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An Analysis of Postgraduate Theses Using Toulmin Argumentation Model in the Field of Mathematics Education in Turkey

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ABSTRACT

The aim of this study is to examine the theses published in the field of mathematics education in Turkey in which Toulmin Argumentation Model was used. It has been analyzed how the prepared theses were distributed in terms of publication year-type, publication language, institute, university, keywords, aims and target audience. In this study conducted by document analysis method, 23 theses in the field of mathematics education, covering a total of 12 years until the end of 2022 without considering any start date, from the theses open to access in the National Thesis Center database of the Council of Higher Education (YÖK) were analyzed. The obtained studies were subjected to descriptive analysis and the analyzed data were presented in tables with frequency and percentage values. Within the scope of the research, some basic results were reached such as; argumentation studies using Toulmin Model in mathematics education have been studied more intensively from the past to the present, Turkish studies are conducted more frequently than English studies in terms of publication language, studies conducted at the doctoral level at the level of institutes are more or equal to the studies conducted at the master's level, university students and secondary school students are preferred more intensively in the selection of the target audience. This study aims to guide the studies that center on the Toulmin Argumentation Model in mathematics education. In addition, it is thought to contribute to the researchers to see the concentration or deficiencies in the field and to encourage the filling of the gap in the field by conducting applications with different study groups rather than working with the same study groups.

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Türkiye'de Matematik Eğitimi Alanında Toulmin Argümantasyon Modeli'nin Kullanıldığı Lisansüstü Tezlerin İncelenmesi

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ÖZET

Bu çalışmanın amacı, Türkiye'de matematik eğitimi alanında yayınlanan Toulmin Argümantasyon Modeli'nin kullanıldığı tezleri incelemektir. Hazırlanan tezlerin; yayın yılı-türü, yayın dili, enstitü, üniversite, anahtar kelimeler, amaçlar ve hedef kitleleri bakımından nasıl dağılım gösterdiği analiz edilmiştir. Doküman analizi yöntemiyle yürütülen bu çalışmada, Yükseköğretim Kurulu (YÖK) Ulusal Tez Merkezi veri tabanındaki erişime açık olan tezlerden, herhangi bir başlangıç tarihi gözetilmeksizin 2022 yılı sonuna kadar toplam 12 yılı kapsayan, matematik eğitimi alanındaki, bu araştırmanın hedeflerine uygun 23 adet tez çalışması analiz edilmiştir. Elde edilen çalışmalar betimsel analize tabi tutulmuş ve çözümlenen veriler tablolarda frekans ve yüzde değerleri ile sunulmuştur. Araştırma kapsamında; matematik eğitiminde Toulmin Modeli'nin kullanıldığı argümantasyon çalışmalarının geçmişten günümüze doğru daha yoğun çalışıldığı, yayın dili açısından Türkçe araştırmaların İngilizce araştırmalara göre daha sık yapıldığı, enstitüler düzeyinde doktora çalışmalarının yüksek lisans düzeyinde yapılan çalışmalardan daha fazla ya da eşit olduğu, hedef kitle seçiminde üniversite öğrencileri ile ortaokul öğrencilerinin daha yoğun tercih edildiği gibi bazı temel sonuçlara ulaşılmıştır. Bu çalışmanın, matematik eğitiminde Toulmin Argümantasyon Modeli'ni merkeze alan çalışmalara yol göstermesi hedeflenmektedir. Ayrıca araştırmacıların alandaki yoğunlaşmayı veya eksikleri görmelerine, aynı çalışma gruplarıyla çalışılmasındansa farklı çalışma grupları ile uygulamalar yapılarak alandaki boşluğun doldurulmasının teşvik edilmesine katkı sağlayacağı düşünülmektedir.

Anahtar Kelimeler: Matematik eğitimi, argümantasyon, Toulmin Argümantasyon Modeli, lisansüstü tez, doküman analizi

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Introduction

In the theses in the field of mathematics education in Turkey, many questions such as "To what extent is the Toulmin Argumentation Model used effectively?" are waiting to be answered in order to strengthen the literature and inspire education policy makers. The importance of argumentation method especially in science education is known. Because the importance of argumentation in science education has been emphasized frequently since the 1990s (Erduran, 2019). It was stated that the term argumentation was rarely used in relation to mathematics until the nineties (Schwarz et al., 2010). Although argumentation is more preferred in the field of science education, argumentation method is also used in mathematics education. Mathematics and science education researchers have reported their research by using the Toulmin Model in the analysis process of argumentation-based education (Hähkiöniemi et al., 2022). Sriraman & Umland (2014) stated that argumentation in mathematics education has two meanings: (i) mathematical arguments produced by both students and teachers in mathematics classes, and (ii) arguments put forward by mathematics education researchers to demonstrate the adequacy of mathematics learning and teaching in various contexts. Unlike experimental disciplines, the validity of an argument in mathematics is assessed according to its logical coherence (Umland & Sriraman, 2014).

In the classroom argumentation process, students are expected to reach the correct knowledge by colliding their arguments. In order to realize this expectation, strong justifications need to be presented. Thanks to the close connection between the data and the claims with the help of strong justifications, the claim put forward is tried to be made consistent and irrefutable. With the contribution of the development obtained from in-class argumentation, students can justify their answers with 'because' while solving the problems they encounter in their individual studies, and they can subject their own thoughts to the argumentation process by talking to themselves. In mathematics education, classroom argumentation practices are seen as a way of developing understanding and comprehension (Schwarz et al., 2010). Reform movements in mathematics education claim that mathematical argumentation has a central role at all grade levels (Francisco, 2022). However, in recent years, it has been stated that the Toulmin Model for the analysis of argumentation in the field of mathematics education has become a magic wand that solves problems in a snap (Cramer & Kempen, 2022). Toulmin's (1958, 2003) argumentation model has been found useful in analyzing arguments in mathematics education studies (Wagner et al., 2014). In addition, Krummheuer (1995) used Toulmin's model in the analysis of mathematical arguments without the refuting and qualifying components of this model and initiated the use of Toulmin's model in mathematics education (Inglis et al., 2007). In a related study, Krummheuer used the Toulmin Argumentation Model to model explanations that emerged during classroom activities (Schwarz, 2009). Toulmin's Argumentation Model is a model in which a person who makes a claim presents reasons to support his/her claim and bases his/her reasons on data (Karbach, 1987). In this model, the speaker should present his/her point of view (claim) on the basis of a justification (warrant) (Rigotti & Greco Morasso, 2009).

Mathematics is recognized as the most objective and rational of academic disciplines (Hannula, 2014). Argumentation, which is considered a natural part of mathematics as a proving science, takes place at the center of proving as mathematical argumentation (Ubuz et al., 2012). Although the quantity and quality of mathematical arguments produced by students and teachers in mathematics courses vary (Sriraman & Umland, (2014), argumentation among mathematicians is often practiced informally to discuss, develop and communicate

mathematical problems and results (Douek, 1999). In some cases, teachers may not have the practical skills to structure classroom discussions so that students can construct or defend complex arguments (Sriraman & Umland, 2014). This, of course, shows how important the teacher's knowledge is in the use of argumentation in the classroom. Because in some cases, the teacher needs to consciously direct the classroom discussion and carry out knowledge construction more effectively. Using the argumentation method in classrooms helps to create fair and equitable learning environments (Francisco, 2022). The effective use of mathematical arguments enables students to demonstrate or explain the truth of a mathematical result in the context of causality through reasoning (Sriraman & Umland, 2014). Based on their studies, Schwarz et al. (2010) stated that argumentation has an extremely important position in the use of argumentation in mathematics education and that argumentation encourages making sense and making assumptions. Although argument can be interpreted with various words such as questioning, persuasion, negotiation, disagreement in daily use, in mathematics classes, it is encountered when students and the teacher try to prove the truth of a claim (Wagner et al., 2014). Arguments play a critical role in the construction of logical knowledge by facilitating individuals to justify their own or others' claims from a positive or negative critical perspective (Msimanga & Mudadigwa, 2019).

Toulmin's (1958, 2003) The Uses of Argument is considered as one of the most influential works in the field of argumentation (Aberdein, 2006) and it is even stated that Toulmin can be considered as the founder of modern argumentation due to his contributions to argumentation theory (van Eemeren et al., 2014). Stephen Toulmin's (1958, 2003) Argumentation Model includes 6 different elements in the argumentation process. These six elements consist of three basic and three auxiliary elements (Toulmin et al., 1984). The basic elements are: (i) claim, (ii) grounds/data, (iii) warrant. Auxiliary elements: (i) backing, (ii) qualifier, (iii) rebuttal. According to Toulmin (1958, 2003), *claim* is the conclusion reached in the discussion, *data* are the facts that form the basis of the argument, *warrant* is the statement that enables the transition from data to argument, *backing* is the statement that increases the credibility of the warrant, *rebuttal* is the exceptional circumstances that may disable the guaranteed conclusion, and *qualifier* is the statement that shows the strength of the argument. Based on this, the elements of the Toulmin Argumentation Model can be summarized as follows: A proposed thesis or conclusion (claim), the information that forms the basis of the claim (data), the statement that makes the inference applied to the claim from the data acceptable and connects it (warrant), additional information presented to increase and strengthen the strength of the warrant (support), evidence used to reject the proposed claim (rebuttal), statements that show the degree of certainty to support the strength of the claim (qualifier). While the basic elements of the model are present in every argument, the auxiliary elements are not necessarily present in every argument (Karbach, 1987; van Eemeren et al., 1996). Therefore, auxiliary elements can be added to the model when necessary (Karbach, 1987). According to Toulmin (1958, 2003), arguments are claims derived from data in accordance with a warrant and are used in the construction of the argumentation method. Through warrants, the data that form the basis of an argument are connected to the claim, which is the main purpose of an argument (Karbach, 1987).

Argumentation, which is seen as an important skill to be learned in many fields such as daily life, politics, science, law and business world (Walton, 2013), refers to the process of drawing conclusions with a chain of reasoning (Umland & Sriraman, 2014). The justifications made in the argumentation method are extremely important as they will affect the credibility of the claims (Benek & Akçay, 2019). Argumentation, which plays an important role in the creation

of models, theories and explanations (Erduran, 2019), is a kind of intervention method in which justifications are presented to change or influence the views and behaviors of the interlocutor (Rigotti & Greco Morasso, 2009). In other words, the argumentation process is carried out with interventions that can be made positively or negatively to the interlocutor's opinion. In order for the argumentation process to be concluded in a healthy way, it is important to reach a common conclusion, even though it is often difficult and time-consuming.

Some difficulties are encountered in the implementation of the argumentation method in the classroom environment. İspir and Yıldız (2021) investigated some of the limitations and difficulties faced by teachers in argumentation practices by subjecting the studies containing findings on the limitations of the argumentation method to document analysis and presented the limitations they encountered by classifying them according to their source. These are: (i) student, (ii) teacher, (iii) group work, (iv) educational environment, (v) method, (vi) curriculum. In addition, students' decreasing mathematics self-efficacy perceptions and increasing mathematics anxieties and concerns (Adal & Yavuz, 2017) should be taken into consideration in the teaching methods applied in the classroom environment.

When the literature is reviewed, we come across various studies in which documents are analyzed: argumentation (Altun & Özsevgeç, 2016; Bağ & Çalık, 2017; Cirit Gül et al. 2021; İnam & Güven, 2019; Kabataş Memiş, 2017; Küçük & Aycan, 2014), argumentation in science education (Çetinkaya & Taşar, 2018; Güven et al. 2016), argumentation in mathematics education (Tekin Dede, 2018; Güneş, 2013; Topuz & Cantürk Günhan, 2021). İnam and Güven (2019) examined the theses conducted in Turkey between 2007 and 2016 in which argumentation method was used in terms of various variables and found two studies in the field of mathematics. Kabataş Memiş (2017) found one study in the field of mathematics in his study in which he examined the theses conducted in Turkey until 2015 and in which argumentation was used according to various variables. Cirit Gül et al. (2021) found fourteen theses in the field of mathematics education in their study in which they examined the theses published between 2000 and 2020 in Turkey and published on the argumentation process. Topuz and Cantürk Günhan (2021) examined argumentation-based research in the field of mathematics between 2011-2019 in Turkey and found eleven postgraduate theses.

The aim of this study is to examine the theses published in the field of mathematics education in Turkey in which Toulmin Argumentation Model was used. In this context, it was analyzed how the studies were distributed within the framework of various variables. Thus, it is aimed to obtain and report important data on how the model, which is frequently used in science education, is applied in mathematics education. This study was conducted in a more specific area by limiting the studies in which Toulmin Argumentation Model was used especially in mathematics education. The aim here is to question the use of the Toulmin Argumentation Model in argumentation studies in the field of mathematics education and to give an idea to researchers who want to work in this field in the light of current data. In addition, the contribution of this study to the field is important in terms of revealing the development of the studies in the related field, providing a basis for researchers who will determine a thesis topic and propose a thesis, facilitating the follow-up of the change in the related literature and providing a one-stop review opportunity. In addition, this study aims to guide studies that center on the Toulmin Argumentation Model in mathematics education. It is thought to contribute to the researchers to see the concentration or deficiencies in the field, to encourage the filling of the gap in the field by conducting applications with different study groups rather

than working with the same study groups, and to serve as a source for future research. In line with the aim of the study, answers to the following questions were sought.

The studies using Toulmin Argumentation Model in mathematics education published in YÖK National Thesis Center in Turkey:

1. How are they distributed according to the *years* and *types*?
2. How are they distributed according to *language of publication*?
3. How are they distributed according to the *institutes* where they were conducted?
4. How are they distributed according to the *universities* where they were conducted?
5. How are they distributed according to the *keywords* used?
6. How are they distributed according to their *aims*?
7. What is their *target audience*?

Methodology

In this study, in which the qualitative research method was adopted, the document analysis method was used. In qualitative research, there are various forms of data collection including interviews, observations and documents (Merriam, 2009). Among these, documents are important data sources used in qualitative research (Yıldırım & Şimşek, 2021) in terms of providing a historical perspective by collecting data on the past (Sözer & Aydın, 2020). With this technique, more in-depth examinations can be made by focusing on the relevant field (Ekiz, 2020). This helps to look from a single window by bringing together the details of the subject under investigation and provides convenience for researchers. Researchers try to understand the studies conducted by examining documents (Sözer & Aydın, 2020). With document review or analysis, the researcher obtains written or electronically recorded documents about the targeted subject and examines them systematically, collects, questions, and analyzes the data (Bowen, 2009; Büyüköztürk et al., 2020; Özkan, 2021; Sönmez & Alacapınar, 2019; Turgut, 2014). In other words, the data contained in the documents accessed in document analysis are found, selected, interpreted, evaluated, and synthesized (Kıral, 2020). The researcher should act very carefully during the review process because this way, very valuable data that has been overlooked can be obtained (Merriam, 2009). In summary, the document analysis method offers the researcher the opportunity to understand and interpret the relevant literature in the light of the data obtained from past studies.

Data Collection

Finding appropriate documents is the first step in document analysis (Merriam, 2009). It is decided which documents will be analyzed within the scope of the research problems (Sözer & Aydın, 2020). Documents can be collected from a wide variety of sources (Özkan, 2021). According to Merriam (2009), the stages of document analysis are as follows: (i) finding documents, (ii) checking their authenticity, (iii) building a systematic for coding and cataloging, (iv) performing data analysis. Within the scope of this study, all the mentioned stages were carried out. Researchers may have to conduct a document analysis due to the research question (Özkan, 2021). In order to find answers to the research questions generated in line with the purpose of the study, theses were first collected. In qualitative research, it is important to select the sample on the basis of the research question, depending on the resources of the researcher (Balcı, 2021). The sample of the study was selected by criterion sampling, one of the purposeful sampling methods. The theses examined within the scope of this study were determined by considering the criteria of (i) conducting the research in

Turkey, (ii) being in the field of mathematics education, (iii) using Toulmin Argumentation Model. What, why and how questions can be answered with documents (Özkan, 2021). In this context, postgraduate theses in the field of mathematics education completed in the National Thesis Center of the Council of Higher Education (YÖK) in which the Toulmin Argumentation Model was used were examined by document analysis method. The theses were accessed by typing *"mathematics" and "argumentation"* in the *enter search term* search section on the main page of YÖK National Thesis Center and clicking the *find button* when *all* options were active. The theses found as a result of the search were checked one by one whether they were related to mathematics education and the Toulmin Model. No start date was considered in the selection of theses. In this way, as a result of the search, 23 theses that were open to access and suitable for the objectives of the research covering a total of 12 years between 2011 and 2022 were included in the document analysis. It has been analyzed how the prepared theses were distributed in terms of publication year-type, publication language, institute, university, keywords, aims and target audience.

Data Analysis

In line with the purpose of the study, 23 theses, which constitute the sources of data analysis, were analyzed under eight different themes using descriptive content analysis, one of the qualitative research methods. In the descriptive analysis, the information obtained from data collection tools such as documents is subjected to analysis under the questions, topics, or themes on the basis of research (Ekiz, 2020). The aim of descriptive analysis is to organize the data obtained and present them in an interpreted manner (Çepni, 2021). In addition, in this type of analysis, it is also aimed to directly illustrate and explain the subject under study (Ekiz, 2020). According to Yıldırım and Şimşek (2021), in the descriptive analysis process, data are summarized and interpreted under predetermined themes.

Firstly, the theses accessed from YÖK Thesis Center in line with the purpose of the study were downloaded to the computer environment and saved, and the data obtained from the relevant sections of each thesis within the framework of the determined variables were processed one by one for each sub-problem and presented in a holistic perspective by transforming them into tables. Master's theses used in the study were coded as (M1, M2, M3...) doctoral dissertations were coded as (D1, D2, D3...). The data obtained during the process were analyzed by an expert lecturer. The data obtained from the theses were presented in frequency and percentage tables.

Validity and Reliability

In ensuring validity and reliability in studies, all threats cannot be prevented but can be minimized (Cohen et al., 2007). In order to ensure the internal consistency of the research, the theses obtained were coded by two different coders and the inter-coder consistency was found to be 0.93 (Miles & Huberman, 1994). Consensus is expected to be at least 80% (Miles & Huberman, 1994; Patton, 2002). In addition, the research process was clearly explained, the results obtained were carefully reported, necessary arrangements were made by taking expert opinion for the data, and the reliability of the research was aimed to be ensured.

Findings

This section presents the findings obtained as a result of document analysis. Describing the findings obtained through document analysis is important for the effective completion of the process (Özkan, 2021). The findings were presented and evaluated for each sub-problem.

1. Findings on the distribution of theses according to years and types

The distribution of studies according to years and types is given in Table 1.

Table 1. *Distribution of Studies According to Years and Types*

	Postgraduate Level			
Year	PhD	Master's Degree	Frequency (f)	Percentage (%)
2011	1	-	1	4
2014	1	-	1	4
2015	1	-	1	4
2016	2	-	2	9
2017	1	-	1	4
2018	1	1	2	9
2019	1	4	5	22
2020	3	-	3	13
2021	1	1	2	9
2022	2	3	5	22
Total	14	9	23	100

When Table 1 is examined; it is seen that 1 (4%) postgraduate thesis was conducted in 2011, 1 (4%) in 2014, 1 (4%) in 2015, 2 (9%) in 2016, 1 (4%) in 2017, 2 (9%) in 2018, 5 (22%) in 2019, 3 (13%) in 2020, 2 (9%) in 2021 and finally 5 (22%) in 2022. It is noteworthy that the most studies were conducted in 2019 and 2022.

2. Findings related to the language of publication of the theses

The distribution of studies according to language of publication is given in Table 2.

Table 2. *Distribution of Studies According to Language of Publication*

	Postgraduate Level			
Publication Language	PhD	Master's Degree	Frequency (f)	Percentage (%)
Turkish	9	8	17	74
English	5	1	6	26

Total	14	9	23	100

When Table 2 is examined, it is seen that 17 (74%) research theses were published in Turkish and 6 (26%) in English.

3. Findings related to the institutes where the theses were conducted

The distribution of the studies according to the institutes to which they were affiliated is given in Table 3.

Table 3. *Distribution of the Studies According to the Institutes Where They were Conducted*

	Postgraduate Level			
Institute Type	PhD	Master's Degree	Frequency (f)	Percentage (%)
Education Sciences	5	5	10	43
Social Sciences	4	3	7	30
Science	4	1	5	22
Postgraduate Education	1	-	1	4
Total	14	9	23	100

When Table 3 was examined, it was seen that a total of 10 (43%) theses were conducted under the Institute of Educational Sciences, 7 (30%) theses under the Institute of Social Sciences, 5 (22%) theses under the Institute of Natural and Applied Sciences and 1 (4%) thesis under the Institute of Postgraduate Studies.

4. Findings related to the universities where the theses were conducted

The distribution of the universities where the studies were conducted is given in Table 4.

Table 4. *Distribution of Studies According to Universities*

	Postgraduate Level			
University	PhD	Master's Degree	Frequency (f)	Percentage (%)
Middle East Technical University	5	-	5	22
Atatürk University	4	-	4	17
Anadolu University	-	2	2	9
Gazi University	1	1	2	9
Adnan Menderes University	-	1	1	4

Balıkesir University	1	-	1	4
Boğaziçi University	-	1	1	4
Dicle University	-	1	1	4
Dumlupinar University	1	-	1	4
Giresun University	-	1	1	4
Hacettepe University	1	-	1	4
Kastamonu University	1	-	1	4
Necmettin Erbakan University	-	1	1	4
Recep Tayyip Erdoğan University	-	1	1	4
Total	14	9	23	100

In Table 4, the universities where the studies were conducted are listed starting from the highest frequency to the lowest frequency and the universities with the same frequency are given in alphabetical order. When the relevant table is examined, it is seen that 5 (22%) studies were conducted at Middle East Technical University, 4 (17%) at Atatürk University, 2 (9%) at Anadolu University, 2 (9%) at Gazi University and 1 (4%) each at other universities (Adnan Menderes University, Balıkesir University, Boğaziçi University, Dicle University, Dumlupınar University, Giresun University, Hacettepe University, Kastamonu University, Necmettin Erbakan University, Recep Tayyip Erdoğan University).

5. Findings related to keywords used in theses

The distribution of the studies according to the keywords used is given in Table 5.

Table 5. *Distribution of Studies According to Keywords Used*

No.	Keywords	Frequency (f)	No.	Keywords	Frequency (f)
1	Argumentation	9	38	Global argumentation	1
2	Toulmin Model	6	39	Computational thinking skills	1
3	Argumentation-based teaching	2	40	Communication skills	1
4	Attitude towards mathematics	2	41	Primary School	1
5	Mathematics	2	42	Proof	1
6	Mathematics education	2	43	Concept cartoons	1

7	Mathematics teaching	2	44	Conceptual understanding	1
8	Mathematical reasoning	2	45	Collective mathematical argumentation	1
9	Mathematical argumentation	2	46	Krummheuer argumentation analysis model	1
10	Mathematical proof	2	47	Local argumentation	1
11	Secondary school mathematics teacher candidates	2	48	Mathematics education technologies	1
12	In-class mathematical applications	2	49	Mathematical argument	1
13	Technology	2	50	Writing mathematical arguments	1
14	Triangles	2	51	Mathematical proof	1
15	Acodesa	1	52	Mathematical modeling	1
16	Argument	1	53	Mathematical process skills	1
17	Argumentation skills	1	54	Model building activities	1
18	The relationship between argumentation and proof	1	55	Probability success	1
19	Quality of argumentation	1	56	Probability teaching	1
20	Argumentation-based learning approach	1	57	Proportional reasoning	1
21	Argumentation-based teaching	1	58	Secondary school mathematics teacher candidates	1
22	Argumentation approach	1	59	Teaching method	1

23	Argumentation structures	1	60	Measurement	1
24	Argumentation method	1	61	Self-assessment	1
25	Argumentative writing	1	62	Self-efficacy	1
26	Scientific process skills	1	63	Reflective thinking skills for problem solving	1
27	Scientific debate	1	64	Classroom teaching experiment	1
28	Cognitive integrity	1	65	Oral argumentation	1
29	Polygons	1	66	Discussion	1
30	Detailing	1	67	Design research	1
31	Dialogical approach	1	68	Design-based research	1
32	Transformation geometry	1	69	Technology assisted instruction	1
33	Function concept	1	70	Technological pedagogical content knowledge	1
34	GeoGebra	1	71	Metacognitive awareness	1
35	Geometric objects and volume measurement	1	72	Assumption-based learning routes	1
36	Geometric construction	1	73	Creative thinking	1
37	Realistic math education	1	74	Mind-problem solving habits	1

When Table 5 is examined, 74 different keywords were identified in the theses examined for the purpose of the study. In the studies, it was seen that the keyword 'argumentation' was used 9 times and the keyword 'Toulmin Model' was used the most with 6 times.

6. Findings related to the aims of the theses

The aims of the studies are summarized in Table 6.

Table 6. *Distribution of Studies According to Their Aims*

Level No	Aims
D1	To examine the collaborative argumentation processes of pre-service mathematics teachers in a technology-enriched environment.
D2	To examine the effect of written and oral arguments on students' mathematics achievement and attitudes as a result of teaching in accordance with the argumentation-based learning approach applied to third grade primary school students.
D3	To examine the effect of argumentation-based instruction on students' problem solving habits and computational thinking skills in mathematics applications course for middle school students.
D4	In this study, which was conducted with several objectives; (i) to develop, test and organize activities for teaching proportional reasoning through conjecture-based learning to seventh grade students, (ii) to examine the development of collaborative reasoning through formal and informal tools through Realistic Mathematics Education with formal tools, (iii) to determine the co-development of ideas and concepts for reasoning.
D5	To examine the effects of sixth grade geometric objects and volume measurement and measurement of liquids on students' (i) academic achievement, (ii) self-efficacy towards mathematical process skills, (iii) knowledge transfer, (iv) willingness to discuss.
D6	In this study, which was conducted with prospective secondary school mathematics teachers in line with several objectives, we aimed to examine (i) the processes of generating and proving conjectures through cognitive integrity-based activities, (ii) the global argumentation structures, components and the refuting component, (iii) the extent to which the participants were able to effectively carry out the approaches they proposed in the activities used with compass and straightedge and GeoGebra.
D7	To examine the effect of teaching probability with argumentation approach applied to prospective mathematics teachers on the participants' probability achievement, retention of knowledge, and argumentation level.
D8	To examine the argumentation structures produced by prospective secondary school mathematics teachers while solving geometry questions in GeoGebra and Paper-Pencil groups.
D9	To reveal the mathematical applications that occur in an instructional environment designed for the subject of triangles.

D10	To examine the argumentation and proof processes of pre-service elementary mathematics teachers in the field of analysis and to reveal the views of the participants.
D11	To determine the effect of teaching the subject of functions with argumentation-based learning approach on students' (i) academic achievement, (ii) attitudes, (iii) science process skills, (iv) conceptual understanding and to compare it with the current teaching method. In addition, it was aimed to reveal the effect of the applied learning approach on argumentation levels and willingness to discuss.
D12	To examine the effect of argumentation-based science learning approach applied to ninth grade students on their creative thinking skills and mathematics achievement.
D13	To identify how the Toulmin discussion model can be used to examine the structure of discussions in lessons and how students interact with each other and with their teachers.
D14	In this study, which was carried out with the participation of pre-service teachers in line with several objectives; (i) to determine the self-assessment levels of transformation geometry and technological pedagogical content knowledge before the application, (ii) to examine the academic achievement, technological pedagogical content knowledge self-assessment levels, conceptual understanding and opinions of the participants after the technology-supported argumentation-based transformation geometry teaching.
M1	To examine the development of arguments produced by third grade primary school students in mathematics lessons, students' mathematical understanding, components of arguments, written and oral discussion levels.
M2	To examine seventh grade students' mathematical reasoning processes about polygons.
M3	To examine students' argument structures, argumentation analysis and its impact on the transition to evidence in technology-supported environments offered to students.
M4	To examine the process of argumentation and proof in mathematical problem solving and to reveal the relationship between them.
M5	To analyze 8^{th} grade students' written and oral mathematical arguments.
M6	To examine the effect of lessons taught with concept cartoons on students' argumentation levels.
M7	To examine the relationship between proof and argumentation skills.

M8	To determine the change in argumentation skills in line with the prepared modeling activities.
M9	The aim of this study is to examine the effect of argumentation-based science learning approach on 9th grade students' (i) academic achievement, (ii) communication skills, (iii) metacognitive skills, (iv) problem solving reflective thinking skills, (v) willingness to discuss, (vi) attitudes towards mathematics.

Table 6 summarizes the aims of the theses subjected to document analysis.

7. Findings related to the target audience of the theses

The distribution of the target audiences of the studies is given in Table 7.

Table 7. *Distribution of Studies According to Target Audience*

		Postgraduate Level		
Level	Target Audience	PhD	Master's Degree	Frequency (f)
Primary school (f=2)	3rd grade	1	1	2
Secondary School (f=10)	6th grade	2	2	4
	7th grade	1	1	2
	8th grade	1	3	4
High school (f=3)	9th grade	2	1	3
University (f=12)	2nd grade	1	1	2
	3rd grade	7	1	8
	4th grade	2	-	2

When Table 7 is analyzed, it is seen that university (f=12) and secondary school (f=10) students make up the majority of the distribution according to the target audience.

Result and Discussion

In this study, the theses conducted using Toulmin Argumentation Model in mathematics education in YÖK National Thesis Center were obtained by document analysis method and subjected to descriptive analysis and the data obtained within the scope of the research problems were given in the findings section with tables. The conclusions based on the findings are presented in this section.

As a result of the document analysis, 23 studies were found in which the Toulmin Model was used in mathematics education. According to the tables in which Cirit Gül et al. (2021) present the findings of the theses related to the subject areas they are related to, more thesis

studies were conducted in the field of science education at the graduate level than in the field of mathematics education. Hafızoğlu and Bahar (2020) found that 102 studies were conducted in science education. From this point of view, it can be said that more argumentation studies have been conducted in postgraduate level science education than in mathematics education. Furthermore, it can be stated that there are fewer studies in mathematics education than in science education. In addition, although it is seen that argumentation studies are more preferred in science education than in mathematics education, it is seen that argumentation method is also used in mathematics education.

When the studies are examined according to the years in which they were conducted, it is seen that the theses were finalized mostly in 2019 and 2022, and the theses written at the doctoral level are more than the theses written at the master's level in total, and this can be interpreted as that studies on the relevant subject at the doctoral level are more preferred. The fact that doctoral studies are more than master's studies coincides with the findings of Topuz and Cantürk Günhan (2021). However, in the field of science, according to the results of Hafızoğlu and Bahar (2020), it was found that argumentation studies conducted at the master's level were more than doctoral studies. From 2014 to 2022, at least one doctoral study was conducted every year, the most doctoral studies were conducted in 2020, the first doctoral dissertation was written in 2011, the first master's thesis was written in 2018, the most master's thesis was finalized in 2019, and as the years progressed from 2011 to 2022, the total number of studies conducted in the last five years was close to three times the total number of studies conducted in previous years. This may indicate that argumentation studies using the Toulmin Model in mathematics education have started to be studied more intensively towards the present day.

When the studies were analyzed according to the language of publication, it was found that the theses were written in two languages, Turkish and English, and it was noteworthy that there were mostly Turkish theses (74%). In addition, it was observed that the English language of publication was used more in doctoral research than in master's research, at least one publication was produced in each language of publication at each level, and one study was conducted in English at the master's level. As a result, it can be said that Turkish research is more frequently used in the language of publication than English research. It shows parallelism with similar studies in the literature by virtue of the greater use of the Turkish language (Atasever, 2019; Sevencan, 2019; Topuz & Cantürk Günhan, 2021).

When the theses were examined according to the institutes, it was seen that the highest number of studies were conducted under the Institute of Educational Sciences, doctoral and master's studies were produced in equal numbers in the Institute of Educational Sciences, the theses produced in the Institute of Natural and Applied Sciences were mostly at the doctoral level, and one study was conducted in the Institute of Postgraduate Studies. In addition, it was noteworthy that doctoral level studies were more than master's level studies in other institutes except the Institute of Educational Sciences. Cirit Gül and others (2021) also concluded that the studies on the argumentation process were mostly conducted in the Institute of Educational Sciences.

When the studies were analyzed according to the universities where they were conducted, it was found that they were prepared in 14 different universities, the most studies were conducted at Middle East Technical University and only as a doctoral thesis, followed by Atatürk University and only as a doctoral thesis. In addition, among the universities where the

studies were conducted, it was noteworthy that only Gazi University conducted both doctoral and master's studies. In the study conducted by Cirit Gül and others (2021), sans limitation in terms of subject area, they stated that postgraduate theses on the argumentation process were mostly made in Gazi University and Marmara University.

It was seen that the studies concentrated on keywords such as 'argumentation' and 'Toulmin Model'. However, it was determined that the frequencies of almost all of the other keywords used were low. By examining the keywords used, an idea can be obtained about the studies in the related field.

When the studies are analyzed in terms of their aims, it can be said that especially doctoral dissertations are conducted with much more number and comprehensive aims. When the objectives are examined, it is seen that some concepts such as academic achievement, attitude, argumentation process, problem solving habits, self-efficacy, knowledge transfer, willingness to discuss, GeoGebra, retention of knowledge, argumentation level, argumentation structure, argumentation and proof, scientific process skills, written and oral argumentation, mathematical reasoning, technology, modeling and argument structures come to the fore. Objectives for new research can be developed by examining the objectives. It has been found that academic success has a higher frequency in the findings of similar studies in the literature regarding the purpose of making theses (Altun ve Özsevgeç, 2016; Kabataş Memiş, 2017; İnam ve Güven, 2019; Hafızoğlu ve Bahar, 2020; Cirit Gül ve diğerleri, 2021). The authors' findings match up with the findings obtained in this study.

When the distribution of the studies according to the target audience was analyzed, it was determined that university students and secondary school students were more preferred in the studies. This result is consistent with the findings of Topuz and Cantürk Günhan (2021) and Cirit Gül et al. (2021). It was observed that doctoral studies were conducted only with 9th grade students at the high school level. In addition, it is seen that university 3rd grade students are more preferred in doctoral dissertations. The diversity of the sample group should be ensured by conducting applications at other grade levels, especially at the grade levels where argumentation studies are not practiced. It has been seen that the argumentation method has been used at various levels in the field of mathematics education, but it has been determined that there are not enough of these studies yet, and it can be said that much more studies should be carried out and finalized in order to contribute to the literature. It is important to conclude the studies in various subject areas at various levels and at various grade levels in order to question the effectiveness of the use of argumentation method in mathematics education. In addition, it is thought that the scientific demonstration of the applicability of the argumentation method, especially starting from the primary school level, will contribute significantly to the future education policies of education policy makers.

Recommendations

According to the results of this research, some recommendations are presented. These are: (i) before starting theses, researchers can benefit from document analysis studies such as this study in addition to conducting a detailed literature review in order to avoid repetition, (ii) more studies in which the Toulmin Argumentation Model is applied can be conducted in different subject areas at both doctoral and master's levels and at grade levels where the study mentioned in this research has not been conducted, (iii) courses implemented with argumentation-based teaching can be increased, especially starting from the primary school

level, (iv) this research was conducted with theses obtained from YÖK National Thesis Center and the same study can be conducted with documents obtained from different sources, (v) this research was conducted in the sample of Turkey and can be compared with different countries, (vi) in-depth content analysis can be conducted by taking into account the results and suggestions of the theses.

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