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# DETERMINATION OF THE BEST SIMPLE MOVING AVERAGE BY STOCHASTIC PROCESSES

**Deniz İLALAN**[*]

**Abstract**

In this study, we consider one of the most popular technical indicators and try to determine the best fitting simple moving average to a given data. Here we utilize from a general mean reverting stochastic process where the mean is time dependent. We propose an identification algorithm which mainly concentrates on the normality of the residual terms after the data is demeaned from simple moving average and also provide evidence that our algorithm works quite well for determination of the "best" simple moving average.

**Keywords:** Stock Returns, Simple Moving Average, Mean Reverting Stochastic Processes, Normality Test.

**Jel Codes:** C02, G12, G17.

# STOKASTİK SÜREÇLERLE EN İYİ BASİT HAREKETLİ ORTALAMANIN BELİRLENMESİ

**Öz**

Bu çalışmada en gözde teknik analiz indikatörlerinden birisi incelenmiş ve veriye en iyi uyan basit hareketli ortalama belirlenmeye çalışılmıştır. Burada, ortalamanın zamana bağlı olduğu genel bir ortalamaya dönen stokastik süreçten faydalanılmıştır. Veri basit hareketli ortalamadan arındırıldıktan sonra kalan terimlerin normal dağılımına odaklanan bir algoritma sunulmuştur. En iyi hareketli ortalamayı belirleyen algoritmamızın çalıştığı bir örnek verilmiştir.

**Anahtar Kelimeler:** Hisse Getirileri, Basit Hareketli Ortalama, Ortalamaya Dönen Stokastik Süreçler, Normallik Testi.

**Jel Kodları:** C02, G12, G17.

[*] Yrd. Doç. Dr., Çankaya Üniversitesi, denizilalan@cankaya.edu.tr

## 1. Introduction

Predictability of stock prices is the major concern of an investor. Academicians usually support the idea originated by Fama (1965) which states that stock market prices are essentially unpredictable. Malkiel (1973) considers some well known trading strategies and concluded that these methods are inferior to even being idle. He, moreover claims that prices cannot always stay above their averages. Contrary to Malkiel, Lo and MacKinlay (2002) find that markets are not completely random after all, and that predictable components do exist in recent stock and bond returns. Practitioners claim that process follow certain patterns based on past market data, primarily price and volume (Kilkpatrick and Julie 2006) called "technical analysis". Technicians use many technical indicators based on charts. All of these indicators are based on Dow Theory (Carlson 2004). There are six basic tenets of Dow Theory:

(i) The market has three movements (major trend, medium swing and short swing).

(ii) Market trends are composed of three phases: an accumulation phase, a public participation (or absorption) phase, and a distribution phase.

(iii) The stock market discounts all news.

(iv) Stock market averages must confirm each other.

(v) Trends are confirmed by volume.

(vi) Trends exist until definitive signals prove that they have ended.

The academic literature on technical analysis is quite shallow on the sense that academicians usually do not "believe" it. Brock et al (1992, pp.1731-1764) examined Dow Jones Industrial Average from 1897 to 1986 and conclude that there are strong for the explored technical analysis strategies. Gençay (1998, pp. 347–359) used the daily Dow Jones Industrial Average Index from 1897 to 1988 to examine the linear and nonlinear predictability of stock market returns with simple technical trading rules. Evidence of nonlinear predictability in stock market returns is found by using the past buy and sell signals of the moving average rules.

Trend is one of the most widely used technical indicator. It mainly indicates a move in a particular direction for a certain time (see Edwards et al 2007 for details). Although there exists other forms of it, linear trend is the most commonly used. Support and resistance levels are essential indicators for a trader as well. They act as an obstacle for prices by either pushing or pressing them.

Moving averages (MA), on the other hand, convey buying or selling signals for a trader. They can also be regarded as dynamic resistance and support levels. Although a consensus has not been reached, there are numerous evidence that MA is a profitable trading technique (see for instance Marshall et al. 2008, pp. 199-210; Zhu and Zhou, 2009, pp. 519-544; Han et al. 2013, pp. 1433–1461). Some alternative approaches are also frequently encountered in the literature. Some prominent recent studies are as follows: Schlüter (2009, pp. 1-21) constructed a quasilinear MA based on the scaling function. Kum et al. (2015, pp. 1131-1150) explored the convergence properties of moving averages on complete metric spaces. Alia et al. (2015, pp. 1756-1761) found evidence that simple moving average (SMA) can be used in cases where information is not shared. Chen et al. (2016, pp. 263-272) took a sample of Taiwan stock market and demonstrate that MA strategy significantly outperforms the buy-and-hold strategy on the portfolio without option issuance. Huang and Ni (2016) compared MAs ın terms of their trading days in other terms lags, and claim that the movement of MAs are closely linked with the board structure of the company.

In this study we refrain ourselves to complex SMA models and examine the problem of selecting the best SMA in terms of trading days. We do this through incorporation of mean reverting stochastic processes. Here, our aim is not to discuss the advantages or disadvantages of SMA. We rather try to find the "best" SMA according to an identification algorithm. In that sense we try to shed light to the investors via choosing the SMA with a reasonable lag conjoined with normally distributed error terms. Moreover our methodology yields an analytically solvable stochastic differential equation. Section 2 briefly describes SMA. In Section 3 we explore mean reverting stochastic processes. Section 4 is devoted to our "identification algorithm" for finding the "best" SMA. Section 5 is the application part. Finally section 6 concludes.

## 2. Simple Moving Average

Among the most popular technical indicators, moving averages are used to smooth out price actions through removing the noise occuring in random price fluctuations. Simple moving average (SMA) is calculated for by adding up the last *n* period prices and then dividing this number to *n*. Formally, for *n* data points $x_1, x_2, \dots, x_n$,

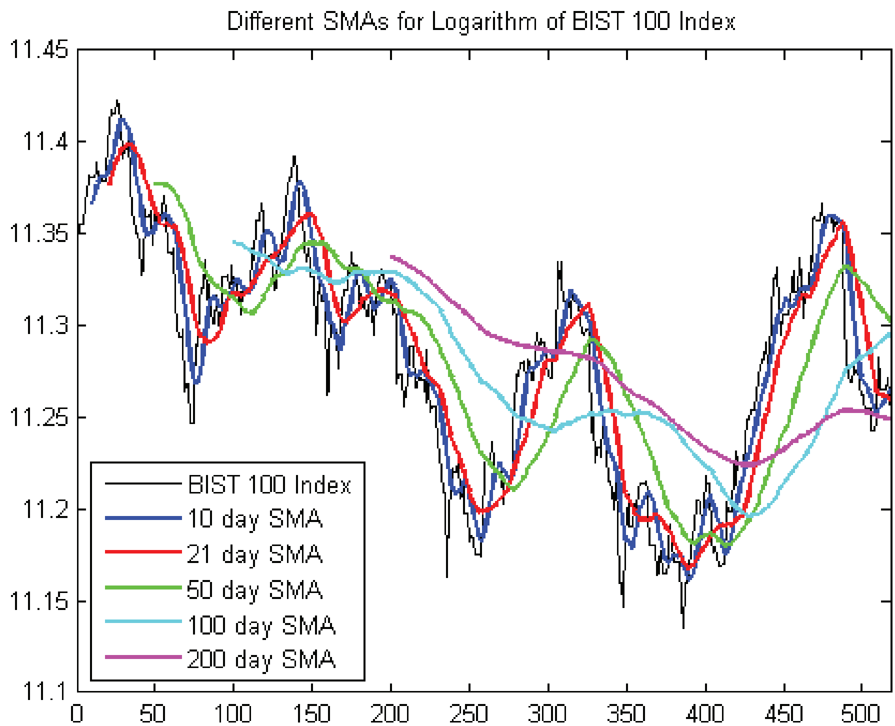
$$SMA = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

However, SMA is computed based on a recursive algorithm. In order to calculate the successive values, a new value is added to the sum while the old value is removed, that is

$$SMA_{today} = SMA_{yesterday} + \frac{x_{n+1}}{n} - \frac{x_n}{n}$$

Choice of *n* depends on the investor. Long term SMAs are called "slow" whereas short term SMAs are called "fast". When more than one SMAs crossover each other then this is deemed as a buying or selling signal which depends on how they intersect. SMAs can also be regarded as dynamic resistance or support levels.

**Figure 1:** 10,21,50,100 and 200 day SMAs for Natural Logarithm of BIST 100 Index between Jan2015 – Jun2016




When only one SMA is used there arises a problem. Technical analysists do not compare SMAs among each other. They rather pick one which fits their interests (long term or short term forecasting) or examine a mixture of them as described above. We, on the other hand, try to determine the "best" SMA through utilization from mean reverting stochastic processes.

### 3. Mean Reverting Stochastic Processes

Mean reverting processes for term structures drew quite a lot of attention in the literature. Benchmark for these is the OU process by Ornstein and Uhlenbeck (1930, pp. 823–841)

$$dS(t) = \theta(\mu - S(t))dt + \sigma dW(t) \tag{3.1}$$

where $\theta, \mu, \sigma \in R^+$ and $W(t)$ is the standard Brownian motion. Vasicek (1977, pp. 177–188) was the first mathematician to use (3.1) for modeling interest rates. Here, the process is assumed to revert back to a constant long term mean $\mu$ with a speed of reversion $\theta$.

Equation (3.1) can be transformed to the following Hull and White (1990, pp. 573–592) Model:

$$dy(t) = (f(t) - y(t))dt + \sigma dW(t) \tag{3.2}$$

where $\sigma > 0$ and $W(t)$ is a standard Wiener process.

The solution of equation (3.2) is done via integration by parts as

$$\begin{aligned}
&dy(t) = (f(t) - y(t))dt + \sigma dW(t) \\
&d(e^t y(t)) = e^t y(t)dt + e^t dy(t) = e^t y(t)dt + e^t \left((f(t) - y(t))dt + \sigma dW(t)\right) \\
&= e^t y(t)dt + e^t f(t)dt - e^t y(t)dt + \sigma e^t dW(t) = e^t f(t)dt + \sigma e^t dW(t) \\
&\Rightarrow e^t y(t) = y(0) + \int_0^t e^s f(s)ds + \sigma \int_0^t e^s dW(s) \\
&\Rightarrow y(t) = e^{-t} y(0) + \int_0^t e^{t-s} f(s)ds + \sigma \int_0^t e^{t-s} dW(s).
\end{aligned}$$

Now the crucial thing here is the normality of the residual terms. If residual terms are not normally distributed then there is no way to convert them to a mean reverting stochastic process. Moreover, fitting a distribution to an arbitrary residual terms is another obstacle. Hence we seek for normally distributed residual terms which is defined in our identification algorithm.

## 4. Identification Algorithm

One day moving average corresponds to the data itself. The averages gets worser when number of days is increased. Here our aim is to find a moving average which corresponds to the time dependent mean $f(t)$ given in (3.2)

Our identification algorithm is based on normal distribution. We, therefore first propose the normality test we use. The test proposed by Jarque and Bera (JB) (1980, pp. 255–259, 1981, pp. 313–318, 1987, pp. 163–172) mainly concentrates on the skewness and kurtosis of the sample data. In order for the sample to be generated from a normal distribution one should expect the skewness and kurtosis to be equal to zero. In JB test having a kurtosis of zero is equivalent to have it less than 3. The JB test statistics is defined as follows:

$$\mathrm{JB} = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right),$$

where $n$ is number of observations, $S$ and $K$ denote skewness and kurtosis given by:

$$S = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^3}{\left(\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2\right)^{3/2}}, \quad K = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^4}{\left(\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2\right)^{2}}.$$

$\overline{x}$ being the sample mean.

The steps of our identification algorithm is as follows:

Step 1) Calculate the SMAs beginning from 2 to the final day.

Step 2) Demean the SMAs from the data and check the normality of the residual terms. If normality test is passed put this SMA aside. Else discard that particular SMA.

Step 3) Choose the SMA from the ones which pass the normality test with the minimum variance and regard it as the "best".

Hence following our algorithm we end up with a moving average acting as a time dependent mean $f(t)$ in (3.2) together with normally distributed residual terms. Of course, predicting the future from the past data is not always possible. Here our aim is to at least model the past data with an analytically tractable fashion. Naturally, if there are more than one mean reverting processes we choose the one which has the minimum variance of residuals. Normality assumption can be quite restrictive in the sense that we may never find an appropriate mean reverting process which represents the data. Therefore, we may face with an existence problem. Nevertheless, we have evidence that our algorithm can sometimes work as stated in Section 5.

## 5. Application Of Our Algorithm To BIST 100 Index

Figure 1 is the natural logarithm of Turkish Stock Exchange BIST 100 between Jan 2015 – Jun 2016. We applied our algorithm here. The results are given in Figure 2 and Figure 3.

**Figure 2:** 21 day and 10 day SMAs and the corresponding residual terms

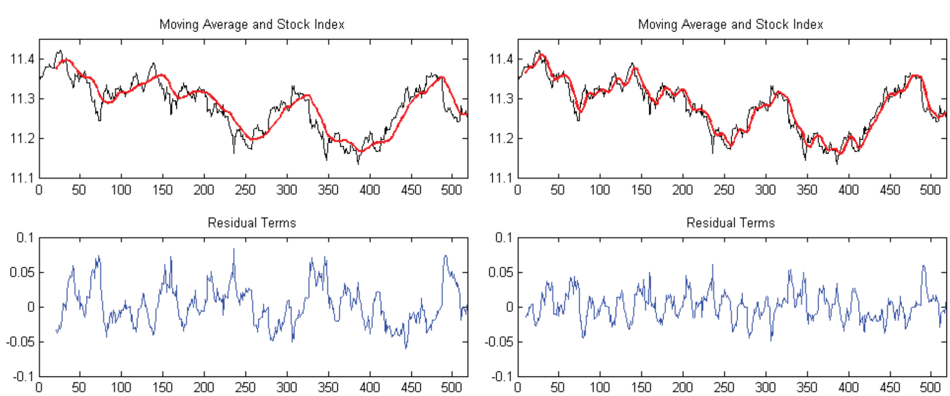
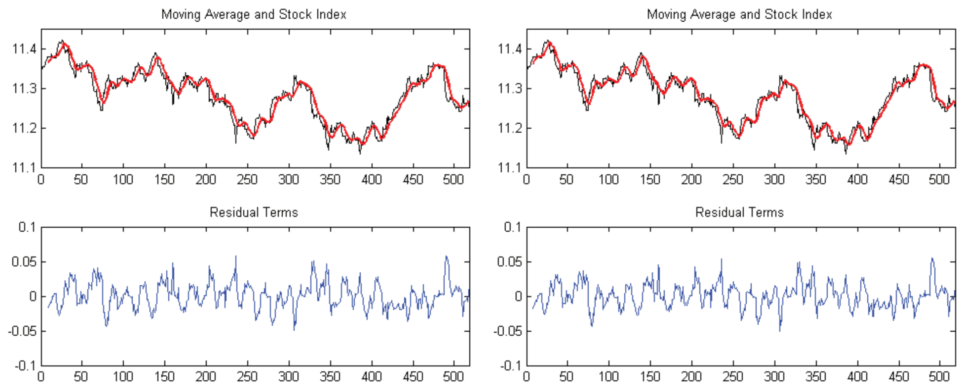


**Figure 3:** 9 day and 8 day SMAs and the corresponding residual terms




We applied JB test and found out that the residuals corresponding to 8,9 and 10 day SMAs are normally distributed (in fact they are the only ones among 518 days)

**Table 1:** Mean and Standard Deviation of the Residual Terms

| Residual for SMA | Mean | Standard Deviation |
|---|---|---|
| 8 | 0.00070379 | 0.017034 |
| 9 | 0.00083064 | 0.018338 |
| 10 | 0.00095144 | 0.019544 |

Hence according to our identification algorithm, 8 day SMA is the best choice which yields the following mean reverting stochastic differential equation

$$dS(t) = \big(SMA_8 - S(t)\big)dt + 0.017 \cdot dW(t)$$

where $W(t)$ is a standard Wiener process. [1]

## 6. Conclusion

Simple moving averages (SMA) are vital indicators for technical analysisists. There is no universally accepted or precise SMA comparison. In this study we try to determine the best SMA for a given data by taking into account a generalized mean reverting stochastic process. It is found out that for certain cases we can find a suitable SMA together with normally distributed residual terms. This in fact can be transformed into a well known model given by Hull and White (1990,

1 All Matlab codes are available upon request.

pp. 573–592). In addition to an analytically solvable framework, our methodology is also practically beneficial. For an investor to model the movements based on past data, our model not only controls the volatility but also renders it to be normally distributed. In that sense we preclude any spike or spike like formations for the error terms which increases the forecasting power. Hence traders can use our identification algorithm in order to determine the SMA which is the best available predictor of the data in question. Although, it may not always exist, we demonstrate a situation where our algorithm actually works.

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