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Analyzing the Structures of Figural Patterns Produced by Middle School Students Based on Number Patterns

Ortaokul Öğrencilerinin Sayı Örüntülerine Dayalı Olarak Oluşturdukları Şekil Örüntülerinin Yapılarının Analiz Edilmesi

Çiğdem KILIÇ

Abstract: Pattern is one of the most important topics of mathematics education in all grades. Some researchers asserted that pattern is the heart and soul of mathematics. Many skills and mathematical knowledge of students can be developed through pattern activities. In this study analyzing the structures of figural patterns/situations produced by middle school students based on number patterns was investigated. In total, 474 middle school students (254 were girls and 220 were boys) attended to study. Data were collected from a pattern task including linear and quadratic (non-linear) number patterns, in which participants were asked to generate figural patterns based on those number patterns. The obtained data were analysed at two levels. The results of the study indicated that participants created 18 linear forms and 33 non-linear forms for linear number pattern and 13 linear forms and 20 non-linear forms for quadratic number pattern. During the study participants preferred to use same geometrical shapes such as circles, triangles and squares, etc. while creating figural patterns. Moreover, participants created many more figural patterns for the 3,5,7,9,11,… number pattern than for the 2,6,12,20,30,… number pattern. Some of the participants had problems while generating figural patterns such as producing non pattern forms, no answer, creating an irrelevant pattern and extending number patterns.
Keywords: Number pattern, figural pattern, linear pattern, quadratic pattern, pattern structures

Öz: Örüntü matematik eğitiminde her düzeyde en önemli konulardan biridir. Bazı araştırmacılar örüntünün matematiğin kalbi ve ruhu olduğunu belirtmektedirler. Öğrencilerin bir çok becerisi ve matematiksel bilgisi örüntü etkinlikleri ile geliştirilebilir. Bu çalışmada öğrencilerin sayı örüntülerine bağlı olarak oluşturdukları şekil örüntülerinin yapıları analiz edilmiştir. Çalışmaya toplam 474 ortaokul öğrencisi (254 kız ve 220 erkek) katılmıştır. Çalışmanın verileri lineer ve lineer olmayan sayı örüntülerinin bulunduğu örüntü görevi ile toplanmıştır. Araştırmadan elde edilen veriler iki düzeyde analiz edilmiştir. Araştırmadan elde edilen sonuçlara bakıldığında katılımcıların 18 lineer yapı ve 33 lineer olmayan yapı lineer sayı örüntüsü için, 13 lineer yapı ve 20 lineer olmayan yapı quadratik örüntüler için üretmişlerdir. Örüntü oluşturma sırasında katılımcılar daire, üçgen ve kare gibi geometrik şekiller kullanmayı tercih etmişlerdir. Ayrıca, katılımcıların 3, 5, 7, 9, 11,.. sayı örüntüsü için oluşturdukları şekiller, 2, 6, 12, 20, 30,..sayı örüntüsü için oluşturdukları şekillerden daha fazladır. Katılımcılardan bazıları şekil örüntüsü oluştururken örüntü olmayan yapılar oluşturma, yanıt verememe, ilgisiz örüntü oluşturma ve örüntüyü devam ettirme gibi sorunlar yaşamışlardır.
Anahtar Kelimler: Sayı örüntüsü, şekil örüntüsü, lineer örüntü, lineer olmayan örüntü, örüntü yapıları

Introduction

Pattern is one of the most important topics in mathematics education. There are many definitions of pattern in the literature. Souviney (1994), for example, defined a pattern as a systematic configuration of geometric figures, sounds, symbols, and actions. McGarvey (2012) defined a pattern as an act of perceiving or imposing structural regularities on physical, behavioural, visual, or symbolic phenomena. A mathematical pattern may be described as any predictable regularity, usually involving numerical, spatial or logical relationships (Mulligan & Mitchelmore, 2009).

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Previous studies have shown many different kinds of pattern types referring to the type of representation system within which the terms are expressed as numerical or figural/geometric forms. Stacey (1989) classified patterns as linear and quadratic patterns according to their nth terms expressed as an+b and an^2+bn+c, respectively, while Orton and Orton (1999) indicated that, for number patterns, if constant differences first appear in the first difference row, then the rule is linear an+b, if it is the second difference row where constants appear, the rule is quadratic an^2+bn+c. Although, 2,4,6,8,10,11,12,14, numbers does not represent a pattern, 2,4,6,8,10,12,14, numbers having a systematic configuration of numbers represent a pattern. For example 2,4,6,8,10,. number pattern is a linear pattern and 2,6,12,20, 30,.is a non-linear. Smith (1997) indicated that patterns can be numerical (involving numbers) or non-numerical (involving shapes, sounds, or other attributes such as colour and position), while Lin, Yang and Chen (2004) classified geometric patterns as linear and quadratic patterns.

Bishop (2000, p.10) indicated that “a number pattern is a sequence of numbers in which there is a well-defined rule for calculating each number from the previous numbers or from its position in the sequence. In a geometric number pattern, the numbers relate to a sequence of geometrical figures in which each figure is derived from the previous figure by some well-defined procedure.”

In some studies, patterns are classified as repeating or growing (Cathcart, Pothier, Vance & Bezuk, 2003; Mulligan & Mitchelmore, 2009; Reys, Suydam, Lindquist & Smith, 1998; Van De Walle, 2004; Warren & Cooper, 2006). For example, Zazkis and Liljedahl (2002) classified patterns into numerical patterns, pictorial/geometric patterns, patterns in computational procedures, linear and quadratic patterns, and repeating patterns. Repeating patterns have a recognisable repeating cycle of elements, referred to as the ‘unit of repeat’ (Zazkis & Liljedahl, 2002). This kind of pattern can have one attribute such as the colour, size, shape, or orientation of objects (Threlfall, 1999). The followings are examples of repeating patterns: alphabetic letters such as A-B-A-B-A-B, and actions such as stand, sit, stand, sit, stand, sit, stand, sit (Warren & Cooper, 2006). Growing patterns change over time (Cathcart et al., 2003) and involve a progression from step to step. Moreover, those patterns are called sequences (Van De Walle, 2004). Growing patterns may be linear such as Y B B Y B B B B Y B B B B B B (in this example, only B’s are growing) (Reys et al., 1998) and 2, 4, 6, (Cathcart et al., 2003) or quadratic such as 1, 4, 9, 16,.number pattern. Repeating patterns are particularly important since they recur in measurement (which involves the iteration of identical spatial units) and multiplication (which involves the iteration of identical numerical units) (Mulligan & Mitchelmore, 2009).

There are many benefits of the pattern activities for students. Pattern activities such as identifying and extending patterns are an important process in algebraic thinking (Van De Walle, 2004) and patterns can also contribute to the development of functional thinking (Souviney, 1994; Van De Walle, 2004; Warren & Cooper, 2006), making generalisations (Cathcart et al., 2003; Rivera and Becker, 2007;Threlfall, 1999), seeing relationships and making connections (Cathcart et al., 2003), problem solving (Bassarear, 2008; Cathcart et al., 2003, Reys et al., 1998), counting (Bassarear, 2008; Frobisher & Threlfall, 1999), using number systems (Frobisher & Threlfall, 1999) and arithmetical operations (Bassarear, 2008; Frobisher& Threlfall, 1999).

Fox (2005) asserted that studying patterns is closely connected to mathematical content areas such as numbers, geometry, measurement, and data. Moreover, Göktepe and Özdemir (2013) indicated that pattern is related to spatial reasoning skills. Patterns are found in arithmetic and geometric sequences problems, as well as in various real situations. Patterns can also be seen in spoken and written words, musical forms and video images, ornamental designs, natural geometry, traffic, and the objects that we create (Reys et al., 1998). Souviney (1994) indicated that patterns can be based on geometric attributes (shapes, regions, and angle), measurement attributes (colour, texture, length, weight, volume, number), relational attributes (proportion, sequence, functions), and affective attributes (values, likes, dislikes, familiarity, heritage, culture).

In Turkey, patterns have been incorporated into the numbers and geometry learning areas of Middle School Mathematics Curriculum since 2005. In these learning areas, students are expected to recognise, identify, describe, and extend repeating and growing patterns and find algebraic rules. In New Middle School Mathematics Curriculum (5^{th} grade to 8^{th} grade), students are expected to extend number and figural patterns and find algebraic rules of patterns (MoNE, 2013).

In the literature, in particular, studies of pattern conducted with especially middle school students have determined their pattern generalization (Akkan, 2013; Amit &Neria, 2008; Bishop, 2002; Jurdak & Mouhayar, 2014; Lannin, 2005; Rivera &Becker, 2008;Walkowiak, 2014) and representation types used in pattern activities (Akkan, 2013; Amit &Neria, 2008; Steele, 2008; Walkowiak, 2014). Bishop (2002), for example, focused on the capacity of students to generalize relationships and the strategies they use to answer questions about linear geometric number patterns. Lannin (2005) indicated that students used different strategies to generalize figural patterns. Rivera and Becker (2008) investigated that the content and structure of generalization involving figural patterns of middle school students and found three forms of generalization involving such constructive standard, constructive nonstandard and deconstructive. Amit and Neria (2008) focused on the generalization methods used by talented pre-algebra students in solving linear and non-linear pattern problems and the students showed mental flexibility, shifting smoothly between pictorial, verbal and numerical representations and abandoning additive solution approaches in favor of more effective multiplicative strategies. Steele (2008) demonstrated that students used different external representations such as drawing diagrams, creating tables, writing verbal generalizations, and constructing generalized symbolic expressions for pictorial growth and change problems. Akkan (2013) aimed to compare and determine 6th-8th graders' efficiencies, strategies and representations of students from different grades (from 6th to 8th) when dealing with problems related to linear and quadratic patterns. According to the results of that study when grade increases, students' efficiencies of generalizing pattern improve in a positive way in all levels and the variety in pattern generalization strategies changed at least in all types of patterns. Moreover, the students of high levels use many types of representation in generalization. Jurdak and Mouhayar (2014) investigated students' (from grades 4 to 11) level of reasoning in pattern generalization tasks. Results show that student level of reasoning exhibited an increasing trend across clusters of grade levels. Type of task (immediate, near, far) and function type (linear, non-linear) seem to mediate the development of level of reasoning across grade level. Walkowiak (2014) asked to students to describe, extend, and generalize pictorial growth patterns and found that their generalizations included informal notation, descriptive words, and formal notation.

In summary, while a number of studies have investigated middle school students' generalization figural or number patterns, analyzing structures of the generated figural patterns based on number patterns requiring knowledge of geometry and spatial reasoning is not studied at all. However, given that structures of generated figural patterns is important for especially measuring middle school students' pattern knowledge, geometry knowledge and their spatial reasoning skills, this present study aimed to fill that gap in the literature. Considering pattern activities are an important process in algebraic thinking and patterns can also contribute to the development of functional thinking and some mathematical skills such as making generalisations, seeing relationships and making connections, problem solving, counting and performing arithmetical operations, it is important to include different pattern activities into Middle School Mathematics Curriculum. For that reason, as a result of this current study, it is aimed to offer some suggestions to the curriculum developers because of the scope of the Middle School Mathematics Curriculum regarding pattern activities.

In this study, the following research questions were addressed:

- What type of figural pattern structures were generated by middle school students based on linear and quadratic number patterns?
- What kind of problems they encountered while creating figural patterns?

Method

In this study, descriptive method was used to collect data. The reason for applying descriptive method was to understand structures of figural patterns produced by middle school students based on number patterns. In the study, a pattern task consisting of two questions (one linear number pattern and one quadratic number pattern) was carried out by all participants**.** The task allowed the participants to produce different figural patterns related to number patterns.

Participants

474 eight-grade students from 4 different schools in Turkey participated in this study in 2014-2015 school years. Of these participants, 254 were girls and 220 were boys and their ages ranged between 14 and 15 years. The reason for choosing eight grades was that all participants already had basic knowledge of pattern since they started to study pattern activities from first grades.

Data collection

Data were collected through a pattern task consisting of one linear number pattern and one non-linear (quadratic) number pattern in a written form. Participants were asked to generate figural patterns individually considering those number patterns. The task is described as below:

Question 1: Could you generate a figural pattern of first five stages regarding the 3,5,7,9,11, number pattern?

Question 2: Could you generate a figural pattern of first five stages regarding the 2,6,12,20,30, number pattern?

Those questions require knowledge on pattern structures that grow according to the number of objects in each stage. To confirm the suitability of this number pattern task, the opinions of a relevant mathematics educator were considered. The educator indicated that the task used in this study was suitable for middle school students. This task was chosen because it allowed us to assess how participants generated figural patterns based on number patterns. After getting opinions of mathematics educator, a pilot study was performed with a similar group consisting of 60 eight grade students. The pilot study aimed to assess its feasibility in terms of language, difficulty, time and number of the questions. In the pilot study, similar group of the participants was given 50 minutes for a number pattern task and two questions were asked. In pilot study, the following questions were addressed;

Question 1: Could you generate a figural pattern of first five stages regarding the 3,5,7,9,11, number pattern?

Question 2: Could you generate a figural pattern of first five stages regarding the 2,6,12,20,30, number pattern?

At the end of the pilot study, those two questions were decided to be asked to the participants and 45 minutes were given for answering the task.

Data analysis

The data obtained from the study were analysed at two levels: (i) semantic analysis and (ii) descriptive analysis. In the semantic analysis, the figural patterns or situations generated by the participants were analysed in accordance with the linear or non-linear number pattern to assess their performance to generate figural patterns. The produced figural patterns were first listed and classified according to their semantic structures. Generated patterns or situations were then coded as linear or quadratic forms and issues were noted. After the semantic analysis of the generated patterns, their frequencies were calculated. The descriptive analysis then provided descriptive information to offer an overall picture of the figural patterns generated by the participants.

In this study, analyzing the structures of the generated figural patterns refers to arranging figures either linear form or non-linear form. In a linear form, figures are arranged either vertical or horizontal in a space following the same sequence in a manner such as

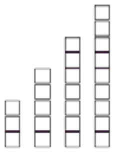
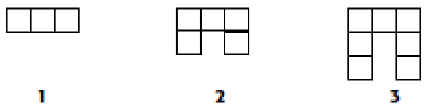
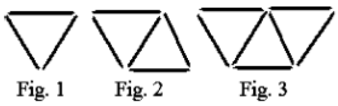
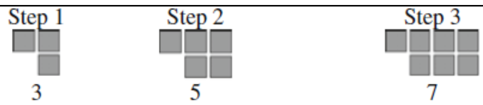
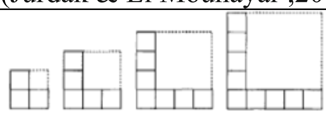
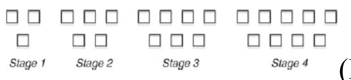
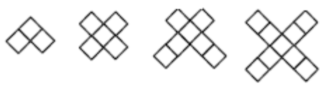
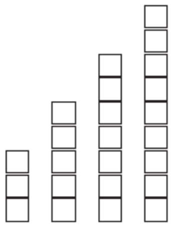
, in a non-linear form, figures are not arranged in a space following the same sequence in a manner such as . The figural representations of the 3,5,7,9,11, and 2,6,12,20,30, number patterns were coded as types (Type 1, Type 2, etc.) separately to analyse the data obtained from the study effectively. Table 1 and Table 2 provide the pattern types related to for both number patterns seen in the literature. In the results section, besides those pattern types from the literature, some of the new pattern types created by the participants for the number patterns are presented.

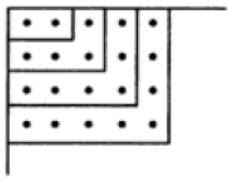
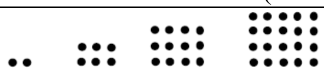
Table 1. *Examples of Figural Representations Of The 3,5,7,9,11, Number Pattern From The Literature*

Types	Structures	Characteristics
Type1	 (Wickett et al., 2002).	In Type 1, three squares ares used in first term of the pattern and pattern is extended adding two squares into corners.
Type 2	 (Radford, 2008).	In Type 2, first term of the pattern is a triangle and has been constructed using three toothpicks and for constructing next terms two toothpicks were added to former one.
Type 3	 (Jurdak & El Mouhayar ,2014).	In Type 3, pattern has been represented by squares.
Type 4	 (Waring et al., 1999).	L- shapes type pattern has been represented by squares reminding L.
Type 5	 (Radford, 2008).	In Type 5, pattern has been represented by circles having two rows of circles.
Type 6	 (Radford, 2010).	In Type 6 each stage has two rows of squares, a top row and a bottom row.
Type 7	 (Warren, 2005).	In Type 7, pattern has been represented by squares.

Type 8	 (Gregg, 2002).	In that Type, pattern has been constructed of circles like V model.
Type 9	 (Gregg, 2002).	In Type 9, pattern has been constructed by squares in a vertical order.

As shown in Table 1, the figural pattern examples given for 3,5,7,9,11, number pattern have different arrangements and shapes (e.g. square, circle, line). However, while, for instance, Types 4,6,7 and 9 use the same geometric shapes, the construction of those patterns are different from each other because of their arrangement.

Table 2. *Examples of figural representations of 2,6,12,20,30, number pattern from the literature*

Types	Structures	Characteristics
Type1	 (Van de Walle, 2004).	That pattern has been represented by circles representing a rectangular.
Type 2	 (Orton et al., 1999).	In overlapping rectangles pattern, the first stage contains 2, second 6, third 12 dots, etc.
Type 3	 (Vogel, 2005).	In Type 3 sequence of rectangular numbers are presented as an array of dot patterns

As shown in Table 2, the representation types of 2,6,12,20,30, seem to differ in terms of arrangement. Geometric figures such as lines, dots and circles are used to represent those types of patterns. Although same geometric shapes have been used for the Type 2 and Type 3, the construction of those patterns differs because of the arrangement.

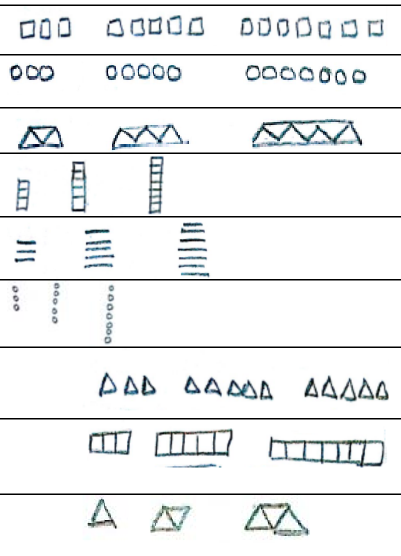
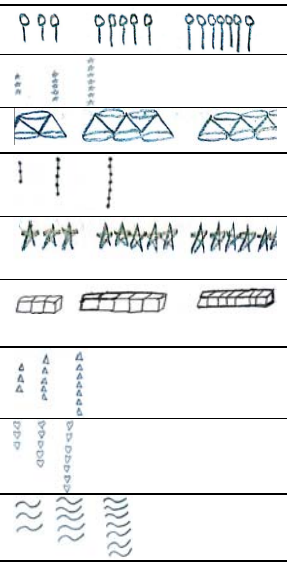
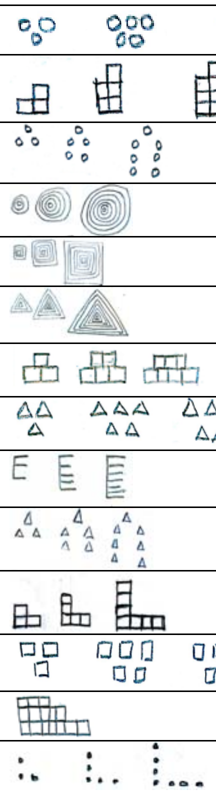
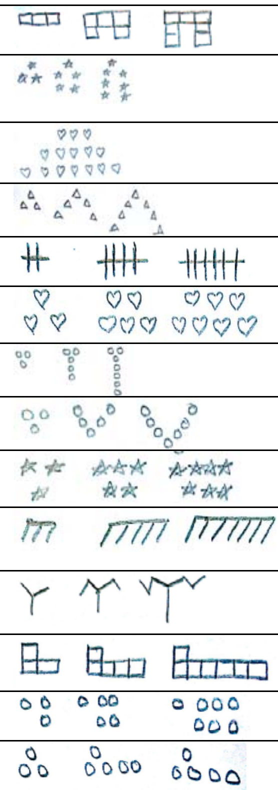
Validity and Reliability

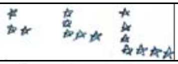
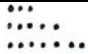
In this study, the researcher asked for the opinion and assessment of one colleague who was blind to the data and unbiased regarding the code list and research findings. In order to examine inter-ratter reliability and increase the reliability of the results, another colleague who has a mathematics education background independently classified the generated figural patterns. The formula of Miles and Huberman (1994) was used to calculate inter-ratter reliability and this was determined to be 95% for question 1 and %94 for question two. The pilot study also contributed to the validity and reliability of the pattern task.In that study all the participants were informed about the aim of the research and asked to attend the study.

Results

The figural patterns generated by middle school students based on linear and non-linear number patterns and their frequencies are presented in the tables in below. While Table 3 indicates the responses of participants related to the 3,5,7,9,11, number pattern, Table 4 shows the responses related to the 2,6,12,20,30,.number pattern based on the structures of responses. Responses were given from high frequency to low frequency.

Table 3. *Figural Pattern Structures Related to the 3,5,7,9,11,.Number Pattern*

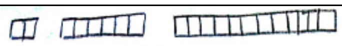
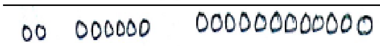
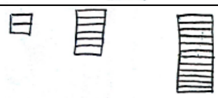
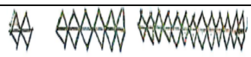
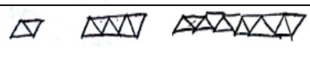
Structures of patterns	Pattern types	f	Pattern types	f
Linear forms		52		2
		36		1
		17		1
		16		1
		14		1
		11		1
		9		1
		7		1
		4		1
Total				176
Non-linear forms		26		1
		9		1
		8		1
		8		1
		6		1
		5		1
		3		1
		3		1
		2		1
		2		1
		2		1
		2		1
		2		1
		1		1

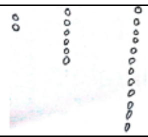
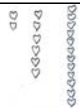
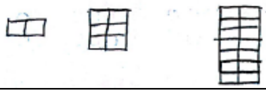
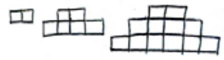
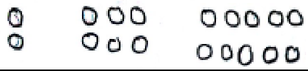
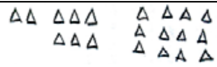
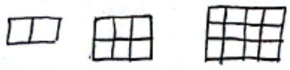
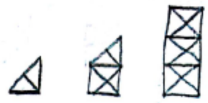
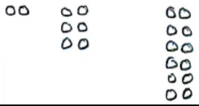
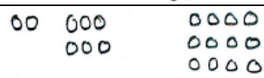
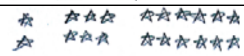
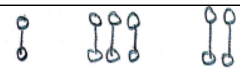
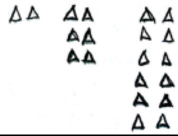
		1		1
		1		1
		1		
Total				98

The participants created 51 different figural patterns for the 3, 5, 7, 9, 11, number pattern using squares, circles, lines, triangles, stars, hearts, prisms, and curves. While 49 patterns were created using only one shape, two figural patterns were created by combining two figures by the participants. Of those pattern examples, eight emerged in the literature, but 43 examples were not; also, Type 7 found in the literature was not produced by any of the participants. For 3, 5, 7, 9, 11, number pattern 18 linear and 33 non-linear forms were created by the participants. 176 participants created linear forms for linear pattern. 52 participants created a figural pattern consisting of squares like a linear model and 36 participants used circles to create a linear form. 17 students used a triangle to create a figural pattern by adding two triangles to each other in every step like a linear form. While 16 participants preferred to create a vertical linear model using squares (Type 9, see in Table 1), 14 of them used lines and 11 preferred squares for same model. 9 participants used triangles to create a linear model and 7 participants used squares while creating a linear model. 4 participants used lines like a linear triangle model (Type 2, see in Table 1) and 2 participants used circles and lines. In this study 8 figural patterns representing linear forms were created by the participants using different geometric structures (circles, squares, prisms, triangles, lines, cubes, curves). Type 1 and Type 8 indicated in Table 1 were created only by one participant.

98 participants produced non-linear forms for linear number pattern. Among non-linear forms, 26 of the participants produced a figural pattern using circles by decomposing figures not being a linear form (upside down form of Type 5, see in Table 1). 9 participants used squares (Rotated 90 degrees to the right form of Type 3) and 8 participants used circles for the vertical model. Other 8 of the participants used circles like intertwined rings. 6 participants used squares and 5 participants used triangles for creating figural patterns like nested models. 3 participants used squares or triangles to produce a figural pattern by decomposing figures. Another 2 participants used lines or triangles and 2 more participants used squares to create a figural pattern (Type 4, see in Table 1). 2 participants used squares to create figural patterns like L-shapes (Type 4, see in Table 1) and 2 participants used squares(Type 6, see in Table1).12 figural patterns were created by the participants using different geometric structures (circles, squares, , lines, stars) or non-geometric structure (hearts).

Table 4. *Figural Pattern Structures Related to the 2,6,12,20,30,.Number Pattern*

Structures of patterns	Pattern types	f	Pattern types	f
Linear forms		45		10
		30		4
		24		3
		13		3
		12		1

		11		1
				1
Total				158
Non-linear forms		16		1
		13		1
		13		1
		7		1
		6		1
		2		1
		2		1
		2		1
		2		1
		1		1
Total				74

For 2, 6, 12, 20, 30,..quadratic number pattern 13 linear forms and 20 non-linear forms were produced by the participants using squares, circles, lines, triangles, rectangles, stars and hearts. 158 participants generated linear forms for quadratic number pattern. 45 of the participants used squares and 30 used circles to create a horizontal linear form for creating figural patterns. 24 participants used lines and 13 used rectangles and 11 used circles to create a vertical linear form. 12 participants used triangles to create a linear form in an incoherent format and the other 10 participants used triangles to ensure that the shapes did not touch each other. Vertical lines were used by 4 participants and horizontal lines by 3 participants. 3 participants created a figural pattern as the bases of the triangles attached to one another. Two participants used triangles and one preferred hearts for creating a figural pattern.

74 participants created non-linear forms for quadratic number pattern. 16 participants used rectangulars and 13 participants used circles or squares to create figural patterns. 7 participants used circles to create a vertical figural pattern and 6 participants created a Type 1 figural pattern. 2 participants used dots and 2 preferred stars to create a figural pattern. 2 participants used triangles, squares or stars to create a figural pattern. Among those patterns,

only one was created by using both circles and lines; the others were created by using only one type of shape. The remaining 11 figural patterns were created by the participants using lines, squares, triangles, stars and rectangles.

Table 5. *Issues that Emerged While Creating Figural Patterns*

Pattern types	Issues				
	Producing non pattern forms	No answer	Irrelevant pattern	Extending the number patterns	Total
3,5,7,9,11,..	184	9	1	6	200
2,6,12,20,30,..	211	30	1	0	242

While producing figural patterns for both pattern types, some problems emerged in the study. Some participants had difficulties during the study. The frequencies of these issues and their structures are given in Table 5**.** For example, producing non pattern forms, providing no answer and providing an irrelevant pattern were common issues for both pattern types. Further, the issue of extending the number patterns emerged only when producing the figural pattern for the 3, 5, 7, 9, 11, number pattern.

Discussion and Conclusion

Pattern activities are very important activities for both educators and students. Pattern activities contribute to the development of algebraic and functional thinking, problem solving, counting and arithmetical operations. In the literature, many pattern studies have focused on students' generalization of figural or number patterns (Akkan, 2013; Amit & Neria, 2008; Bishop, 2002; Jurdak & Mouhayar, 2014; Lannin, 2005; Rivera & Becker, 2008; Steele, 2008, Walkowiak, 2014), whereas studies of analyzing structures of figural patterns were generated by middle school students following number patterns are scarce. Considering structure of generated figural patterns is important for especially to measure participants' knowledge of the subject areas of mathematics such as algebra, geometry and number and knowledge of pattern, this study aimed to analyze structures of figural patterns which were generated by middle school students based on growing number patterns.

Data obtained from the current study indicated that different figural patterns were generated by the participants according to both types of growing number patterns. Most of the students generated different types of figural patterns that followed the given linear number pattern and for the non-linear number pattern; they used different figures (either geometric or non-geometric forms). Figural patterns were created by using not only geometric structures such as squares, lines, circles, triangles, prisms, stars and curves but also non-geometric structure such as hearts.. Participants placed those structures either vertical, horizontal or both of them during the generating activity for the 3,5,7,9,11, and 2,6,12,20,30, number patterns. Although, in the study different non-linear forms emerged for 3,5,7,9,11, and 2,6,12,2,0,30,.number patterns, the frequency is high for linear forms for both types of patterns. During the study, participants created many more figural patterns for the 3,5,7,9,11, number pattern than for the 2,6,12,20,30, number pattern. Although, in the study different non-linear forms emerged for 3,5,7,9,11, and 2,6,12,2,0,30,.number patterns, the frequency is high for linear forms for both types of patterns. Most of the figural patterns created by the participants using geometric shapes are predominantly circles, triangles and squares. In the study participants created all pattern types except Type 7 placed in Table 1 for 3,5,7,9,11, number pattern and Type 1 placed in Table 2 for 2,6,12,20,30, number pattern. The performance of participants here can be related to their geometry thinking ability. For that reason students should be encouraged to use both different geometric forms and non-geometric forms while creating figural patterns based on the number patterns. Furthermore, students should be encouraged to use more than one figure while creating figural patterns.

In this study, participants encountered problems during producing figural patterns based on the number patterns. Some of the participants could not create any figural patterns. Although they used either geometrical or non-geometrical structures for number patterns during generating figural patterns, they did not place those structures correctly. While most chose the correct figures, they were not aware that figural patterns are formed of objects that convey positions in a structural relationship and resemble each other in some way. They did not recognize the regularities in the forms that they produced. Because pattern activities do not include generating figural patterns based on number patterns in the mathematics curriculums in Turkey, this lack of familiarity with the topic may affect their pattern-generating performance and also Fox (2005) found that patterns are closely connected to mathematical content areas such as numbers, geometry, measurement, and data, which can be affected by participant performance. Furthermore, participants had issues while creating figural patterns for the 2,6,12,20,30, number pattern much more than for the 3,5,7,9,11, number pattern. It can be concluded that the type of pattern affected participants' pattern generating performance. For that reason, in order to develop middle school students' figural pattern generating performance, different pattern generating activities including different pattern types should be included in the activities.

In future studies, students should be motivated to produce different figural patterns based on the number patterns using different geometrical structures including two dimensional and three dimensional shapes (linear or non-linear forms) or non-geometrical structures. Those structures can be vertical, horizontal or mixed. Then, different figural patterns regarding the number patterns in the literature should be taught to extend their pattern knowledge. Furthermore, the findings of this study indicated that participants were more successful at creating figural patterns according to a linear number pattern compared with a non-linear (quadratic) number pattern. The reasons why they produce linear and non-linear forms for both pattern types can be investigated. Most participants tried to use geometric shapes to create figural patterns. For that reason, future correlation studies should assess attitudes towards geometry. In addition, other variables related to learners' performance such as spatial reasoning and mathematical thinking style might have influenced their performance. In order to understand participants' performance more in depth, correlation studies and mixed method research may be conducted. Moreover, students can be given the repeating number patterns being another type of patterns and be asked to create figural patterns considering those repeating number patterns and find the structures of the generated figural patterns. Moreover, strategies that participants used while creating figural patterns, the processes and the reasons can be investigated. Creating figural pattern activities based on number patterns should be integrated into the middle school mathematics curriculums. Similar studies can be conducted with teachers to measure their performance for generating figural patterns following number patterns.

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Uzun Öz

Giriş

Örüntü, matematik eğitiminde yer alan önemli konulardan biridir. Örüntünün tanımının ne olduğuna bakıldığında, alanyazında farklı tanımlara rastlamak mümkündür. Örneğin, Souviney (1994) örüntüyü geometrik şekiller, sesler, semboller ve eylemlerin sistematik bir yapılandırması olarak tanımlarken, matematiksel bir örüntünün desen genellikle, sayısal mekansal ya da mantıksal ilişkileri içeren herhangi öngörülebilir düzen olarak tarif edilebileceği de belirtilmiştir (Mulligan ve Mitchelmore, 2009). Örüntüleri sınıflandıran çeşitli araştırmacılar da bulunmaktadır. Bu araştırmacılardan, Stacey (1989) örüntüyü n.inci terimin ifade ediliş biçimine göre $an+b$ lineer ve an^2+bn+c quadratic olarak sınıflamıştır. Smith (1997) ise örüntüyü sayısal ve sayısal olmayan biçiminde ikiye ayırmıştır. Bazı çalışmalarda örüntüler tekrarlayan yada genişleyen biçiminde sınıflandırılmıştır (Cathcart ve diğerleri, 2003; Mulligan ve Mitchelmore, 2009; Reys ve diğerleri, 1998; Van De Walle, 2004; Warren ve Cooper, 2006). Tekrarlayan örüntüye, A-B-A-B-A-B, yada ▼●▼●▼● örnek olarak verilebilinirken (Warren ve Cooper, 2006), genişleyen örüntüye de Y B B Y B B B B Y B B B B B B (bu örnekte B genişliyor) (Reys vd., 1998) ve 2,4,6, (Cathcart vd., 2003) verilebilinir.

Zazkis ve Liljedahl (2002) örüntünün matematiğin kalbi ve ruhu olduğunu ifade etmektedirler. Örüntü etkinliklerinin öğrencilere pek çok yararı bulunmaktadır. Örüntü etkinlikleri sayesinde öğrenciler, cebirsel etkinliklere dahil olur ve cebiri öğrenirler, fonsiyonel düşünmeleri gelişir ve genelleme yapabilirler (Souviney, 1994; Van De Walle, 2004; Warren ve Cooper, 2006). Örüntülerle çalışmanın sayılar, geometri, ölçme ve veri konuları ile yakından ilişkili olup (Fox, 2005), uzamsal yetenekler ile de ilgili olduğu belirtilmektedir (Göktepe ve Özdemir, 2013). Örüntüler, aritmetik ve geometrik dizi problemlerinde ve günlük yaşam durumlarında, sözlü ve yazılı kelimelerde, müzik yapılarında ve video görsellerinde, doğal geometride, trafikte ve bizlerin oluşturduğu diğer nesnelerde karşımıza çıkmaktadırlar (Reys, Suydam, Lindquist ve Smith,1998).).

Türkiye'de örüntüler konusuna 2005 yılında sayılar ve geometri öğrenme alanlarında yer verilmeye başlanmıştır. 2013 yılında yenilenen ortaokul matematik dersi öğretim programında ise örüntüler konusu ile ilgili olarak öğrencilerden, sayı ve şekil örüntülerini genişletme ve örüntülerin cebirsel kurallarını bulmaları beklenmektedir.

Alanyazına bakıldığında, ortaokul öğrencileri ile yapılan örüntü ile ilgili çalışmaların öğrencilerin örüntüleri genellemeleri (Akkan, 2013; Amit ve Neria, 2008; Bishop, 2002; Jurdak ve Mouhayar, 2014; Lannin, 2005; Rivera ve Becker, 2008; Walkowiak, 2014) ve örüntü etkinliklerinde kullandıkları temsiller üzerine odaklandığı görülmektedir (Akkan, 2013; Amit ve Neria, 2008; Steele, 2008; Walkowiak, 2014). Örüntülerle ilgili yapılan çalışmalarda sayı örüntülerine dayalı olarak şekil örüntüsü oluşturma çalışmalarına yer verilmediği göze çarpmaktadır. Sayı örüntülerine dayalı olarak şekil örüntüsü oluşturma etkinlikleri, bize öğrencilerin geometri, sayı ve cebir ve örüntüler konusundaki bilgilerini ortaya çıkarma konusunda yardımcı olacaktır. Bu nedenle bu çalışma ile alandaki bu boşluk doldurulmak istenmektedir. Bu çalışmada aşağıdaki sorulara yanıt aranmıştır;

1.Öğrencilerin lineer ve lineer olmayan sayı örüntülerine dayalı olarak oluşturdukları şekil örüntülerinin yapısal özellikleri nasıldır?

2.Öğrenciler şekil örüntülerini oluştururken ne tür sorunlarla karşılaşmışlardır?

Yöntem

Bu çalışma betimsel bir çalışma olup, çalışma 2014-2015 eğitim öğretim yılında gerçekleştirilmiştir. Araştırmaya, ortaokul 8. sınıfa devam eden toplam 474 öğrenci (254 kız ve 220 erkek) katılmıştır. Öğrencilerin yaşları 14 ve 15 yaşları arasında değişmektedir. Çalışmaya 8. sınıf öğrencilerinin seçilmesinin nedeni, bu öğrencilerin öğrenim yaşantıları boyunca tekrarlayan ve genişleyen sayı ve şekil örüntülerini tanıma, açıklama ve devam ettirme ve örüntülerin cebirsel kurallarını bulma gibi çalışmalara yer vermiş olmaları ve dolayısıyla örüntüler konusuyla ilgili temel bilgilere sahip olmalarıdır. Çalışmada öğrencilere lineer sayı örüntüsü olan 3,5,7,9,11,. ve lineer olmayan sayı örüntüsü olan 2,6,12,20,30,.sayı örüntüleri verilmiş olup, bu örüntülere dayalı olarak şekil örüntüleri oluşturmaları istenmiştir. Araştırmadan elde edilen veriler, ilk olarak semantik olarak analiz edilmiş ve daha sonra da bu veriler betimsel olarak analiz edilmiştir.

Bulgular

Araştırmadan elde edilen bulgulara bakıldığında, öğrencilerin 3, 5, 7, 9, 11, lineer sayı örüntüsüne yönelik olarak 18 farklı lineer ve 33 lineer olmayan yapı oluşturdukları belirlenmiştir. 2,6,12,20,30, sayı örüntüsü için ise 13 farklı lineer ve 20 lineer olmayan yapı oluşturdukları saptanmıştır. Çalışmada 3,5,7,9,11, sayı örüntüsü için 176 katılımcı kare, daire, üçgen, yıldız çizgi gibi geometrik modeller kullanarak lineer yapı oluştururken, 98 katılımcı da hem geometrik hem de geometrik olmayan modelleri kullanarak lineer olmayan yapılar oluşturmuşlardır. 2,6,12,20,30,.. sayı örüntüsü için 158 katılımcı üçgen, kare, daire ve çizgi gibi geometrik modeller kullanarak ve kalp gibi geometrik olmayan model kullanarak lineer yapılar oluştururken, 74 katılımcı da kare, üçgen, daire, yıldız ve çizgi gibi geometrik yapılar kullanarak lineer olmayan yapılar oluşturmuşlardır. Her iki örüntü türünü oluştururken katılımcılardan bazıları bir takım sorunlarla karşılaşmışlardır. Bu sorunlar; yanıt verememe, örüntü olmama, ilgisiz örüntü oluşturma ve örüntüyü devam ettirme sorunlarıdır.

Tartışma ve Sonuç

Araştırmadan elde edilen sonuçlara bakıldığında, öğrencilerin kendilerine verilen her iki sayı örüntüsüne yönelik farklı şekil örüntüleri oluşturdukları saptanmıştır. Öğrencilerin oluşturdukları örüntülerin bazıları Tablo 1 ve Tablo 2’de yer alan örüntülerdir. 3,5,7,9,11, ve 2,6,12,2,0,30, sayı örüntülerine yönelik olarak oluşturulan lineer olmayan yapılar daha çok çeşitlilik göstermesine rağmen, her iki örüntü türü için de lineer olan yapıların frekansı daha yüksektir. Bunun yanı sıra, öğrencilerin lineer sayı örüntüsüne yönelik şekil örüntüsü oluşturmada daha başarılı oldukları da saptanmıştır. Öğrenciler lineer ve lineer olmayan sayı örüntülerine yönelik şekil örüntüleri oluşturulurken kare, üçgen, çizgi, prizma, yıldız gibi geometrik yapıların yanı sıra, kalp gibi geometrik olmayan yapı da kullandıkları görülmektedir. Öğrenciler ağırlıklı olarak kare, üçgen ve daire gibi geometrik yapılara yer vermişlerdir. Bu durum onların geometriye olan yatkınlıkları ve geometrik şekil bilgileri ile açıklanabilir. Bu nedenle öğrenciler, bu tür etkinliklerde farklı geometrik şekiller kullanmaları konusunda cesaretlendirilmelidirler.

Sayı örüntüsüne bağlı olarak şekil örüntüsü oluşturmada öğrencilerin bir takım sorunlar yaşadıkları belirlenmiştir. Bu durum onların daha önceden böyle bir çalışma ile karşılaşmamış olmaları ile açıklanabileceği gibi, onların örüntü bilgileri, uzamsal düşünmeleri ve geometrik düşünmeleri ile de ilgili olabileceği de düşünülmektedir. İleride yapılacak olan bu çalışmaya benzer çalışmalarda öğrencilere farklı sayı örüntüleri verilip farklı şekil örüntülerini oluşturmaları konusunda motive edilmelidirler. Öğrencilerin şekil örüntüsü oluştururken ne tür stratejiler kullandıkları ve nasıl bir süreç yaşadıkları da ayrıca bir araştırma konusu olarak ele alınabilir. Bu çalışmada genişleyen sayı örüntülerine dayalı olarak öğrencilerden şekil örüntüsü oluşturmaları istenmiştir, Ancak, daha sonra bu konu ile ilgili yapılacak olan çalışmalarda, öğrencilere tekrarlayan sayı örüntüleri verilerek öğrencilerden bu sayı örüntülerine bağlı olarak şekil örüntüleri oluşturmaları da istenebilir. Benzer çalışma öğretmenlerle de yapılabilir.