alphanumeric
https://alphanumericjournal.com/

Volume 12
Issue 01
Year 2024

Research Article

A Novel Game-Theoretical Approach for The Possibilistic Mean - Variance Model

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Abstract

Possibility theory is a significant tool to deal with imprecise probability and benefit from expert knowledge. Thus, the possibilistic mean-variance (MV) model is a considerable alternative for the portfolio selection problem. In this study, we propose an extension of the possibilistic MV model to the multiple market strategies where we assume that the possibility distributions of asset returns are given with triangular fuzzy numbers. The proposed extension related to the game theory is provided with a linear optimization problem. Thus, it can be solved with the Simplex algorithm as in this study. After giving the theoretical points, we illustrate it by using a numerical example.

Keywords Fuzzy Set, Game Theory, Linear Optimization, Portfolio Selection, Possibility Theory

Jel Codes C61, C72, G11

Contents

2024.12.01.OR.01

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Timeline

Submitted Jan 23, 2023
Accepted Jul 01, 2024
Published Jul 20, 2024

Citation

Göktaş, F. (2024). A Novel Game-Theoretical Approach for The Possibilistic Mean - Variance Model, *alphanumeric*, 12 (1), 1-12. https://doi.org/10.17093/alphanumeric.1244061

1. Introduction

The portfolio selection problem is a vital decision-making problem. Mathematical models based on the probability theory are commonly used to solve this problem. The MV model introduced by Markowitz (1952) is the most known of these models. Its possibilistic counterpart introduced in Carlsson et al. (2002) enables us to model the asymmetry in the past data and incorporate the subjective judgements into the portfolio selection problem, unlike Markowitz's MV model. Thus, the possibilistic MV model provides flexibility (Taş et al., 2016). In this study, we focus on it, and the reader may refer to Zhang et al. (2017) and Fullér & Harmati (2017) for further information about the possibilistic portfolio selection.

The solution of the possibilistic MV model is studied by Zhang et al. (2009) and Taş et al. (2016). Its mathematical analysis is studied by Corazza & Nardelli (2019) and Göktaş & Duran (2020). Let the possibility distributions be given with triangular fuzzy numbers, as in this study. Then, the possibilistic correlation between any two assets equals 1, which is a perfect correlation (Taş et al., 2016). That is, this model is linearized, and thus, it can be easily solved. On the other hand, diversified optimal portfolios cannot be uniquely found due to this linearization (Göktaş & Duran, 2020). To deal with this issue, Göktaş (2024) considers an ellipsoidal uncertainty set for the possibilistic mean vector and forms a convex optimization problem for the portfolio selection problem. In this study, we prefer a discrete uncertainty set for the possibilistic mean and standard deviation vectors pair. Then, we propose an extension of the possibilistic MV model to the multiple market strategies. The proposed extension related to the game theory is given with a linear optimization problem. Thus, it can be solved with the Simplex algorithm as in this study.

The rest of the paper is organized as follows. Section 2.1 examines the possibilistic MV model, assuming that the possibility distributions are given with triangular fuzzy numbers. Section 2.2 provides the theory of the proposed extension under the same assumption. Section 3 illustrates the proposed extension with a numerical example. Section 4 concludes the paper.

2. Methods

2.1. The Possibilistic Mean – Variance Model

The concept of possibility is related to plausibility, referring to the tendency of events to occur. In the possibility theory, there are two different measures, which are not self-dual, unlike the probability measure. The possibility measure represents the plausibility of an event, whereas the necessity measure represents the certainty of the event. The possibility (necessity) measure gives the upper (lower) bound for the imprecise probability (Dubois, 2006). In the possibility theory, possibility distributions are usually provided with fuzzy numbers (Souliotis et al., 2022). Figure 1 gives the graph of the membership function of the triangular fuzzy number (-0.5, 0, 1).

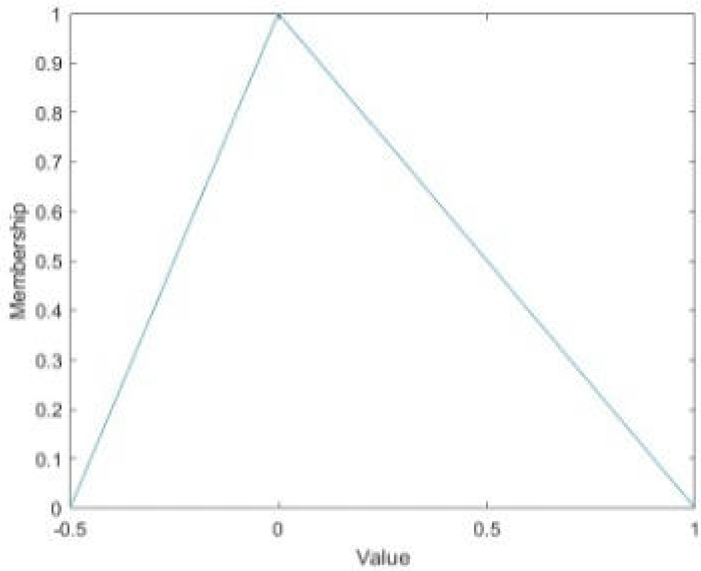

Figure 1. The graph of the membership function of the triangular fuzzy number

Like Carlsson et al. (2002), the feasible set (S) in the portfolio selection problem is below, where w is the weight vector of assets, the weight of the i^{th} asset is equal to w_i, and n is the number of the assets.

$$S = \left\{ w : \sum_{i=1}^{n} w_i = 1 \text{ and } w_i \geq 0, \forall i \right\} \tag{1}$$

Let the random vector of the asset returns be shown with r. Let (b_i, c_i, e_i) be the possibility distribution of r_i. Then, the possibility distribution of portfolio return is found below based on Zadeh's Extension Principle.

$$\sum_{i=1}^{n} w_i(b_i, c_i, e_i) = \left(\sum_{i=1}^{n} w_i b_i, \sum_{i=1}^{n} w_i c_i, \sum_{i=1}^{n} w_i e_i \right) \tag{2}$$

Let $E_p()$ and $STD_p()$ be the possibilistic mean and standard deviation operators, respectively. Based on Eq. 2, the portfolio's possibilistic mean and standard deviation are found below, respectively (Carlsson & Fullér, 2001; Carlsson et al., 2002; Taş et al., 2016). μ_i and α_i correspond to the possibilistic mean and standard deviation of the i^{th} asset, respectively. That is, the possibilistic mean (standard deviation) of any portfolio is equal to the weighted average of the possibilistic mean (standard deviation) of the assets, and the possibilistic correlation between any two assets equals 1 (Corazza & Nardelli, 2019; Göktaş & Duran, 2020).

$$\begin{aligned} E_p\left(\sum_{i=1}^{n} w_i r_i\right) &= \sum_{i=1}^{n} w_i \mu_i = \sum_{i=1}^{n} w_i \left(\frac{b_i + 4c_i + e_i}{6}\right) \\ STD_p\left(\sum_{i=1}^{n} w_i r_i\right) &= \sum_{i=1}^{n} w_i \sigma_i = \sum_{i=1}^{n} w_i \left(\frac{e_i - b_i}{2\sqrt{6}}\right) \end{aligned} \tag{3}$$

As in this study, the portfolio selection problem usually depends on bi-objective, where the first (second) objective is to maximize (minimize) the return (risk). Based on Eq. 3 and the weighted sum method, the possibilistic MV model can be defined with the following linear maximization problem where the weight of the first objective (β) is in $[0, 1]$. Its optimal solution is called an efficient portfolio (EP) (Göktaş & Duran, 2020). Eq. 4 is a return maximization problem when β equals 1. Eq. 4 is a risk minimization problem when β equals 0.

$$\max_{w \in S} \sum_{i=1}^{n} (\beta\mu_i - (1-\beta)\sigma_i) w_i \tag{4}$$

The portfolio (MaxP) that maximizes the performance is defined below (Göktaş & Duran, 2020). By definition, it is an efficient portfolio. We assume that at least one portfolio's performance is positive.

$$\max_{w \in S} \left(P(w) := \frac{\sum_{i=1}^{n} (w_i \mu_i)}{\sum_{i=1}^{n} (w_i \sigma_i)} \right) \tag{5}$$

Eq. 5 is a linear fractional maximization problem. Since the performance does not depend on the amount of any portfolio, the optimal solution of Eq. 5 is found when the optimal solution of Eq. 6 is standardized (Goldfarb & Iyengar, 2003; Tütüncü & Koenig, 2004). Standardization means that a vector is divided by the sum of its elements to provide that the sum of its elements is equal to 1.

$$\begin{aligned} &\max \sum_{i=1}^{n} w_i \mu_i \\ &\text{s.t.} \sum_{i=1}^{n} w_i \sigma_i = 1 \\ &w_i \geq 0, \forall i \end{aligned} \tag{6}$$

Due to the Corner Point Theorem, we have the following results (Göktaş & Duran, 2020).

a) If $\beta\mu_i - (1-\beta)\sigma_i$ expression is uniquely maximized by only one asset, then there is a unique optimal solution, and EP consists of only this asset.
b) If $\beta\mu_i - (1-\beta)\sigma_i$ expression is maximized by two or more assets, then there are alternative optimal solutions, and EP equals any convex combinations of these assets.
c) If $\frac{\mu_i}{\sigma_i}$ expression is uniquely maximized by only one asset, then there is a unique optimal solution, and the MaxP consists of only this asset.
d) If $\frac{\mu_i}{\sigma_i}$ expression is maximized by two or more assets, then there are alternative optimal solutions, and MaxP equals any convex combinations of these assets.

2.2. The Proposed Extension of The Possibilistic Mean – Variance Model

The possibility of distributions can be determined in different ways, such as using the descriptive statistics of the past data and/or expert knowledge. For example, the parameters of triangular fuzzy numbers can be determined by using the sample minimum, mean and maximum statistics, respectively (Taş et al., 2016). Here, the first (third) parameters of the triangular fuzzy numbers correspond to the worst-case (best-case) predictions for future returns, whereas the second parameters of the triangular fuzzy numbers correspond to the base-case predictions for future returns.

In this study, we assume that the possibility distributions are determined in the possibilistic MV model as in Taş et al. (2016). In our proposed extension of the possibilistic MV model, we call this situation the past market strategy. We assume that the translation $(\theta_{i,j})$ and ambiguity $(\pi_{i,k})$ parameters are determined by the m experts based on their future perspectives. Then, we determine the m^2 possible market strategies as below.

Here, the left side depends on the definition of triangular fuzzy numbers, and the right side corresponds to the zth market strategy in the proposed extension where $z = ((j-1) \cdot m + k))$. For example, when j=2, k=3 and m=4, the translation parameters determined by the second expert and the ambiguity parameters determined by the third expert are combined to form the seventh market strategy. Rustem et al. (2000) use a similar approach for extending Markowitz's MV model. It is called the M-m rival return and rival risk scenarios.

$$-\pi_{i,k}\sqrt{6} < \min(c_i - b_i, e_i - c_i) \Rightarrow r_i = \left(b_i + \theta_{i,j} - \pi_{i,k}\sqrt{6}, c_i + \theta_{i,j}, e_i + \theta_{i,j} + \pi_{i,k}\sqrt{6}\right) \tag{7}$$

Remark: When the triangular fuzzy number is symmetrical, the inequality given in Eq. 7 reduces to $-\pi_{i,k} < \sigma_i$. If the j^{th} expert is more optimistic (pessimistic) about the future return of the i^{th} asset than its past return, then $\theta_{i,j}$ is positive (negative). If the k^{th} expert is more uncertain (certain) about the future return of the i^{th} asset than its past return, then $\pi_{i,k}$ is positive (negative).

Example: Suppose that the past strategy for the first asset is equal to the symmetric triangular fuzzy number $(-0.1, 0, 0.1)$. Let $\theta_{1,j}$ is equal to 0 for all j. Then, it is clear that $-\pi_{1,k}\sqrt{6}$ should be smaller than 0.1 for all k. Otherwise, $\left(-0.1 - \pi_{1,k}\sqrt{6}, 0, 0.1 + \pi_{1,k}\sqrt{6}\right)$ will not be a triangular fuzzy number. Its possibilistic standard deviation in the past strategy is equal to $0.2 \div 2\sqrt{6}$. That is, $-\pi_{1,k}$ should be smaller than $0.2 \div 2\sqrt{6}$.

Based on Eq. 3 and Eq. 7, we calculate the ith asset's possibilistic mean in the z^{th} market strategy as below where μ_i corresponds to the possibilistic mean in the past market strategy.

$$\begin{aligned} E_p\left(\left(b_i + \theta_{i,j} - \pi_{i,k}\sqrt{6}, c_i + \theta_{i,j}, e_i + \theta_{i,j} + \pi_{i,k}\sqrt{6}\right)\right) &= \frac{b_i + \theta_{i,j} - \pi_{i,k}\sqrt{6} + 4(c_i + \theta_{i,j}) + e_i + \theta_{i,j} + \pi_{i,k}\sqrt{6}}{6} \\ &= \frac{b_i + 4c_i + e_i}{6} + \frac{6\theta_{i,j}}{6} \\ &= \mu_i + \theta_{i,j} \end{aligned} \tag{8}$$

Based on Eq. 3 and Eq. 7, we calculate the i^{th} asset's possibilistic standard deviation in the z^{th} market strategy as below where σ_i corresponds to the possibilistic standard deviation in the past market strategy. Clearly, the possibilistic mean and standard deviation are independent from each other in the z^{th} market strategy.

$$\begin{aligned} STD_p\Big(\big(b_i+\theta_{i,j}-\pi_{i,k}\sqrt{6}, c_i+\theta_{i,j}, e_i+\theta_{i,j}+\pi_{i,k}\sqrt{6}\big)\Big) &= \frac{\big(e_i+\theta_{i,j}+\pi_{i,k}\sqrt{6}\big)-\big(b_i+\theta_{i,j}-\pi_{i,k}\sqrt{6}\big)}{2\sqrt{6}} \\ &= \frac{e_i-b_i}{2\sqrt{6}}+\frac{\pi_{i,k}2\sqrt{6}}{2\sqrt{6}} \\ &= \sigma_i+\pi_{i,k} \end{aligned} \tag{9}$$

Based on Eq. 3, Eq. 8 and Eq. 9, we find the possibilistic mean and standard deviation of portfolio in the z^{th} market strategy as below respectively.

$$\begin{aligned} E_p\left(\sum_{i=1}^{n} w_i r_i\right) &= \sum_{i=1}^{n} w_i\big(\mu_i+\theta_{i,j}\big) \\ STD_p\left(\sum_{i=1}^{n} w_i r_i\right) &= \sum_{i=1}^{n} w_i\big(\sigma_i+\pi_{i,k}\big) \end{aligned} \tag{10}$$

Let β be in $[0,1]$. Like Eq. 4, we propose the possibilistic MV model with the multiple market strategies as below based on Eq. 10. We call its optimal solution as a new efficient portfolio (NEP).

Here, the y variable is associated with the worst-case scenario. We call its optimal solution as the new efficient portfolio (NEP). If there are alternative optimal solutions of Eq. 11, the optimal solution set is convex and compact (Raghavan, 1994).

$$\begin{aligned} &\max\ y \\ &\text{s.t. } y \le \sum_{i=1}^{n}\big[\beta\big(\mu_i+\theta_{i,j}\big)-(1-\beta)\big(\sigma_i+\pi_{i,k}\big)\big]w_i \text{ for all } z \\ &\sum_{i=1}^{n} w_i = 1 \\ &w_i \ge 0, \forall i \end{aligned} \tag{11}$$

Let the elements of the payoff matrix (R) be as in Eq. 12 where $z=((j-1)\cdot m+k))$. Then, Eq. 11 corresponds to a zero-sum game with two players where the investor is the first (row) player and the market is the second (column) player. The optimal y value in Eq. 11 is called the game value. The assets are the pure strategies for the investor whereas the other portfolios are the mixed strategies for the investor (Sikalo et al., 2022). The i^{th} row of R corresponds to the i^{th} asset, whereas the z^{th} column of R corresponds to the z^{th} marker strategy.

$$R_{i,z}=\beta\big(\mu_i+\theta_{i,j}\big)-(1-\beta)\big(\sigma_i+\pi_{i,k}\big) \tag{12}$$

Eq. 11 is mathematically similar to portfolio selection model by Young (1998). Unlike our approach, it only depends on past data. Eq. 13 is equivalent to Eq. 11 since t and u variables are independent from each other and $y=(t-u)$. The t variable is related to the worst-case possibilistic mean. u variable is related to the worst-case possibilistic standard deviation. In practice, Eq. 13 should be preferred to Eq. 11 for ease of use.

$$\begin{aligned} &\max\ t-u \\ &\text{s.t. } t \le \beta \sum_{i=1}^{n} w_i(\mu_i+\theta_{i,j}) \text{ for all } j \\ &\quad u \ge (1-\beta)\sum_{i=1}^{n} w_i(\sigma_i+\pi_{i,k}) \text{ for all } k \\ &\quad \sum_{i=1}^{n} w_i = 1 \\ &\quad w_i \ge 0, \forall i \end{aligned} \tag{13}$$

We define the portfolio (MaxWP) that maximizes the worst-case performance (WCP) as below. It is a NEP by definition. We assume that at least one portfolio's WCP is positive.

$$\max_{w\in S}\left(WCP(w) := \min_{j,k} \frac{\sum_{i=1}^{n} w_i(\mu_i+\theta_{i,j})}{\sum_{i=1}^{n} w_i(\sigma_i+\pi_{i,k})}\right) \tag{14}$$

Eq. 14 is a max–min linear fractional programming problem and can be solved with special algorithms like those used by Jiao & Li (2022). It is also equivalent to the following linear maximization problem except for the standardization issue (Goldfarb & Iyengar, 2003; Tütüncü & Koenig, 2004). Thus, we find MaxWP by standardizing the optimal solution of Eq. 15.

$$\begin{aligned} &\max\ t \\ &\text{s.t. } t \le \sum_{i=1}^{n} w_i(\mu_i+\theta_{i,j}) \text{ for all } j \\ &\quad \sum_{i=1}^{n} w_i(\sigma_i+\pi_{i,k}) \le 1 \text{ for all } k \\ &\quad \sum_{i=1}^{n} w_i = 1 \\ &\quad w_i \ge 0, \forall i \end{aligned} \tag{15}$$

We know from the theory of zero-sum games that the optimal solution of Eq. 11 for the investor is probably a mixed strategy (Raghavan, 1994). That is, Eq. 11 probably gives a diversified optimal portfolio, unlike the possibilistic MV model. This information is also valid for MaxWP since it is a NEP by definition. Thus, we achieve the main aim of this study with the help of game theory.

3. Results and Discussion

In this section, we illustrate and compare the possibilistic MV model and its proposed extension based on the *Example III* of Göktaş & Duran (2020). We take $\beta = 0.5$ for EP and NEP. Two objectives in the portfolio selection problem (to maximize the return and minimize the risk) are equally weighted. We also call some special optimal portfolios as follows. When β equals 0, we get MinPS (MinWPS) in the possibilistic MV model (proposed extension). When β equals 1, we get MaxPM (MaxWPM) in the possibilistic MV model (proposed extension).

Let A1, A2, A3, A4 and A5 be five different assets where their past returns' possibility distributions are symmetrical triangular fuzzy numbers. In Table 1, we give the information about them in the possibilistic MV model.

Table 1. The information about the possibilistic MV model

Assets	P. Mean	P. Std. Dev.	Performance	(5) for $\beta = 0.5$
A1	-0.05	0.05	N/A	-0.05
A2	0.2	0.15	1.3333	0.025
A3	0.15	0.1	1.5	0.025
A4	0.05	0.2	0.25	-0.075
A5	0.05	0.15	0.3333	-0.05

We find MaxPM, MinPS, MaxP and EP as in Table 2. MaxPM (MaxP) consists of only A2 (A3) since A2 (A3) uniquely maximizes the possibilistic mean (performance). MinPS consists of only A1 since A1 uniquely minimizes the possibilistic standard deviation. EP is equal to any portfolio that consists of only A2 and A3 since both A2 and A3 maximize the objective function of Eq. 4 for $\beta = 0.5$. In this study, we take their weights be equal in EP.

Table 2. Some optimal portfolios in the possibilistic MV model

Assets	MaxPM	MinPS	MaxP	EP
A1	0	1	0	0
A2	1	0	0	0.5
A3	0	0	1	0.5
A4	0	0	0	0
A5	0	0	0	0

Let the translation (θ) matrix be formed based on the future perspectives of four different experts as in Table 3. Expert 1 is neither optimistic nor pessimistic about the future return of A3 than its past return whereas Expert 2 (Expert 4) is optimistic (pessimistic) about it.

Table 3. The translation (θ) matrix

Assets	Expert 1	Expert 2	Expert 3	Expert 4
A1	0	0.15	0.15	0.1
A2	0	-0.1	-0.05	-0.05
A3	0	0.05	0.05	-0.1
A4	0	0.05	0	0.15
A5	0	0.05	0.15	0

Let the ambiguity (π) matrix be formed based on the future perspectives of these experts as in Table 4. Because the possibility distributions are given with the symmetrical triangular fuzzy numbers, it is sufficient to check that $-\pi_{i,k}$ is smaller than σ_i for all i and k. (Based on Eq. 10, this condition also guarantees that the possibilistic standard deviations are positive in each market strategy.) Expert 1 is neither more certain nor more uncertain about the future return of A3 than its past return whereas Expert 2 (Expert 4) is more certain (more uncertain) about it.

Table 4. The ambiguity (π) matrix

Assets	Expert 1	Expert 2	Expert 3	Expert 4
A1	0	0	0.1	0.05
A2	0	0	−0.05	0
A3	0	−0.05	0.1	0.1
A4	0	0	−0.1	−0.15
A5	0	−0.1	−0.05	0.05

By combining the j^{th} column of Table 3 and the k^{th} column of Table 4, we form the z^{th} market strategy in our example where $z = ((j-1) * m + k))$. Clearly, the first market strategy is equal to the past market strategy. We uniquely find MaxWPM, MinWPS, MaxWP and NEP as in Table 5. NEP is equal to MaxWP in our example, where β is taken as 0.5 for NEP. These optimal portfolios are more diversified than their counterparts in Table 2.

Table 5. Some optimal portfolios in the proposed extension

Assets	MaxWPM	MinWPS	MaxWP	NEP
A1	0	0.375	0.1429	0.1429
A2	0.3333	0.5	0.4286	0.4286
A3	0.3333	0	0.2286	0.2286
A4	0.3333	0.125	0.2	0.2
A5	0	0	0	0

Figure 3 gives the exact efficient frontier of the possibilistic MV model. Since the possibilistic mean and standard deviation are the linear functions of w, the efficient frontier of the possibilistic MV model consists of several consecutive line segments.

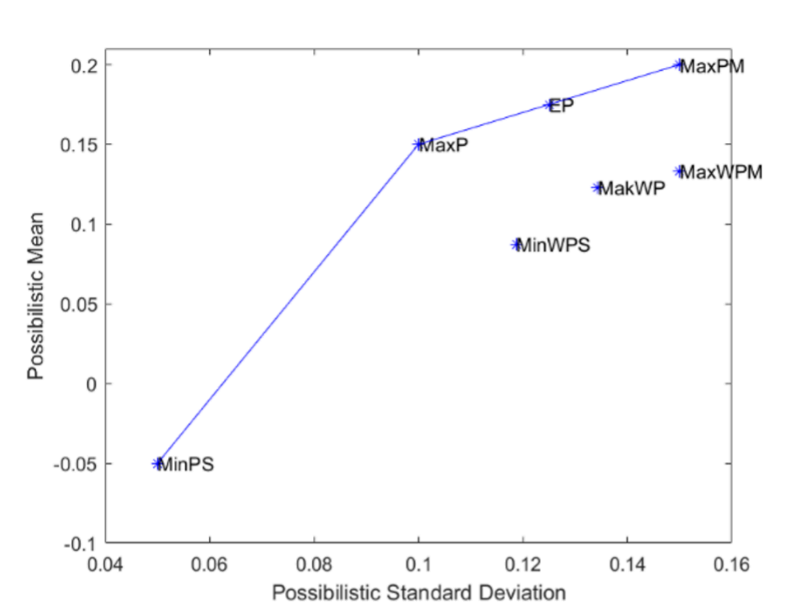

Figure 2. The possibilistic MV model's efficient frontier

Figure 3 approximates the proposed extension's efficient frontier. We see that the worst-case possibilistic means of the new efficient portfolios are all above a certain level. That is, there are mixed strategies for the investors, which give satisfactory results even against the most effective market strategy.

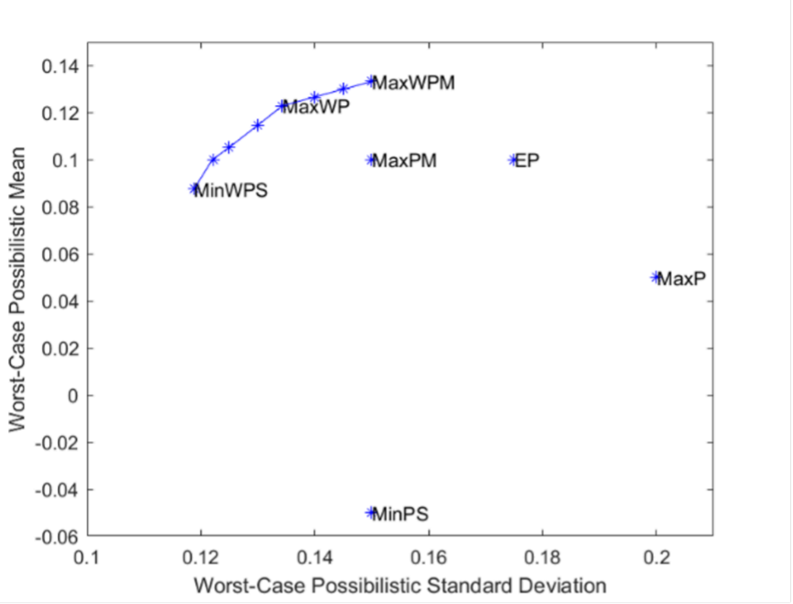

Figure 3. The approximated efficient frontier of the proposed extension

We give the (worst-case) performance values and ranks of the optimal portfolios in Table 6. TThe Spearman's rank correlation between the performance and the worst-case performance is −0.1071. These two measures give the negatively correlated results in our example. The most effective market strategy against MaxWPM, MinWPS and MaxWP is our example's first (past) market strategy. Although their performances against the past market strategy are not too high, they perform satisfactorily against all market strategies, unlike MaxPM, MinPS, MaxP and EP.

Table 6. The (worst-case) performance comparisons

Portfolios	Per. Value	Per. Rank	WCP Value	WCP Rank
MaxPM	1.3333	3	0.6667	4
MinPS	N/A	7	N/A	7
MaxP	1.5	1	0.25	6
EP	1.4	2	0.5714	5
MaxWPM	0.8889	5	0.8889	2
MinWPS	0.7368	6	0.7368	3
MaxWP	0.9149	4	0.9149	1

Based on the information given in Table 2, Table 5 and Table 6, we remark that we will derive more diversified and robust optimal portfolios with the proposed extension. This may be because our approach depends on game theory, a solid mathematical tool in decision-making (Sikalo et al., 2022).

4. Conclusions

This study extends the possibilistic MV model to multiple market strategies under certain assumptions. It is known that the possibilistic MV model provides flexibility to practitioners, and the model's success depends on predicting and correctly modelling future returns. In this regard, we believe we are one step ahead of the possibilistic MV model by considering the multiple market strategies. By definition, our approach is expected to give more diversified and robust optimal portfolios, as in our illustrative example. For these reasons, we believe it may be a good alternative to the possibilistic MV model. On the other hand, the success of our approach highly depends on determining the possible market strategies correctly. Furthermore, the real market conditions can differ from the most effective market strategy. Moreover, our approach is applicable only under the assumption that the possibility distributions are given with triangular fuzzy numbers. As a result, our approach has some strong properties under severe limitations.

Declarations

Conflict of interest The authors have no competing interests to declare that are relevant to the content of this article.

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