EXPLICIT CALIBRATION OF PURE JUMP PROCESSES: THE BIST30 EUROPEAN OPTION CASE

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ABSTRACT

Purpose- The study has two main purposes. First, it aims to show the efficiency of pure jump processes, more specifically Variance Gamma (VG) and Normal Inverse Gaussian models (NIG), in option pricing by comparing with the Black Scholes (BS) option pricing mode. Second, it aims to calibrate the European options written on BIST30 index.

Methodology- We introduce an alternative derivation of option pricing formulas under the VG and NIG model assumption. We analyze the VG and NIG models' pricing performance by comparing their pricing result with the classical BS model for the BIST30 index. Our data includes the BIST30 index daily price and European options written on it from 05 May 2018 to 05 May 2020 with a maturity of 90 days. In the given period, the European call options' strike prices range from 1200 to 1650, and the European put options' strike prices range from 1000 to 1400. To compare their efficiency, first, the models are calibrated by minimizing the sum of squared deviations between the historically recorded and theoretical European option prices. Second, the theoretical option prices are computed, and the results are compared with the historically recorded option prices.

Findings- Our significant contribution is the calibration of pure jump processes, VG and NIG processes, with the help of characteristic functions, the BS model prices for the BIST30 index, and the computation of European options prices traded in BIST. The study showed that the NIG process outperforms both VG and BS models, and the BS model is the worst model in option pricing for BIST.

Conclusion- The VG and NIG processes can be calibrated by the help of their characteristic functions, and European option price estimations with these models superior to BS model option prices for BIST. Therefore, these processes are more efficient in BIST than the classical BS model.

Keywords: Variance Gamma Process, Normal Inverse Gaussian Process, Black-Scholes Model, Options Pricing; Calibration
JEL Codes: C63, G12, D46

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