

# ***Discharge Coefficient of Sharp-Crested Trapezoidal Labyrinth Weirs***

***Omer BILHAN***<sup>1</sup>

***M. Emin EMIROGLU***<sup>2</sup>

## **Abstract**

Labyrinth weirs provide an effective means to increase the spillway discharge capacity of dams and are often considered for renovation projects required due to an increase in expected flood inflow to the reservoir of an existing dam. Free crest spillways are hydraulically efficient and safe in operation. Since their discharge capacity is directly proportional to the crest length several types have been developed with the purpose to increase the length of the latter. In recent years many research investigations have considered the hydraulic performance of labyrinth weirs, particularly as dependent on the geometric features. The previous work has improved the design basis for such weirs. In the present study, discharge coefficients were experimentally determined for sharp crested trapezoidal labyrinth weirs of varying side wall angle ( $\alpha$ ). The experimental results of 21 physical models were used to develop a hydraulic design and analysis method for labyrinth weirs. The present research primarily aims at evaluating various characteristics of a flow-over labyrinth weir by conducting experimentations at wider range of values for important parameters.

**Keywords:** *Fluid mechanics, weir, discharge coefficient, trapezoidal, labyrinth weir*

## **1. Introduction**

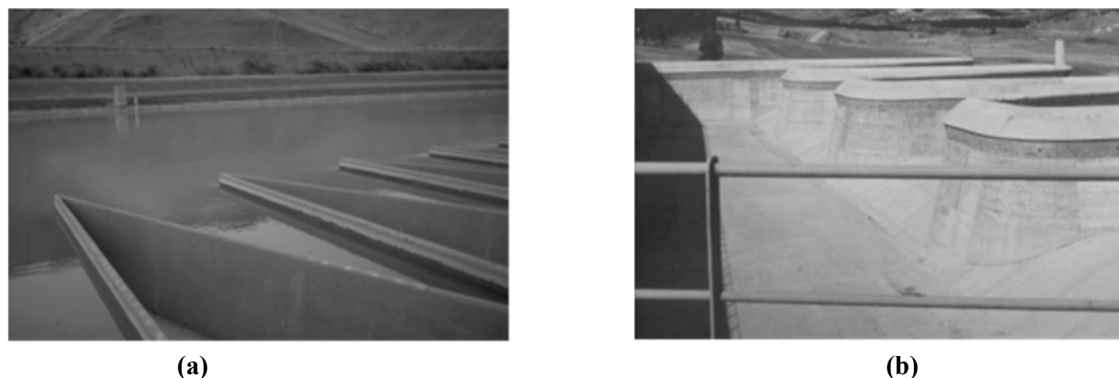
Spillways play a major role in ensuring the flood safety of dams. Insufficient spillway capacity has been the cause of one-third of all dam failures. Labyrinth weirs provide higher discharge capacity than conventional weirs, with the ability to pass large flows at comparatively low heads. A labyrinth weir is a linear weir that is 'folded' in plan-view to increase the crest length for a given channel or spillway width. Due to the complex design of the overflow structure, the labyrinth spillway discharge capacity is affected by many factors including weir geometry and approach channel conditions [1]. There is great flexibility in the geometric design of labyrinth

weirs. Yet, optimizing the many geometric variables in the hydraulic design of a labyrinth weir can be challenging. For example, the sidewall angle ( $\alpha$ ), total crest length ( $L_c$ ), crest shape, number of cycles ( $N$ ), the configuration of the labyrinth cycles, and the orientation and placement of a labyrinth weir must all be determined. A labyrinth weir is a linear weir that is 'folded' in plan-view to increase the crest length for a given channel or spillway width [2]. Figure 2 provides the key details of the labyrinth weir. The distinguishing characteristic of labyrinth spillways is that the plan shape is not linear but varies using a repeating planform as U shape (eventually rectangular), V or triangular shape (Figure 1 (a)) and trapezoidal shape (Figure 1 (b)).

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<sup>1</sup> Nevsehir H.B.V. University, Nevsehir, Turkey, [obilhan@nevsehir.edu.tr](mailto:obilhan@nevsehir.edu.tr)

<sup>2</sup> Firat University, Elazig, Turkey



**Figure 1.** (a) Triangular labyrinth weir of the hydroelectric power plant Ohau C in New Zeland. (b) Trapezoidal labyrinth weir of Cimia dam in Italy.

Optimizing the many geometric variables in the hydraulic design of a labyrinth weir can be challenging. For example, the sidewall angle ( $\alpha$ ), total crest length ( $L_c$ ), crest shape, number of cycles ( $N$ ), the configuration of the labyrinth cycles, and the orientation and placement of a labyrinth weir must all be determined. Furthermore, the geometry of a labyrinth weir causes complex 3-dimensional flow patterns that must be considered. The flow rate passing over the labyrinth is dependent on the crest length, which can be controlled by modifying the number of folds. The relationship between length and discharge is not linear, however, except for very small heads. As the water level above the labyrinth weir increases, four stages of nappe shape occur: fully aerated, partially aerated, transition and submerged. The thickness of nappe and depth of the tailwater do not affect the discharge capacity of the labyrinth weir in the fully aerated flow condition. In this case, the labyrinth weir acts as a vertical cross section of the linear weir. As the water level above the labyrinth weir increases and the tailwater rises, the nappe becomes partially aerated (adhering to the weir wall) and the discharge coefficient is reduced [3-4].

In recent years, extensive research on the influence of geometric and hydraulic parameters on the hydraulic behavior of labyrinth weirs, particularly on the discharge capacity, has been completed. Taylor (1968) [5] presented initial studies on the behavior of labyrinth weirs and presented the hydraulic performance as it compares with that of sharp-crested weirs. Hay and Taylor [6] followed up on Taylor's

work and developed design criteria for labyrinth weirs. Based on their research findings, they suggested Eq. (1) for the discharge coefficient of labyrinth weirs.

$$C_d = 3.22 + 0.40 \frac{h}{P} \quad (1)$$

where  $C_d$  is the discharge coefficient,  $h$  is the depth of flow over the weir crest and  $P$  is the weir height.

Additional work by Darvas [7] utilized the results from physical model studies to expand on the theory and develop a family of curves to evaluate spillway performance. Extensive physical model studies were performed by Houston [8] to evaluate various labyrinth geometries and approach conditions. The U.S. Bureau of Reclamation (USBR) tested a model of labyrinth spillway for Ute Dam and Hyrum Dam [8-9]. They found that the discrepancy between their observations and those of Hay and Taylor (1970) [6] were caused by difference in head definition. has also investigated model studies of the labyrinth weir and Eq. (2) is his suggested equation for calculation of discharge over labyrinth weirs.

$$Q = C_d \left( \frac{\frac{W_c}{P}}{\frac{P}{W_c}} \right) W_c H_t \sqrt{g H_t} \quad (2)$$

where  $Q$  is the discharge over labyrinth weir,  $C_d$  is the discharge coefficient,  $H_t$  is the total upstream head

measured relative to the weir crest,  $W_c$  is the channel width and  $P$  is the weir height.

Magalhaes and Lorena [11] calculated discharge coefficient ( $C_d$ ) of labyrinth weirs as function of  $L/w$  and  $H_t/P$  parameters. They defined discharge capacity of labyrinth weirs with Eq. (3).

$$Q = C_d W_T 2gH_t^{1.5} \quad (3)$$

Tullis et al. [12] carried out extensive experimental work on the performance of the labyrinth weir. They proposed a flow equation for the labyrinth weir that is identical to the basic equation applicable to a linear weir, but with modification of the coefficient of discharge. They also presented experimental data of the variation of discharge coefficient of labyrinth weir with a head to weir height ratio ( $H_t/P$ ) for side wall angles ( $\alpha$ ) of  $6^\circ$  to  $18^\circ$ . Additional curves for weir side angles of  $25^\circ$  and  $35^\circ$  were obtained by extrapolation. Tullis et al. (2007) [13] extended this work by providing a dimensionless head-discharge relationship for submerged labyrinth weirs. Using a physical model of the labyrinth weir of Dog River Dam in Georgia, Savage et al. [14] showed that the method of Tullis et al. [12] produced a discharge error up to  $\pm 25\%$ .

Labyrinth weirs are also used as side weirs to increase the outflowing discharge. Emiroglu et al. [15] carried out extensive experimental work on the performance of the labyrinth side weirs and presented coefficient of discharge curves in a simplified way as compared to previous investigators. Further work on triangular labyrinth side weirs was completed by Bilhan et al. [16] using Artificial Neural Network (ANN) techniques to calculate the discharge coefficient under critical flow conditions.

Khode et al. [17] carried out flume studies on trapezoidal labyrinth weirs for side wall angles  $6^\circ$ ,  $8^\circ$ ,  $10^\circ$ ,  $16^\circ$ ,  $21^\circ$ ,  $26^\circ$  and  $30^\circ$ . Khode et al. [18] extended these studies for a wider range of flow conditions.

Anderson and Tullis [19] used laboratory-scale physical models to compare the hydraulic efficiency

of the Piano Key (PK) weir design with that of a geometrically similar rectangular labyrinth weir, with and without sloping floors installed in the inlet and outlet keys. The test data showed that the PK weir was more efficient than the geometrically comparable rectangular labyrinth weir, a fact likely attributable to a reduction in entrance losses associated with the PK weir inlet key geometry.

Crookston and Tullis [20] published labyrinth weir design equations that are applicable to in-channel labyrinth weir applications in which the approach flow is oriented normal to the weir axis. Consequently, some uncertainty exists regarding the hydraulic performance of labyrinth weir configurations that deviate from the experimental conditions associated with the empirical determinations.

Anderson and Tullis [19] investigated 9 laboratory-scale four-cycle PK weir configurations to develop a better understanding of the effects of PK weir geometry on discharge efficiency. The appropriateness of the recommended head-discharge equation specific to the recommended design was evaluated, and the relative head-discharge efficiency of trapezoidal labyrinth and PK weirs with respect to footprint restrictions and crest length were compared in this study.

Information regarding nappe aeration conditions (clinging, aerated, partially aerated, and drowned), nappe instability, and nappe vibrations for trapezoidal labyrinth weirs on a horizontal apron with quarter- and half-round crests ( $6^\circ \leq \alpha \leq 35^\circ$ ) was presented by Crookston and Tullis [21]. In this study, hydraulic behaviors associated with nappe aeration conditions were recommended to aid in labyrinth weir design, related to nappe behavior (e.g., crest shape, crest roughness, vents, nappe breakers, notches, and staged cycles) in future studies.

While all these documented studies have provided significant insights to the behavior of labyrinth weirs under specific conditions, the general theory remains: the capacity of labyrinth weir is a function of the upstream total head, the effective crest length, and the coefficient of discharge. The discharge coefficient

depends on the total head, weir height, thickness, crest shape, apex configuration, and angle of side wall. While viscosity and surface tension are also significant variables, their influence is limited at velocities of sufficient magnitude and by appropriate model geometries [19]. The purpose of this study is to systematically investigate the discharge capacity of sharp-crested trapezoidal labyrinth weir using a broad range of experiments, and considered together with the

other effective dimensionless parameters. Most of the design and performance information regarding labyrinth weirs has been developed from physical model studies, often for a specific prototype installation (e.g., Avon, Dungo, Hyrum, Keddara, Lake Brazos, Lake Townsend, Ute, and Woronora). A selection of notable research studies that have provided hydraulic design guidance for labyrinth weirs is presented in Table 1.

**Table 1.** Labyrinth Weir Design Methods

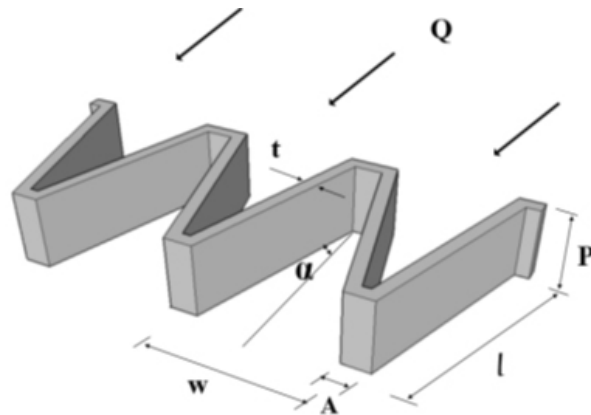
	<b>Authors</b>	<b>Labyrinth Cycle Type</b>	<b>Crest Shape</b>
1	Taylor (1968), Hay and Taylor (1970) [5-6]	Tri, Trap, Rect	Sh, HR
2	Darvas (1971) [7]	Trap	LQR
3	Hinchliff and Houston (1984) [23]	Tri, Trap	Sh, QR
4	Lux and Hinchliff (1985) [24]	Tri, Trap	QR
5	Magalhães and Lorena (1989) [11]	Trap	WES
6	Tullis et al. (1995) [12]	Trap	QR
7	Melo et al. (2002) [25]	Trap	LQR
8	Tullis et al. (2007) [13]	Trap	HR
9	Lopez et al. (2008) [26]	Trap	LQR

Tri = Triangular, Trap = Trapezoidal, Rect = Rectangular, QR –Quarter-round ( $R_{crest} = tw/2$ ), LQR – Large Quarter-round ( $R_{crest} = tw$ ), HR – Half-round, Sh – Sharp, WES – Truncated Ogee

**2. Experimental Method**

21 physical models of labyrinth weirs were fabricated and tested at the Firat University Hydraulic Research Laboratory, located in Elazig, Turkey [22]. The experimental set-up includes sump, pumping system, discharge tank, rectangular flume, digital flowmeter and labyrinth weir. Water is recirculated through 250 mm diameter of supply line using two 75 HP pumps. Water for experimental setup is taken from the supply line by means of a pipe with 150 mm diameter. The discharge was measured by means of a Siemens electromagnetic flow-meter installed in the supply line. Water was supplied to the main channel (2 m wide and 0.80 m

height this channel length is 3.0 m) through a supply pipe from the sump (volume of 15 m<sup>3</sup>) with flow controlled by a gate valve. For damping the water surface waves and reducing turbulence, baffle wall and wood surface dampener is provided. In the experiments, the upstream elevation was built higher than the downstream elevation so that free flow conditions occur downstream of the weir. Sheet metal materials which have 4 mm thickness (*t*) were used for labyrinth weirs. Labyrinth weirs designed as three-cycles. Schematic view of trapezoidal labyrinth weirs is given in Fig. (2). Each trapezoidal labyrinth weir models with a sharp crested shape was tested in the experiments (Examples shown in Fig. (3)).



**Figure 2.** Schematic view of the trapezoidal weirs located on straight channel



**Figure 3.** Experimental set-up for Trapezoidal labyrinth weirs

To measure the nappe height, water depth was measured accurately using Mitutoyo digital point gauges (accurate to  $\pm 0.01$  mm) just upstream of the weirs. Level measurements were taken at a distance from the weir equal to five times the nappe height. For flow rate measurements, Nortek brand acoustic three-axis velocimeter was used.

In the experiments, the weir heights were taken as 100 mm, 150 mm and 200 mm and apex width ( $A$ ) was taken as 80 mm. Sharp-crested shapes is provided for all models. All experiments were performed according to free flow conditions.

The flow over labyrinth weir is three dimensional and does not readily fit into mathematical description and

hence the discharge function is found through experimental studies and analysis. The crest coefficient depends on the total head, weir height, thickness, crest shape, apex configuration and angle of side wall. To simplify the analysis, the effect of viscosity and surface tension could be neglected by selecting model and velocity of sufficient magnitude. The discharge over labyrinth weir can be expressed as:

$$Q = \frac{2}{3} C_d \sqrt{2g} H_t^{1.5} L \quad (4)$$

Where  $Q$  is the discharge over a labyrinth weir;  $C_d$  is the discharge coefficient of the labyrinth weir;  $L$  is the effective length of labyrinth weir;  $H_t$  is the total head ( $V_0^2/2g + h$ ) and  $g$  is the gravitational acceleration constant (Fig 4).

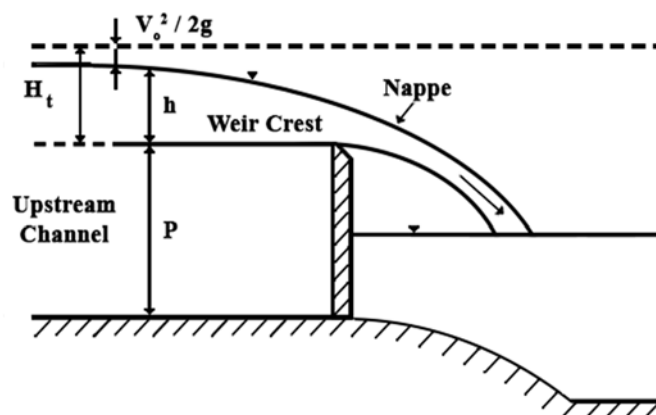


Figure 4. Definition sketch for flow over a sharp crested weir

Head over labyrinth weir was measured for different value of discharges in the range of 14.7 L/s to 136.9 L/s. In this range, the head over the labyrinth weir varied from 10 to 90 mm. The model of linear weir is also tested in the same flume for the purpose of comparison.

In an effort to develop a better understanding of the hydraulic characteristics of trapezoidal labyrinth weirs, with this experimental research was undertaken. Table 2 gives the description of different types of the trapezoidal labyrinth weirs tested.

Table 2. Physical model test program in the present study

Model	$W_c$ (cm)	$P$ (cm)	$L$ (cm)	$N$	$A$ (cm)	$L_c/w$	Type of Weir
1	196	10	196	-	-	-	Linear Weir, $\alpha=90^\circ$
2	196	10	294	3	8	1.50	Trapezoidal Labyrinth Weir, $\alpha=37^\circ$
3	196	10	345	3	8	1.76	Trapezoidal Labyrinth Weir, $\alpha=30^\circ$
4	196	10	427	3	8	2.18	Trapezoidal Labyrinth Weir, $\alpha=23^\circ$
5	196	10	534	3	8	2.73	Trapezoidal Labyrinth Weir, $\alpha=18^\circ$
6	196	10	621	3	8	3.17	Trapezoidal Labyrinth Weir, $\alpha=15^\circ$
7	196	10	774	3	8	3.95	Trapezoidal Labyrinth Weir, $\alpha=12^\circ$
8	196	15	196	-	-	-	Linear Weir, $\alpha=90^\circ$
9	196	15	294	3	8	1.50	Trapezoidal Labyrinth Weir, $\alpha=37^\circ$
10	196	15	345	3	8	1.76	Trapezoidal Labyrinth Weir, $\alpha=30^\circ$
11	196	15	427	3	8	2.18	Trapezoidal Labyrinth Weir, $\alpha=23^\circ$
12	196	15	534	3	8	2.73	Trapezoidal Labyrinth Weir, $\alpha=18^\circ$
13	196	15	621	3	8	3.17	Trapezoidal Labyrinth Weir, $\alpha=15^\circ$
14	196	15	774	3	8	3.95	Trapezoidal Labyrinth Weir, $\alpha=12^\circ$
15	196	20	196	-	-	-	Linear Weir, $\alpha=90^\circ$
16	196	20	294	3	8	1.50	Trapezoidal Labyrinth Weir, $\alpha=37^\circ$
17	196	20	345	3	8	1.76	Trapezoidal Labyrinth Weir, $\alpha=30^\circ$
18	196	20	427	3	8	2.18	Trapezoidal Labyrinth Weir, $\alpha=23^\circ$
19	196	20	534	3	8	2.73	Trapezoidal Labyrinth Weir, $\alpha=18^\circ$
20	196	20	621	3	8	3.17	Trapezoidal Labyrinth Weir, $\alpha=15^\circ$
21	196	20	774	3	8	3.95	Trapezoidal Labyrinth Weir, $\alpha=12^\circ$

### 3. Experimental Results

Experiments are carried out on six trapezoidal labyrinth weir models having side wall angles of 12°, 15°, 18°, 23°, 30° and 37° and a linear weir models having sharp crested shape similar to labyrinth weirs models. On all these models, head-discharge measurements are taken for weir height of  $P=10, 15$  and 20 cm.

Discharge coefficient for trapezoidal labyrinth weirs was computed using equation (Eq. (4)). Discharge coefficients of trapezoidal labyrinth side weirs have much higher values than the conventional weirs. The effect of crest shape on the discharge coefficient is very significant for the same channel width and crest length.

From these experiments, the variation of  $C_d$  for trapezoidal labyrinth weirs with  $H_t/P$  is plotted for

$P=10, 15$  and 20 cm in Fig.5. It is noted that discharge coefficient for labyrinth weirs is computed using equation (Eq. (4)). It is apparent from the results in Fig. 5 that discharge capacity of the labyrinth weirs is much higher than the conventional weirs. The primary reason for this is that the crest length of the labyrinth weir is much longer than that of the conventional weir. Similarly, to establish a relationship between  $L_c/w$  with  $C_d$  the observed data are plotted and shown in Fig.6. The layouts of labyrinth weir should be normally designed for maximum value of  $C_d$  max and the design curve developed from this present experimental study would help the designer to design trapezoidal labyrinth weir. The coefficient of discharge  $C_d$  is minimum for side wall angle of 12 degree and increases with increase in side wall angle approaching the value of linear weir.

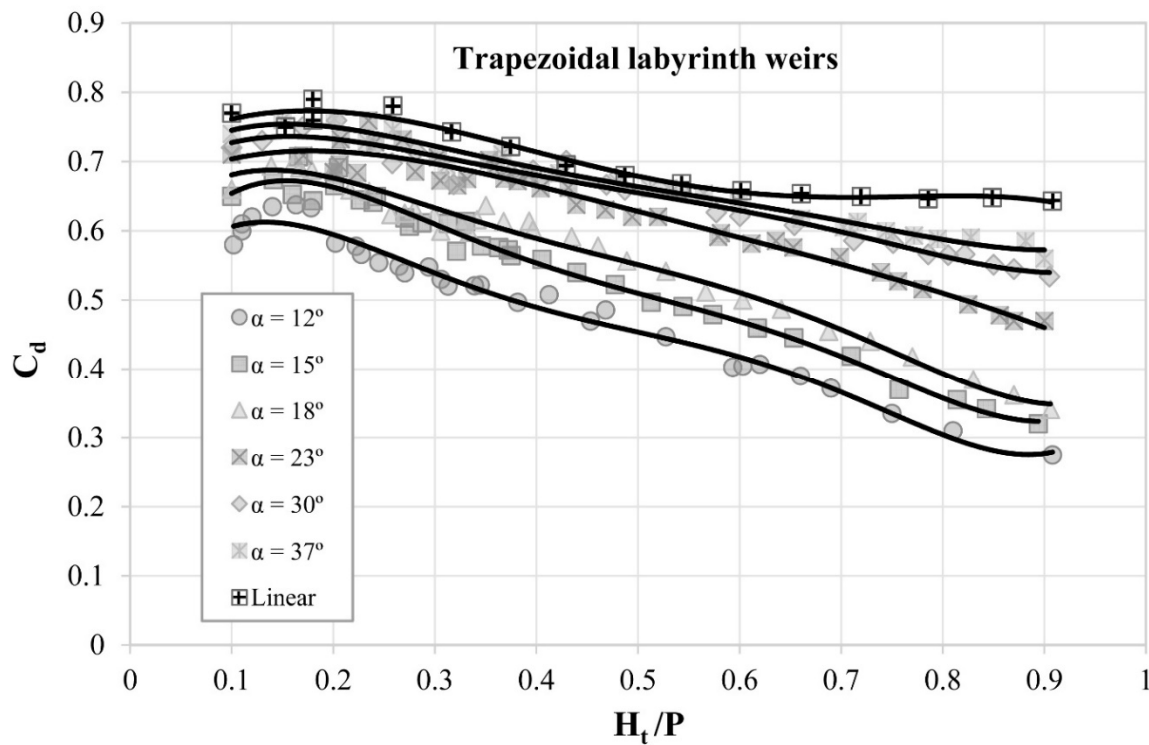


Figure 5. Variation of discharge coefficient ( $C_d$ ) with head to weir height ( $H_t/P$ ) for trapezoidal labyrinth weirs

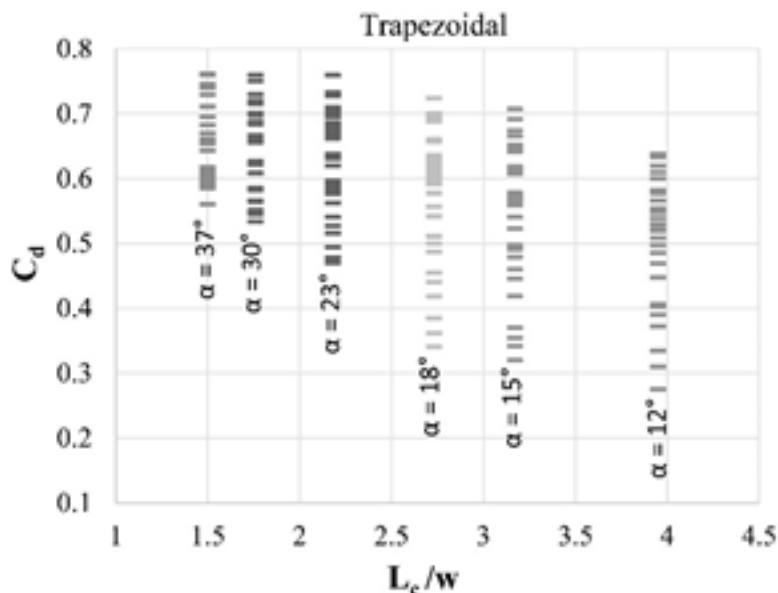


Figure 6. Variation of discharge coefficient ( $C_d$ ) with  $L_c/w$  for trapezoidal labyrinth weirs

To represent the data of the equation form, correlation analysis is carried out for the observed data for each model, separately. The 5<sup>th</sup> degree polynomial provides

a reasonable fit between  $C_d$  and  $H_t/P$ . Thus, discharge coefficient ( $C_d$ ) of sharp-crested labyrinth weir is expressed as:

$$C_d = A_0 + A_1 \left( \frac{H_t}{P} \right) + A_2 \left( \frac{H_t}{P} \right)^2 + A_3 \left( \frac{H_t}{P} \right)^3 + A_4 \left( \frac{H_t}{P} \right)^4 + A_5 \left( \frac{H_t}{P} \right)^5 \tag{5}$$

The values of  $C_d$ ,  $A_0$  to  $A_5$  and  $R^2$  are shown in Table 3.

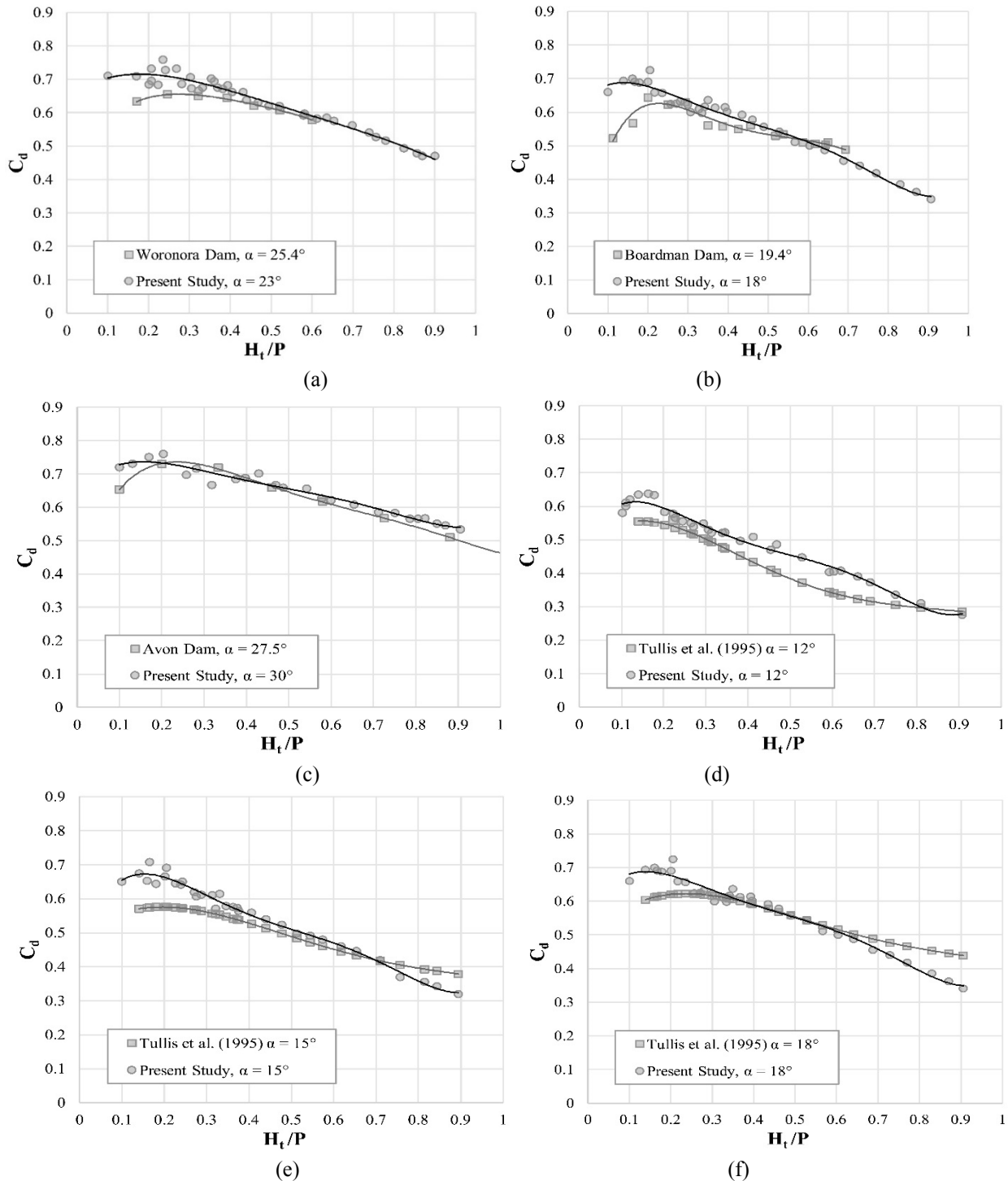
Table 3. Coefficient of discharge per unit length of trapezoidal labyrinth weir

Model	$A_0$	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$R^2$
Trapezoidal, $\alpha = 12^\circ$	0.4598	2.8255	- 17.296	40.718	- 43.59	17.273	0.9799
Trapezoidal, $\alpha = 15^\circ$	0.4600	3.4773	- 19.171	42.242	- 43.039	16.438	0.9848
Trapezoidal, $\alpha = 18^\circ$	0.5638	2.1933	- 12.876	29.283	- 30.857	12.090	0.9738
Trapezoidal, $\alpha = 23^\circ$	0.6417	0.9673	- 4.0152	5.7463	- 3.8338	0.8922	0.9608
Trapezoidal, $\alpha = 30^\circ$	0.6395	1.5467	- 8.3233	17.937	- 18.184	6.9522	0.9620
Trapezoidal, $\alpha = 37^\circ$	0.6537	1.6113	- 8.6152	18.076	- 17.719	6.5955	0.9790
Linear	0.6991	0.9370	- 3.4166	2.4939	1.8340	-1.9528	0.9665

The discharge coefficient values of labyrinth weir compared well with those of Woronora Dam, Boardman Dam, and Avon Dam. Moreover, the results of the present study compared well with those of Tullis

et al. [12], as shown in Fig.7. Although the data values are different, due to the variation in nappe shape and side wall angle for each study, the trends are similar to the findings of Tullis et al. [12] and prototype data.





**Figure 7.** Variation of discharge coefficient ( $C_d$ ) with head to weir height ( $H_t/P$ ) for present study compared to other findings: (a) Woronora Dam [7], (b) Bordman Dam [27], (c) Avon Dam [7]; (d–f) data from Tullis et al. [12]

#### 4. Conclusions

Labyrinth weirs can pass large flows at comparatively low heads. The crest shape is one of the most important factor which affects the discharge capacity for labyrinth weirs. According to this experimental study, it was found that the trapezoidal labyrinth weirs are hydraulically more efficient than the linear weirs from the perspective of ease of construction and the discharge capacity. The values for coefficient of discharge can be suitably obtained from the design curves and the regression equation generated through this study for trapezoidal labyrinth weirs. The values for discharge coefficient of trapezoidal labyrinth weirs can be suitably obtained from the design curves and the regression equations generated through this study for  $\alpha$  between 12 to 37°. The coefficient of discharge  $C_d$  is minimum for side wall angle of 12 degree and increases with increase in side wall angle approaching the value of linear weir. Of course, given unlimited width, greater efficiencies (discharge per head) will be obtained for a linear weir. However, the trapezoidal provides much greater weir length in confined space with only limited reductions in efficiency (reduction in  $C_d$ ).

A problem for labyrinth weirs is matching the outflow to downstream flow limitations. An example would be where previous water rights limit releases from the reservoir at floods below the hundred-year flood. If the labyrinth is to be added to an existing reservoir where the downstream requirements limit the flows at low water-surface elevations, a labyrinth with a small angle may provide more capacity than can be tolerated. For such an installation, a large angle labyrinth may better fit the outflow requirements. If the low flow requirement is extremely small, a short section of weir at a lower elevation could be used to pass small flows. The full labyrinth would not activate until the flood exceeds some predetermined level, such as the hundred-year flood. It is recommended that a labyrinth weir design be verified with a physical or numerical model study, as it would include site-specific conditions that may be outside the scope of this study and may provide valuable insights into the performance and operation of the labyrinth weir.

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#### NOMENCLATURE

$A$	Apex width;
$C_d$	Discharge coefficient;
$g$	Acceleration constant of gravity;
$h$	Depth of flow over the weir crest;
$H_t$	Total upstream head measured relative to the weir crest;
$H_t/P$	Headwater ratio;
$l$	Centerline length of weir sidewall;
$N$	Number of labyrinth weir cycles;
$P$	Weir height;
$Q$	Discharge over weir;
$V$	Average cross-sectional flow velocity upstream of weir;
$W_c$	Channel width;
$w$	Width of a single labyrinth weir cycle;
$L$	Total crest length of labyrinth weir;
$L_c$	Total crest length for a single labyrinth weir cycle;
$R^2$	Determination coefficient
$t$	Weir wall thickness

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