Kastamonu Education Journal, 2023, Vol. 31, No:1, 70-86
doi: 10.24106/kefdergi.1246426

| Research Article / Araştırma Makalesi |

The Process of Abstraction of Contextual Limit Knowledge

Bağlam Temelli Limit Bilgisinin Soyutlanma Süreci

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Keywords
1. Abstraction
2. Construction
3. Contextual learning
4. Limit concept
5. RBC+C model

Anahtar Kelimeler
1.Bağlamsal öğrenme
2.Limit kavramı
3.RBC+C modeli
4.Soyutlama
5.Yapılandırma

Received/Başvuru Tarihi
09.09.2022

Accepted / Kabul Tarihi
07.12.2022

Abstract

Purpose: This study aims to reveal the abstraction processes developed in the minds of pre-service mathematics teachers in problems in which the Limit is designed contextually. The abstractions developed by the pre-service teachers in this process will contribute to teaching the concept of Limit. In this respect, it was aimed to reveal which epistemic actions of the RBC+C abstraction model exhibited in the process of abstracting the concept of Limit while solving the contextually arranged Limit problems.

Design/Methodology/Approach: The case study was used in this study. The research participants are 52 (30 female and 22 male) primary school mathematics teacher candidates. Interviews were conducted with three participants selected by the purposive sampling method. In these interviews, five questions were asked by the researchers to reveal the mathematical thinking levels and abstraction processes of the participants. Semi-structured interviews and observation methods were used to collect data. The data were videotaped and transcribed. Transcripts were analyzed and interpreted according to the cognitive actions of the RBC+C model.

Findings: It was determined that the pre-service teachers who were interviewed developed an understanding of limits and did not experience any difficulties in using this concept in mathematical operations. However, they needed help in transferring the limit information presented in context to new situations, explaining and proving them mathematically. In addition, it was determined that they had difficulty adapting the formal definition to the new situation when faced with the contextual situation in which they could make the formal definition of the Limit. However, it has been observed that a specific conceptual schema is formed in their minds. This shows that they cannot entirely create limit information in their minds. Pre-service teachers structure the formal definition by rote. It was observed that the participants in the excellent category performed the epistemic action of Construction compared to the others.

Highlights: Contextually designed Limit problems show that pre-service teachers have memorized the formal definition of Limit in their minds. They needed help transferring the concept of Limit to different situations and finding solutions to limit problems in contextual situations using the formal definition of the Limit concept. The RBC+C abstraction model is an effective tool in the analysis of contextually designed problem-solving processes.

Öz

Çalışmanın amacı: Bu çalışmada, limitin bağlamsal olarak tasarlandığı problemlerde matematik öğretmen adaylarının zihinlerinde geliştirdikleri soyutlama süreçlerinin ortaya çıkarılması amaçlanmıştır. Öğretmen adaylarının bu süreçte geliştirdikleri soyutlamaların limit kavramının öğretimine katkı sağlayacağı düşünülmektedir. Bu bakımdan öğretmen adayları bağlamsal olarak düzenlenmiş Limit problemlerini çözümlerken limit kavramını soyutlama sürecinde RBC+C soyutlama modelinin hangi epistemik eylemlerini sergiledikleri ortaya çıkarılmak istenmiştir.

Materyal ve Yöntem: Bu çalışmada nitel araştırma türlerinden biri olan durum çalışması kullanılmıştır. Bu araştırmanın katılımcılarını 52 (30 kadın ve 22 erkek) ilkokul matematik öğretmen adayı oluşturmaktadır. Amaçlı örnekleme yöntemi ile seçilen üç katılımcı ile görüşmeler yapılmıştır. Bu görüşmelerde araştırmacılar tarafından katılımcıların matematiksel düşünme düzeylerini ve soyutlama süreçlerini ortaya çıkarmak için beş soru sorulmuştur. Veri toplamak için yarı yapılandırılmış görüşme ve gözlem yöntemleri kullanılmıştır. Veriler videoya kaydedilmiş ve yazıya dökülmüştür. Transkriptler, RBC+C modelinin bilişsel eylemlerine göre analiz edilmiş ve yorumlanmıştır.

Bulgular: Görüşme yapılan öğretmen adaylarının limit anlayışı geliştirdikleri ve bu kavramı matematiksel işlemlerde kullanmakta herhangi bir zorluk yaşamadıkları belirlenmiştir. Ancak bağlam içinde sunulan limit bilgilerini yeni durumlara aktarmada, matematiksel olarak açıklamakta ve ispatlamakta zorlanmışlardır. Ayrıca limitin biçimsel tanımını yapabilecekleri bağlamsal durumla karşılaştıklarında biçimsel tanımı yeni duruma uyarlamakta zorlandıkları belirlenmiştir. Ancak zihinlerinde belirli bir kavramsal şemanın oluştuğu gözlemlenmiştir. Bu da limit bilgilerini zihinlerinde tam olarak oluşturamadıklarını göstermektedir. Öğretmen adaylarının formal tanımı ezber olarak yapılandırdıkları söylenebilir. Öğretmen adayları bağlamsal olarak tasarlanan limit problemlerinin çözümünde limitin tanımı ile ilgili farklı temsiller gösterebilseler de tanımdaki değişkenler hakkında yorum yapamamakta ve özgün bir matematiksel yapı oluşturamamaktadırlar.

Önemli Vurgular: Bağlamsal olarak tasarlanan Limit problemleri göstermektedir ki öğretmen adayları Limitin formal tanımını zihinlerinde ezber olarak yapılandırmışlardır. Farklı durumlara Limit kavramını transfer etmede ve Limit kavramının formal tanımını kullanarak bağlamsal durumlardaki limit problemlerine çözüm üretmede zorlanmışlardır. RBC+C soyutlama modeli bağlamsal olarak tasarlanan problem çözme süreçlerinin analizinde etkili bir araçtır.

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Citation/Alıntı: Gürbüz, M.Ç., Özdemir, M.E., & Erkek, K. (2023). The Process of Abstraction of Contextual Limitt knowledge, *Kastamonu Education Journal, 31*(1), 70-86. doi: 10.24106/kefdergi.1246426

INTRODUCTION

Limit is among the most basic concepts of calculus, as it is directly related to many important concepts such as derivatives, integrals, continuity, and approximation theory (Cornu, 1991; Gürbüz et al., 2018). Although Limit has the characteristics of a basic concept, only a tiny part of individuals in mathematics education can adequately understand the concept of Limit (Tall & Vinner, 1981; Sierpinska, 1987). Limit is a word of Latin origin used with various meanings; the number at which the images under the boundary, Limit, maximum scope, endpoint, and function are stacked in numbers close to this number for a certain number. In other words, knowing the Limit is a preliminary knowledge of many important mathematical concepts such as derivatives, integrals, continuity and convergence. For this reason, it has an important place in mathematics teaching and programs. For non-mathematics majors, the lack of understanding of the concept of limits may not be a severe problem for the study of advanced mathematics courses; however, mathematics students and teachers' understanding of Mathematics will significantly affect their ability to study further Analysis courses because all of these courses require Calculus as a prerequisite (Liang, 2016). Teaching Limit knowledge in calculus recently is viewed in two ways: the theoretical calculus used in applications, often based on the symbolism of Leibniz, enhanced by the work of Cauchy using infinitesimal techniques, and the formal mathematical analysis of Weierstrass based on quantified set theory (Tall and Katz, 2014).

As in the emergence of other mathematical concepts, the concept of Limit emerged in producing solutions to real-life problem situations and was developed by various mathematicians. Euclid and Archimedes are the first users of this concept. Archimedes used the concept of Limit while calculating the area of the circle. Archimedes began the process by placing regular polygons with increasing sides inside and outside the circle. As evidence, he uses the method of consumption, like the Ancient Greek geometers (this method can be seen as a primitive form of calculus). He continued with the calculation of the area of the triangle in any polygon. In his rationale, the area of the circle is associated with the area of a right triangle whose base is equal to the circumference of the circle and whose height is equal to the radius of the circle (Høyrup, 2019), and Archimedes calculated the lower and upper limits in this process (Cajori, 2014). Studies shaped the current use of the Limit in the 17th century. Fermat explains the reciprocal relationship between the Limit of a curve at a point on the graph and the tangent of that point of the curve (Baki, 2008, p. 147). In the following years, Newton and Leibniz used limits to calculate integrals. It has been realized that differential equations were born from Limit (Baki, 2014, p. 145). Cavalieri showed that the area under the curve in the range is the Limit (Baki, 2008, p. 146). Then, the current application in mathematics is the Limit definitions specified by Weierstrass and Cauchy (Arslan & Çelik, 2015). The epsilon-delta definition of the Limit is the formalization of the limit concept (Balcı, 2014).

The mathematical definition of the concept of Limit was expressed by Augustin-Louis Cauchy (1780–1857) as follows: “When values successively assigned to the same variable, when approaching a constant value forever, end up differing from it as little as desired, that constant is called the limit of all the others" (Dugac, 1973). Although Cauchy's definition of continuity and derivative in terms of Limit is relatively modern, it would be entirely wrong to say that "he gave the first true mathematical description of the limit and never needed modification." Another definition was made by Karl Weierstrass (1815–1897): “if for each there is at least one real number for the real numbers providing the inequality of such that, then is called the limit of the function at a point and is indicated by =” (Nakane, 2014).

Limits are not a simple matter, especially since they involve different operations involving infinity (Baştürk & Dönmez, 2011; Tangül, Barak and Özdaş, 2015; Gürbüz et al., 2018). The existence of infinite processes in the concept of Limit makes it difficult for students to understand the concept of Limit. Most students have difficulty making sense of Limit knowledge in their minds (Çıldır, 2012). Durmus (2004); Gürbüz, Toprak, Yapıcı and Doğan (2011); In Tatar, Okur, and Tuna (2008) mathematics lessons, the subject of Limit comes first among the subjects that students perceive as complex (Durmuş, 2004; Gürbüz et al., 2011). Revealing the difficulties in understanding the concept of Limit is valuable in terms of its relationship with other mathematical concepts, and there is a tendency in this direction (Baştürk & Dönmez, 2011). Teaching the concept of Limit within the scope of the Mathematics Lesson aims to provide students with skills such as calculating the limit values of various functions at a certain point and understanding the relationship between graph and graph. He uses the Limit of a function, the theorems of the concept of Limit, and can determine whether the Limit exists by formal definition. Undoubtedly, the ability of students to achieve these achievements depends on their having a sufficient conceptual understanding of the formal definition of the concept of Limit (Güven, Baki & Çekmez, 2012).

Studies on limit knowledge (Barak, 2007; Baştürk & Dönmez, 2011; Denbel, 2014; Juter, 2006; Szydlik, 2000; Tangül et al., 2015; Memnun et al., 2017; Gürbüz et al., 2018; Zollman, 2014) identify students' difficulties and misconceptions in limit learning. The misconceptions identified regarding the concept of limit were identified by Barak (2007) as epsilon-delta definition, limit definition, the limit of a function at a point, limits approaching from the right and left, the relationship between limit and continuity concepts, a function defined at a single point, graphing functions, understanding the concept of infinity and using the limit in equations. Denbel (2014) stated that university students have similar misconceptions that they see a limit point as unreachable, an approach, a limit or a dynamic process. Studies in the literature show that very few individuals adequately understand the formal definition of the concept of limit through instruction (Ervynck, 1981; Quesada et al., 2008). The fact that the definition of limit is too full and difficult to understand has led students to develop the idea of highlighting an informal definition and pushing the formal definition into the background in teaching the concept of limit (Fernandez, 2004; Gass, 1992). On the other hand, many researchers think differently and formally define the concept of limit as the main point of transition to abstract thinking, make inferences about formal mathematical expressions and use formal proof techniques (Ervynck, 1981; Swinyard & Lockwood, 2007). Novak (1993) stated that the purpose of education is to direct students to meaningful learning. He defined meaningful learning as the necessity of making connections between newly introduced concepts and previous knowledge. Concept learning is defined differently for each approach. Some studies (Biber & Argün, 2015; Bukova-Guzel, 2007; Çıldır, 2012; Dönmez & Baştürk, 2010; Kula & Bukova-Guzel, 2015; Roh, 2007; Gürbüz et al., 2018) investigated how students learn about limits, and they included some teaching studies in order to eliminate the difficulties and misconceptions about limit knowledge. In the teaching of limit knowledge, teaching strategy (Akbulut & Işık, 2005; Dönmez & Baştürk, 2010), activity-based teaching (Akkoyunlu et al., 2003; Kula & Bukova- Güzel, 2015), technology use (Çıldır, 2012) and various mathematical It has been stated that attribution (Biber & Argün, 2015; Tangül et al., 2015) processes have a positive effect.

Our approach here is to explain the underlying nature of human perception and how our human aptitude for language and symbolism (the language of mathematics) leads to forms of mathematical abstraction in (unique) contextual situations. Recent theories of the development of mathematical thinking (Dubinsky, 1991; Gray & Tall, 1994; Sfard, 1991) focus on how humans carry out mathematical operations, such as addition, sharing, calculating the limit of a sequence, differentiation and integration. At each stage, we perform a process that occurs in time to produce an output that may also be conceived as a mental entity independent of time. Counting gives rise to the concept of number; the process of adding 3 + 2 gives rise to a mental concept, the sum, which is also written using the same symbol 3 + 2; sharing gives the concept of fraction; calculating a trigonometric ratio such as sin A = opposite/hypotenuse gives the concept of sine; the process of differentiation gives the derivative; and the process of integration gives the integral. In every case, a symbolic notation represents both a desired process and the resulting concept, such as 3 + 2, a(b + c). Gray and Tall (1994) refer to this conception of a symbol that dually represents a process or concept as a precept, with the additional flexibility that different symbols with the same output, such as a(b + c) and ab + ac, represent the same precept. Analyzing a sequence () given by a specific formula involves calculating a set of terms and observing how the process leads to a particular value - limit. Cornu (1981, 1991) described how students think of the shrinking process as producing an object that is itself arbitrarily small but not 0. Tall (1988) defined it as a general limit. What is meant to be expressed here is different from what the limit is, what it does or how it should be taught. The brain activity underlying this process is more fundamental. The connection from the perceptual world of our human experience to the computational world of arithmetic and algebra was initiated by Descartes and others. This paved the way for Newton and Leibniz to calculate naturally perceived phenomena, such as the measurement of length, area, volume, time, distance, velocity, acceleration and their rates of change and growth, using the computational and manipulable symbolism of calculus. Cantor and Weierstrass took the matter further by interpreting the number line in terms of number, arithmetic, order, and completeness symbolism. However, what radically changed the way we think about mathematical ideas was Hilbert (1900)'s introduction of formal axiomatic mathematics. He shifted attention from natural phenomena that we perceive physically and mentally to properties of phenomena. Axioms determine a mathematical structure, and deductions are made with mathematical proof. This frees mathematical thinking from the limitations of human perception to the possibilities of formally defined systems and their outcome properties (Tall and Katz, 2014).

Tall (2004, 2008) formulated three fundamentally different ways mathematical thinking develops, which are related both to the historical development of ideas and to the cognitive development of the individual from childhood to the mathematician: embodied, symbolic and formal. Mathematical ideas begin with perceiving the embodied world (Tall & Katz, 2014). Thought is refined through language's subtle experimentation to develop abstract concepts in an increasingly complex world of conceptual organization. Núñez et al. (1999) stated that the formal epsilon-delta continuum differs from the natural continuum based on human arrangement and perception. Lakoff and Núñez (2000), rejecting the views that mathematics is universal, absolute and precise, explained that this situation would make mathematics inaccessible even to students with other fundamental interests and skills.

This study aims to understand how pre-service teachers mathematically abstract the concept of limit in the face of problem situations that require solving with the limit expressed in life situations. In this respect, it is desired to examine the abstraction processes of pre-service teachers about how the examples of the limit given in living environments will be defined in the mathematical world. In the literature, processes such as didactically explaining the meaning of the concept of limit (Juter, 2006), examining the structures related to the concept of limit from an epistemological point of view (Bergsten, 2006), and constructing the limit in the mind of the individual (Gürbüz et al., 2018; Memnun et al., 2017) have been examined. In this study, the abstraction process of mathematical knowledge is examined. Previous mathematical knowledge is the tendency of an individual to respond to a perceived problem situation in a social context by constructing or organizing mathematical processes and objects in his mind to cope with the situation (Cottrill et al., 1996).

The theoretical basis adopted for this study and terms used in terminologies such as construction and schema is reflected. “The coordinated process scheme of a mathematical structure is difficult, and not every student can create it right away. As a result, a contextual limit problem design was created that focused on enabling students to construct "specific mental constructs" that are important for understanding the concept of limit. The research method is a cyclical process in which a genetic divergence is developed through an epistemological analysis. After extensive observation and student consultation, a renewed cycle can lead to design and, eventually, theory changes. In this way, it is desired to bring the research approach into a robust epistemological component that interacts with the cognitive approach.

It is influenced by PISA, TIMSS…**Etc** educational research in recent years; prospective teachers interested in contextually organized problems that have become more common in Turkish national ranking exams (required for secondary and higher education entry) such as LGS, AYT, TYT **Etc**. His skills are intriguing. It is desired to investigate the abstraction skills of the pre-service teachers regarding the contextually organized problems of the concept of limit. The abstraction process of the limit concept will be examined with the Abstraction in Context (AIC) theory for contextual problems. Among the abstraction theories, AIC was preferred because of its compatibility with contextual situations (Dreyfus, 2007; Gürbüz et al., 2018).

Contextual Limit/mathematic Concept

The role of contextualization in the understanding of science cannot be denied today. Various reform movements in education, such as mathematical literacy, science for all, critical thinking, constructivism and contextual teaching, can be seen as a reaction against the current way of presenting and organizing education. These movements often try to teach science content as well as science. Teaching science means teaching how science is developed and how one concept relates to another, not just the factual content of science. The idea of contextualizing the material taught has become a kind of consciousness for various reform movements.

A remarkable feature of his education is that, to a lesser extent known in other fields, it was conducted through textbooks, especially works written for students. Even books that focus on a single subject differ mainly in level and pedagogical detail, not in content or conceptual structure (Kuhn, 1963). Expressed for the constructivist approach, "Knowledge is not a commodity that is somehow outside the knower and can be transferred or instilled with a diligent perception, but the result of the constructive activity of an individual subject." (Glasersfeld, 1990, p. 37). According to this view, learning is a meaning-making activity in which the learner tries to place new information into existing mental structures. Where conceptual overlap or context is the dominant factor, new information is always linked to similar information. With this way of reasoning, the importance of context in the learning process comes to the fore, since information cannot exist alone in long-term memory, and even in the reasoning process, there are constant attempts to establish connections between concepts.

A context-based approach has its advantages (Tabach and Friedlander, 2008): It facilitates learning processes by giving real or concrete meaning to an abstract concept or algorithm. Provides reference points; when work is done at a more abstract level, it can create models that allow students to refer back at a later stage of learning. It increases student motivation and the student's willingness to participate in learning. It strengthens the potential to use algebraic models and skills in other fields. It is both a starting point for understanding the performance of concepts and operations in learning context-based mathematics, as opposed to simply applying instructions in the process of solving a problem. Context is the main process of this situation. Wiest (2001) defined the context as an important element in the construction of meaning from beginning to end and likened it to a ground that facilitates the meaning of parts. Gravemeijer and Doorman (1999) stated that a pure math problem can be contextual, in any case a contextual task must be experiential, real for the student, and serve as the basis from which a mathematical concept that is important can be built. Context plays a role in the learning process by creating real or concrete meanings for an abstract concept.

The act of applying conceptual knowledge learned in a context to solve a problem in a new (unfamiliar) context is called transfer or transfer (Ross, 1987). Studies have shown that transfer is a difficult process (Pan and Rickard, 2018). Students who are faced with new problem scenarios manage to realize their applicability in only 10-30% of the time (Norman et. al. 2007). Successful transfer process requires recognizing the structure of the abstract concept underlying the problem (Ross & Kennedy, 1990). Research on transfer should move away from the analogic reasoning paradigm to adopt a more ecologically valid use of analogies and context diversity during learning (Kulasegaram. et al. 2012). It aims to draw attention to the rich mathematical thinking generated when students work on contextual mathematization tasks. We use the RBC+C abstraction approach to create reporting methods that increase the visibility of this rich mathematical thinking and preserve and respect its complexity.

RBC+C Abstraction Model

Studies emphasizing conceptual learning more than procedural learning in limit teaching highlight mathematical abstraction (Gürbüz et al., 2018). In addition, examining the abstraction processes in teaching limit knowledge has gained importance (Memnun et al., 2017). In this respect, conceptual learning of a limit requires the abstraction process. For this reason, structuring issues, which require the examination of the abstraction processes of a concept in the minds of individuals, are important research topics in the learning field of mathematics. In addition, mathematics is a science of abstraction, and most mathematical concepts are obtained through abstraction (Altun, 2014, p. 5). This makes it essential to overcome the abstraction process of concepts, especially in mathematics education. For this reason, it is essential to determine how to develop abstractions and generalizations that students will realize with the help of their prior knowledge (Bukova-Guzel, 2006, 2007). For all these reasons, this study examines how pre-service mathematics teachers mathematize real-life knowledge in terms of the concept of limit and how they structure and abstract it in their minds for the same concept.

Abstraction is a structural process; It is the process of transforming mental structures in context into mathematical structures or constructing mathematical structures from mental structures. It isolates a concept based on its characteristics and directs a comprehension process from the context to a series of correct (Dreyfus, 2007; Mitchelmore, 2002; Sierpinska, 1994; Tall, 1988; Yılmaz, 2011). In mathematical abstraction, mathematical concepts are obtained as a result of a mental structuring process

(Gürbüz, 2021); in this respect, a transition between concepts is ensured, and understanding is provided through conceptualization rather than memorizing procedural knowledge (Can, 2011). For this reason, research on mathematical abstraction has an important place in mathematics education. Therefore, the theme of abstraction has an important place in mathematics education. Since the abstraction process is not directly observable (Dreyfus, 2007), it has become necessary to identify observable actions that provide information about the abstraction process. This model was defined as RBC (Recognizing, Building-with, and Constructing) Abstraction Theory by Hershkowitz, Schwarz, and Dreyfus (2001) to analyze mathematical abstraction processes. The Nested RBC Abstraction Model (Recognize, Construct, Build) Model emphasizes the need for abstraction and encouragement (incentive) in the process of creating (forming) the concept in mind. Hershkowitz et al. (2001) defined the epistemic actions of the abstraction process as recognizing, co-constructing and constructing. In RBC, it has been observed that when a person forms a concept in his mind, epistemic actions are ordered non-sequentially (Dreyfus, 2007). It is emphasized that the actions have a structure that can be intertwined and accommodate each other (Özmantar, 2005).

RBC+C (recognizing, building-with, construction and consolidation) abstraction model, which was developed and used to reveal the mathematical abstraction process from the context in the mind of the individual, reorganizes the old structures to reach a new structure, establishes connections and relations between them and establishes connections and relations between them and requires integration into a single thought process.

“Recognizing” in the RBC+C model is to attribute meaning to the mathematical structures in the learning environment by using the formal or informal information available in the individual's repertoire (Hershkowitz et al., 2001). This includes recognizing the structures that individuals are familiar with about the mathematical structure studied (Bikner-Ahsbahs, 2004; Hassan & Mitchelmore, 2006); in other words, it is the use of structures when necessary (Dreyfus, 2007).

In the process of "building-with" in the RBC+C model, individuals who need familiar structures to generate new knowledge create a viable solution to the problem by using their existing structural knowledge. This process, in which the act of building together is intertwined with the epistemic act of recognition, requires associating the known with the new content (Bikner-Ahsbahs, 2004; Hershkowitz et al., 2001). The construction of the individual with his action is a critical point in the abstraction process, and when it is not observed, it can give a clue that will activate him (Dreyfus, 2007).

In the RBC+C model, “construction” is the process of restructuring the known structures by making a partial change and creating new structures/meanings based on this action (Bikner-Ahsbahs, 2004). The reason is that the individual cannot construct a new structure without performing other cognitive actions using his knowledge and experience. Construction occurs due to the realization of the other two cognitive actions (Dreyfus, 2007). If the individual contemplates only a mathematical subject intensely, a structure will be formed (Dreyfus, Hershkowitz & Schwarz, 2001).

Depending on the problem, the same problem enables a student to perform the knowledge construction action, while another student can perform this process recognizing action (Gürbüz, 2021). In other words, the occurrence of these actions is not clear and precise. This depends on the student's past experiences, personal skills, and whether stimuli trigger the student's knowledge (Dreyfus et al., 2001). However, Dreyfus (2007) stated that new structures constructed using abstraction are fragile, making it difficult to maintain the new structure. Consolidation can occur when one person associates structures with another, uses structures to create a new structure and reflects on these structures. The act of combining can occur when students study well-known mathematical topics and, at the same time, use a situation or concept they have just abstracted for further abstraction (Dreyfus & Tsamir, 2004).

Hershkowitz et al. (2001) found that abstraction occurs during problem-solving in their study with ninth-grade students. Özmantar and Monaghan (2007) stated the factors affecting the abstraction process in their study of the absolute value function. Yeşildere and Türnüklü (2008) examined the effects of different mathematical forces on the abstraction process. The RBC + C model has been studied in different studies using various mathematical concepts: most enormous integer function (Altun & Yılmaz, 2008), properties of algebraic and arithmetic operations (Dreyfus et al., 2001; Gürbüz, 2021), ratios and proportions (Hassan & Mitchelmore, 2006), and probability (Dreyfus, Hadas, Hershkowitz & Schwarz, 2006; Hershkowitz, 2004; Schwarz, Dreyfus, Hadas & Hershkowitz, 2004). In addition, functions (Hershkowitz et al., 2001), absolute value (Özmantar, 2005), linear equations (Sezgin-Memnun & Altun, 2012), infinity (Tsamir & Dreyfus, 2002), and triangles (Yeşildere & Türnüklü, 2008) have also been investigated. In addition, the RBC+C abstraction model has been compared with Sfard's theoretical model, and the issues that the RBC+C model is effective in this process have been revealed (Gürbüz et al., 2018). It was decided to choose RBC during the analysis period since vital situations where the reason for the emergence of RBC, and the original theory was an abstraction from the context.

The Study Aim

First, it is an undeniable reality that as an experienced instructor of calculus courses for more than 32 years, understanding the concept of Limit is the basis for teaching concepts such as Continuity, Definite Integral, Differential Calculus, Etc. Higher Mathematics is more than reconsidering the epistemic actions of Recognizing and Construction in terms of continuity, integration, differentiability, Etc. and finding new properties and connections in the process of going from Recognizing to Construction or how mathematical thinking works in this process. In this context, it is another fact that if this critical concept is not fully learned, it will not be adequately understood in advanced concepts or only operationalized will be prioritized. This study is based on our previous study, which investigated how the concept of Limit changes operationally and conceptually in the individual's mind (Gürbüz et al., 2018); unlike operational thinking, is to motivate students to draw attention to their conceptual context-based understanding of the concept of "limit" and to reveal abstraction processes in terms of RBC+C from this point of view. In a way, this study is a continuation of our previous study.

This study aims to reveal the realization processes developed by pre-service mathematics teachers for problems in which the Limit is organized contextually. It is thought that the abstractions developed by the pre-service teachers in this process will contribute to teaching the concept of Limit. In this context, the research problem is “How do prospective teachers abstract the concept of limit from contextually organized Limit problems?” has been determined. Answers were sought for the following sub-problems:

1. What is the abstraction of prospective mathematics teachers in contextually organized limit problems?
2. What are the situations that pre-service teachers have difficulty abstracting in contextually organized limit problems?

METHOD/MATERIALS

Lester (2005) reasons why educational research needs to be pursued within a scaffolding framework. A framework is here seen as “a basic structure of the ideas (i.e., abstractions and relationships) that serve as the basis for a phenomenon that is to be investigated” (p. 458), representing its relevant features as determined by the adopted research perspective, and serving as a viewpoint to conceptualize and guide the research. A research framework thus “provides a structure for conceptualizing and designing research studies,” including the nature of research questions and concepts used and how to make sense of data. A case study, one of the qualitative research types, was used in this study. Since the research has descriptive and explanatory features (Yin, 2003) expected in a well-designed case study, it was concluded that this method is suitable for use. Case studies are suitable for examining a particular phenomenon in detail, such as a person, a process, an operation, or an institution. In qualitative case studies, the researcher(s) investigates a process, questions can address a situation description, and themes can be derived from it (Creswell & Creswell, 2017, p. 130). The situation in this research is a detailed examination of pre-service teachers' concept abstractions in contextually designed limit problems. The data obtained in case studies are vital in reality, and the findings are open to interpretation (Cohen & Manion, 1994). The data was collected with the course's instructor and a participant. Participant interviews were thought to help interpret mathematical thinking and abstraction during the interview. Therefore, it can be considered a descriptive case study that requires detailed information about a phenomenon to provide data for future comparative studies (Merriam, 2002).

The participants of this study are pre-service mathematics teachers at a university in the big city of Marmara, the geographical region with the most Universities in Turkey. 52 (30 female and 22 male) primary school mathematics teacher candidates constitute the sample of this research, which was carried out to reveal the ability of pre-service teachers to abstract from the context related to limit problems prepared in the contextual format during the Analysis-1 lesson. It was determined that the participants were first-year students studying in the primary school mathematics education program, and all the participants regularly attended the Analysis-1 course. Among all students, students from three success levels were determined for interviews with the purposeful sampling method. All three students are women. Evaluating the GPAs of the participants and the opinions of the course teacher, they were classified into three categories: good, moderate and mediocre. A random student was selected from each category, and interviews were conducted. In addition, these students participated in the study voluntarily. They had already learned the series and functions necessary to acquire limited knowledge. The students were given the nicknames Ayse, Fatma and Hayriye. The students in the study group of the research completed the calculus courses.

Data Collection Instrument

Although not all problems alone lead to an abstraction, the abstraction of a concept in the mind of the individual can only be realized in the problem-solving process (Hershkowitz et. al. 2001). For this reason, four contextual problems were designed by the researchers in order to determine the epistemic abstraction actions of pre-service teachers in the process of abstracting the concept of limit from the context. These problems are derived from the contexts that the participants may encounter in daily life in order to reveal the basic features of the limit concept, for example:

Question-1: Two people approaching a wall are talking, and they are telling each other about the numbers on the number line on the floor. How can they know where the wall separating these people is, this is an example of what mathematical concept? Briefly write the name of the concept and explain it.

i. How would you explain this situation in a mathematical language?
ii. When you consider the mathematical concept mentioned in this question, what is the advantage of the approximation method used to find the number under the wall? Why is this approximation method used mathematically?

Question-2: While traveling by car, the speedometer reads 85km/h. It shows the way to go in an hour. However, in the real world, your speed is constantly changing over time, and the only real information you have is that you have traveled 150 km from A to B in exactly 2 hours. The number your speedometer gives you is the limit at $t_0$moment of your trip.

$$v(t_0) = \lim_{\Delta t \to 0}\left(1 + \frac{s(t_0) - (s(t_0 - \Delta t))}{\Delta t}\right)$$

Here it shows the distance traveled until the time. How would you explain the mathematical expression shown here to someone who doesn't understand? Please explain.

Question 3: Two friends are playing the game of keeping score. Ali kept the number c in his mind and his friend told him the number c+ δ. Ali, on the other hand, told his friend that the value of f(c+ δ) is "ϵ" higher than the value of f(c), which is the function value of the number it holds. At this time, his friend said the number c-δ, while Ali stated that the number he kept was as little as "ϵ" from the function value.

i. Express the situation expressed here with the limit using the δ- ϵ technique.
ii. Draw the limit graph showing your approach method.

Question 4: It is desired to make a rectangle with the largest area with 36 meters long wire.

Perimeter of Rectangle= 2w+2l=36

Length(w)	8	8.5	9	9.5	10
Area(a)	80	80.75	81	80.75	80

i. The limit value of the field will be 81 as the length "w" value approaches 9. Express the expression with the limit using the δ- ϵ technique.
ii. Explain why the "w" side must take the value 9 for the area of the rectangle to be the largest, using at least two of the graphs or expressions of f(w)=area,f(w).

All the designed contextual limit problems are related to the mathematics teaching department Analysis 1 course learning outcomes. Therefore, the problems are directed towards the abstraction of the limit concept.

The aim of the first problem is to know that the concept of limit can be approached from two different directions in the construction of epistemic knowledge, to explain the approach method with a mathematical language and to measure the conceptual reflection of the approach method, which is expressed mathematically. Pre-service teachers are expected to reflect their knowledge of the concept of limit in order according to the three epistemic actions of RBC.

The purpose of the second problem requires that the Limit information be explained with different representation tools. In this respect, pre-service teachers' demonstrations will show which of the RBC epistemic actions they will exhibit regarding the concept of Limit.

The purpose of the third problem is to enable them to apply the epsilon-delta technique to a contextual situation and to derive a complete mathematical definition from the contextual form of the limit. In this respect, the epistemic actions of the candidates with their solutions will be determined.

The aim of the last problem is to adapt the epsilon-delta technique from the contextual situation to the mathematical form, to show the epsilon-delta technique in mathematical form, and finally to associate the epsilon-delta technique with the context by showing it in geometric form. The purpose of the problems and the RBC+C epistemic actions that teacher candidates are expected to show in the problem-solving process was presented for expert opinion. The difficulty, sequencing, and epistemic indicators of the problems were rearranged in line with their suggestions and justifications.

Procedure

At the center of the research is the process of abstracting the concept of limit from the context of pre-service mathematics teachers. Semi-structured interviews and participant observations were used as data collection tools in determining the abstraction processes of the candidates. In accordance with the academic calendar, within the scope of the Analysis Course, education continued in accordance with the achievements until the midterm. Afterward, the purpose and scope of the research were explained to the participants in detail before the application was made. Then, the volunteers were determined, and the working group was determined. Interviews were held without any additional training for the candidates. The interviews were recorded with a voice recorder. Their responses to semi-structured forms and supplementary papers were collected. All records were analyzed as a whole. This is to allow the researcher to review them as often as he wishes, delaying a final decision until he is sure of which points to emphasize in the analysis process.

Data Analysis, Reliability and Validity

The data were analyzed and interpreted using descriptive analysis of students' answer sheets, audio recordings, and observer field notes. The purpose of the analysis is to organize the obtained data and present it by interpreting it within the theoretical framework. The data obtained for this purpose are described systematically and clearly (Yıldırım and Şimşek, 2016). At this stage, the dialogues recorded during the interview were listened to several times and translated into expressions/texts in the study.

These data were analyzed, and the findings were divided into meaningful categories. Accordingly, the RBC+C model was applied to the expressions in the abstraction process and these expressions were analyzed through the cognitive actions of this abstraction model. In this context, the themes used and determined in the analysis of written interview expressions are recognition, co-construction, construction, and reinforcement. In other words, these cognitive actions were observed and recorded collectively by two researchers independently in the solution of each problem. Definitions of these cognitive actions for the problems in this research and examples of these actions are given in Table 1. The analysis template prepared by the researchers for the interview questions was adapted from the study (Gürbüz, 2021).

Table 1. Cognitive Actions of the RBC+C Abstraction in Context Model

Epistemic Action	Indicator	Example
Recognizing	It includes formal or informal information that an individual has previously acquired. They are attributing meaning to mathematical elements in the teaching environment. It creates individual awareness about what kind of prior knowledge is needed to learn new information.	Recognition of limit information, The meaning of the right and left approaches in finding the limit, Relating the meaning of limit from context to epsilon-delta
Building- with	It is seen in the problem-solving process. It is an example of personal use of pieces of information that one already knows to create a workable solution to the problem. This is intertwined with cognitive epistemic actions.	The use of limit information to solve the first problem item i, Using the limit information in the equation to solve the second problem, In the third problem, the processes before the formation of the structure, Procedures before installing δ- ϵ technique to solve the fourth problem
Construction	It is to partially change the known structures or to create a new mathematical structure based only on preliminary information.	Constructing a new mathematical structure contextualized by using the epsilon-delta technique in the first items of the third and fourth problems, the formation of the formal definition of the limit.
Consolidation	It requires developing a new structure in the mental process. It is the elimination of the fragility of the new structure.	In the second item of the third problem, the formal definition of the limit is reinforced because a different notation is needed. In the second item of the fourth problem, the formal definition is reinforced in explaining a new situation.

Students use their prior knowledge to solve the limit problems presented in context. With this preliminary information, it is expected to perform building-with action for different purposes. Afterward, the thinking styles used in the process of constructing the limit knowledge of the students are revealed. The extent to which the students have created their limit information and the necessary preliminary structures are reported. Finally, interpretations were made according to the data in order to make sense of the findings explained. The relationships between these findings and the conclusions drawn are presented in detail.

In this study, triangulation and inter-interpretive reliability methods were preferred to ensure reliability (Lincoln & Guba, 1985). In order to provide triangulation, the suitability of the categories to the steps of the theory was determined comparatively by using the transcripts of the participants' written texts and audio recordings. While data from written texts were used predominantly to define the categories, audio recordings were used to validate the categories. For example, based on written texts, the appropriateness of the RBC model for epistemic actions was determined. The classifications took their final form after being confirmed with audio recordings. For inter-interpretive reliability, participants' predictions, observations and explanations were evaluated as raw data by two independent researchers. Researchers have assigned them to independent thematic categories in connection with the steps of the theory. After the coding, it was seen that the consensus of the researchers was over 85%. Researchers debated inconsistencies in these categories until a consensus was reached.

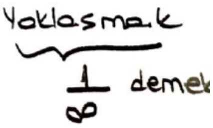
FINDINGS

The interview, which is the primary data source of the research, was carried out on pre-service mathematics teachers with three different success levels who participated in the research. During this 60-minute process, these participants worked on four different problems.

The first problem (approaching from the left and right to find the limit at a point) was prepared for finding the limit at a point. Participants are expected to recognize the limit in the contextual situation, and to remember an approach that uses finding the limit at some point. In item i, the remembered mathematical structure is expected to be expressed in mathematical language, that is, they are expected to exhibit the B epistemic action. In item ii, the answers of the participants determine the epistemic action they exhibit. This is the nature of RBC+C. If the advantage of the approach method used in the process of finding the limit is determined and the reasons for this approach are expressed mathematically, a C epistemic action can be observed.

Abstraction epistemic actions exhibited by students according to their problem solutions differed from each other.

Ayse 5: I think this concept is approximation. Approaching is for me the same thing as a half infinity.



Approaching = $\frac{1}{\infty}$ $mean$

Ayse 8: They are approaching point 33 from the right and left.

Fatma 3: Once the two say the numbers closest to the wall, it will be easy to guess the number in between. It is an example of approaching the limit from the right to the left.

…

Fatma 6: The two of them are very close to that number, but they cannot see that number and they get support from each other in this regard.

Hayriye 2: Instead of the wall, we can find it by saying the numbers as close to the wall as possible by the people on the right and left.

…

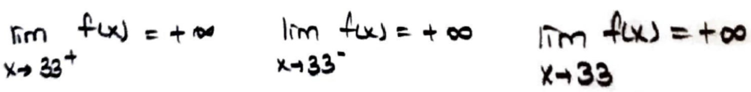
Hayriye 5: because we try to get it from the closest place as possible, not from a far place at the limit, so that the result can be found.

…

Hayriye 9: This is an example of the concept of limit in mathematics. If a limit variable x approaches a fixed-point c and f(x) approaches a value of L, it is said that the limit of "f" at point c is L.

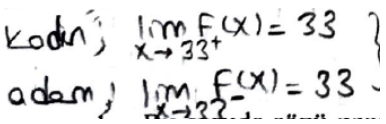
Three participants showed that they had limit knowledge. In addition, they all exhibit the epistemic action of R. While Ayse exhibited a simpler R epistemic action, Hayriye showed a more layered epistemic action regarding limit knowledge. Ayşe has defined the approach method more shallowly. This shows that his approximation method thinks that the function $f(x)$ will be meaningful only for the "∞" value of x. Hayriye, on the other hand, expressed a mathematical definition for the value of the Limit at a point. It gives rise to the idea that there is a dynamic point for all participants in finding the limit with the approach method. In item i of the same question, the epistemic actions of the participants in using limit knowledge were investigated.

Ayse 14: There is a limit at 33 points.



Ayse made the above-mentioned definitions without any explanation. Fatma did not show any mathematical expressions. Hayriye, on the other hand, made a limit definition based on the $f(x)$ function.

Hayriye16: These should be equal to each other so that we can reach a conclusion, but an important point is that the function does not have to be defined at that point, that is, 33 point in this question, I would like to point out that.



Participants were able to exhibit the Building-with epistemic action for finding the Limit with a right and left approach. However, only the good Hayriye exhibited a full Building-with epistemic action, while the participant in the mediocre category showed a deficient Building-with epistemic action.

In item ii of the same question, the structure expected to be revealed by the participants was investigated. However, the participants could not exhibit a full Construction epistemic action. They just exemplified this approach from their own repertoire.

Hayriye 20: There are some functions whose advantage cannot be found as a result. For example, when you write these functions, the value may not come, or there may be functions that go to infinity. The limit makes it very easy for us to find them.

Although a full Construction epistemic action has not been revealed, the participants stated that an existing known structure can be partially changed and its results. However, an action at this level was not observed for the participants in the average and middle category. In general, the participants made experiential connections with their past knowledge in the process of constructing limit knowledge.

The second problem is the velocity-time variation in a given function$f(x)$. They were expected to find the limit of the function at the time t_0 expressed in the problem. The focus of the problem is the mathematical explanation of how to find the limit operation of the speed-time variation of a car in real life situation. Although the prominent abstraction process is seen as the Building-with epistemic action, a Construction action is expected in the reflective explanation. Naturally, this process is directly dependent on the answers given by the participants.

Ayse 30: To explain this to someone who doesn't understand, we need to simplify it. But I think it is at a level that cannot be simplified. I cannot simplify that expression in my head.

Hayriye 42: Usually we use the limit to find the current speed. While using the limit, we try to keep the velocity, that is, the time, close to zero so that we can calculate the current (it talks about t_0) as much as possible so that we can find the current result/speed.

$$current\ speed\ of\ the\ car\ =\ as\ time\ approaches\ zero(1+\frac{distance\ traveled\ so\ far\ -\ current\ distance}{time\ change}$$

Hayriye 48: The speedometer gives us the speed we are going at that moment and while doing this, it uses the closest time interval.

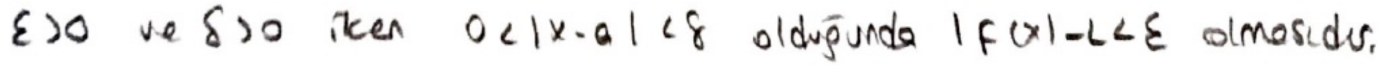
…

Hayriye 52: For this reason, when calculating the limit, Δt is indicated by an arrow as going to zero.

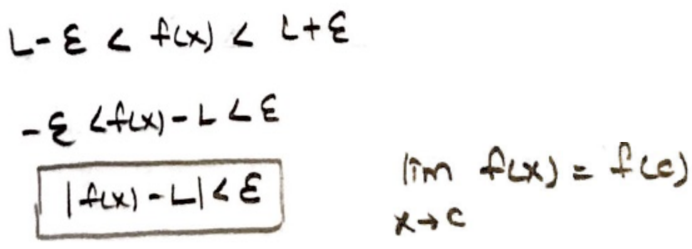
In this problem, only the participant in the good category was able to show a full Construction epistemic action. In other words, it is necessary to differentiate the function in the instantaneous velocity calculation. An explanation of how the function is derived using the limit is given by the participant. This shows that limit information can be used to create derivatives and can perform abstraction. Only the participant in the mediocre category could not reach this level.

In the third problem, one is expected to arrive at a formal definition of a limit using the $\delta - \epsilon$ technique from a contextual situation. Constructing a definition is a Construction epistemic act. However, if the preliminary information is written without expressing it in accordance with the contextual situation, only action B is exhibited. In the second item of the same question, they were asked to express the limit in a different form. The mathematical structures developed or constructed in the questions answered so far need to be brought together and expressed in a different mathematical notation. In this respect, consolidation action is expected depending on the answers of the participants.

When the answers given to the first item of the third question were analyzed, it was seen that the participants generally drew graphs in accordance with the known limit definition and tried to make a formal definition using the $\delta - \epsilon$ technique. For example, Fatma, who is in the middle category, defined a $f(c)$ function only for the c value in the abstraction process.



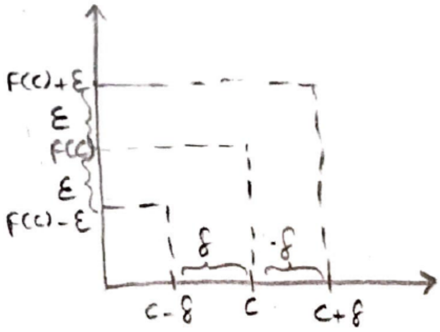
However, it is seen that he could not integrate the data related to the limit state, which was expressed contextually, in accordance with the definition. The process was completed as memorization by giving a definition. For this reason, it is seen that the limit information is only in the epistemic action dimension of using. Participant Construction, who was at the middle level in the abstraction of the contextually designed limit information, could not abstract in the epistemic action. Therefore, no Consolidation was observed. For this question, although only the participant in the mediocre category could do an appropriate Building-with to develop an appropriate abstraction, he did not exhibit an epistemic action at the Construction level and a complete abstraction did not occur.



Ayse 40: c is a point on the x-axis. As c - and c+, they are approached from the right and left. Δ is such a small number that one could say 0.0001. Afterwards, Ayse could not associate the contextual situation with the formal definition.

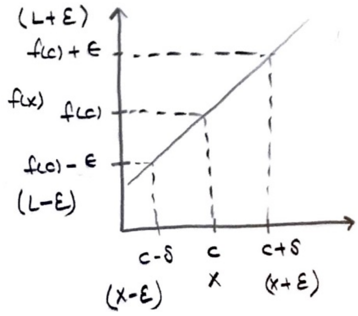
In general, the participants made solutions at the level of Building-with epistemic action and were able to express the formal definition. However, because they could not take an epistemic action at the Construction level, an abstraction did not emerge.

When the answers given to the second item of the third question were analyzed, it was seen that the participants were generally able to make an abstraction similar to the first item of the same question. In summary, it has been observed that they express the formal definition but cannot abstract from the contextual situation, that is, they show improvement at the operational level. A graphical representation of Hayriye is shown in the figure below.



Hayriye 64: I used deltas from the right and left of c. I said $c+\delta$ to the right of C. By the way, I wrote c on the x-axis. In short, I wrote $c+\delta$ on the c axis to the right of c and c-δ on the x axis to the left, and the value of $c-\delta$ corresponds to $f(c)-\in$.

When the answers given to the second item were analyzed, it was seen that they could make a clearer abstraction compared to the first item. Geometrically, the abstraction of the $\delta-\epsilon$ technique from the contextual situation was observed more clearly than the algebraic one. Ayse answered the same question with an approach that contains some errors in its content.

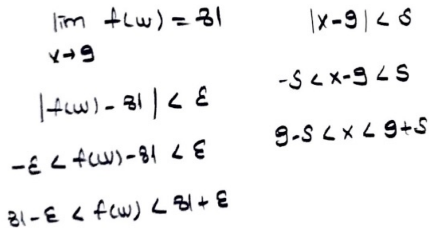


Although Ayse was able to construct the formal definition of the limit geometrically, she could not fully express the change in the $f(c)$ function in the $c-\delta$, c and $c+\delta$ intervals during the construction process.

Ayse 49: c is a point on the x-axis. As c - and c+, they are approached from the right and left. Δ is such a small number that one could say 0.0001.

Although Ayse has gone through a complete construction process, it is seen that she has done this with some knowledge deficiencies.

In the first item of the fourth and final question, they were asked to use a technique that was an application of the limit. In this process, the desired derivative is obtained with the limit. However, since the participants were able to operate with memorized information, a construction process did not emerge, and therefore an abstraction did not occur.



Ayse 69: I may have mixed up the δ and $\in$ values. But in the end, both are very close to zero, so I don't think anything will change.

Ayse 79: I remember w and l must be close to each other for it to be the largest, after all, we found this using derivative.

Ayse could not make an abstraction in obtaining an implementation of the derivative using the limit. It has been observed that he remembers some of his prior knowledge correctly, thus exhibiting the epistemic action in the δ and $\in$ techniques, although he could not obtain a function as in the second problem, despite placing the data correctly. In this respect, it is seen that he remained at the B level and could not reach the Construction level. On the other hand, Hayriye made similar transactions with Ayse. Similarly, epistemic action could be exhibited at the Building-with level.

In the second item of the fourth question, how they will relate the concept of derivative and limit and what kind of structure will emerge in this process are examined. Participants tried to reach the solution by finding the vertex of the function through the applications of the derivative. In general, instead of constructing limit information, they turned to applications of derivatives, which are memorized information.

Fatma 98: I went with the derivative first so that the area is the largest. I said that the derivative of f should have w=0.

…

Fatma 104: I took the derivative of the value in the field. In this derivative, w=9.

Ayse 90: Here we can take the derivative to find the maximum value of the field. coming out of here.

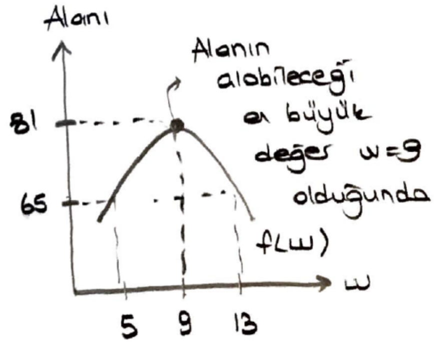
$$f(w) = -w^2 + 18w$$

$$f'(w) = -2w + 18 \quad w=9 \text{ tepe deger}$$

Ayse 96: At the peak value, namely 9, the area will have the largest value.

…

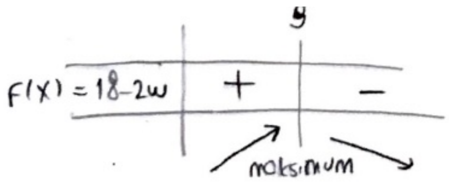
Ayse 100: When we show this graphically, we see that the peak is the highest at the point of the peak, that is, at the point 9, the area will have the largest value as 81.



Hayriye 108: The derivative of the function is usually used to find its maximum value. So this helps us with big and small area calculations. Its derivative became 18-2w. I set this equal to zero. 2w, 18 is out and w is 9 out of here.

$$\text{Alanı} = 18w - w^2$$
$$f(x) = 18w - w^2$$
$$f'(x) = 18 - 2w = 0$$
$$2w = 18$$
$$w = 9$$

Hayriye 112: However, it does not tell us that w is the maximum value. To prove that it has a maximum value, we need to draw a graph.



As can be seen from the answers of the participants in the last question, there was no abstraction for limit information. In general, the participants used derivatives and applications to reach the result.

DISCUSSION

In this study, the processes of abstracting the concept of limit from the contextual situation of pre-service mathematics teachers from three levels (good, medium and mediocre) were examined using the RBC+C abstraction model. The participants' processes of abstraction from context were examined using four main contextual research problems and sub-problem items on cognitive actions of recognition, co-construction, construction and reinforcement. These problems, designed by the researchers to reveal students' mathematical thinking levels and abstraction processes, enabled students to use their prior knowledge and to have a new structure.

The pre-service teachers who could construct the concept of limit in their minds in contextually designed problems were asked to form the approach method for the limit point obtained at one point and to explain the situation mathematically. At this stage, the step of knowledge came to the fore. However, in explaining the situation mathematically, examples from B+ and C steps are seen. In general, it was observed that the participants could not provide an appropriate explanation for the C level, while they could easily do B+ at this stage. The general reason is that they do not build a complete abstraction of limit knowledge, and the experientially acquired knowledge needs to be assimilated.

The second question was asked in a form in which both the approximation method and the derivative were obtained from the limit information. The answers to the question showed that only the participants with good B+ knowledge, that is, able to practice more through experience could realize an abstraction. Although the participants' ability to practice a lot to perform abstraction has similar characteristics to the literature (Gürbüz et al., 2018; Bergsten, 2006; Sezgin-Memnun, 2017), it is seen that their ability to form a complete structure is weak. In the second item of the same question, all participants were able to explain the situation.

In the third problem, a well-known limit metro is tried to be abstracted. Although all participants completed their tasks successfully, it was observed that there were some information deficiencies in the explanation part, which affected the abstraction process. This incomplete information showed that in constructing the concept, the participants caused errors in transforming algebraic information into geometric information. The RBC+C abstraction model helped analyze and explain students' thoughts when some answers were unexpected or inconsistent with previous answers. It seemed natural to serve as an explanatory tool (Ron, Dreyfus, & Hershkowitz, 2010) for correct answers partially based on incorrect information and incorrect answers mainly based on correct information. In short, the participants exhibit some lack of knowledge even when they work on a well-known concept in the knowledge creation process. This situation affects the abstraction process. In the last contextual problem, all participants could abstract from the contextual situation using the technique. However, it has been observed that no C or C+ specific to their disclosure has occurred. In general, they construct a C in the form of the development of R and B+ epistemic actions. In the second item of the same problem, only the participant in the excellent category could perform an abstraction.

In contrast, the others were able to exhibit the b+ epistemic action. Mainly, the participants turned to concepts such as derivative and peak value in constructing limit information. One reason is that they use implementations of the derivative, which is an easier way of constructing limit information. While the participants were creating the mathematical structures they were expected to construct in the problem-solving process, they tended to create new structures based on the structures they knew by heart rather than forcing their minds.

The RBC+C abstraction model proved to be helpful in the process of explaining the path to soft information as opposed to just learning. Hayriye, who has a high level of mathematical success, structured her limit knowledge as a result of the analysis of this abstraction model's cognitive actions of recognizing, co-constructing, constructing and consolidating. Here, the structuring process of knowledge is an abstraction. Participants at all levels could demonstrate appropriate epistemic actions regarding using limit information presented in context. However, there has yet to be a complete success in writing this contextual limit information in mathematical form, and thus in the opportunities that allow abstraction away from memorized information. This shows that the development of students' mental structures confirms these cognitive actions. In related studies (Altun & Yılmaz, 2008; Sezgin-Memnun et al., 2017; Gürbüz et al., 2018) that focused on the abstraction of limit information in Turkey, students generally stated that they formed the concept within themselves. In addition, it can be stated that the participants of this study do not have misconceptions about limit knowledge. This confirms that pre-service mathematics teachers have had an effective learning process for limit knowledge.

However, the participants had difficulties transferring the contextually designed limit information to a new situation, giving a complete mathematical explanation, and explaining the limit information mathematically beyond memorization. For this reason, except for Hayriye, who is at a reasonable level, the others could not achieve a complete abstraction. Although similar results were obtained in Roh's (2007) study, participants needed help in a complete abstraction of limit knowledge in learning processes organized with contextual problem-solving activities. Although most participants were successful in the questions containing the active part of the limits, they could not achieve the same success in the questions containing the conceptual part. This research result is like the literature studies (Barak, 2007; Queseda et al., 2008; Güven et al., 2012; Gürbüz et al., 2018). It was observed that the participants exhibited the building with epistemic action better, but they had much difficulty constructing epistemic action. This may be because conceptual learning and procedural learning are not balanced in mathematics, there is more conceptual learning than procedural learning, and therefore students cannot apply the concepts or definitions they learn in mathematics lessons (Soylu & Aydın, 2006).

CONCLUSION

Under its nature, it was used to investigate the process of structuring limit knowledge in the minds of pre-service teachers through the RBC+C model of the process of abstracting mathematical knowledge from a contextual situation. In this process, it was repeated that the RBC+C model was helpful in determining the components of limit knowledge in the minds of pre-service teachers. It was determined that the pre-service teachers interviewed developed an understanding of limits and needed help using this concept in mathematical operations. However, they had difficulties transferring the limited information presented in context to new situations and explaining and proving it mathematically. In addition, it was found that they had difficulty adapting the formal definition to the new situation when faced with a contextual situation in which they could make a formal definition of the limit. Pre-service teachers understand the formal definition of memorization. However, it has been observed that a specific conceptual schema is formed in their minds. This shows that they cannot form the limited information in their minds entirely. In summary, although the pre-service teachers could show different representations of the definition of the limit in the solution of the limit problems designed contextually, they could not comment on the variables in the definition. They could not construct a unique mathematical structure.

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The authors received no financial support for the research, author-ship, and/or publication of this article.

Statements of publication ethics

We hereby declare that the study has not unethical issues and that research and publication ethics have been observed carefully.

Researchers' contribution rate

The first author played an active role in writing the conceptual framework and discussion conclusions of the research, and the second and third authors played an active role in the data collection and analysis process.

Ethics Committee Approval Information

Istanbul Aydin University Education Sciences Ethics Committee, Number: E-45379966-020-55230.

REFERENCES

Akbulut, K., & Işık, A. (2005). Limit kavramının yaklaşımında yaklaşımlı öğretmek ile ilgili kavramların incelenmesi ve benzerleri ile ilgili olarak. *Kastamonu Eğitim Dergisi, 13*(2), 497-512.

Akkoyunlu, A., Güler, M., Uğurel, İ., & Alan, E. (2003). Ortaöğretimde limit kavramının kullanımına yönelik bir çalışma. Retrived from www.matder.org.tr

Altun, M. (2014). *Liselerde matematik öğretimi* [Mathematics teaching in high schools]. Bursa, Turkey: Aktüel.

Altun, M., & Yılmaz, A. (2008). Lise öğrencilerinin tam değer fonksiyonu bilgisini oluşturma süreci. *Ankara Üniversitesi Eğitim Bilimleri Fakültesi Dergisi, 41*(2), 237-271.

Arslan, S. & Celik, D. (2015). Two concepts assumed to be difficult: Limits and continuity. In I.O. Zembat, M. F. Özmantar, E. Bingölbali, H. Şandır, & A. Delice (Eds.), *Mathematical concepts with definitions and historical development* (pp. 463–487). Ankara, Turkey: PegemA.

Baki, A. (2008). *Kuramdan uygulamaya matematik eğitimi* [Mathematics education from theory to practice]. Ankara, Turkey: Harf Publishing.

Baki, A. (2014). *Matematik tarihi ve felsefesi* [Mathematics history and philosophy]. Ankara, Turkey: PegemA.

Balci, M. (2014). *Genel matematik* [Calculus I]. Ankara, Turkey: Sürat Publishing.

Barak, B. (2007). Determination of the misconceptions about limit subject [Unpublished master's thesis]. Balikesir University. Retrieved from https://tez.yok.gov.tr/UlusalTezMerkezi/

Baştürk, S. & Dönmez, G. (2011). Mathematics student teachers' misconceptions on the concepts of limits and continuity. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 5*, 225–249.

Bergsten, C. (2006). Trying to reach the limit. The role of algebra in mathematical reasoning. In Novotna, J., Moraova, H., Kratka, M. & Stehlíkova, N. (Eds.) *Proceedings of the 30th conference of the international group for the psychology of mathematics education (PME), 2*, 153-160. Retrieved from http://files.eric.ed.gov

Biber, A. C., & Argün, Z. (2015). The relations between concept knowledge related to the limits concepts in one and two variables. *Bartin University Journal of Faculty of Education, 4*, 501-515.

Bikner-Ahsbahs, A. (2004). "Towards the Emergence of Constructing Mathematical Meanings". *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 2*:119-126.

Bukova-Güzel, E. (2007). The effect of a constructivist learning environment on the limit concept among mathematics student teachers. *Kuram ve Uygulamada Egitim Bilimleri, 7*(3), 11-29.

Cajori, F. (2014). Matematik tarihi [A history of mathematics] (trans. D. İlalan). Ankara, Turkey: ODTU Publishing.

Can, M. (2011). Matematiksel soyutlama ve soyutlamanın indirgenmesi [Mathematical abstraction] (Master's thesis, Yildiz Technical University, Istanbul, Turkey). Retrieved from https://tez.yok.gov.tr/UlusalTezMerkezi

Çıldır, S. (2012). Visualization of limit in computer environment and the views of prospective teachers on this issue. *Hacettepe University Journal of Education,42*, 143–153.

Cohen, L., & Manion, L. (1994). *Research methods in education*. London, UK: Routledge.

Cornu, B. (1991): "Limits". In D. Tall (Ed.) *Advanced Mathematical Thinking*, Dordrecht, Kluwer, 133–166.

Cottrill, J., Dubinsky, E., Nichols, D., Schwingendorf, K., Thomas, K., & Vidakovic, D. (1996). Understanding the limit concept: Beginning with a coordinated process scheme. *Journal of Mathematical Behavior, 15*(2), 167–192.

Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches*. Sage publications.

Denbel, D. G. (2014). Students' misconceptions of the limit knowledge in a first calculus courses. *Journal of Education and Practice, 5*, 24-40.

Donmez, G., & Basturk, S. (2010). Pre-service mathematical teachers’ knowledge of different teaching methods of thelimit and continuity concept. *Procedia-Social and Behavioral Sciences, 2*(2), 462-465.

Dreyfus, T. (2007). Processes of abstraction in context the nested epistemic actions model. Retrieved from from https://pdfs.semanticscholar.org/d190/0be9d6a043ac815c81344caa8c2713dcc329.pdf.

Dreyfus, T., & Tsamir, P. (2004). Ben's consolidation of knowledge structures about infinite sets. *The Journal of Mathematical Behavior, 23*(3), 271-300.

Dreyfus, T., Hadas, N., Hershkowitz, R., & Schwarz, B. (2006). Mechanisms for consolidating knowledge constructs. In J. Novotná, H. Moraová, M. Krátká, & N. Stehliková (Eds.), *Proceedings of the 30th Conference of the International Group for Psychology of Mathematics Education: Vol. 2* (pp. 465–472). Prague, Czech Republic: PME.

Dubinsky, E. (1991). Reflective abstraction in advanced mathematical thinking. In D. Tall (Ed.) *Advanced Mathematical Thinking*, Dordrecht, Kluwer, 95–123.

Dugac, P. (1973). Eléments d'analyse de Karl Weierstrass. *Archive for history of exact sciences, 10*(1), 41-174.

Durmus, S. (2004). Matematikte öğrenme güçlüklerinin saptanması üzerine bir çalışma [A research on the determination of learning difficulties in mathematics]. *Kastamonu Education Journal, 12*, 125–128.

Ervynck, G. (1981). Conceptual difficulties for first year university students in the acquisition of the notion of limit of a function. Proceedings of the Fifth Conference of the International Group for the Psychology of Mathematics Education, Berkeley, pp. 330-333.

Fernández, E. (2004). The Students’take on the Epsılon-Delta Definition of a Limit. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 14*(1), 43-54.

Gass, F. (1992). Limits via graphing technology. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 2*(1), 9-15.

Glasersfeld, E. von. (1990). Environment and communication. In L. P. Steffe & T. Wood (Eds.). Transforming children’s mathematics education: international perspectives (pp. 30–38). Hillsdale, NJ: Lawrence Erlbaum Associates.

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational studies in mathematics*, *39*(1), 111-129.

Gray, E. M., & Tall, D. O. (1994). Duality, ambiguity, and flexibility: A “proceptual” view of simple arithmetic. *Journal for research in Mathematics Education*, *25*(2), 116-140.

Gürbüz, M. Ç., Ağsu, M., & Özdemir, M. E. (2018). An Analysis of How Preservice Math Teachers Construct the Concept of Limit in Their Minds. *European Journal of Education Studies, 5*(6), 103–126. https://doi.org/10.5281/zenodo.1490240

Gürbüz, M.Ç. (2021). Ortaokul öğrencilerinin cebir kavramlarını soyutlama süreçlerinin incelenmesi [Investigation of secondary school students' abstraction processes of algebra concepts] (Unpublished doctoral thesis, Bursa Uludag University, Bursa, Türkiye). Retrived from https://tez.yok.gov.tr/UlusalTezMerkezi/

Gürbüz, R., Toprak, Z., Yapıcı, H., & Doğan, S. (2011). Ortaöğretim Matematik Müfredatında Zor Olarak Algılanan Konular ve Bunların Nedenleri. *Gaziantep University Journal of Social Sciences, 10*(4).

Guven, B., Baki, A., & Cekmez, E. (2012). Using dynamic geometry software to develop problem solving skills. *Mathematics and Computer Education, 46*(1), 6.

Hassan, I., & Mitchelmore, M. (2006). The role of abstraction in learning about rates of change. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), *Identities, cultures and learning* spaces: Vol. 1 (Proceedings of the 29th Annual Conference of the Mathematics Education Research Group of Australasia, pp. 278–285). Adelaide, Australia: MERGA.

Hershkowitz, R. (2004). From diversity to inclusion and back: Lenses on learning (Plenary lecture).In M. J. Hoines, & A. B. Fuglesad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology ofMathematics Education: Vol. 1* (pp. 55–68). Bergen, Norway: PME

Hershkowitz, R., Schwarz, B. B., & Dreyfus, T. (2001). Abstraction in context: Epistemic actions. *Journal for Research in Mathematics Education*,*32* 195-222. http://dx.doi.org/10.2307/749673

Høyrup, J. (2019). *Selected essays on pre-and early modern mathematical practice*. Springer International Publishing.

Juter, K. (2006). Students' attitudes to mathematics and performance in limits of functions. *Mathematics Education Research Journal, 17*, 91–110. http://dx.doi.org/10.1007/BF03217417

Kuhn, T. (1963). The function of dogma in scientific research. In A.C. Crombie (Ed.), *Scientific change* (pp. 347–369). New York: Basic Books.

Kula, S. & Bukova-Güzel, E. (2015). Reflections of Mathematics Student Teachers' Knowledge Related to the Purposes of the Curriculum on Their Limit Teaching. *Journal of Theoretical Educational Science/Kuramsal Eğitim Bilim Dergisi, 8*(1).

Kulasegaram, K., Min, C., Ames, K. *et al.* The effect of conceptual and contextual familiarity on transfer performance. *Adv in Health Sci Educ* **17,** 489–499 (2012). https://doi.org/10.1007/s10459-011-9326-z

Lakoff, G., & Núñez, R. (2000). *Where mathematics comes from* (Vol. 6). New York: Basic Books.

Liang, S. (2016). Teaching the concept of limit by using conceptual conflict strategy and Desmos graphing calculator. *International Journal of Research in Education and Science (IJRES), 2*(1), 35-48.

Lincoln, Y. S. & Guba, E. G. (1985). *Naturalistic inquiry* (Vol. 75). SAGE.

Memnun, D. S., & Altun, M. (2012). RBC+ C modeline göre doğrunun denklemi kavramının soyutlanması üzerine bir çalışma: özel bir durum çalışması. *Cumhuriyet Uluslararası Eğitim Dergisi, 1*(1), 17-37.

Merriam, S. B. (2002). Introduction to qualitative research. *Qualitative research in practice: Examples for discussion and analysis*, *1*(1), 1-17.

Mitchelmore, M. (2002). The role of abstraction and generalization in the development of mathematical knowledge. *Proceeding of the 9th East Asia Regional Conference on Mathematics Educationand the Southeast Asian Conference on Mathematics Education* (pp. 27–31). Singapore.

Nakane, M. (2014). Did Weierstrass's differential calculus have a limitavoiding character? His definition of a limit in # – δ style, *BSHM Bulletin: Journal of the British Society for the History of Mathematics, 29:*1, 51-59, DOI: 10.1080/17498430.2013.831241.

Norman, G. R., Dore, K., Krebs, J., & Neville, A. J. (2007). The power of the plural: Effect of conceptual analogies on successful transfer. *Academic Medicine, 87*(Supp), 16–18.

Novak, J. D. (1993). How Do We Learn Our Lesson? *Science Teacher, 60*(3), 50-55.

Núñez, R. E., Edwards, L. D., & Filipe Matos, J. (1999). Embodied cognition as grounding for situatedness and context in mathematics education. *Educational studies in mathematics*, *39*(1), 45-65.

Özmantar, M. F. (2005). An investigation of the formation of mathematical abstractions through scaffolding (Doctoral dissertation, University of Leeds, Leeds, United Kingdom). Retrieved from http://etheses.whiterose.ac.uk/11271/

Ozmantar, M. F., & Monaghan, J. (2007). A dialectical approach to the formation of mathematical abstractions. *Mathematics Education Research Journal, 19*(2), 89-112.

Pan, S. C., & Rickard, T. C. (2018). Transfer of test-enhanced learning: Meta-analytic review and synthesis. *Psychological Bulletin, 144*(7), 710–756. https://doi.org/10.1037/bul0000151.

Quesada, A., Einsporn, R. L., & Wiggins, M. (2008). The Impact of the Graphical Approach on Students' Understanding of the Formal Definition of Limit. *International Journal for Technology in Mathematics Education, 15*(3). 95-102.

Roh, H. K. (2007). An Activity for Development of the Understanding of the Concept of Limit. In *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 105-112).

Ross, B. H. (1987). This is like that: The use of earlier problems and the separation of similar effects. *Cognitive Psychology, 13*, 629–638.

Ross, B. H., & Kennedy, P. T. (1990). Generalizing from the use of earlier examples in problem solving. *Journal of Experimental Psychology. Learning, Memory, and Cognition, 16*, 42–55.

Schwarz, B., Dreyfus, T., Hadas, N., & Hershkowitz, R. (2004). Teacher guidance of knowledge construction. In M. J. Hoines, & A. B. Fuglesad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of MathematicsEducation: Vol. 4* (pp. 169–176). Bergen, Norway: PME. Retrieved from https://www.emis.de/proceedings/PME28/RR/RR175_Schwarz.pdf

Sezgin Memnun, D., Aydın, B., Özbilen, Ö., & Erdoğan, G. (2017). The abstraction process of limit knowledge. *Educational Sciences: Theory & Practice, 17*, 345–371. http://dx.doi.org/10.12738/estp.2017.2.0404

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics, 22*, 1–36.

Sierpińska, A. (1987). Humanities students and epistemological obstacles related to limits. *Educational studies in Mathematics, 18*(4), 371-397.

Sierpińska, A. (1994). The diachronic dimension in research on understanding in niathematics-usefulness and limitations of the concept of epistemological obstacle. *Didactica Mathematicae*, *16*(01).

Soylu, Y., & Aydin, S. (2006). A study on importance of the conceptual and operational knowledge are balanced in mathematics lessons. *Journal of Education Faculty, 8*(2), 83-95.

Swinyard, C. A., & Lockwood, E. (2007). Research on students' reasoning about the formal definition of limit: An evolving conceptual analysis. In Proceedings of the *10th Conference on Research in Undergraduate Mathematics Education* (CRUME2007).

Szydlik, J. E. (2000). Mathematical beliefs and conceptual understanding of the limit of a function. *Journal for Research in Mathematics Education, 31*(3), 258–276. http://dx.doi.org/10.2307/749807

Tabach, M., & Friedlander, A. (2008). The role of context in learning beginning algebra. *Algebra and algebraic thinking in school mathematics*, *70*, 223.

Tall, D. (1988). The nature of advanced mathematical thinking. *Discussion paper for the working group on advanced mathematical thinking.* PME-XII, Vezprém, Hungary. Retrived from https://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot1988i-nature-of-amt-pme.pdf

Tall, D. (2008). The transition to formal thinking in mathematics. *Mathematics Education Research Journal*, *20*(2), 5-24.

Tall, D. O. (2004), The three worlds of mathematics. *For the Learning of Mathematics, 23*(3), 29–33.

Tall, D., & Katz, M. (2014). A cognitive analysis of Cauchy's conceptions of function, continuity, limit and infinitesimal, with implications for teaching the calculus. *Educational Studies in Mathematics, 86*(1), 97-124.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics, 12*(2), 151-169. https://doi.org/10.1007/bf00305619

Tangül, K., Barak, B., & Özdaş, A. (2015). Students' concept definitions and concept images about the limit knowledge. *Anadolu Journal of Educational Sciences International, 5*(1), 88–114. http://dx.doi.org/10.18039/ajesi.48087

Tatar, E., Okur, M., & Tuna, A. (2008). A study to determine learning difficulties in secondary mathematics education. *Kastamonu Education Journal*, *16*(2), 507-516.

Tsamir, P. & Dreyfus, T. (2002). Comparing infinity sets–A Process of Abstraction: The case of Ben. *Journal of Mathematical Behavior, 21*, 1–23.

Wiest, L. R. (2001). The role of computers in mathematics teaching and learning. *Computers in the Schools*, *17*(1-2), 41-55.

Yeşildere, S., & Türnüklü, E. B. (2008). İlköğretim sekizinci sınıflamanın bilgi oluşturmasını analiz etmekle ilgili güvenirliklerinin incelenmesi. *Uludağ Üniversitesi Eğitim Fakültesi Dergisi, 21*(2).

Yıldırım, A., & Şimşek, H. (2016). Sosyal bilimlerde nitel araştırma yöntemleri [Qualitative research methods in the social sciences] (Vol. 10). *Ankara: Seçkin Yayıncılık*.

Yılmaz, R. (2011). Matematiksel soyutlama ve genelleme süreçlerinde görselleştirme ve rol [Visualization and role in mathematical abstraction and generalization process] (Unpublished doctoral thesis, Gazi University, Ankara, Türkiye). Retrived from https://tez.yok.gov.tr/UlusalTezMerkezi/

Yin, R. K. (2003). Designing case studies. *Qualitative research methods*, *5*(14), 359-386.

Zollman, A. (2014). University students' limited knowledge of limits from calculus through differential equations. The mathematics education for the future project: Proceedings of the 12th International Conference, (pp. 693-698). Retrieved from https://www.researchgate.net