Available online at www.academicpaper.org
Academic @ Paper

International Journal of Energy Applications and Technologies
Vol. 3, Issue 2, pp. 37 – 40, 2016

www.academicpaper.org/index.php/IJEAT

Research Article

LAMINAR NATURAL CONVECTION IN TRIANGULAR ENCLOSURES

Emin Fuad Kent *

Mechanical Engineering Faculty, Istanbul Technical University, Gümüşsuyu, Istanbul, Turkey.

Received 23rd September 2016, Accepted 6th November 2016

Abstract

Natural convection in non-rectangular enclosures is numerically analyzed in this study. Streamlines and isotherms are presented for different triangular enclosures with different boundary conditions and Rayleigh numbers. Mean Nusselt numbers on hot walls are also calculated in order to make comparisons between different cases.

Keywords- *Natural convection, laminar flow, Nusselt number, Rayleigh number, triangular enclosure.*

1. Introduction

One of the first studies about natural convection in rectangular enclosures was a solely experimental study made by Flack et al. [1] in 1979. After this, in 1982 Akinsete and Coleman [2] analysed natural convection of air in a two dimensional laminar right triangular enclosure using numerical methods. Steady state solutions are made for the conditions in which the aspect ratio (A.R.) is between 0.0625 and 1, and Grashof number varies in a range of 800 to 64000. Poulikakos ve Bejan [3] analysed right triangular enclosures in which the upper wall is cold and the lower wall is hot. The conditions that the aspect ratio is closing up to zero are analysed with Rayleigh number varying from 10^3 to 10^5.

Lam et al. [4] analysed the natural convection in right triangular, trapezoidal and rectangular enclosures by experimental and numerical methods. In all the cases, bottom wall is hot and side walls are adiabatic. Aspect ratio is kept constant at 4 and the horizontal angle is changed between 0° and 25°.

Tabbarrok and Lin [5] analyzed natural convection in various geometries by finite element method. Square and quarter circle are among the examined geometries. The results that have been found with different Rayleigh numbers are similar with the previous researches.

* Corresponding Authors;
kente@itu.edu.tr

Note: This paper has been presented at the International Conference on Advanced Technology & Sciences (ICAT'16) held in Konya (Turkey).

Karyakin et al. [6] studied laminar natural convection inside isosceles triangular enclosures. Salmun [7,8] used air and water in right triangular geometries. The calculations are made for different Rayleigh number values between 10^2 and 10^5 and aspect ratios between 0.1 and 1.

Asan and Namli [9] studied the natural convection during the winter time heating of an attic space. They used finite volume method and analysed cases where Rayleigh numbers vary between 0.25 and 1. Multi-cellular structure is obtained in every enclosure when Rayleigh number equals to 10^6. As aspect ratio decreases multi-cellular streamlines are observed at lower Rayleigh numbers.

Kent et al. [10, 11] investigated finite element and finite volume solutions of natural convection in triangular enclosures.

2. Mathematical Model

For a steady state heat transfer with constant thermal and physical properties, laminar and steady incompressible flow with no viscous dissipation, the partial differential equations take the following vectorial forms.

Continuity,

$$\vec{\nabla} \cdot \vec{V} = 0 \qquad (1)$$

Momentum,

$$\rho(\vec{V}.\vec{\nabla})\vec{V} = \eta\nabla^2\vec{V} - \vec{\nabla}p - \rho\vec{g} \qquad (2)$$

Energy,

$$\rho c_p(\vec{V}.\vec{\nabla})T = k(\vec{\nabla}.\vec{\nabla})T \qquad (3)$$

Two-dimensional form of these partial different equations in Cartesian coordinates can be written in primitive variable formulation as follows.
Continuity,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4}$$

x-momentum,

$$\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho vu)}{\partial y}=\eta\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\partial p}{\partial x} \tag{5}$$

y- momentum, with the Boussinesq approximation for the buoyancy term,

$$\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(\rho vv)}{\partial y}=\eta\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\partial p}{\partial y}+\rho g\beta(T-T_{ref}) \tag{6}$$

energy,

$$\rho c_p\left(\frac{\partial(\rho uT)}{\partial x}+\frac{\partial(\rho vT)}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{7}$$

Rayleigh number,

$$Ra_h=\frac{g\beta\Delta Th^3\rho^2 c_p}{k\eta} \tag{8}$$

3. Physical Model

The triangular grid structure is given in Fig. 1. In this work, natural convection of air with Prandtl number of 0.72 is analysed. The change of the streamlines and isotherms at different Rayleigh numbers ranging from 10^3 to 10^5 are obtained numerically and presented as graphics.

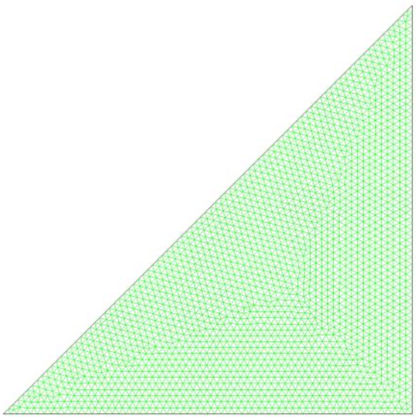

Fig. 1. Triangular grid structure

The four different cases considered in this study is given in Table 1. A.R. is abbreviation of Aspect Ratio.

Table 1. Thermal boundary conditions and cases.

Cases	Hypotenuse	Horizontal	Vertical	A.R.
Case I	Cold	Adiabatic	Hot	1
Case II	Adiabatic	Hot	Cold	1
Case III	Cold	Hot	Adiabatic	1, 0.5
Case IV	Cold	Adiabatic	Hot	1

4. Numerical Model

Navier Stokes governing equations which are discretized with finite volume method are solved by Fluent 6.0.12 commercial software [12] by using SIMPLE and Upwind Difference methods. Iterations are continued until the convergence is obtained in a 60x60 grid structure.
Before starting to solve the problem, trial calculations are made in 30x30, 45x45 and 60x60 grid structures in order to prove that the problem is grid independent. Solution method is applied to these grid structures and then the calculated Nusselt numbers, maximum values of stream functions and the coordinates of these stream function values in the enclosure are compared. All the comparisons have shown that the change of these values are under 1% between 45x45 and 60x60 grid structures. So the problem is accepted to be grid independent and calculations are made by using 60x60 grid structure in order to get more precise results.
In Case-1, hypotenuse is cold, vertical wall is hot and horizontal wall is adiabatic. Streamlines and isotherms of this case are shown in Fig. 2. As the Rayleigh number is raised, isotherms start to get non-uniform shapes.

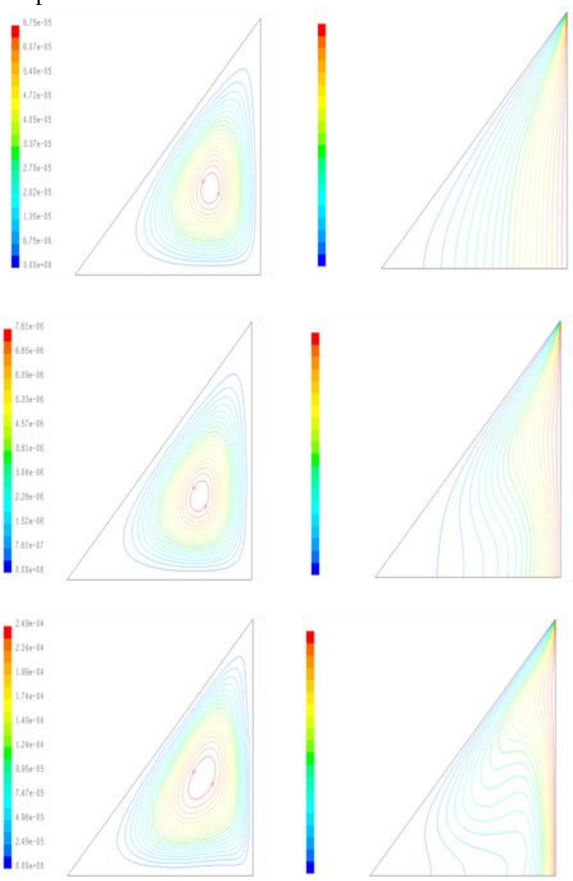

Fig. 2. Streamlines and isotherms inside the right triangular enclosure with the Case I boundary conditions for Rayleigh numbers 10^3,10^4 and 10^5, respectively

For Case II, when Rayleigh number is 10^3, there is a conductive heat transfer inside the triangle and isotherms have an absolute symmetry. But when Rayleigh number is raised to 10^5, heat transfer turns out to be convective as isotherms get non-uniform shapes. The results are given in Fig. 3.

For Case III, streamlines and isotherms for the case with A.R.=1 is shown in Fig. 4. Stream function values increase from the side to the inner part of the triangle and gets its maximum value in the central cell. If we compare this case with Case II, we can see that the triangle that we analyzed in Case III has lower stream function values when Rayleigh number is equal to 10^3 and 10^4. But when Rayleigh number is raised to 10^5, maximum stream function value in Case III becomes higher than Case II. This shows that convection effect on the heat transfer in Case III is more than Case II.

For A.R.=0.5, streamlines and isotherms are shown in Fig. 5. Multicellular streamlines are also obtained in the present study when Rayleigh number is raised to 10^5 and aspect ratio is below 1. While Rayleigh number is 10^3, streamlines are very similar to those in the right triangular enclosure with A.R.=1. But, when Rayleigh number is raised to 10^5, streamlines transform into a three cells structure.

For Case IV, streamlines and isotherms are shown in Fig. 6. While Rayleigh number is 10^3, isotherms seem to be symmetrical and uniform. But, with the increase of Rayleigh number, isotherms get a non-uniform structure.

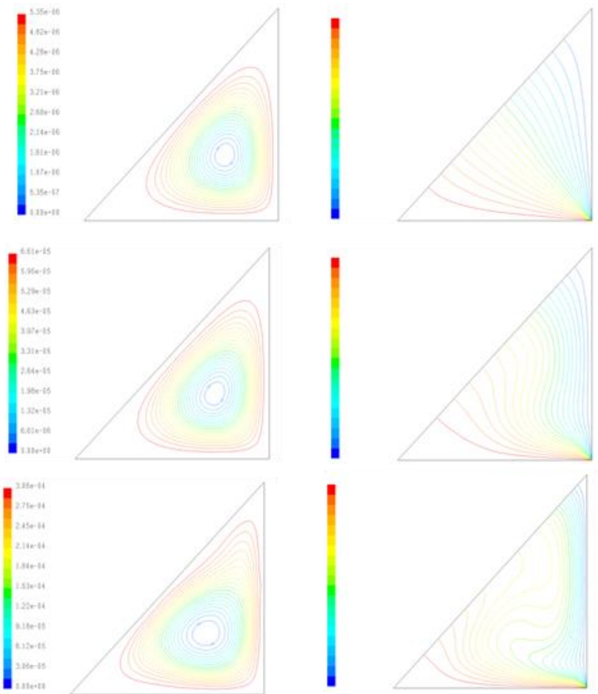

Fig. 3. Streamlines and isotherms inside the right triangular enclosure with the Case II boundary conditions for Rayleigh numbers 10^3,10^4 and 10^5, respectively.

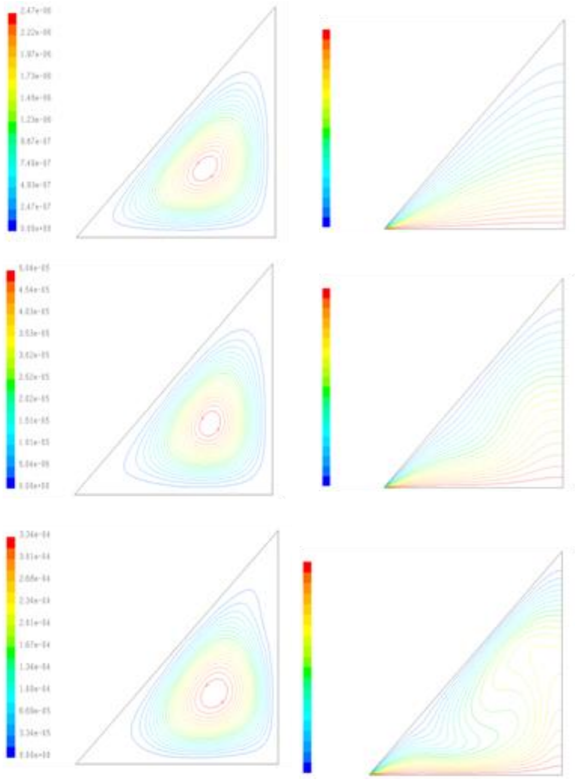

Fig. 4. Streamlines and isotherms inside the right triangular enclosure (A.R.=1) with the Case III boundary conditions for Rayleigh numbers 10^3,10^4 and 10^5, respectively.

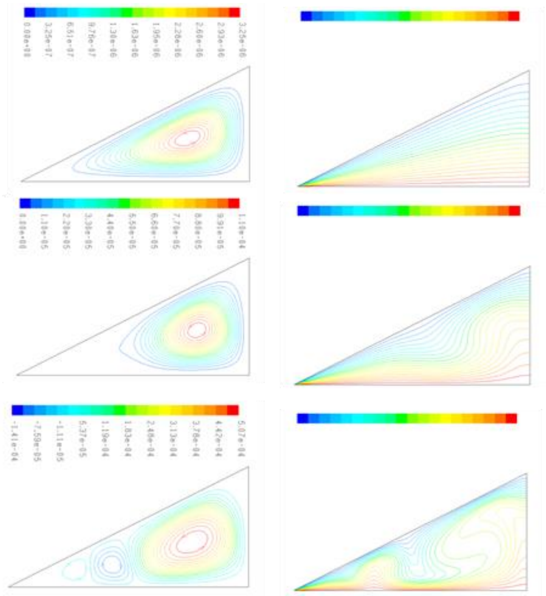

Fig. 5. Streamlines and isotherms inside the right triangular enclosure (A.R.=0.5) with the Case III boundary conditions for Rayleigh numbers 10^3,10^4 and 10^5, respectively.

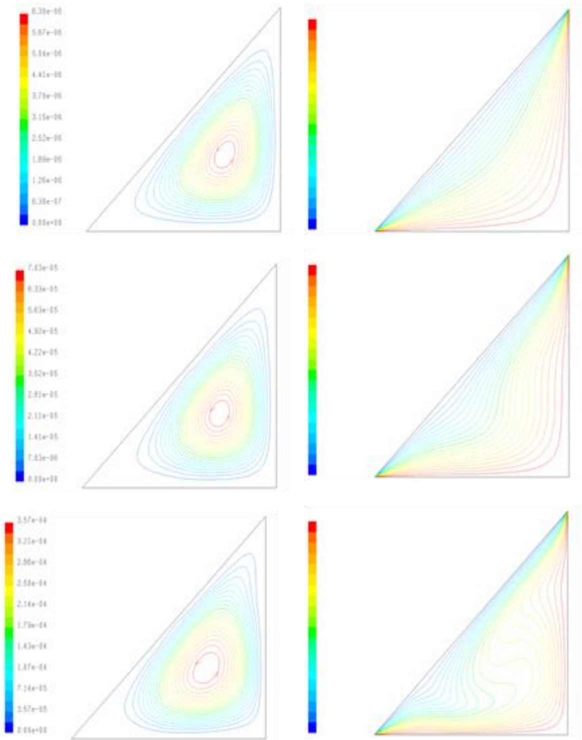

Fig. 6. Streamlines and isotherms inside the right triangular enclosure with the Case VI boundary conditions for Rayleigh numbers 10^3,10^4 and 10^5, respectively.

5. Conclusions

In this study, steady state laminar natural convection in right triangular enclosures is analyzed. Finite volume method is used to discretize the Navier Stokes equations. SIMPLE algorithm and upwind difference methods are applied to the governing equations. In order to investigate the effects of aspect ratio on the streamline patterns and isotherms, calculations are made on two different triangular enclosures.

References

[1] Flack D., Konopnicki, T.T. and Rooke J.H., The measurement of natural convective heat transfer in triangular enclosures, ASME J. Heat Transfer, 101, 648-654, 1979

[2] Akinsete, V.A. and Coleman, T.A.., 1982. Heat transfer by steady laminar free convection in triangular enclosures, Int. J. Heat Mass Transfer, 25, 991-998.

[3] Poulikakos, D. and Bejan, A., The fluid dynamics of an attic space, J. Fluid Mech., 131, 251-269, 1983

[4] Lam S.W., Gani R. and Symons J.G. 1989, Experimental and numerical studies of Natural Convection in Trapezoidal Cavities, ASME J. Heat Transfer, 111, 372-377

[5] Tabarrok B. and Lin R.C. 1977, Finite element analysis of free convection flows, Int. J. Heat Mass Transfer, 20, 945-952.

[6] Karyakin YU. E., Sokovishin YU. A. and Martynenko O. G. 1988, Transient natural convection in triangular enclosures, Int. J. Heat Mass Transfer, 31, 1759-1766.

[7] Salmun H., 1995, The stability of a single-cell steady-state solution in a triangular enclosure, Int. J. Heat Mass Transfer, 38, 363-369.

[8] Salmun H., 1995, Convection patterns in a triangular domain, Int. J. Heat Mass Transfer, 38, 351-362.

[9] Asan H. and Namli L., 2001, Numerical simulation of buoyant flow in a roof of triangular cross-section under winter day boundary conditions, Energy and Buildings, 33, 753-757.

[10] Kent E.F., Asmaz E. and Ozerbay S., 2005, Finite element solution of natural convection in triangular enclosures, IV. International Conference on Computational Heat and Mass Transfer, Paris, 17-20 May, p. 45-47.

[11] Fuad Kent, E., Asmaz, E. & Ozerbay, S., Laminar natural convection in right triangular enclosures, Heat Mass Transfer (2007) 44: 187.

[12] FLUENT 6.0.12 2001, Fluent Inc. Cavendish Court Lebanon, NH 03766, USA.