GU J Sci 30(1): 395-400 (2017)

Gazi University

Journal of Science

http://dergipark.gov.tr/gujs

Vague Vector Spaces on Sub Graphs

V. Amarendra BABU[1,*], Kanhaiya JHA[2], K.V.Rama RAO[3]

[1]*Department of Mathematics, Achraya Nagarjuna University,Nagarjuna Nagar-522510, India.*

[2]*Mathematics Group, Department of Natural Science, Katmandu University, Katmandu, Nepal.*

[3]*Department of Mathematics, S.R.R and C.V.R Degree and P.G. college, Vijayawada, India.*

Article Info

Received: 22/11/2016
Accepted: 29/12/2016

Keywords

Vague additive groups
vague rings
vague fields
vague vector spaces
sub graphs

Abstract

We introduce the concepts of vague additive groups and vague rings on sub graphs. Vague fields on Galio's groups. Also we define the concept of vague vector space on sub graphs.

1. INTRODUCTION

The concept of fuzzy set was introduced by Zadesh. Since then this idea has been applied to other algebraic structures such as groups, rings etc. With the development of fuzzy set, it is widely used in many fields. Meanwhile, the deficiency of fuzzy sets are also attract attention. The attenction such as fuzzy set is single function, it cannot express the evidence of supporting and opposing. Based on this reason, the concept of vague set [4] introduced by Gau in 1993. Vague sets as a extension of fuzzy sets, the idea of vague sets is that the membership of every element can be divided into two aspects including supporting and opposing. The notion of fuzzy groups defined by Rosen field [8] is the first application fuzzy set theory in algebra. Ranjit Biswas [6] initiated the study of vague algebra by studying vague groups. We introduced the concepts of vague additive groups, vague rings, vague fields and modules[8], [9]and [10].

Graph theory was born in 1736 with Euler's paper in which he solved the Kongsberg Bridge's problem in 1847. G.R. Kirchoff developed the theory of tress to applications in electrical networks. Mobius solved the four color problem. Graph theory has a surprising number of applications in many developed areas. Graph theory serves as a mathematical model for any system involving a binary relation.

Modern abstract algebra is a powerful tool in the theory as well as in the applications of graphs. It is necessary to represent a graph algebraically and wishes to onlist the aid of a computer in solving graph theory problems.

Now we introduce the concepts of vague additive groups and vague rings on sub graphs. Vague fields on Galio's groups. Also we define the concept of vague vector space on sub graphs.

2. PRELIMINARIES

In this section we collect important results which were already proved for our use in the next section**.**

Definition2.1: [4]A vague set A in the universal of discourse X is characterized by two membership functions given by:

*Corresponding author, e-mail: amarendravelisela@ymail.com

A truth membership function $t_A : X \rightarrow [0,1]$ and

A false membership function $f_A : X \rightarrow [0,1]$,

Where $t_A(x)$ is a lower bound of the grade of membership of x derived from the "evidence for x", and $f_A(x)$ is a lower bound on the negation of x derived from the "evidence against x" and $t_A(x) + f_A(x) \leq 1$.Thus the grade of membership of x in the vague set A is bounded by subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. The vague set A is written as $A =\{\langle x, [t_A(x),f_A(x)] \rangle / x \in X \}$.

Where the interval $[t_A(x), 1 - f_A(x)]$ is called the value of x in the vague set A and denoted by $V_A(x)$.

Definition2.2:[4] A vague set A of a universe X with $t_A(x) = 0$ and $f_A(x) = 1$ for all $x \in X$, is called the zero vague set of X.

Definition2.3: [4] A vague set A of a universe X with $t_A(x) = 1$and $f_A(x) = 0$ for all $x \in X$, is called the unit vague set of X

Defenition2.4[1]: Let (X, +) be a group. A vague set A of X is called a vague additive (briefly VAG) group of X if the following conditions is satisfies:

(1). $V_A(x + y) \leq \text{imax}\{V_A(x) , V_A(y)\}$, for all $x, y \in X$;

(2). $V_A(-x) \leq V_A(x)$, for all $x \in X$.

Definition 2.5:[2] Let X be a ring and R be a vague set of X. Then R is a vague ring of X if the following conditions are satisfied:

(1). $V_R (x + y) \leq \text{imax}\{ V_R (x) , V_R (y)\}$, for all $x, y \in X$;

(2). $V_R (- x) \leq V_R (x)$, for all $x \in X$;

(3). $V_R (xy) \geq \text{imin}\{ V_R (x) , V_R (y)\}$ for all $x, y \in X$.

Definition2.6:[2] Let X be a field and F be a vague set of X. Then F is a vague field of X if the following conditions are satisfied:

(1). $V_F (x + y) \leq \text{imax}\{ V_F (x) , V_F (y)\}$, for all $x, y \in X$;

(2). $V_F (- x) \leq V_F (x)$, for all $x \in X$;

(3). $V_F (xy) \geq \text{imin}\{ V_F (x) , V_F (y)\}$, for all $x, y \in X$;

(4). $V_F (x^{-1}) \geq V_F (x)$ for all $x \in X$.

Defenition2.7:[7] Let V be a vector space over a field F and A be a vague set of V. Then A is a vague vector space of V if the following conditions is satisfies:

(1). $V_A(x + y) \leq \text{imax}\{V_A(x) , V_A(y)\}$, for all $x, y \in V$;

(2). $V_A(ax) \leq V_A(x)$, for all $x \in F$ and $x \in V$;

(3). $V_A(0) = 0$.

Defenition2.8: A linear graph G(V,E) consists of a nonempty set of objects $V = \{v_1, v_2, v_3\}$ called vertices and another set $E = \{e_1, e_2, e_3\}$ called edges such that each edge e_k is indentified with an unordered pair $\{v_i, v_j\}$ of vertices.

Defenition2.9: A graph g is said to bea sub graph of a graph G if all the vertices and all the edges of g are in G and each edge of g has the same end vertices in g as in G. We denote this fact by $g \subset G$.

3.VAGUE VECTOR SPACES ON SUB GRAPHS

Defenition3.1: Let g_1 and g_2 are two sub graphs of G. Clearly $g_1 \oplus g_2$ is again a sub graphs of G. The set of all sub graphs od g forms an abeliean group with resepect to ring sum.

Here the null graph φ acts as the identity element. Every sub graph as its own inverse.

The following example shows tha existance of the vague additive group on sub graphs.

Example3.2: Let S = g_1, g_2, g_3 and g_4 be the set of sub graphs of G, where g_1 = (0,0), g_2 = (1,0), g_3 = (0,1) and g_4 = (1,1). Define addition module2 on S as follows:

$+_2$	g_1	g_2	g_3	g_4
g_1	g_1	g_2	g_3	g_4
g_2	g_2	g_1	g_4	g_1
g_3	g_3	g_4	g_1	g_2
g_4	g_4	g_3	g_2	g_1

Clearly (S, $+_2$) is an abelian group. Let A be a vague set of S defined by

$t_A(x)$ = 0.6 if x = g_1 and $f_A(x)$ = 0.3 if x = g_1

= 0.7 if x = g_2 = 0.2 if x = g_2

= 0.8 if x = g_3, g_4 = 0.1 if x = g_3, g_4

Then A is a vague additive group on subgraphs.

The following example shows tha existance of the vague ring on sub graphs.

Example3.3: In representing graphs we are concered only with modulo 2. It consists of R = {0,1} and the addition madulo 2 and multiplicatiom madulo 2 operations are as follows:

$+_2$	0	1
0	0	1
1	1	0

$\cdot_2$	0	1
0	0	0
1	0	1

Clearly (R, $+_2$, $\cdot_2$) is a ring.

Let B be a vague set of R defined by

$t_B(x)$ = 0.7 if x = 0 and $f_B(x)$ = 0.2 if x = 0

= 0.8 if x = 1 = 0.1 if x = 1

Then B is vague ring on subgraphs.

The following example shows tha existance of the vague field on sub graphs.

Example3.4: Let F = {0,1} and the addition madulo 3 and multiplicatiom madulo 3 operations are as follows:

$+_3$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

$\cdot_3$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Clearly (F, $+_3$, $._3$) is a field.

Let C be a vague set of R defined by

$t_C(x)$ = 0.7 if x = 0 and $f_C(x)$ = 0.2 if x = 0

= 0.8 if x = 1,2 = 0.1 if x = 1,2

Then C is vague field on subgraphs.

Example3.5: Let $G_F(5)$ = {0, 1, 2, 3,4} be a Galois field. Let D be a vague set of $G_F(5)$ defined by

$t_D(x)$ = 0.6 if x = 0 and $f_D(x)$ = 0.3 if x = 0

= 0.7 if x = 1 = 0.2 if x = 1

= 0.8 if x = 2 = 0.1 if x = 2

= 0.8 if x = 3,4 = 0 if x = 3,4

Then D is a vague field on subgraphs.

A vector space W_G associated with a graph G consists of

Galois field modulo 2 that is the set {0,1} with operations addition madulo 2 and multiplicatiom madulo 2.

2^e vectors where e is the number of edges in G.

An addition opration between two vectors x, y in this defined as the vector sum, that is

$X \oplus Y = (x_1 + y_1, x_2 + y_2, x_3 + y_3 \ldots\ldots x_e + y_e)$ where $X = (x_1, x_2, x_3 \ldots\ldots x_e)$,

$Y = (y_1, y_2, y_3 \ldots\ldots y_e)$ and being the addition modulo2.

A sclar multiplication between a sclar c ϵ Z_2 and a vector X be defined as

$cX = (cx_1, cx_2, cx_3 \ldots\ldots cx_e)$.

Example3.6: Let $W_G = \{ g_1, g_2, g_3, \ldots\ldots g_8\}$ be a vector space of subgraphs, where

g_1 = (0,0,0), g_2 = (1,0,0), g_3= (0,1,0), g_4= (0,0,1),

g_5 = (1,1,0), g_6 = (1,0,1), g_7= (0,1,1), g_8= (1,1,1).

The operation ring sum on W_G defined as follows:

$+_2$	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
g_1	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
g_2	g_2	g_1	g_5	g_6	g_3	g_4	g_8	g_7
g_3	g_3	g_5	g_1	g_7	g_2	g_8	g_4	g_6
g_4	g_4	g_6	g_1	g_7	g_8	g_2	g_3	g_5
g_5	g_5	g_3	g_2	g_8	g_1	g_7	g_6	g_4
g_6	g_6	g_4	g_8	g_2	g_7	g_1	g_5	g_3
g_7	g_7	g_8	g_4	g_3	g_6	g_5	g_1	g_2
g_8	g_8	g_7	g_6	g_5	g_4	g_3	g_2	g_1

Clearly W_G is a vector space on sub graphs together with ring sum and scalar multiplication over the field $G_F(2)$.

Now W_G is associated with vague values then the vector space of sub graphs is called a vague vector space on sub graphs.

Definition3.7: Let G be vector space on graphs. W_G be a vector space on subgraphs. A vague vector space on subgraphs of G called V_{W_G} defined as follows:

(1). V_{W_G} (x + y) ≤ imax{ V_{W_G} (x) , V_{W_G} (y)}, for all x, y ϵ W_G;

(2). V_{W_G} (ax) ≤ V_{W_G} (x), for all x ϵ F and x ϵ W_G;

(3). V_{W_G} (0) = 0.

Example3.8: Let W_G is a vector space on subgraphs in the example3.6. Let E be a vague set of W_G defined by

$t_E(x) = 0$ if $x = g_1$	and	$f_E(x) = 1$ if $x = g_1$
$= 0.1$ if $x = g_2$		$= 0.8$if $x = g_2$
$= 0.2$ if $x = g_3, g_5$		$= 0.7$ if $x = g_3, g_5$
$= 0.3$ if $x = g_4, g_6, g_7, g_8$		$= 0.6$ if $x = g_4, g_6, g_7, g_8$

Then E is a vague vector space on subgraphs.

Theorem3.9: Intersection of two vague subgroups of subgraph of a vague graph G is also a vague sub group of sub graphs.

Proof: Let G_1 and G_2 be any two vague subgroups of subgraph of a vague graph G. Let $g_1, g_2 \in G_1 \cap G_2$

Clearly $g_1, g_2 \in G_1$ and $g_1, g_2 \in G_2$. Then we have

$$\begin{aligned} t_{G1\cap G2}(g_1 - g_2) &= \min\{t_{G1}(g_1 - g_2), t_{G2}(g_1 - g_2)\} \\ &\leq \min\{\max\{t_{G1}(g_1), t_{G1}(g_2)\}, \max\{t_{G2}(g_1), t_{G2}(g_2)\}\} \\ &\leq \min\{\max\{t_{G1}(g_1), t_{G2}(g_1)\}, \max\{t_{G1}(g_2), t_{G2}(g_2)\}\} \\ &\leq \max\{\min\{t_{G1}(g_1), t_{G2}(g_1)\}, \min\{t_{G1}(g_2), t_{G2}(g_2)\}\} \\ &\leq \max\{t_{G1\cap G2}(g_1), t_{G1\cap G2}(g_2)\} \end{aligned}$$

Similarly, we can prove that $1- f_{G1\cap G2}(g_1 - g_2) \leq \max\{1-f_{G1\cap G2}(g_1), 1- f_{G1\cap G2}(g_2)\}$.

Therefore $G_1 \cap G_2$ is a vague sub group of sub graphs.

Theorem3.10: If G_1 and G_2 are two vague subgroups of subgraph of a vague additive group on sub graphs then $G_1 \cup G_2$ is also a vague additive subgroup of sub graphs.

Proof: Let G_1 and G_2 be any two vague subgroups of subgraph of a vague additive group and $g_1, g_2 \in G_1$, $g_1, g_2 \in G_2$. Then we have

$$\begin{aligned} t_{G1\cup G2}(g_1 - g_2) &= \max\{t_{G1}(g_1 - g_2), t_{G2}(g_1 - g_2)\} \\ &\leq \max\{\max\{t_{G1}(g_1), t_{G1}(g_2)\}, \max\{t_{G2}(g_1), t_{G2}(g_2)\}\} \\ &\leq \max\{\max\{t_{G1}(g_1), t_{G2}(g_1)\}, \max\{t_{G1}(g_2), t_{G2}(g_2)\}\} \\ &\leq \max\{t_{G1\cup G2}(g_1), t_{G1\cup G2}(g_2)\} \end{aligned}$$

Similarly, we can prove that $1- f_{G1\cup G2}(g_1 - g_2) \leq \max\{1-f_{G1\cup G2}(g_1), 1- f_{G1\cup G2}(g_2)\}$.

Therefore $G_1 \cup G_2$ is a vague subgroup of sub graphs.

Theorem3.11: The set of all vague subgraph vectors in V_{W_G} froms a vague sub space vector space V_{W_s}.

Proof: Let $g_1, g_2 \in V_{W_s}$ and a ϵ F.

Then $g_1 \oplus g_2 \in V_{W_s}$ and a $g_1 \in V_{W_s}$.

By definition of vague vector space it follows that

(1). t_{W_s} (x + y) ≤ imax{ $t_{W_{sG}}$ (x) , t_{W_s} (y)}:

(2). t_{W_s} (ax) ≤ t_{W_s} (x), for all x ϵ F ;

(3). t_s (0) = 0.

Similarly

(1).1- f_{W_s} (x + y) ≤ imax{1- $f_{W_{sG}}$ (x) ,1- f_{W_s} (y)}:

(2).1- f_{W_s} (ax) ≤ 1- f_{W_s}(x), for all x ϵ F ;

(3). 1- f_{W_s} (0) = 0.

Hence the result.

4. CONCLUSION

In this paper the concept of vague vector space on sub graphs has been introduced and it is expected that seceral results from circuits and cutsets can be extened. It is hoped that that the concept of vague vector space on subgraphs will give rise to the notations like vague normal linear spaces on subgraphs.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors

REFERENCES

[1] V. Amarendra Babu, K.V. Ramarao, T. Anitha, Vague additive groups, Universal journal of applied mathematics, 3 (3) (2015) 50 – 52.

[2] V. Amarendra Babu, K.V. Ramarao, Vague rings anf vague fields, Journal of multidisciplinary Engineering Science and Technology, 2 (7) (2015) 1943-1945.

[3]. M.Demirci, Vague groups, Jou. Math. Anal. Appl. 230(1999)142 – 156.

[4] Gai W. L., Buehrer D. J., Vague sets, IEEE Transactions on Systems, Man and Cybernetics, 23(20)(1993), 610 – 614.

[5]Hakimuddin Khan, Musheer Ahmad and Biswas.R.,On vague groups, International journal of computational congnition 5(1)(2007) 27 – 30.

[6] Ranjit Biswas , Vague Groups, Int. J. Comput. Cngition, 4 (2) (2006), 20 – 23.

[7] K.V. Ramarao, V. Amarendra Babu, Vague vector spaces and vague modules,International Journal of Pure and Applied Mathmatics, 111 (2) (2016) 179 - 188.

[8] Rosenfield, A., Fuzzy groups, Jou. Math. Anal. Appl.35(1971)512 – 517.

[9] L. A. Zadesh, Fuzzy sets, Information and Control 8 (1965), 338 – 353.