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Assessing constructive heuristics for solving hydro unit commitment and loading problem

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Abstract:

Solving Hydro Unit Commitment problems is a challenge for real systems due to substantial number of variables and non-linear equations. The use of Constructive Heuristics is a robust strategy to solve this kind of problems. This present paper proposes a Constructive Heuristic that uses a Non-Linear Programming (NLP) solver and a Sensitivity Factor during its solution process. The methodology is applied and compared to a case study. The results are evaluated with variations of the original case.

Keywords: *Constructive heuristics, Hydro unit commitment, Non-linear programming*

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1. INTRODUCTION

The daily schedule of hydroelectric power plants purposes to determine which units will be operating and their respective generation levels for the following day. In the formulation of the problem, a set of operational constraints is considered, as well as generation goals, which are guidelines based on system's operation planning.

The works of [1] and [2] aim to improve the operating point considering an hourly energy generation time schedule for a set of hydroelectric plants in cascade. As the generation is predetermined, the improvement in performance comes from using the least amount of water to generate the same amount of energy. For this intent, a detailed representation of each plant was considered, with individualization of turbines and generators. This problem is classified as a hydro unit commitment and loading (HUCL).

In the works mentioned, the Objective Function (OF) target is to minimize the sum of the total turbined flow in the plants. Additionally, nonlinear functions are considered to represent the water and energy mass balance equations, as well as operational constraints. This structure leads to large problems that require Mixed Integer Nonlinear Programming to solve. In this sense, the present paper proposes a Constructive Heuristic (CH) that uses a Non-Linear Programming (NLP) solver during its solution process. This solution strategy allows the resolution of problems with plants that have different sets of units, in which a sensitivity factor is used to determine the best group of units for each operating condition.

2. MATERIAL AND METHODS

The structure used in this work to mathematically represent the problem is inspired by [1]. The formulation contemplates the hydraulic, mechanical, and electrical losses present in the energy production process considering individual turbines, as well as the maximum and minimum flow limits of the turbine. However, the spillway discharged flows are not considered, as it works with discharge values lower than the capacity of the plant's reservoirs. Additionally, the turbined outflows are propagated by travel times. The mathematical formulation is presented in sequence through the equations numbered (01) to (14). The variable description is presented in Table 1.

$$Min\ Z = \sum_{H,U,t} A_t^{H,U} Q_t^{H,U} \tag{1}$$

$$V_{t+1}^H = V_t^H + \alpha * \left(I_t^H + \sum_U A_t^{H,U} Q_t^{H,U} + \sum_{H\in\beta} \sum_U A_{t-\tau}^{H,U} Q_{t-\tau}^{H,U} \right) \tag{2}$$

$$D_t^H \leq \sum_U A_t^{H,U} * Pg_t^{H,U} \tag{3}$$

$$Pmin^{H,U} \leq Pg_t^{H,U} \leq Pmax^{H,U} \tag{4}$$

$$Pg_t^{H,U} = Pst_t^{H,U} - Pgg_t^{H,U} - Pmt_t^{H,U} \tag{5}$$

$$Pst_t^{H,U} = \eta * Q_t^{H,U} * HH_t^{H,U} * \rho_t^{H,U} \tag{6}$$

$$Pmt_t^{H,U} = f1\left(Pg_t^{H,U}\right) \quad Pgg_t^{H,U} = f2\left(Pg_t^{H,U}\right) \tag{7}$$

$$HH_t^{H,U} = Up_t^{H,U\prime} - Down_t^{H,U} - lo_t^{H,U} \tag{8}$$

$$Up_t^{H,U} = f3(V_t^H) \tag{9}$$

$$Down_t^{H,U} = f4(A_t^{H,U} * Q_t^{H,U}) \quad (10)$$

$$lo_t^{H,U} = f5(A_t^{H,U} * Q_t^{H,U}) \quad (11)$$

$$\rho_t^{H,U} = f6(A_t^{H,U} * Q_t^{H,U}, HH_t^{H,U}) \quad (12)$$

$$f7(HH_t^{H,U}) \leq Q_t^{H,U} \leq f8(HH_t^{H,U}) \quad (13)$$

$$A_t^{H,U} = \{0,1\} \quad (14)$$

Table 1. Variables Description

Symbols	Item Description
Z	Objective function
$Q_t^{H,U}$	Turbined outflow in each unit at each stage
$A_t^{H,U}$	Boolean matrix to represent the operative state of the turbine
H	Set of hydroelectric plants
U	Set of generating units
t	Set of temporal steps or steps considered in the problem
α	Conversion term from cubic meters per second to hectometers per hour
τ	Travel time between hydroelectric plants
β_r	Set of hydroelectric power plants located upstream of hydroelectric power plant "r"
V_t^H	Reservoir volume at each stage
D_t^H	Energy requirement per reservoir at each stage
$Pg_t^{H,U}$	Net power generated by turbine at each stage
$Pmin^{H,U}$	Lower turbine power limit
$Pmax^{H,U}$	Maximum turbine power limit
$Pst_t^{H,U}$	Gross power generated by the turbine at each stage
$Pgg_t^{H,U}$	Electrical losses of the generator at each stage
$Pmt_t^{H,U}$	Mechanical losses of the generator at each stage
$HH_t^{H,U}$	Net hydraulic head per step
$Up_t^{H,U}$	Upstream level at each step
$Down_t^{H,U}$	Downstream level at each step
$lo_t^{H,U}$	Hydraulic and mechanical losses at each stage
$\rho_t^{H,U}$	Efficiency of the turbine at each stage
η	Constant that depends on the gravity and density of the water [kg m-2 s-2]
f1, f2, f3, f4, f5, f6, f7 e f8	Polynomial or exponential functions

The OF (1) is the minimization of turbine flows from all the power plants. As the generation is individualized and pre-defined, the only way to improve the system's performance is to improve its efficiency, which is, reducing the use of water to generate the same amount of energy. It is noteworthy that the choice of meeting an individualized generation goal per plant significantly restricts the system's decision-making freedom. This choice is associated to a strategy in which the plant operator does not optimize the complete problem; he only chooses machines to comply with a certain goal.

Equation (2) deals with the mass balance of reservoirs. Equation (3) is the energy balance, in which the generation goal is guaranteed. Equation (4) deals with the maximum and minimum operating limits of the generator. In turn, equations (5) and (7) represent the mechanical and electrical losses of the generator. Equation (6) relates turbine's mechanical power output with turbine's efficiency and net head. Equation (8) deals with the transformation from gross head to net head. Equations (9) and (10) are upstream and downstream polynomials, respectively. Equation (11) estimates the hydraulic losses. Equation (12) represents the turbine's hydraulic performance curve. Equation (13) defines the maximum and minimum limits for turbine flow. Finally, Equation (14) is the turbine drive integrity constraint.

The main difference between the equations presented and those used in [1] is the consideration of the matrix $A_t^{H,U}$ as a multiplier factor in all equations where there are variables $Q_t^{H,U}$ and $Pg_t^{H,U}$. In [1], the matrix $A_t^{H,U}$ appears only in equations (04) and (13). Equation (04) is written as $A_t^{H,U} * Pmin^{H,U} \le Pg_t^{H,U} \le A_t^{H,U} * Pmax^{H,U}$, while equation (13) is written as $A_t^{H,U} * f7\left(HH_t^{H,U}\right) \le Q_t^{H,U} \le A_t^{H,U} * f8(HH_t^{H,U})$.

The problem (1) to (14) is non-linear in nature with integer and continuous variables. In this sense, a solution strategy based on CH is proposed. To apply this technique, the integrity constraints applied to variable "A" are relaxed, which changes from binary variable to continuous variable. This new variable is represented by the letter "a". The variable indicates whether the unit is operating or not, with an upper limit of 1 and a lower limit of 0. This consideration affects two variables: the turbined flow (Q) and the net power generated (Pg). Table 2 presents this new treatment for the problem.

Table 2. Equation Modifications

	Original problem	*Modified problem*
Turbine flow	$A_t^{H,U} * Q_t^{H,U}$	$a_t^{H,U} * Q_t^{H,U}$
Net power generated by turbine	$A_t^{H,U} * Pg_t^{H,U}$	$a_t^{H,U} * Pg_t^{H,U}$

The CH scheme was inspired by [3] and [4]. In these problems, the technique was used to plan the expansion of transmission systems. Initially, the number of new lines and circuits was not subject to an integrality constraint. So, the solution could contemplate a fraction of the line to be built, which, in practice, is unrealistic. The purpose of the CH was to manipulate the results to transform the fraction results into integer results.

In this work, as the variable "a" is simultaneously multiplied by outflow and power generated in the turbine. The generation target obliges that the variable "a" needs to be greater than zero. The main assumption of this work is that the NLP optimization process itself can indicate the best solution. The optimization process naturally leads to small "a" values for unnecessary turbines and "a" values equal to 1 for turbines essential to the energy supply.

Thus, with these modifications, the problem is represented by equations (15) to (28).

$$Min\ Z = \sum_{H,G,U,t} a_t^{H,U} * Q_t^{H,U} \tag{15}$$

$$V_{t+1}^H = V_t^H + \alpha * \left(I_t^H + \sum_{G,U} a_t^{H,U} * Q_t^{H,U} + \sum_{H \in \beta} \sum_{G,U} a_{t-\tau}^{H,U} * Q_{t-\tau}^{H,U} \right) \tag{16}$$

$$D_t^H \le \sum_U a_t^{H,U} * Pg_t^{H,U} \tag{17}$$

$$Pmin^{H,U} \le Pg_t^{H,U} \le Pmax^{H,U} \tag{18}$$

$$Pg_t^{H,U} = Pst_t^{H,U} - Pgg_t^{H,U} - Pmt_t^{H,U} \tag{19}$$

$$Pst_t^{H,U} = \eta * Q_t^{H,U} * HH_t^{H,U} * \rho_t^{H,U} \tag{20}$$

$$Pmt_t^{H,U} = f1\left(Pg_t^{H,U}\right) \quad Pgg_t^{H,U} = f2\left(Pg_t^{H,U}\right) \tag{21}$$

$$HH_t^{H,U} = Up_t^{H,U} - Down_t^{H,U} - lo_t^{H,U} \tag{22}$$

$$Up_t^{H,U} = f3(V_t^H) \tag{23}$$

$$Down_t^{H,U} = f4(a_t^{H,U} * Q_t^{H,U}) \tag{24}$$

$$lo_t^{H,U} = f5(a_t^{H,U} * Q_t^{H,U}) \tag{25}$$

$$\rho_t^{H,U} = f6(a_t^{H,U} * Q_t^{H,U}, HH_t^{H,U}) \tag{26}$$

$$f7(HH_t^{H,U}) \le Q_t^{H,U} \le f8(HH_t^{H,U}) \tag{27}$$

$$0 \le a_t^{H,U} \le 1 \tag{28}$$

In this case, as all variables of the problem are continuous, it can be solved by an NLP optimization package. In this sense, CONOPT [5] is used to solve the NLP problem.

However, directly applying the optimization package does not solve the original problem (1) to (14), but its relaxed version. Thus, the proposed CH strategy solves problem (15) to (28) in steps to find the solution to the original problem. The solution scheme consists of two steps. The first one aims to reduce the dimension of the problem by setting the maximum possible amount of “a” values as integer with few solvers runs. In turn, in the second part, all configurations of units are exhaustively checked, and the best result is chosen.

The first step starts with solving problem (15) to (28). After the first resolution of the problem with NLP, “a” values of the sets of turbines (represented by “G”) are summed for the hydroelectric plant and for each time step. If there is more than one group of units in a hydropower plant at this time step, a Sensitivity Factor (SF) is used. The SF is a fraction between the sum of “a” factor in this group and the total number of units in this group. The SF value can be interpreted as a percentage of units in operation.

Equation (29) represents the calculation of SF. The Ng variable represents the number of units per generator set.

$$SF_t^{H,G} = \frac{\sum_{U \subset G} a_t^{H,G,U}}{Ng_t^{H,G}} \tag{29}$$

SF is used to choose which group should be privileged in the very first iteration. For example, considering there are two distinct groups in a power plant, the X group with three turbines and the Y group with two. For a defined time, step, if the sum of “a” for Group X is 1.5 and 0.8 for Y, the SF will be 0.5 and 0.4, respectively. In this case, group X must be indicated for the first iteration.

Furthermore, in this first iteration, "a" is used as an indicator of the number of units that should be set to 1. For example, if there are three units in the group and all of them have "a" equal to 0.5, then the sum is 1.5. This value indicates that at least one turbine must be set to 1. The number also indicates that at least one turbine must be set to 0. Thus, the CH defines unit U1 must be as on (1) and unit U3 off (0). The “a” value of the middle unit is not defined yet, as the first iteration does not indicate a clear value for it.

At the end of the first part, for each period and each group, there must be only one unit without a fixed “a” value. Thus, the second step of the algorithm consists of testing free “a” value of each set of turbines. The testing order follows the ascending order of the indices of time steps, plants and generator sets. Tests of combinations of different “a” values were not considered.

The problem is solved with turbines with free “a” values set to 0 and then to 1. The option that results in the smallest objective function is the solution. If one of the alternatives is unfeasible, the other possibility is chosen. Usually, the lack of solution is related to a fixed value of 0, since there is a generation target for each plant that must be satisfied.

Continuing with the previous example, first, the value "a" of U2 should be set to 0 and then the value should be set to 1. Assuming that the OF result is 8 for "a" set to 0, and the OF value is 9 when “a” is set to 1, the best choice is the former option - 8. Therefore, the value of “a” should be set to 0.

3. THEORY

The HUCL problem requires a strategy to deal with nonlinear equations and integer variables related to unit commitment. For purposes of planning the operation of a large electrical system, as analyzed by [6], the linearization of equations and disregarding integer variables for hydroelectric plants can be perfectly justified, given the other complexities involved.

Thus, one way to consider at least hydro unit commitment is to use Mixed Integer Linear Programming (MILP). In this case, the problem equations must also be linearized, but the individual turbines are represented. This method is more accurate but can be very time-consuming for large-scale systems [7,8].

In turn, the consideration of nonlinearities and hydro unit commitment requires the Mixed Integer Nonlinear Programming (MINLP). However, despite recent improvements to optimization packages, these are only suitable for small problems compared to industry-relevant problems. Even today, practical problems must be simplified and reduced in size to obtain treatable formulations in the process, limiting the benefits of greater mathematical detail in the representations of the problems [9].

Some research such as [10] and [11] propose strategies combining different solvers to deal with the problem. [11] consider the resolution in two stages. The first phase solves the relaxation of a mixed integer nonlinear program to obtain the turbined water flow, the reservoir volume and the number of units operating in each period of the planning horizon. The second stage solves a linear mixed-integer scheduling problem to determine which combination of turbines to use in each period. In turn, [10] divide the problem into three parts. Initially, unfeasible and undesirable solutions are discarded. Then, dynamic programming is used to solve the unit commitment problem of the ideal static unit for a given generation of plants, viable unit combinations and current hydraulic conditions. Finally, the HUCL problem is formulated and solved as a large network problem with boundary constraints.

These strategies are robust ways to create fast and efficient decision support systems to plan real-time unit generation schedules. However, both methods require linearization of equations. [1] assesses that, from the standpoint of the Independent System Operator (ISO), the considerable number of reservoirs prevents it from considering the complex modeling associated with hydroelectric units. For this reason, for [6], the hydroelectric units are modeled by a piecewise-linear function, and the unit commitment constraints of the hydroelectric units are not considered. Therefore, a discrete mixed nonlinear intrinsic modeling is replaced by a continuous linear modeling.

[1] proposed a mathematical cascade model formulated to minimize, in each plant and time stage, the turbine flow required to supply the hourly generation goal defined by ISO. This approach allows using non-linear equations, but does not aim to solve the problem of the complete system. Therefore, it is being considered to complement the ISO perspective. The resolution method applied in [1] is divided into two phases. The first is to apply the Lagrangian Relaxation to obtain the optimal dual solution, which is unfeasible in relation to the primal variables, as it solves a convexified form of the HUCL problem. Later, this solution is used as a starting point to recover the viability of the primal solution, through the inexact Augmented Lagrangian.

It should be noted that the problem presented by [1] is non-linear and non-convex, therefore, it is mathematically impossible to guarantee the optimal solution. Under these conditions, the use of a well-established strategy, such as the proposal, is coherent to give a sufficiently good operating point for hydroelectric plants. The use of Constructive Heuristics is a feasible method to deal with this type of problem.

4. RESULTS AND DISCUSSION

The methodology is applied to solve the test problem presented by [1]. In this test system, a four hydroelectric powerplants cascade is considered. Each plant must be able to supply a generation goal for 24 hourly time steps. Equations (15) to (28).

In this context, the application of the methodology leads to a problem of 16 blocks of restrictions and 13 blocks of variables with 13593 non-null elements, 3547 unique variables and 4247 equations. In addition, there are 7225 non-linear array entries in the model. For the first solution, with a free "a" value for all units, the CONOPT requires 1381 iterations to find the solution. Also, the solver needs to run a few more times for the first step of CH and many times for the second step. However, after the first solution of the problem, the calculation time decreases drastically, due to the improvement of the starting point. In the first step of the solution strategy, less than 60 solver iterations are required for convergence. In turn, for the second part of the method, 30 interactions are enough for convergence.

The first solution, without "a" integrality restriction, has a value of 51302 m3/s. In the first stage, this value is increased by 0.25% and, at the end of the second stage, the value is 51745 m3/s, an increase of 0.86%. Tables 3 and 4 illustrate how the method works for plants H1 and H4 in the fifth time step.

Table 3. Results of "A" by the Heuristic for t = 5 and H=1

t=5 H=1	Solver output	First Step	Second Step
g1 u1	0.801	1	1
g1 u2	0.801	1	1
g1 u3	0.801	0.46	1

Table 3 shows, in practice, how the method works when there is only one generator set. The sum of the values of "a" from the solver is equal to 2,403, which causes turbines U1 and U2 to be fixed at 1 in the first step, while U3 remains free. In the second step, the solution is evaluated for value of "a" for U3 set to zero and set to one, and the best one is chosen. In this case, there is no solution to the problem if "a" is set to 0. For this reason, the "a" of U3 is set to 1.

Table 4 shows the CH operation for two generator sets. By the initial output of the solver, there is an indication that the units of group 1 are more appropriate than those of group 2. The Sensitivity Factor numerically indicates this solver result and directs the first step starting with group 1. When starting for group 1, all units U1, U2 and U3 were set at 1, as the sum value was 3. For the second generator set, the sum of the values of "a" was 0.3, which indicates that U2 must be set at 0 and U1 must remain free for the second stage. In the second stage, through the exhaustive test, it is concluded that the smallest value of the Objective Function occurs when the U1 of G2 is equal to 0.

Table 4. Results of "A" by the Heuristic for t = 5 and H=4

t=5 H=4	Solver output	First Step		Second Step
		Fixing the G with higher SF	Fixing the other G	Testing the free values
g1 u1	1	FS G1 = 1	1	1
g1 u2	1		1	1
g1 u3	1		1	1
g2 u1	0.156	FS G2 = 0.16	0.15	0.3
g2 u2	0.156		0.15	0

The results were compared with the values of [1]. The OF value found in this article (51745 m^3/s) is compatible with the value of 51868.6 m^3/s found by [1], with a difference of 0.24% approximately. Furthermore, the number of turbines in operation for each time step was the same in both papers. In this sense, the results indicate that the methodology can solve the HUCL problem for the case of four hydropower plants in cascade.

Complementarily, the coherence of the algorithm was evaluated for different operating conditions, with the variation of the initial volume of the reservoirs and variation of the generation goals. The Table 5 presents the cases tested with different percentages of initial volume in relation to the useful volume and different percentages of target in relation to the base case value.

Table 5. Tests Considered for Algorithm Validation

Generation goal	Initial volume of reservoirs 30%	40%	50%	60%	70%
50%	Case 1a	Case 1b	Case 1c	Case 1d	Case 1e
100%	Case 2a	Case 2b	Case 2c	Case 2d	Case 2e
105%	Case 3a	Case 3b	Case 3c	Case 3d	Case 3e

The results of these tests are compiled in Fig. 1, where the value of the Objective Function is presented for each case tested.

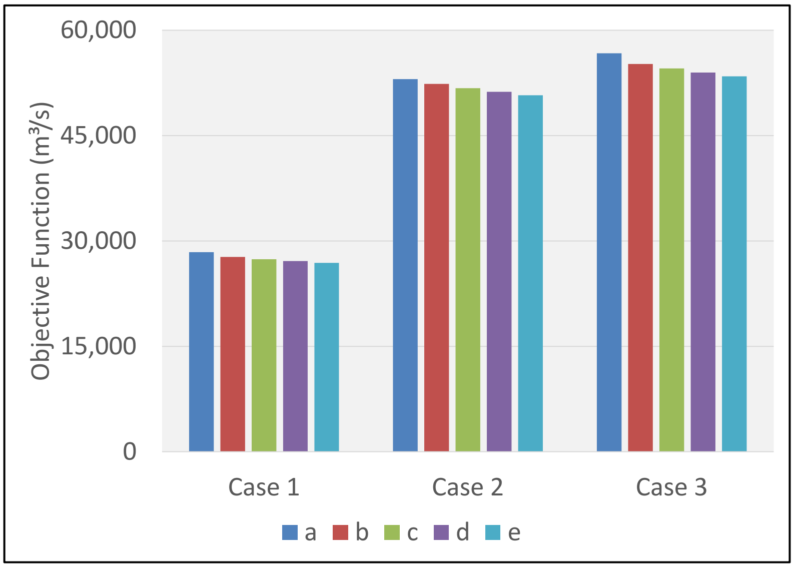

Figure 1. Algorithm results for different initial conditions

By observing Fig. 1, it is possible to see that the algorithm can represent the expected behavior for the system. For example, when the upstream reservoir level decreases, the flow requirement increases to ensure the same generation goal. This occurs because, with a smaller head, it is necessary to increase the turbined flow to obtain the same power. On the other hand, reducing the generation target, the turbined flow also decreases. With less need for generation, there is, consequently, less need to turbine water.

Finally, it is worth noting that the structure of the algorithm allows testing slight changes in the model's considerations, which incurs in the possibility of continuing the work from some alternatives.

5. CONCLUSIONS

The approach proposed from Constructive Heuristics solves a simple problem previously postulated in the literature. The design of the proposal is related to the definition of the main restrictions for solving the problem.

The methodology is not guaranteed to be the most efficient or effective in the search for the global minimum, however it allows the monitoring, by the algorithm, of the search for the minimum value based on the Sensitivity Factors. In this sense, other Sensitivity Factors and other sets can be included to complement the optimization monitoring.

Finally, it is highlighted that the use of algorithms based on sets with the activation of NLP solvers and support of Constructive Heuristics is a promising approach to deal with complex optimization problems such as those observed in the electricity sector. The construction of the problem and the solution strategy are key factors for the success of the analysis.

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