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The Investigation of Pre-service Elementary Mathematics Teachers' Subject Matter Knowledge About Probability

İlköğretim Matematik Öğretmeni Adaylarının Olasılık Alan Bilgilerinin İncelenmesi

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Abstract: Because of the change in the middle school mathematics curriculum in Turkey, it is necessary to examine subject matter knowledge of pre-service elementary mathematics teachers about probability. This study is significant in terms of the Turkish mathematics education literature since it contributes to the future curriculum efforts for elementary mathematics education programs. Data were collected through face-to-face interviews which were focused on explanations about basic probability concepts and an instrument which was developed for evaluating content knowledge for probability of elementary mathematics teachers. Since mathematics teachers should have both procedural and conceptual knowledge regarding the concept taught, researcher aimed to understand to what extent pre-service elementary mathematics teachers are capable of conceptual and procedural knowledge needed for probability teaching. Findings showed that the 23 participants needed to develop their conceptual knowledge regarding probability and they tended to behave computational oriented while solving probability problems which showed also their higher procedural understanding. Besides, it was concluded that they couldn't make expected connections between probability and statistics concepts.
Keywords: Probability, subject matter knowledge, conceptual understanding, procedural understanding, pre-service mathematics teachers

Öz: Türkiye'de Ortaokul matematik programında gerçekleşen değişiklikle, ilköğretim matematik öğretmenliği adaylarının olasılık konusunda alan bilgilerinin değerlendirilmesi ihtiyacı ortaya çıkmıştır. Gelecek ilköğretim matematik öğretmenliği program çalışmalarına katkı sağlayabileceği gerçeğiyle, Türk matematik eğitimi alanyazınının geliştirilmesi açısından bu çalışma önemlidir. Üçüncü ve dördüncü sınıfta okuyan ilköğretim matematik öğretmenliği öğrencileriyle yüz-yüze görüşmeler düzenlenerek temel olasılık kavramları hakkında açıklamalar istenmiştir. Görüşmenin ikinci kısmında her katılımcı olasılık alan bilgilerini değerlendiren bir teste tabi tutulmuştur. Matematik öğretmenlerinin öğretilecek konu hakkında işlemsel ve kavramsal bilgilerinin olması gerektiği gerçeğiyle, katılımcıların olasılık kavramlarını öğretmek için ne ölçüde işlemsel ve kavramsal bilgiye sahip olduklarının incelenmesi amaçlanmıştır. 23 katılımcıdan elde edilen bulgular, onların olasılık konusunda kavramsal bilgilerinin geliştirilmeye ihtiyaçları olduğunu ve olasılık problemlerini çözerken çoğunlukla hesaplamaya dayalı zihinlere sahip olduklarını ve dolayısıyla işlemsel bilgi düzeylerinin daha yüksek olduğunu göstermektedir. Aynı zamanda, katılımcıların istatistik ve olasılık konuları arasında yeterince ilişki kuramadıklarını ortaya çıkarmıştır.
Anahtar Kelimeler: Olasılık, matematik alan bilgisi, kavramsal anlama, işlemsel anlama, matematik öğretmen adayları

Introduction

Subject matter knowledge of mathematics teachers was accepted as an important component of what teachers should know in order to teach mathematics. However, what teachers' subject matter knowledge covers is not clear yet. Current discussion mostly goes on with the course requirements, grade point averages, major fields of study, as such of pre-service mathematics

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teachers (Ball, 1990). As Ball (1990) pointed out that prospective teachers' understandings, how they understand the subjects they will teach, how they know them and how they think about them, were less-focused issues by the researchers.

In revised school mathematics curriculum, which started to be instructed in middle-level schools in Turkey in September 2013, the density of probability was reduced compared to previous curriculum, and its instruction is placed into the 8th grade level only with a superficial understanding of probability, such as determining the probable cases of an event, determining the cases whose probabilities are more probable, less probable or equally likely probable, understanding that the probability of an event is between 0 and 1, and that of certain and impossible events, and computing the probability of a basic event. These can be called as 'basic concepts of probability'. Moore (1997, as cited in Biehler, Ben-Zvi, Bakker, & Makar, 2012) recommends some changes from the statistical point of view, in that of content (more key concepts, and data analysis, and less probability), pedagogy (fewer lectures, more active learning) and technology (for data analysis and simulations). So, the new curriculum could be identified a well-reflection of Moore's recommendation that it enhances more statistics and less probability while leaving the deeper conceptual knowledge to the high-school level, as compared with previous curricula with an integration of use of technology where available for teachers.

What earlier studies showed that pre-service mathematics teachers have a less comprehension of probability compared with the other learning areas of curriculum; that is, they found probability subjects difficult to teach especially because of their lack of content knowledge related with it (Quinn, 1997; Stohl, 2005). Contemporary efforts are addressing the same issue as well so that teacher education should be enhanced while giving an attention to teaching probability of mathematics teachers (Stohl, 2005; Jones & Thornton, 2005; Batanero & Díaz, 2012). Moreover, Batanero and Diaz (2012) argued that it should be different than the enhancing teaching mathematics because of the difference of mathematics and stochastic in their nature. Change in the middle school curriculum necessitates the study of examination of knowledge of pre-service elementary mathematics teachers about the highlighted subject, namely probability. Whether pre-service elementary teachers have both conceptual and procedural understandings of probability in order to teach it has been understood (Star, 2005). Therefore, this study is significant in the above needs of the Turkish mathematics education literature as well as it contributes to the consequences of curriculum efforts and will be a light for future considerations of this issue.

Since mathematics and stochastic differ in nature, consequently their way of teaching differs (Batanero & Diaz, 2012). Although there is a course named as methods of teaching mathematics in every mathematics education program in Turkish education faculties, only a few of them offers a course which was specially designed to teach methods of statistics and probability in Turkey. Therefore, this study is significant that it should be needed to investigate the subject matter knowledge of preservice mathematics teachers regarding probability and their abilities to connect relationships among stochastic concepts in order to provide a background for designing courses in order to teach specific methods of statistics and probability. Moreover, the examination of the conceptual and procedural knowledge of Turkish preservice mathematics teachers in different probability concepts might also affect the design of these courses in order to enhance mathematics teacher education in Turkey.

This study aims to investigate the subject matter knowledge of pre-service elementary mathematics teachers regarding probability through a lens of procedural and conceptual understanding. The research questions in this study are as follows: (a) To what extent are pre-service elementary mathematics teachers capable of conceptual and procedural knowledge of probability subjects held in elementary mathematics curriculum in Turkey? (b) What are the feelings of pre-service elementary mathematics teachers towards teaching probability?

Review of Related Literature

Ball (1990) approached to the procedural and conceptual understandings of prospective teachers while analyzing their subject matter knowledge through a longitudinal study performed with 252 pre-service teachers (217 elementary candidates and 35 candidates majoring mathematics) with a focus on division with fractions. She concluded that subject matter knowledge of teachers has two major dimensions. First one is substantive knowledge of mathematics that includes knowledge of concepts and procedures, understanding of underlying principles and meanings, and understanding the connections among mathematical ideas. Second dimension of subject matter knowledge is the knowledge about mathematics. Ball (1990) explained this as the 'understanding the nature of mathematical knowledge and of mathematics as a field' (p. 458).

Very-well known definitions for conceptual and procedural knowledge types were introduced by first Scheffler (1965), but expanded by Hiebert and Lefevre (1986) and Star (2005) tried to describe them in his study. Hiebert and Lefevre (1986) defined conceptual knowledge as "[it] is characterized most clearly as knowledge that is rich in relationships, like a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information" (p. 3). They also categorized the conceptual knowledge as primary and reflective. Apart from conceptual knowledge, Hiebert and Lefevre (1986) explained the procedural knowledge in two types: "one kind of procedural knowledge is a familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols; the second kind of procedural knowledge consists of rules or procedures for solving mathematical problems" (p. 7).

Star and Stylianides (2013) discriminated the views of both mathematics education and psychological research communities regarding conceptual and procedural knowledge. They emphasized that the disagreement stems from the way of handling the issue. While mathematics education community presumes conceptual and procedural knowledge in terms of qualities of knowledge; psychological research community sees them as knowledge types. Knowledge quality and knowledge type could be simply distinguished as in the following: The former one means how well something is understood, with a superficial or a deep-level understanding, for instance. However, knowledge type refers to what is known (Star & Stylianides, 2013). Based on Ball's (1990) description of subject matter knowledge, conceptual and procedural knowledge could be described as knowledge types under the subject matter knowledge which mathematics teachers should have for teaching. Consequently, "the adjectives 'conceptual' and 'procedural' demarcate what type of knowledge is being characterized. Thus, conceptual knowledge would refer to knowledge of concepts, including principles and definitions; procedural knowledge would refer to knowledge of procedures, including action sequences and algorithms used in problem solving" (Star & Stylianides, 2013, 174).

Regarding the conceptual and procedural knowledge of mathematics teachers, Ball's (1990) study could be given as an example as it showed the discrimination between them, although the subject was fractions which participants studied. Ball (1990) concluded that prospective teacher candidates either they are elementary candidates or majoring mathematics had mostly a procedural understanding since most of them saw mathematics as a body of rules and procedures and most of the participants couldn't explain the reason of a specific fact or principle. This point of view might be applicable for all subjects in elementary mathematics curricula in fact. Many teachers or teacher candidates treat mathematics as a body of rules, having only wrong or right (true or false) results and this was resulted in a computational mind (Thompson, 1984; Thompson, Philipp, Thompson, & Boyd, 1994, as cited in Stohl, 2005). As a result of this computational view regarding teaching mathematics, it can also be deduced that elementary mathematics classes mostly include rules, procedures and how to apply them while solving questions, but not the meaning of the facts or principles. Consequently, the participants of Ball's (1990) study couldn't explain the meaning of division algorithm with fractions, for instance. Hence, it could also be inferred that conceptual understanding of her participants was weak regarding division with fractions.

Probability and statistics were embraced together and were named as stochastic. Stochastic as a subject began to be treated with an increasing interest and importance for elementary level of mathematics curricula nearly 20-25 years (Stohl, 2005). However, it was already concluded that most university students and adults have little understanding about probability and they have some misconceptions about it (e.g. Fischbein & Schnarch, 1997; Konold et al., 1993; Shaughnessy, 1977, as cited in Stohl, 2005). Again, most of the studies also recommended that prospective teachers and in-service teachers (as well as teacher educators) should have an understanding of probability subjects (Stohl, 2005).

Stohl (2005) investigated the teachers with a computational orientation and she concluded that they mostly handle teaching probability with a deterministic view. This means that, teachers often see teaching probability as a use of procedures to calculate theoretical probabilities when their real-world examples are absent. This view in fact stems from the difference between the areas of mathematics. Stohl (2005) explains this difference as in the following:

> *The theoretical field of mathematics called "probability theory" has as many procedures and structures as any other field of mathematics. However, directly linking this structure (and accompanying theoretical exercises) to real situations, like rolling dice or predicting the weather, is not nearly as straightforward as in other areas of mathematics studied in school. (p. 347)*

Therefore, Stohl (2005) explained why teaching probability should be different in terms of its theoretical character which differs from that of other areas of mathematics. Regarding teachers' conceptual knowledge about probability, Stohl (2005) also emphasized the disconnection between statistics and probability subjects since probability mostly is specified as a subset of statistics and the connections between probability and data analysis or descriptive statistics were not highlighted in school mathematics.

Related with teachers' content knowledge of probability with a 22 pre-service and 12 in-service elementary teachers, Begg and Edward (1999) concluded that teachers had a weak understanding about probability concepts. The participants of this study also specified also that they had a less confidence on teaching probability rather than graphing or statistical calculations. Nicholson and Darnton (2003, as cited in Stohl, 2005) found in their study that teachers have more procedural knowledge than conceptual knowledge since they mostly tend to focus on calculations rather than trying to explain the inferences from probabilistic concepts.

On the whole, studies related with teachers' subject matter knowledge of probability summarized above showed both of its dimensions which are conceptual and procedural knowledge (Ball, 1990; Begg & Edward, 1999; Stohl, 2005). Moreover, they concluded that mathematics teachers mostly have the ability to execute procedures and calculations in probability, but not have a deeper understanding behind the probabilistic concepts and cannot make fulfilling explanations regarding them. This result mostly stems from their way of learning stochastic, their inability to connect statistical and probabilistic concepts, their lack of subject matter knowledge about probability, and their unconfident feelings about teaching probability. However, it was already established that pre-service elementary mathematics teachers should develop their understanding of stochastic; they must have both conceptual and procedural knowledge (Hiebert & Lefevre, 1986).

Methodology

This study uses qualitative approaches in order to answer its research questions and is a part of the research which aimed to investigate pre-service teachers' subject matter knowledge of both probability and statistics subjects. Here, in this part, researcher outlined methodology used in the main research. First involvement of the participants into this research was explained and secondly the interview as the main data collection tool was described below.

Participants were determined from elementary mathematics education departments in İstanbul where researcher was able to reach. Since the courses related with teaching methods were placed at the beginning of 3rd year of elementary mathematics teacher education program in Turkey, 3rd or 4th grade university students were planned to involve in this study. Researcher announced her study and the way of data collection to these students via their instructors; then, 23 participants volunteered for the study. 12 of them are 4^{th} year students and the rest are in their 3^{rd} year in the elementary mathematics teacher education program. Later, researcher made appointments with the participants according to their availability for the interview.

Data Collection

Researcher collected data through face-to-face interviews. During the interview, participants were directed some questions regarding their background education, the subjects which they think they are capable most and least regarding all grades of elementary mathematics curriculum, their teaching expectations/strategies/techniques regarding probability and statistics and the technological tools which they could use in teaching probability and statistics. Secondarily, they were posed some questions regarding basic definitions of statistics and probability. These questions were: *What does the probability of an event mean? What are certain, equally likely and impossible events? What is the measure of the probability of any event? How is the probability of an event calculated?*

At the end of the interview, participants were requested to respond to an instrument. Since interviews were audio-recorded, the participants were asked to respond it as orally. This provided coding their answers as well. The instrument was prepared through the use of questions named as Diagnostic Teacher Assessments in Mathematics and Science, and developed by CRiMSTeD- Center for research in Mathematics and Science Teacher Development at University of Louisville. These diagnostic assessment tests were generated according to subjects and aimed “(1) to describe the breadth and depth of mathematics content knowledge so that researchers and evaluators can determine teacher knowledge growth over time, the effects of particular experiences (courses, professional development) on teachers' knowledge, or relationships among teacher content knowledge, teaching practice, and student performance and (2) to describe elementary school teachers' strengths and weaknesses in mathematics knowledge so that teachers can make appropriate decisions with regard to courses or further professional development” (Center for Research in Mathematics and Science Teacher Development, 2008).

Researcher contacted with CRiMSTeD and they sent two tests regarding the subjects of probability and statistics and they gave permission to use to the researcher. Researcher then selected the items related with statistics and probability in these two tests and translated into Turkish language. The instrument involves 22 items which are open-ended questions as well as multiple-choice items. Together with the first part of the interview, each one took approximately 45-60 minutes for each participant.

Data Analysis

Collected data were transcribed verbatim and analyzed through coding techniques with the usage of qualitative data analysis techniques as Creswell (2007) outlined in his book. Before collecting data, researcher generated possible themes and codes for the data. While coding the data, researcher coded the related words or phrases as specified in the themes and codes table. At the end, data analysis was performed through the incidence of these codes. The responses of participants to the instrument were assessed through a pre-formed rubric. While multiple-choice items were assessed as correct or wrong response; open-ended items were assessed as correct, wrong or partial responses. Partial responses mean partially correct responses.

In this article, only items regarding probability were selected to analyze the research questions. There are 8 items related with probability in the test, 3 of them are open-ended and the rest are multiple-choice items. Specifically, they are related to probability of a basic event; certain, impossible and equally-likely events; theoretical and experimental probability, types of events and sample space. During the first part of the interview, participants specifically were asked questions about probability as mentioned before. The 8 questions related with probability concepts in the instrument were attached as Appendix A to the end.

The assessments of CRiMSTeD have also established high levels of reliability and validity (Bush, et al, under review, as cited in Jacobbe, 2007). Since the test was applied during the interview and requested to respond as orally, researcher had the chance to observe whether the items were understood clearly by the participants. Moreover, researcher never directed the participants to right or wrong responses. After the participant said that s/he completed the solution, researcher asked which item s/he wanted to respond. After the responses to the all items were completed, researcher asked to the participant whether they could finish the interview. These efforts provide the trustworthiness of the data collection, as well.

Findings

During data collection period, participants were directed questions related with probability terminology such as the definition of probability, definitions of certain, impossible and equally likely events and calculation of probability of an event as well as they were asked to solve the test including 8 items related with the above subjects and additionally the difference/relation of theoretical and experimental probability. 5 of the items in the test are multiple-choice (choosing 1 among 4 alternatives) and the rest are open-ended questions. Achievement ratio per each item regarding the subject asked was given in the Table 1 below.

During the interview, some of the participants defined probability as giving a method for calculation of it, it was not a complete definition, and some gave explanations with synonym words for probability. All of them knew that the measure of probability was between 0 and 1, which was another question directed through interview. They gave also complete explanations for certain and impossible events. For the definition of equally likely events, nearly half of the participants had a misconception that the probabilities of equally likely events are the same and ½. Related with the question how a probability of an event is calculated, most of the participants did not use the expected terminology, such as the word 'sample space'.

It is also worth to mention here that most of the participants have identified probability as the most troublesome topic for themselves; some said 'I know probability, but I don't know what I do in class while I am teaching it, since I don't know the logic behind it'. Most of them mentioned also that they found probability and statistics as the least known topic by themselves, and when the researcher asked the reason for that, probability was the topic which was accepted as dealing with abstract issues more with respect to other subjects in the elementary mathematics curriculum, according to responses of participants. They pointed that they learned probability without knowing in their elementary school years, like memorization. For this reason, nearly all of the participants considered the change in the curriculum related with the probability subject as meaningful and stressed that probability was early to teach in elementary school because of its abstract nature.

For the secondary data for this research, i.e. the test, evaluation of open-ended items was performed through a previously prepared rubric such that a full response means that participant talked about all the expected terminology and provided all the aspects of the topic covered in it; an incomplete response means that participant did not provide all of the expected discussion and did not make a satisfactory response as expected; a wrong response means that participant responded irrelevantly and did not mention about any of the expected aspects of the topic covered in the item. The findings were summarized based on these data as in the following table:

Table 1. *Findings Based on the Items in the instrument*

Item #	Type	Related Topic	Ratio of achievement
Item1	Multiple Choice	Impossible Event	22 of 23 are correct
Item2	Multiple Choice	Finding the probability of an event	23 of 23 are correct
Item3	Multiple Choice	Theoretical vs. Experimental Probability	17 of 23 are correct
Item4	Open-ended	Finding the probability of an event	6 of 23 made a full response, 17 of 23 responded completely wrong.
Item5	Open-ended	Sample space	12 of 23 made a full response, 4 of 23 responded wrong. 7 of 23 responded partial.
Item6	Open-ended	Theoretical vs. Experimental Probability	6 of 23 responded wrong or gave no response. 6 of 23 have responded partial. 11 of 23 made a full response.
Item7	Multiple Choice	Types of events	17 of 23 are correct.
Item8	Multiple Choice	Sample space	20 of 23 are correct.

The items 1 and 2 were analyzing the procedural knowledge related with impossible events and finding the probability of an event. While all participants responded correct to the second item, only one participant had a mistake in her response for the first item. Another item, which participants had higher achievement with respect to the others, was the last one, i.e. 20 participants responded correct to it. The results of fifth item were not resulted with similar as in the last item, although they cover the same topic. Nearly half of the participants (12 of 23) responded full, the rest answered incomplete or wrong to this item.

17 participants responded correct to the third item, which is related with the relation of theoretical and experimental probability. Similar success ratio can be seen in the sixth item, which is related with the same subject. In the sixth item, participants were directed to describe a class activity showing the difference between theoretical and experimental probability. While 15 participants responded full, the rest gave incomplete or wrong answers. Some of them had no idea about the difference between theoretical and experimental probability, some gave irrelevant examples. The participants who made a full response mostly gave the example of coin tossing, or taking a specific colored ball from a bag of different colored balls. Some of them proposed using virtual manipulations. In these class activities, teacher chose some students to make the experiment and students make this experiment as much as possible. They concluded mostly that the experimental probability for these experiments would approach to the theoretical probabilities as the number of experiments increases.

Seventh item is another item which has a higher achievement ratio among all of the items, and it was questioning the types of events, like certain events, impossible events and equally likely events. 17 of participants correctly answered to this question.

Fourth item was dealing with the predicting the catfish population in a river through two consecutive hunts, i.e. in the first hunt biologists caught 138 catfish and they marked them and in the second one, they caught 241 catfish, 16 of them are pre-marked. The condition is that 138 marked catfish intermingled freely in the river with the unmarked ones, and during the period between these two hunts, neither new catfish added nor existing catfish died. This item was the most challenging one in the test, although the related multiple-choice item had a higher achievement, most of the participants (17 of 23) answered completely wrong, only 6 of them gave a full response. There was no partial response for this item.

The findings based on the instrument tend to be similar to the findings based on interview obviously since their achievement ratios to the items regarding the types of items as conceptual or procedural knowledge for probability. There were 3 paired (one for procedural and one for conceptual) items for three subjects: sample space, finding the probability of an event and difference/relation between theoretical and experimental probability. When these pairs are compared with each other, it can be seen that achievement ratio of items for procedural knowledge are higher than their pairs for conceptual knowledge.

Discussion

The findings of this study show similar aspects mentioned in the above framework for subject matter knowledge while emphasizing the discrimination between conceptual and procedural knowledge. Based on the above explanations for procedural and conceptual knowledge, all the three of the items directed as open-ended in the test could be described as dealing with conceptual knowledge; and the rest are dealing with the procedural knowledge and all of them are multiple-choice items.

In general, it can be claimed that pre-service elementary mathematics teachers have a high achievement in procedural level of knowledge for probability subjects. They mostly know some basic definitions, such as definition of probability, types of events, definition of sample space. However, most of the participants have difficulty in answering the questions necessitating conceptual knowledge, which are related with the subjects of finding the probability of an event (catfish problem), sample space, and theoretical and experimental probability relationship. It can be claimed that the participants for this study have not an ability to connect what they know about probability and have not a higher-order comprehension needed for knowledge answering to the questions (Ball, 1988; Hiebert & Lefevre, 1986; Stohl, 2005).

Based on the findings through interviews, definition of the probability of an event was performed procedurally; most of them used the sentence such as 'it means the number of wanted events divided by number of all events' although this definition has some terminological mistakes. For example, none of them used the word 'ratio' as defining it or the term 'sample space' as Green (1987) stated as one of the conditions of having an understanding of probability conceptually. Watson (2001) also concluded that teachers felt more confident in the concept of 'average' rather than the concept of 'sample'. She explained this finding as not giving enough importance to the concept of 'sample'. Moreover, another reason could be teachers' computational orientation so that the participants in Watson's (2001) study could underestimate the importance of the conceptual understanding. In this study, the participants showed similar tendency towards not using the expected terminology. As Stohl (2005) stated before, their computational or procedural oriented minds couldn't notice the concepts.

For the definitions of certain and impossible events, all of them explained that a certain event has a probability of 1, and the probability of an impossible event is 0. Some of them provided examples for their definitions additionally and their examples were also appropriate. However, for the definition of equally likely events, nearly half of them explained that their probability is ½ and they mostly supported their explanations with the example of coin tossing, such as having a tail and having a head are equally likely events. Begg and Edward (1999) concluded in their study that some of their participants couldn't explain the equally likely events because of having a misconception related with independence of events. The participants of this study only gave an example of experiment which resulted as two different events and most of them said that the probabilities of equally likely events equal to each other and is ½. However, their success rate is much higher in the seventh item from the test. It was seen that more than half of the participants gave a correct response to this item; it was related with impossible events specifically. Therefore, overall, findings show that participants lack of conceptual knowledge about probability since they could not use the concept in different situations and they could not relate it with other concepts using higher order thinking abilities as Ball (1988) stated.

Findings related with considering probability as one of the most abstract issues in mathematics show that pre-service teachers have an understanding of probability as a subject roughly, not deeply. Although most of them used some real-world examples while giving explanations for the questions in both interview and instrument, they found probability mostly abstract and they saw that it is the thing that makes probability difficult to teach. This finding could be explained as their lack of ability to make connections among the probability and statistical concepts as Stohl (2005) suggested beforehand. This inability to understand probability and feeling inadequate in teaching probability was explained with teachers' misunderstandings about probability as being a subset of mathematics (Stohl, 2005). Therefore, it could be claimed that the reason they found probability as abstract is their lack of conceptual knowledge about probability. This finding is also remarkable in that elementary mathematics teachers found probability as abstract although mostly some other subjects were found as abstract, for instance geometry.

Third item related with difference/relation between theoretical and experimental probability is asking the correct alternative based on the results of an experiment, in which colored spinner is used. Using an elimination method among the alternatives, 17 of participants made correct decision on this item. However, the item in the instrument related with the same subject was searching for a class activity which can help the students in order to distinguish the relation between theoretical and experimental probability. Most of the participants had difficulty in describing an activity which includes specifically increasing the number of experiment. The participants responded wrong to this item, had no idea about the difference about them. Stohl (2005) identified this issue as lacking of knowledge about law of large numbers among mathematics teachers. Hence, this lack of knowledge causes to a misconception that experimental probability is approaching to theoretical probability as number of experiments increases. She explained that this is due to an incorrect interpretation of law of large numbers since experimental probability could be different than the theoretical probability although a large number of trials were made. Stohl (2005) further explained this issue as a result of misconception or lack of understanding in the concepts of limit as presented in mathematics lessons or in textbooks. Therefore, it can again be claimed that preservice elementary mathematics teachers lack of conceptual thinking, they prefer to solve procedurally, not deepening their comprehension process (Ball, 1988; Hiebert & Lefevre, 1986).

When we consider the possible reasons of why conceptual knowledge of preservice elementary mathematics teachers have been less-developed compared with procedural knowledge, the courses offered for teacher candidates during their university education are like 'recipe-type' or 'rule-bound' courses which only deal with the calculations and lead preservice teachers to memorize the subjects while underestimating the logic behind it, as Shaughnessy (1992) stressed out previously (p.466). He also claims that preservice teachers lack of opportunity to develop their stochastic reasoning in university courses with their misunderstandings about probability. Nearly half of the participants have stressed that they feel themselves not knowing very well about probability although they have taken a course namely as probability and statistics. The other half of the students mentioned that they have a course related with teaching probability and statistics in elementary level, however, unless they learned about probability very well, they cannot teach, so first they need to know it, as they expressed and therefore correspond with the arguments by Shaughnessy (1992). During the probability and statistics courses they took in their second or third year of teacher education, they already emphasized that it covered mostly the theories and their proofs. The participants also specified that they need to learn about how to teach probability and statistics included in the course of 'methods of teaching mathematics' or as a separate course. They also mentioned that they first need to learn probability and statistics before teaching it.

So on the whole, this study discussed the subject matter knowledge for probability held by preservice elementary mathematics teachers from the conceptual and procedural knowledge dimensions. Findings implied that subject matter knowledge assessed by the items in the test and questions directed through interviews have two dimensions, procedural and conceptual knowledge, as discussed clearly by the researchers previously (Hiebert & Lefevre, 1986; Ball, 1988; Stohl, 2005) and correspond to the framework which was bounded above.

Recommendations

The implications of this study will be enlightening for the future research of the subject matter preparation of preservice elementary mathematics teachers in Turkey. The discussion of the findings can have an impact on teacher education programs in the universities in order to revise their course objectives and develop content knowledge of preservice mathematics teachers in terms of statistics and probability. This study can have positive influences on the development of elementary mathematics education programs in nationwide, and might affect the perspectives of teacher educators, who are responsible for training the teachers, as well.

Moreover, research also needed to develop content knowledge and pedagogical content knowledge of preservice elementary teachers regarding statistics and probability. Their conceptual knowledge could be developed as well as their procedural knowledge. Research also needed to understand why conceptual knowledge of preservice teachers was less-developed compared with their procedural knowledge. Some professional learning environments could be designed in order to enhance content knowledge needed for statistics and probability for preservice elementary mathematics teachers.

References

Ball, D.L. (1988). *The subject matter preparation of prospective mathematics teachers: Challenging the myths*. East Lansing, MI: National Center for Research on Teacher Education.

Ball, D.L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449-466.

Batanero, C., & Díaz, C. (2012). Training teachers to teach probability: reflections and challenges. *Chilean Journal of Statistics*, *3*(1), 3–13.

Begg, A., & Edwards, R. (1999, December). *Teachers' ideas about teaching statistics.* Paper presented at the combined annual meeting of the Australian Association for Research in Education and the New Zealand Association for Research in Education. Melbourne, Australia.

Biehler, R., Ben-Zvi, D., Bakker, A., & Makar, K. (2012). Technology for enhancing statistical reasoning at the school level. In K. Clements, A. Bishop, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Third international handbook of mathematics education*. New York: Springer.

Bush, W. et. al. (under review). Diagnostic mathematics assessment for middle school teachers. *Journal of Mathematics Teacher Education*.

Center for Research in Mathematics and Science Teacher Development University of Louisville. (2008). Diagnostic mathematics assessments for elementary school teachers. Retrieved 18/11/2016, from https://louisville.edu/education/centers/crmstd/diag-math-assess-elem.

Creswell, J.W. (2007). *Qualitative inquiry and research design: Choosing among five traditions* (2nd ed.). Thousand Oaks, CA: Sage.

Green, D. (1987). Probability concepts: putting research into practice. *Teaching Statistics*, *9*(1), 8-14.

Jacobbe, T. (2007). *Elementary school teachers' understanding of essential topics in statistics and the influence of assessment instruments and a reform curriculum upon their understanding* (Doctoral dissertation, Clemson University).

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ: Erlbaum

Jones, G.A., & Thornton, C.A. (2005). An overview of research into the teaching and learning of probability. In *Exploring Probability in School* (pp. 65-92). Springer US.

MONE (2013). *Ortaokul Matematik Dersi (5, 6, 7 ve 8. Sınıflar) Öğretim Programı (Middle-School Mathematics Curriculum (Grades 5, 6, 7 and 8)*. Ankara: MEB.

Moore, D.S. (1997). New pedagogy and new content: The case of statistics. *International statistical review, 65*(2), 123-137.

Nicholson, J.R., & Darnton, C. (2003). Mathematics teachers teaching statistics: What are the challenges for the classroom teacher? In *Proceedings of the 54th Session of the International Statistical Institute*. Voorburg, The Netherlands: International Statistical Institute.

Quinn, R.J. (1997). Effects of mathematics methods courses on the mathematical attitudes and content knowledge of preservice teachers. *The Journal of Educational Research, 91*(2), 108-114.

Scheffler, I. (1965). Conditions of knowledge: An introduction to epistemology and education. Chicago: Scott, Foresman, and Company.

Shaughnessy, J.M. (1992). Research in probability and statistics: Reflections and directions. In Grouws, D.A. (Ed) *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics.* (pp. 465-494). New York, NY, England: Macmillan Publishing Co. Inc.

Star, J.R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education, 36*(5), 404-411.

Star, J.R., & Stylianides, G.J. (2013). Procedural and conceptual knowledge: Exploring the gap between knowledge type and knowledge quality. *Canadian Journal of Science, Mathematics, and Technology Education, 13*(2), 169-181.

Stohl, H. (2005). Probability in teacher education and development. In *Exploring probability in school* (pp. 345-366). Springer US.

Thompson, A.G. (1984). The relationship of teachers' conceptions of mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*, 105-127.

Thompson, A.G., Philipp, R.A., Thompson, P.W., & Boyd, B.A. (1994). Calculational and conceptual orientations in teaching mathematics. In A. Coxford (Ed.) *Profesional development for teachers of mathematics: 1994 Yearbook* (pp. 79-92). Reston, VA: National Council of Teachers of Mathematics.

Watson, J.M. (2001). Profiling teachers' competence and confidence to teach particular mathematics topics: The case of chance and data. *Journal of Mathematics Teacher Education 4*(4), 305-337.

Uzun Öz

Değişikliğe uğrayarak Eylül 2013'te uygulanmaya başlanan güncel ortaokul matematik dersi öğretim programında olasılık öğrenme alanının ağırlığının önceki programa göre azaltıldığı gözlenmiştir. 'Olasılığın temel kavramları' olarak adlandırabileceğimiz yeni içeriği ile olasılık öğrenme alanının işlenişi sadece 8. sınıf düzeyine bırakılmıştır. Bu haliyle güncel ortaokul matematik programı Moore'un (1997) önerilerinin iyi bir yansımasıdır denebilir, çünkü Moore (1997, akt. Biehler, Ben-Zvi, Bakker ve Makar, 2012) istatistiksel bakış açısıyla, alan bilgisi (daha çok kavram ve veri analizi ve daha az olasılık), pedagoji (daha az ders anlatımı, daha çok aktif öğrenme) ve teknoloji (veri analizi ve simülasyonlar için) açısından önerilerde bulunmuştu.

Önceki çalışmalar matematik öğretmenliği adaylarının olasılık öğrenme alanında diğer öğrenme alanlarına nazaran daha az bir anlayışa sahip olduklarını göstermektedir. Öğretmen adayları olasılık konularını öğretmekte zorlandıklarını, çünkü olasılık alan bilgisinde eksiklikleri olduğunu belirtmişlerdir (Quinn, 1997; Stohl, 2005). Ortaokul matematik programında hayata geçirilen değişiklik, ilköğretim matematik öğretmenliği adaylarının olasılık alan bilgilerinin incelenmesi ihtiyacını doğurmuştur. Ayrıca, öğretmen adaylarının olasılık konusundaki anlayışlarının işlemsel ya da kavramsal düzeyde olup olmadıkları da araştırılmalıdır. Dolayısıyla, bu çalışma gelecek program çalışmalarına katkıda bulunuyor olması ve öğretmen adaylarının alan bilgilerinin olasılık kapsamında inceleniyor olması açılarından değerlidir ve Türk matematik eğitimi alan yazınına katkı sağlamaktadır. Bu çalışmanın amacı, ilköğretim

matematik öğretmenliği adaylarının olasılık öğretimine dair bakış açılarını ve olasılık konusunda alan bilgilerini işlemsel ve kavramsal bilgileri bağlamında incelemektir.

Kavramsal bilgi "ilişkiler açısından zengin bilgiler olarak karakterize edilir, bilgiler ağı gibidir, öyle bir ağ ki, bilgiler arası ilişkiler, ayrık bilgi parçaları kadar önemlidir" (Hiebert ve Lefevre, 1986, 3). Bunun yanı sıra, Hiebert ve Lefevre (1986), işlemsel bilgiyi iki tipte tanımlamışlardır: "işlemsel bilginin bir türü, sembollerin kabul edilebilir alternatifleri için sözdizimsel bir gelenekle, sistemin bireysel sembollerine olan tanışıklıktır; diğer türü ise, matematiksel problemleri çözmeyi sağlayan kurallar ve yöntemlerdir" (s.7). Ball (1990) çalışmasında, matematik öğretmenliği adaylarının çoğunlukla işlemsel bilgiye sahip oldukları, çünkü onların matematiği bir kurallar ve işlemler bütünü olarak gördükleri sonucuna varmıştır. Aslında bu sonuç, sadece kesirler için değil, matematik programında yer alan diğer öğrenme alanları için de geçerli olabilir. Araştırmacılar, matematik öğretmenlerinin veya matematik öğretmenliği adaylarının, matematiği salt doğru ya da yanlış sonuçlara götüren bir kurallar bütünü olarak gördüklerini ve dolayısıyla da hesaplamaya dayalı bir zihne sahip olduklarını söylemektedirler (Thompson, 1984; Thompson, Philipp, Thompson ve Boyd, 1994, akt. Stohl, 2005). Bu sonuç, aynı zamanda, ortaokul matematik derslerinin de benzer hesaplamaya dayalı bakış açısıyla, tanımlar, kurallar ve işlemler açısından zengin fakat kavramlar açısından yetersiz olabileceği fikrine götürür.

Yöntem

Bu çalışma, öğretmen adaylarının istatistik ve olasılık alan bilgilerini ölçmeye çalışan araştırmanın bir parçasıdır ve amacına yönelik olarak nitel yöntemler uygulanmıştır. Bu çalışmada yer alan 23 katılımcı gönüllülük esasına dayalı olarak belirlenmiştir ve İstanbul'da öğrenim gören 3. ve 4. Sınıf ilköğretim matematik öğretmen adaylarıdır. Araştırmacı, katılımcılarla yüz-yüze görüşmeler yapmıştır. Görüşmenin ilk kısmında, katılımcılara en iyi ve en zayıf öğreteceklerini düşündükleri öğrenme alanları, olasılık ve istatistik öğretimleri hakkında beklentileri, stratejileri, teknikleri ve bunların öğretiminde kullanabildikleri teknolojiler sorulmuştur. Bunların yanında, bazı temel olasılık kavramları sorulmuştur: bir olayın olma olasılığı ne demektir? Kesin, eşit olasılıklı ve imkânsız olaylar nelerdir? Bir olayın olma olasılığının ölçüsü nedir? Bir olayın olma olasılığı nasıl hesaplanır?

Görüşmenin sonunda, katılımcılardan olasılık ve istatistik konularındaki alan bilgilerini ölçmeye yönelik olarak hazırlanmış 22 soruluk bir testi cevaplamaları istenmiştir. Bu test, açık uçlu soruları da içermesi açısından, görüşme başında alınmaya başlanan ses kaydı durdurulmamış ve cevaplarını sözel olarak vermeleri istenmiştir. Dolayısıyla, her katılımcı, bireysel olarak ve sözel ifade ederek testi cevaplandırmışlardır. Her görüşme yaklaşık olarak 45-60 dakika sürmüştür. Toplanan veri, daha sonra kelimesi kelimesine yazıya aktarılmış, kodlanmış ve nitel yöntemlerle analiz edilmiştir (Creswell, 2007).

Bu makaleye konu olan çalışma, bu görüşmenin ve bu testin olasılıkla ilgili maddelerine verilen cevapları analiz etmiştir. Testte yer alan 8 soru sadece olasılıkla ilgilidir (Appendix A)

Bulgular ve Tartışma

Katılımcılar, görüşmenin ilk kısmında olasılığın temel kavramlarıyla ilgi yöneltilen sorulara ilişkin olarak sadece hesaplamaya dayalı açıklamalar yapmışlardır. Bütün katılımcılar, bir olayın olma olasılığının 0 ile 1 arasında bir değer alabileceğini söylemişler ve kesin ve imkânsız olayları doğru bir şekilde tanımlamışlardır. Katılımcıların neredeyse yarısı, eşit olasılıklı olaylar konusunda bir kavram yanılgısına sahiptiler ve ½ şeklinde açıkladılar. Hatta bir olayın olma olasılığına ilişkin beklenen terminolojiyi çoğunlukla kullanmamışlardır, örneğin, 'örnek uzay' terimi çoğunlukla katılımcılar tarafından kullanılmamıştır.

Yine, görüşmenin ilk kısmında, katılımcıların çoğu olasılığı öğretmekte en çok zorlanacakları konu olarak belirtmişler ve kendilerinin de en az bildiklerini düşündükleri öğrenme alanı olarak seçmişlerdir. Bazıları, 'olasılığı biliyorum, ama sınıfta öğretirken ne yapacağım konusunda fikrim yok, çünkü arkasındaki mantığı bilmiyorum' diye ifade etmiştir. Birçok katılımcı, olasılığı en az bildikleri konu olarak belirtmelerinin, olasılık konularının soyut kavramlarla ilişkili olması sebebiyle olduğunu söylemişlerdir. Yine, kendi ortaokul ve lise yıllarında olasılığı 'bilmeden' öğrendiklerini, bir şekilde ezberlediklerini ifade etmişlerdir.

Teste ilişkin bulgularda, açık uçlu ve kapalı uçlu sorulara verilen yanıtlar aşağıda Tablo.1'de verilmiştir:

Tablo 1. Test Sorularının Konusu, Tipi ve Başarı Oranı

Madde #	Soru tipi	İlgili konu	Başarı oranı
Madde 1	Çoktan seçmeli	İmkansız olay	23'te 22 doğru
Madde 2	Çoktan seçmeli	Bir olayın olma olasılığını bulma	23'te 23 doğru
Madde 3	Çoktan seçmeli	Teorik ve deneysel olasılık	23'te 17 doğru
Madde 4	Açık uçlu	Bir olayın olma olasılığını bulma	6 katılımcı doğru cevapladı, 17 katılımcı yanlış cevapladı.
Madde 5	Açık uçlu	Örnek uzay	12 katılımcı doğru cevapladı, 4 katılımcı yanlış cevapladı, 7 katılımcı ise kısmen doğru cevapladı.
Madde 6	Açık uçlu	Teorik ve deneysel olasılık	6 katılımcı yanlış ya da doğru vir cevap veremedi, 6 katılımcı kısmen doğru cevapladı, 11 katılımcı doğru cevapladı.
Madde 7	Çoktan seçmeli	Olay türü	23'te 17 doğru
Madde 8	Çoktan seçmeli	Örnek uzay	23'te 20 doğru

Bu çalışmanın bulguları, işlemsel ve kavramsal bilgiler arasındaki ayrıma vurgu yaparak, mevcut alanyazına göre benzer yönelimler sergilemektedir. Genel olarak, ilköğretim matematik öğretmenliği adaylarının olasılıkta yüksek bir işlemsel bilgiye sahip oldukları söylenebilir. Buna rağmen, katılımcıların örneğin kedibalığı probleminde olduğu gibi kavramları anlamayı gerektiren bir olayın olasılığını bulma probleminde, teorik ve deneysel olasılık ilişkisi ve örnek uzayı açıklamaya yetecek düzeyde kavramsal bilgiye sahip olmadıkları görülmüştür.

'Bir olayın olması olasılığı nedir?' sorusuna ilişkin katılımcıların hiçbiri, 'oran' ya da 'örnek uzay' terimlerini kullanmamışlardır, bu Green'in (1987) belirttiği şekilde olasılıkta kavramsal bilgiye işaret eden önemli şartlardan biridir. Watson (2001) ise bu durumun öğretmen adaylarının çoğunlukla 'örneklem' kavramıyla daha az güvende hissetmeleriyle alakalı olduğunu açıklar. Stohl (2005) daha önce de belirtildiği şekilde, öğretmen adaylarının kavramları fark edememelerinin sebebi olarak, onların hesaplamaya dayalı zihinlere sahip olduğunu öne sürer. Eşit olasılıklı olaylarda kaydedilen kavram yanılgısıyla ilgili olarak, Begg ve Edward (1999) bunun 'olayların bağımsızlığı' ile ilgili kavram yanılgısından kaynaklandığını söylemektedir. Benzer konuyu soran çoktan seçmeli maddedeki başarıları dikkate alınırsa, katılımcıların kavramsal bilgilerinin yeterince iyi olmadığı ve kavramı farklı durumlarda ele alamadıkları ve diğer kavramlarla ilişki kuramadıkları iddia edilebilir (Ball, 1988).

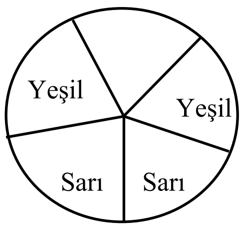
Olasılığı soyut bir konu olarak ele almaları ve öğretmekte de bu açıdan zorlanacaklarını düşünmeleri, onların olasılık anlayışlarının yeterince derin olmadığını söyleyebilir. Stohl'un (2005) sonuçlandırdığı üzere, bu bakış açısı, onların istatistik ve olasılık kavramları arasında yeterince iyi ilişkiler kuramamalarının sebebidir. Bu da, olasılığı matematiğin bir alt konusu olarak görmelerinden kaynaklanır. Dolayısıyla, onların kavramsal bilgilerinin yeterli olmadığı sonucuna varılabilir.

Öğretmen adaylarının işlemsel bilgilerine nazaran kavramsal bilgilerinin daha az gelişmiş olmasının olası sebepleri şunlar olabilir: üniversitede verilen derslerin çoğunlukla, "reçete gibi" ve "kurala bağlı" olarak verilmesi ve bunun da öğrencileri, arkasında yatan mantığı göz ardı ederek ezberlemeye yöneltmesi olarak açıklanabilir (Shaughnessy, 1992, 466). Katılımcıların neredeyse yarısı, olasılık ve istatistik dersini almış olmalarına rağmen, olasılığı yeterince iyi bilmediklerini hissettiklerini söylemişlerdir. Bazılarının 'istatistik ve olasılık öğretimi' dersi almalarına rağmen, kendilerini öğretmede yeterince iyi hissetmedikleri, çünkü öğretebilmek için öncelikle bilmek gerektiğini söylemişlerdir.

Sonuç olarak, katılımcıların işlemsel bilgileri, kavramsal bilgilerinin ötesinde olduğu ve kavramsal bilgilerini geliştirmeye ihtiyaçları olduğu söylenebilir. Bulgular ve yukarıda çevrelediğimiz teorik çerçevemize göre, bulgular aynı zamanda matematik öğretmenliği programına ilişkin gelecek çalışmalarda katkıda bulunabilir. Programda yer alan dersler, öğretmen adaylarının kavramsal bilgilerini de geliştirecek düzeyde yeniden şekillendirilebilir ve istatistik ve olasılığın öğretimine özel olarak yeni dersler önerilebilir. Öğretmen eğitimcilerinin matematik öğretmen adaylarının hangi olasılık kavramlarında ne düzeyde kavramsal ya da işlemsel bilgiye sahip oldukları, onlara bu derslerin tasarımında yardımcı olabilir.

Appendix A – The items from the instrument included in this study (Testin bu çalışmaya dahil edilen soru maddeleri)

<table>
<tr><td colspan="2">Matematik ve Fen için Tanısal Öğretmen Değerlendirme (DTAMS)
(Louisville Üniversitesi'nin izniyle alınmış ve Türkçe'ye çevrilmiş
istatistik ve olasılık sorularıdır.)
İsim ____________ Tarih ___________

Açıklama: Aşağıdaki testte ilköğretim matematik müfredatında bulunan istatistik ve olasılık konularıyla ilgili sorular bulunmaktadır. Sorulara mümkün olduğunca açık cevaplar veriniz ve gerekirse cevaplarınıza kısa açıklamalar ekleyiniz. Zaman ayırdığınız için çok teşekkür ederim.</td></tr>
<tr><td>Soru</td><td>Cevap</td></tr>
<tr><td>1. Aşağıdaki olasılıklardan hangisi bir olayın kesinlikle olmayacağını gösterir?
a. 1
b. 100
c. 0
d. ½</td><td></td></tr>
<tr><td>2. Bir torbada 3 kırmızı, 2 mavi ve 5 yeşil bilye vardır. Bu torbadan kırmızı bilye çekme olasılığı kaçtır?
a. 1/3
b. 3/10
c. 3/7
d. 1/10</td><td></td></tr>
<tr><td>3. Aşağıdaki çarkın (5 eşit parçaya bölünmüştür) döndürülmesiyle elde edilen verilerin tablodaki gibi olduğunu varsayalım.


<table><tr><td>Sonuç</td><td>Sıklık</td></tr><tr><td>Kırmızı</td><td>9</td></tr><tr><td>Yeşil</td><td>18</td></tr><tr><td>Sarı</td><td>17</td></tr></table>
Aşağıdaki cümlelerden hangisi her zaman doğrudur?
a. Bu olay deneysel olasılığa sahip değildir.
b. Sonuçların sıklığı teorik olasılıkla uyum sağlamadığı için, veri toplama hatası yapılmıştır.
c. Bu çark yeniden döndürülürse, sonuç sarı olacaktır.
d. Bir olayın teorik olasılığı ile deneysel olasılığının tam olarak aynı</td><td></td></tr>
</table>

çıkması nadir rastlanan bir durumdur.	
4. Biyologlar bir nehirdeki kedi balığı popülasyonunu araştırıyorlar ve 138 kedi balığı avlayıp işaretliyorlar. Üç ay sonra, ikinci avda yakalanan 241 kedi balığından 16 tanesinin önceden işaretlenmiş olduğunu farkediyorlar. Buna göre aşağıdaki soruları cevaplandırınız. a. 138 işaretlenmiş kedi balığının, işaretlenmemiş kedi balıklarıyla beraber nehre karıştırıldığını ve üç ay boyunca başka bir kedi balığının eklenmemiş ya da çıkarılmamış olduğunu varsayarak, nehirdeki kedi balığı sayısını tahmin ediniz. b. Tahmininiz için bir savunma yapınız.	
5. Bir öğretmen sınıfına örnek uzayı öğretmektedir. Öğretmen sınıfa şöyle söyler: "Sizden, bir kutudan iki bilyeyi seçme deneyinin örnek uzayını listelemenizi istiyorum. Kutuda bir kırmızı ve bir mavi bilye vardır. Kutudan bir bilye seçeceksiniz, çektiğiniz bilyeyi tekrar kutuya koyup ikinci bilyeyi seçeceksiniz." A öğrencisi örnek uzayın KK, KM, MK ve MM'den oluştuğunu söyler. B öğrencisi, A öğrencisiyle aynı fikirde olmadığını ve örnek uzayın KK, KM ve MM'den oluştuğunu, KM ve MK'nın aslında aynı sonuç olduğunu iddia eder. a. Hangi öğrencinin doğru söylediğini belirleyiniz ve nedenini açıklayınız. b. Hatalı olan öğrencinin durumu doğru anlamasına yardımcı olmak için bir etkinlik tarif ediniz.	
6. 8. sınıfların deneysel ve teorik olasılık arasındaki farkı anlayabilmesini sağlamak için bir etkinlik tarif ediniz.	
7. Aşağıdakilerde hangisi bir olayın büyük olasılıkla gerçekleşmeyeceğini ifade eder? a. Kesin b. Daha fazla olası c. İmkansız d. Daha az olası	
8. Engin, iki farklı kutudan birer bilet çekilen bir karnaval oyunu oynamaktadır. Her kutu biri 'kedi' biri de 'köpek' yazılmış iki bilet içermektedir. Kazanması için, eş biletleri seçmelidir. Aşağıdakilerden hangisi, bu deney için örnek uzayı gösterir? a. (kedi, köpek), (kedi, kedi), (köpek, köpek), (köpek, kedi) b. kedi, köpek c. kedi, köpek, kedi, köpek d. (kedi, köpek), (kedi, kedi), (köpek, köpek)	