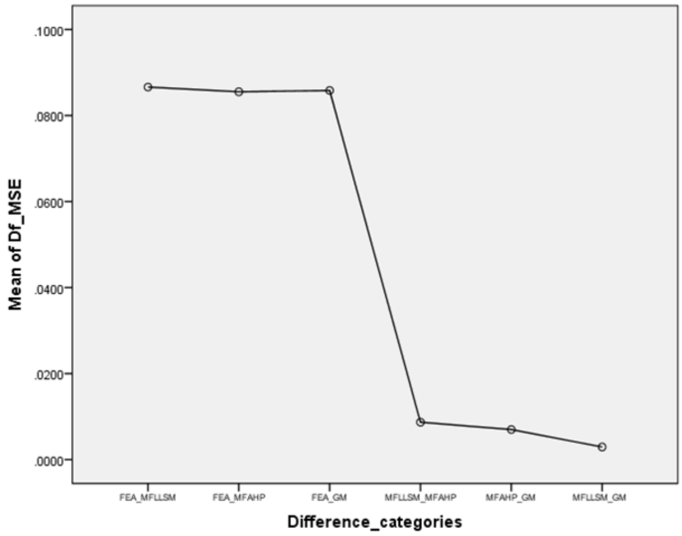

Figure 2. Graphical representation of mean differences of weights.

Table 6. *CPU time of the implementations(in seconds)(Sorted in ascending order according to the values of the MFAHP)*

Ref_No	FEA	MFLLSM	MFAHP	GM
(Isaai et al., 2011)	1.82E-05	1.580142	1.05E-05	1.500E-04
(Büyüközkan & Çifçi, 2011)	1.70E-05	3.219868	1.16E-05	1.421E-04
(Praščević & Praščević, 2016)	2.40E-05	1.792041	1.37E-05	2.049E-04
(Dong, Li & Zhang, 2015)	2.11E-05	1.797778	1.39E-05	1.918E-04
(Dong, Li & Zhang, 2015)	2.16E-05	1.793572	1.42E-05	2.103E-04
(Sehra, Brar & Kaur, 2013)	2.24E-05	1.812290	1.43E-05	2.353E-04
(Jaganathan, Erinjeri & Ker, 2007)	3.42E-05	2.298624	1.94E-05	3.886E-04
(Aydogan, Delice & Papajorgji, 2013)	2.77E-05	2.300450	1.97E-05	3.294E-04
(Arikan & Dağdeviren, 2013)	5.28E-05	6.117062	4.05E-05	5.963E-04
(Celik, Er & Ozok, 2009)	6.23E-05	7.209692	4.24E-05	6.851E-04
(Aydogan, Demirtas & Dagdeviren, 2015)	7.78E-05	9.279611	5.55E-05	8.203E-04
(Tyagi et al., 2017)	11.06E-05	11.689248	7.62E-05	13.782E-04
(Kahraman, Cebeci & Ruan, 2004)	10.09E-05	13.257331	7.80E-05	14.571E-04
(Yuen & Henry, 2008)	12.28E-05	12.222307	9.49E-05	12.977E-04
(Arif et al., 2021)	26.79E-05	21.035452	16.61E-05	33.436E-04

In addition, the Kruskal-Wallis test for independent samples, presented in Table 7, and Mann-Whitney post hoc tests in Table 8 confirm that there is no difference between MFAHP and FEA in terms of running time among these four approaches, whereas GM is significantly different from MFAHP and FEA, too.

Table 7. *Kruskal-Wallis test result for independent samples for running times.*

Total N	60
Test Statistic	49.318[a]
Degree Of Freedom	3
Asymptotic Sig.(2-sided test)	0.000
[a] The test statistic is adjusted for ties.	

When MFAHP is preferred, the decision process seems to overcome the problems of FEA, and a similar decision is made without the high computational cost as in MFLLSM and GM. All approaches were programmed in C# language.

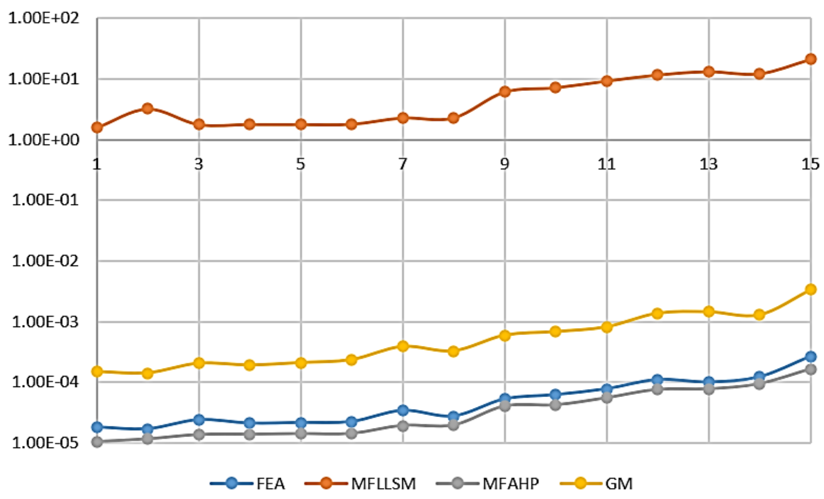

Figure 3. Graphical representation of CPU time of the implementations.

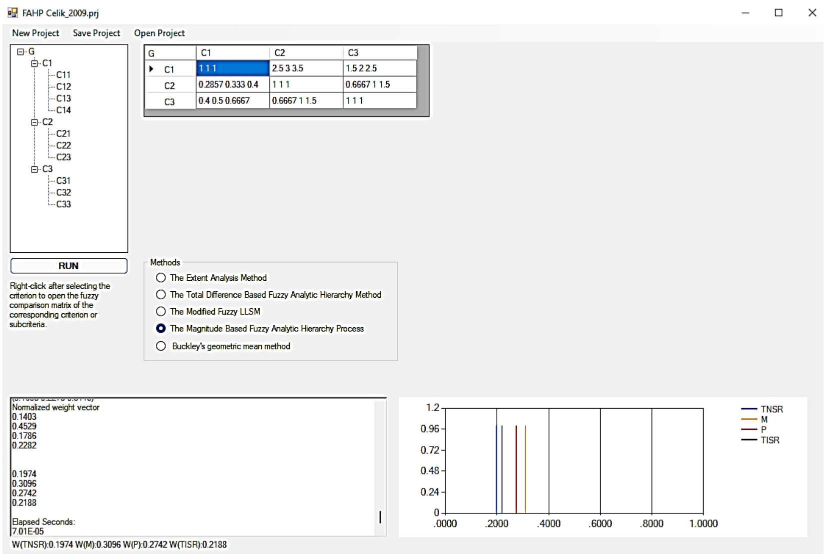

Figure 4. A screenshot of the application.

Table 8. *Pairwise comparisons of running times.*

	Test Statistic	Std. Test Statistic	Sig.	Adj. Sig.[a]
MFAHP-FEA	5.267	0.826	0.409	1.000
MFAHP-GM	-24.333	-3.816	0.000	0.001*
MFAHP-MFLLSM	39.867	6.252	0.000	0.000*
FEA-GM	-19.067	-2.990	0.003	0.017*
FEA-MFLLSM	-34.600	-5.426	0.000	0.000*
GM-MFLLSM	15.533	2.436	0.015	0.089

Each row tests the null hypothesis that the Samp-1 and Samp-2 dist.s are the same. Asymptotic significances (2-sided tests) are displayed. The significance level is .05.

[a] Significance values have been adj. by the Bonferroni correction for multiple tests.

* Absolute value of weight differences is significant at the 0.05 level.

The CPU of the computer, where all the experiments were performed, is an AMD ThreadRipper 1950x clocked @3.7 GHz and the main memory consists of 64 GB of DDR4 RAM.

An application that has implementations of all above-mentioned algorithms, and the data of all examples can be downloaded from https://github.com/baristezel/FAHP. A screenshot of the application is shown in Fig.4. We hope that it may aid in the understanding of the MFAHP method and its contributions.

Also, the global weight values obtained by these methods for the example in (Celik, Er & Ozok, 2009), as shown in Fig.5. In this figure, it can be seen that MFAHP results are very close to the midpoints of MFLLSM results and GM results.

5. DISCUSSION AND CONCLUSION

In this study, instead of the FEA method, which is frequently used in the solution of complex FMCDM problems and leads to incorrect results, the MFAHP method, which allows for obtaining appropriate and efficient results, has been evaluated. FEA, MFLLSM, MFAHP, and GM are compared by using numerical examples. As a result, it has been shown that MFAHP can solve problems arising from FEA and has similar center value results that are more easily calculated compared to MFLLSM and GM results. Even though the GM appears to produce results that are more similar to MFLLSM in some instances, pairwise comparison tests revealed no significant differences between MFAHP, GM, and MFLLSM results. On the other hand, when the running time values are examined, it is seen that the MFAHP gives results in a shorter time than MFLLSM and GM, while there is no statistically significant difference between MFAHP and FEA. While it has been observed that there is no statistically significant difference between MFAHP, GM, and MFLLSM according to global weight differences between methods, the MFAHP approach is both faster and much simpler than others.

It can be argued that the main contribution of this study is to demonstrate, through statistical significance, that the MFAHP method is capable of producing results that are as accurate as those obtained by both the GM method and the more challenging-to-understand and -implement MFLLSM. It was observed that there was no comparison of the methods with MFLLSM in the literature. In this sense, the comparison of the efficiency of the MFAHP method with the MFLLSM, which solves the problem as a constrained nonlinear optimization model and gives accurate results, is a prominent feature of this study. Moreover, statistical analysis demonstrates that the MFAHP method achieves the result values in a shorter time than both the GM and the MFLLSM. In other words, MFAHP method results can be obtained as accurately as MFLLSM and GM results and as fast as FEA. Finally, to aid other researchers in conducting research or applications in this field, we have developed software that calculates all the methods discussed in this study.

Since triangular fuzzy numbers are generally used in comparison matrices in FMCDM problems, the limitation of this study is that the examples used to compare the results of the methods in our study contain only such fuzzy numbers. However, it will be an important contribution to examine the effectiveness of the MFAHP method for other types of fuzzy numbers, especially in future studies.

Overall, the MFAHP algorithm is comparable to the MFLLSM and GM methods in terms of weight calculation accuracy for triangular fuzzy numbers, while also demonstrating superior performance in computational efficiency.

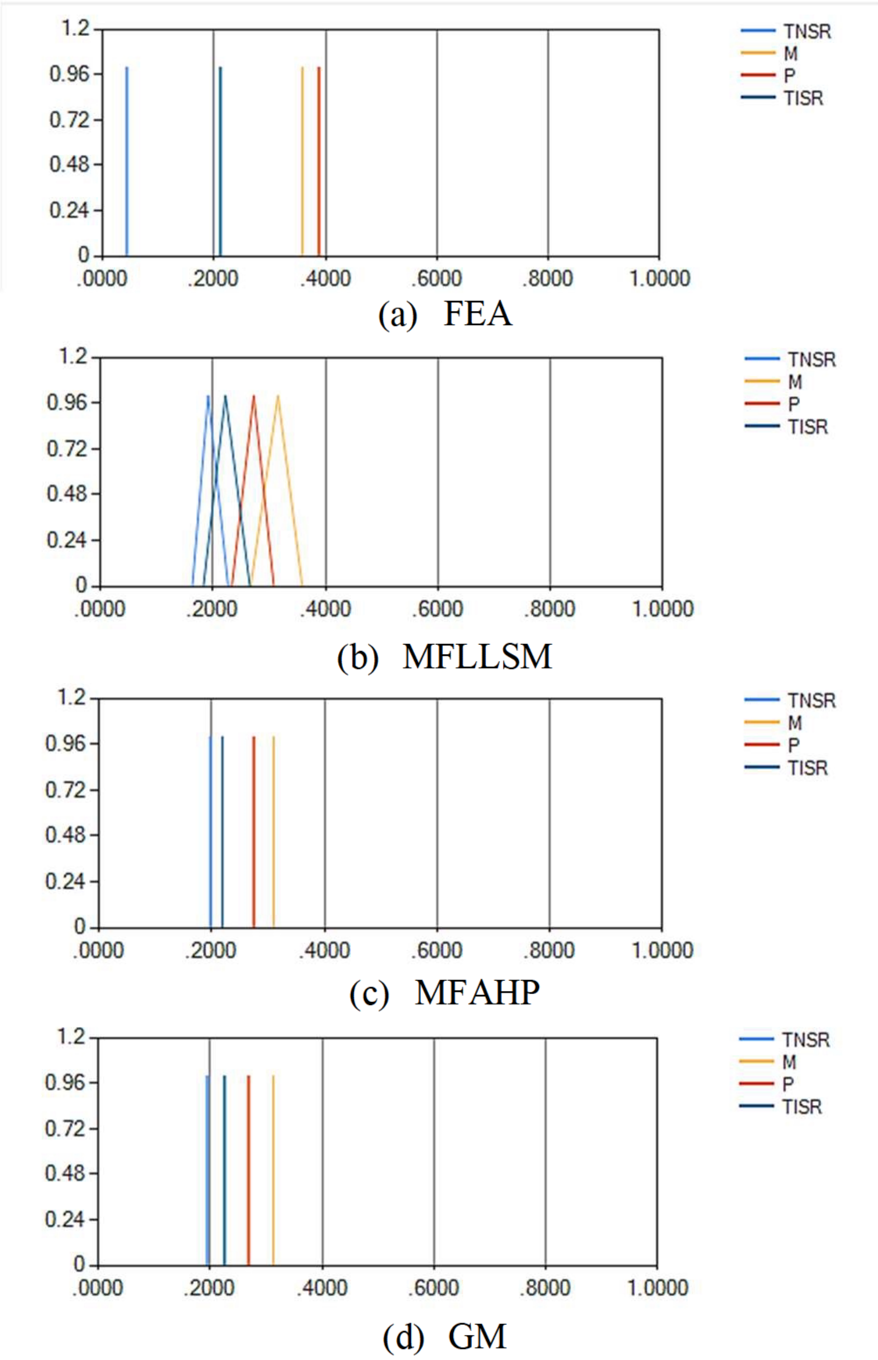

Figure 5. Graphical representation of the global weights for the example in (Celik, Er & Ozok, 2009).

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REFERENCES

Abbasbandy, S., & Hajjari, T. (2009). A new approach for ranking of trapezoidal fuzzy numbers. *Computers & Math. with Applications, 57*(3), 413-419.

Ahmed, F., & Kilic, K. (2019). Fuzzy Analytic Hierarchy Process: A performance analysis of various algorithms. *Fuzzy Sets and Systems, 362*, 110-128.

Amiri, M. P. (2010). Project selection for oil-fields development by using the AHP and fuzzy TOPSIS methods. *Expert systems with applications, 37*(9), 6218-6224.

Arif, J. M., Ab Razak, M. F., Mat, S. R. T., Awang, S., Ismail, N. S. N., & Firdaus, A. (2021). Android mobile malware detection using fuzzy AHP. *Journal of Information Security and Applications, 61*, 102929.

Arikan, R., Dağdeviren, M., & Kurt, M. (2013). A fuzzy multi-attribute decision making model for strategic risk assessment. *International Journal of Computational Intelligence Systems, 6*(3), 487-502.

Aydogan, E. K., Delice, E. K., & Papajorgji, P. (2013). An effective approach for evaluating usability of Web sites. In Enterprise Business Modeling, Optimization Techniques, and Flexible Information Systems (pp. 97-107). IGI Global.

Aydogan, E. K., Demirtas, O., & Dagdeviren, M. (2015). A New Integrated Fuzzy Multi-Criteria Decision Model for Performance Evaluation. *Business and Management Studies, 1*(1), 38-55.

Ballı, S., & Karasulu, B. (2013). Bulanık karar verme sistemlerinde paralel hesaplama. *Pamukkale Üniversitesi Mühendislik Bil. Dergisi, 19*(2), 61-67.

Boender, C. G. E., De Graan, J. G., & Lootsma, F. (1989). Multi-criteria decision analysis with fuzzy pairwise comparisons. *Fuzzy Sets and Systems, 29*(2), 133-143.

Bortolan, G., & Degani, R. (1985). A review of some methods for ranking fuzzy subsets. *Fuzzy sets and systems, 15*(1), 1-19.

Buckley, J. J. (1985). Fuzzy hierarchical analysis. *Fuzzy sets and systems, 17*(3), 233-247.

Büyüközkan, G., Kahraman, C., & Ruan, D. (2004). A fuzzy multi-criteria decision approach for software development strategy selection. *International journal of general systems, 33*(2-3), 259-280.

Büyüközkan, G., Çifçi, G., & Güleryüz, S. (2011). Strategic analysis of healthcare service quality using fuzzy AHP methodology. *Expert systems with applications, 38*(8), 9407-9424.

Celik, M., Er, I. D., & Ozok, A. F. (2009). Application of fuzzy extended AHP methodology on shipping registry selection: The case of Turkish maritime industry. *Expert Systems with Applications, 36*(1), 190-198.

Chutia, R., & Chutia, B. (2017). A new method of ranking parametric form of fuzzy numbers using value and ambiguity. *Applied Soft Comp.*, 52, 1154-1168.

Dong, M., Li, S., & Zhang, H. (2015). Approaches to group decision making with incomplete information based on power geometric operators and triangular fuzzy AHP. *Expert Systems with Applications, 42*(21), 7846-7857.

Dweiri, F., Kumar, S., Khan, S. A., & Jain, V. (2016). Designing an integrated AHP based decision support system for supplier selection in automotive industry. *Expert Systems with Applications, 62*, 273-283.

Isaai, M. T., Kanani, A., Tootoonchi, M., & Afzali, H. R. (2011). Intelligent timetable evaluation using fuzzy AHP. *Expert systems with App., 38*(4), 3718-3723.

Jaganathan, S., Erinjeri, J. J., & Ker, J. I. (2007). Fuzzy analytic hierarchy process based group decision support system to select and evaluate new manufacturing technologies. *The International Journal of Advanced Manufacturing Technology, 32*(11), 1253-1262.

Kahraman, C., Cebeci, U., & Ruan, D. (2004). Multi-attribute comparison of catering service companies using fuzzy AHP: The case of Turkey. *International journal of production economics, 87*(2), 171-184.

Kinay, A. O., & Tezel, B. T. (2022). Modification of the fuzzy analytic hierarchy process via different ranking methods. International Journal of Intelligent Systems, 37(1), 336-364.

Korkmaz, I., Gökçen, H., & Çetinyokuş, T. (2008). An analytic hierarchy process and two-sided matching based decision support system for military personnel assignment. *Information Sciences, 178*(14), 2915-2927.

Kubler, S., Robert, J., Derigent, W., Voisin, A., & Le Traon, Y. (2016). A state-of the-art survey & testbed of fuzzy AHP (FAHP) applications. *Expert Systems with Applications, 65*, 398-422.

Kwiesielewicz, M. (1996). The logarithmic least squares and the generalized pseudoinverse in estimating ratios. *European Journal of Operational Research, 93*(3), 611-619.

Liu, Y., Eckert, C. M., & Earl, C. (2020). A review of fuzzy AHP methods for decision-making with subjective judgements. *Expert Systems with Applications, 161*, 113738.

Saaty, T. L. (1977). A scaling method for priorities in hierarchical structures. *Journal of mathematical psychology, 15*(3), 234-281.

Saaty, T.L. (1980) *The Analytic Hierarchy Process: Planning, Priority Setting, Resources Allocation.* Mcgraw-Hill, New York.

Vaidya, O. S., & Kumar, S. (2006). Analytic hierarchy process: An overview of applications. *European Journal of operational research, 169*(1), 1-29.

Xu, Z., & Liao, H. (2013). Intuitionistic fuzzy analytic hierarchy process. *IEEE transactions on fuzzy systems, 22*(4), 749-761.

Wang, Y. M., Elhag, T. M., & Hua, Z. (2006). A modified fuzzy logarithmic least squares method for fuzzy analytic hierarchy process. *Fuzzy Sets and systems, 157*(23), 3055-3071.

Chang, D. Y. (1996). Applications of the extent analysis method on fuzzy AHP. *European Journal of Operational Research, 95*(3), 649-655.

Mardani, A., Jusoh, A., & Zavadskas, E. K. (2015). Fuzzy multiple criteria decision-making techniques and applications–Two decades review from 1994 to 2014. *Expert Systems with Applications, 42*(8), 4126-4148.

Van Laarhoven, P. J., & Pedrycz, W. (1983). A fuzzy extension of Saaty's priority theory. *Fuzzy sets and Systems, 11*(1-3), 229-241.

Ruoning, X., & Xiaoyan, Z. (1996). Fuzzy logarithmic least squares ranking method in analytic hierarchy process. *Fuzzy Sets and Systems, 77*(2), 175-190.

Wang, Y. M., Luo, Y., & Hua, Z. (2008). On the extent analysis method for fuzzy AHP and its applications. *European Journal of Operational Research, 186*(2), 735-747.

Zadeh, L. A. (1965). Fuzzy sets. *Information and control, 8*(3), 338-353.

Wang, X., & Kerre, E. E. (2001). Reasonable properties for the ordering of fuzzy quantities (I). *Fuzzy sets and systems, 118*(3), 375-385.

Wang, X., & Kerre, E. E. (2001). Reasonable properties for the ordering of fuzzy quantities (II). *Fuzzy sets and systems, 118*(3), 387-405.

Wang, Y. M., & Elhag, T. M. (2006). On the normalization of interval and fuzzy weights. Fuzzy sets and systems, 157(18), 2456-2471.

Praščević, N., & Praščević, Ž. (2016). Application of fuzzy AHP method based on eigenvalues for decision making in construction industry. *Tehnički vjesnik/Technical Gazette, 23*(1), 57-64.

Sehra, S. K., Brar, D., Singh, Y., & Kaur, D. (2013). Multi criteria decision making approach for selecting effort estimation model. arXiv preprint arXiv:1310.5220.

Tyagi, S., Agrawal, S., Yang, K., & Ying, H. (2017). An extended Fuzzy-AHP approach to rank the influences of socialization-externalization-combination-internalization modes on the development phase. *Applied Soft Computing, 52*, 505-518.

Yuen, K. K., & Lau, H. C. (2008). Software vendor selection using fuzzy analytic hierarchy process with ISO/IEC 9126. IAENG International journal of computer science, 35(3).

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