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Scheduling parallel batch processing machines: A case study in the semiconductor industry

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Abstract

This paper presents a mathematical programming-based solution approach for the scheduling problem of batch-processing parallel machines with eligibility constraints. A case study has been presented in the semiconductor industry, where the ovens are scheduled for the underfill cure operation of products. The case includes constraints, such as oven-product eligibility restrictions, loading constraints for the batching of products for ovens, daily production requirements, and oven capacity constraints. In this study, we also assess the difference between creating batches of a single product type or different product types to be allocated to the ovens. The case study results have shown that the proposed models, in comparison to the current situation, increase the occupancy rate of ovens. The execution of the models aids the company in gaining visibility on the scheduling of ovens and successfully managing the production plan and order commitment. The proposed models have been effective and supportive of the semiconductor company.

1. Introduction

In today's competitive global markets, fast growth and competitiveness are the key pillars of the industrial sector. For this reason, companies try to achieve high quality, short production time, and low cost, and they try to use all resources efficiently and provide the conditions for rapid and high-quality production.

The semiconductor industry is one of the most rapidly growing industries on the market today. This industry must manufacture many products with high volumes since many digital products in our daily lives, such as smartphones, computers, TVs, household appliances, and cars, use semiconductors. Semiconductor manufacturing has the most complex production systems because of the short life cycles of the products and the complicated processing steps. Therefore, companies in the semiconductor industry must manage their resources effectively to overcome many challenges, such as supply chain constraints, time, customer pressure, and new products.

This study examines the problem of scheduling parallel batch-processing ovens with eligibility constraints within a semiconductor company. The problem involves scheduling daily production orders for different product types on these ovens. In the current situation, five ovens are used for the underfill cure process, and magazines loaded with production orders are placed in these ovens, considering the eligibility requirements and capacity restrictions of ovens. Product batches loaded into ovens can consist of single or multiple product types.

Two novel mixed-integer programming (MIP) models are formulated to provide an optimal solution by either loading ovens, creating batches of only one product type, or different product types. The proposed models minimize the maximum cycle number run among all parallel ovens daily. The rest of the paper is organized as follows. Section 2 provides a research background on parallel batch processing scheduling problems. Section 3 describes the problem addressed in the case study. Following, section 4 provides a detailed description of the mathematical models proposed. Section 5 consists of the computational results and discussion. Finally, section 6 concludes by summarizing the gains that have been made.

2. Literature Review

Ikura and Gimple's (1986) study is probably the first to deal with batch processing machines (BPMs) scheduling. A BPM can process several jobs simultaneously as a batch. Once batch processing is started, the machine cannot be stopped or interrupted, which means that no product can be added or removed. The processing time of the batch is the longest processing time of all jobs in the batch. Ahmadi, Ahmadi, Dasu, and Tang (1992) addressed a class of two or three-stage flow shops in which one of the machines is a BPM. They assumed that the processing time of the BPM is the same for all jobs and provided algorithms for minimizing the mean flow time and makespan.

One of the earliest studies belongs to Lee, Uzsoy, and Martin-Vega (1992). They first presented efficient dynamic programming-based heuristics for a single BPM to minimize maximum tardiness and the number of tardy jobs and then developed heuristics for identical parallel BPMs to minimize makespan. Chandru, Lee, and Uzsoy (1993) studied the scheduling problem for minimizing total completion time on a single BPM in which the set of jobs to be scheduled belongs to several job families, and the jobs in the same family have the same processing time. They first developed a polynomial-time dynamic programming algorithm for the single-machine case and then developed heuristics for the parallel-machine case.

There is an increasing focus on scheduling problems for BPMs due to many applications in different industries, such as aircraft, textile, and food (Kempf, Uzsoy, and Wang, 1998; Baker and Trietsch, 2009; Beldar and Moghtader, 2022); a greater number of applications are found in the semiconductor industry. For instance, Uzsoy (1994) studied scheduling a single BPM driven by burn-in operations in the manufacture of semiconductors to minimize the total completion time. He developed a branch-and-bound algorithm and several heuristics and showed that heuristics could quickly find solutions close to the optimal. Kempf, Uzsoy, and Wang (1998) considered the same problem with job families, unequal job sizes, and resource constraints. They developed integer programming formulations and heuristics to minimize the total completion time and makespan. Lee and Uzsoy (1999) were interested in the scheduling problem on a single BPM with dynamic job arrivals to minimize the makespan. They developed polynomial and pseudopolynomial time algorithms for specific cases in burn-in operations. In addition, they developed efficient heuristics for the general case. Chou (2007) considered a single BPM problem for the burn-in operation where jobs have dynamic release times and unequal sizes. The author proposed a combined genetic algorithm (GA) and dynamic programming approach where GA determines the job sequence and dynamic programming identifies the batches.

Mathirajan and Sivakumar (2006), in their review work, mainly focused on semiconductor manufacturing due to the wealth of research in this industry. They examined many papers on scheduling BPMs in the semiconductor industry and classified them based on problem configurations and methodology to provide a comprehensive and categorized resource for future studies. According to their study, BPMs usually represent relevant process bottlenecks due to expensive equipment or long uninterrupted processing times; therefore, manufacturers must optimize the scheduling of these machines initially. Their review paper also notes that heuristic approaches are preferred over simulation and mathematical programming to schedule batch processing operations in the semiconductor industry.

In this paper, we are also concerned with the scheduling problem of parallel BPMs in the semiconductor industry. In particular, we give past research on scheduling parallel BPMs. Chang, Damodaran*, and Melouk (2004) developed a simulated annealing (SA) approach to minimize makespan for identical parallel BPMs with unequal job size and processing time. Kashan, Karimi, and Jenabi (2008) proposed a hybrid GA to minimize makespan on identical parallel BPMs. Their results highlighted that the hybrid GA is more efficient than the SA algorithm.

Damodaran and Chang (2008) scheduled the parallel BPMs to test Printed Circuit Boards (PCBs) for which the processing time and the PCB dimensions are given. Each batch is formed so that the overall size of all PCBs in the batch is not larger than the capacity of the machine. The processing time per batch is the longest PCB processing time in the batch. To minimize the makespan, the authors proposed several heuristics to schedule PCBs on these parallel BPMs. The research conducted by Li, Qiao, and Wu (2009) was motivated by a scheduling problem in the dry strip operations at a semiconductor wafer manufacturing facility. The parallel BPMs with incompatible job families and dynamic job arrivals were scheduled with sequence-dependent setup time and qual-run requirements for advanced process control. They developed an ant colony optimization algorithm to achieve a satisfactory solution in a reasonable computation time.

Wang and Chou (2010) addressed the parallel BPMs scheduling problem in manufacturing environments such as the burn-in operation in the manufacture of semiconductors and the aging test operation in the manufacture of thin film transistor-liquid crystal displays (TFT-LCDs); they provided an MIP model, and metaheuristics based on GA and SA. Tai and Lai (2011) considered the liquid crystal injection scheduling problem (LCISP) for identical parallel BPMs involving constraints on limited maximum waiting times, unequal ready times, machine

setup times, and incompatible product families. The authors proposed an MIP model and efficient solution procedures. Chang, Yang, Hu, and Chen (2012) studied the scheduling problem of uniform parallel BPMs with non-identical job sizes. They provided the MIP models for minimizing makespan and total completion time; then, they provided a polynomial-time algorithm with worst-case ratio analysis.

Chou (2013) solved the scheduling of unrelated parallel BPMs with job release times and unequal job sizes to minimize the total weighted tardiness. This study proposed an MIP model and several heuristics based on dispatching rules, dynamic programming methods, and SA algorithms. Zhou, Liu, Chen, and Li (2016) addressed the scheduling problem of parallel BPMs having different speeds and capacities and arbitrary job sizes. Their paper proposed an MIP model first, then developed a differential evolution-based hybrid algorithm for solving large problem instances.

Liu, Yuan, and Li (2016) studied online scheduling of equal-length jobs on unbounded parallel BPMs to minimize makespan with the limited restart feature; that means a running task may be interrupted, losing all the work done on it, and the jobs are rescheduled from scratch with only one restart option. Arroya and Leung (2017) discussed the identical parallel BPMs with unequal job sizes and non-zero ready times and developed an MIP model and a heuristic approach. In the paper of Jia, Huo, Li, and Chen (2019), the jobs have identical processing times, non-identical sizes, and unequal weights. The jobs are delivered to the customers by the vehicles after processing. Two heuristic algorithms and an algorithm based on ant colony optimization were presented to minimize the total weighted delivery time of the jobs. Ozturk (2020) presented a decomposition method that uses column generation to schedule parallel batches of jobs on identical parallel machines with different job release dates, processing times, and sizes while machines have limited capacity. In Song's (2022) paper, the unrelated parallel BPM scheduling problem is addressed to minimize the total energy consumption and makespan. A self-adaptive multi-objective differential evolution algorithm was put forward. Recently, Nguyen and Sheen (2023) considered the parallel BPMs scheduling problem to minimize the makespan under constraints of unequal lot sizes, start time windows, and incompatible families. They developed a decomposition-based heuristic to obtain a near-optimal solution for large-scale problems. Also, Ji, Xiao, Yu, and Wu (2023) proposed a hybrid large neighborhood search utilizing the tabu strategy and local search for scheduling unrelated parallel BPMs, where the batches are limited to the jobs from the same family.

Fowler and Mönch (2022) recently surveyed the literature on parallel BPMs scheduling problems and focused on deterministic scheduling. They presented a classification of parallel batching problems, distinguishing the cases where all jobs can be used to form a batch and the others where only jobs of the same family can form a batch. Measures related to makespan, flow time, and due date were also examined, and future research directions were discussed.

This paper differs from the studies in the literature in two points. First, in the literature, studies on scheduling batch processing in semiconductor industries are mainly related to burn-in operation or wafer fabrication. This study includes a case study for the scheduling of ovens used for an assembly process called underfill cure. Second, the proposed modeling approach is different from the existing models. Initially, an MIP model was formulated that creates batches of a single product type to be placed in the ovens. Then, this model was extended to schedule the ovens, considering the creation of batches from different product types. The developed model assigns batches loaded into the magazines to ovens. The jobs according to different product types have unequal sizes; therefore, the capacity of magazines varies according to the job sizes. The defined problem can be described as the scheduling problem of batch-processing parallel ovens with eligibility constraints. The ovens have eligibility restrictions, meaning that each job can only be processed in an eligible oven. Since the processing of batches takes equal time, the time horizon is divided into 3-h time buckets, which is the same duration for the underfill cure operation of each oven. These time buckets are considered as 3-h cyclic runs of an oven, and the ovens can operate a limited number of cycles during the day. The developed MIP models form the product batches and assign these batches to the determined cycles during the day. In this respect, the modeling approach differs from the existing MIP models. The models also determine the required number of magazines to be processed to satisfy the daily production requirements for each product. The objective function differs from the available MIP models by minimizing the maximum cycle number scheduled among the ovens with the aim of minimizing the makespan. The objective function also guarantees the consecutive scheduling of cycles for an oven throughout the day by minimizing the maximum run cycle of an oven for each batch of products.

3. Problem Description

The company where the case study was conducted is one of the biggest European semiconductor manufacturers and designers. It is a global high-technology company with high-volume production and produces electronic

components, such as microcontrollers, integrated circuits (IC), and sensors for automotive, consumer electronics, health, communication, computer, peripheral, and industrial automation industries.

The case study focuses on the production line of Flip Chip Ball Grid Array (FCBGA) products, a mid-cost and high-performance semiconductor package. This production line produces five FCBGA products and consists of a series of the following processing steps illustrated in Figure 1.



Figure 1. The processing steps of the production line

Wafers and substrates constitute the raw materials of this production line. "Wafer Preparation" is the first step in the production process and involves several operations, such as sawing, mounting, and laser grooving, to make it suitable for the following steps. The other raw materials, substrates, are placed on boats individually in the "Tray-to-Boat" step; then, the boats are placed on magazines for transportation throughout the remaining steps of the process. The image of an empty boat, a loaded boat, and a magazine is illustrated in Figure 2, while the "Tray-to-Boat" step is given in Figure 3.

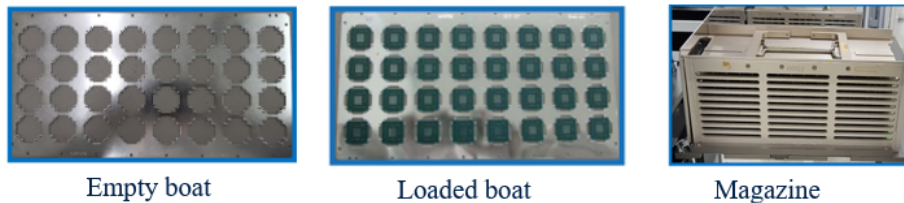


Figure 2. The image of boats and magazine

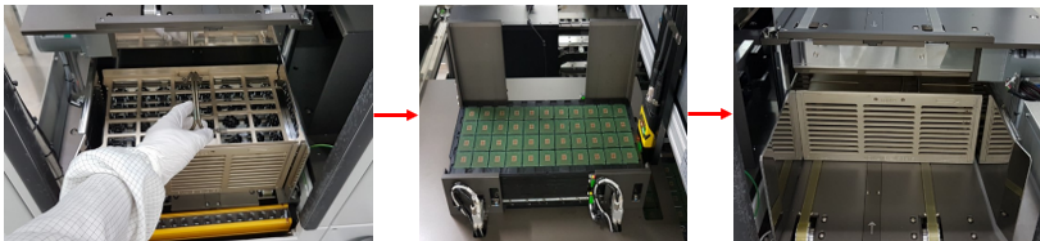


Figure 3. The illustration of the Tray-to-Boat step

Surface Mounted Device (SMD) components are soldered on the pads of substrates in the SMD step. Then, the "Plasma" step removes epoxy, soldering, flux, and all organic contaminants from the substrate surface. Subsequently, the "Die Attach" step attaches a silicon chip (die) from the wafer skeleton to the die pad of the substrate. Next, the "Underfill" step dispenses the glue along the edge of the die attached to the substrate to create a strong bond between the die and the substrate. Afterward, the substrates proceed through the "Underfill Cure" (UC) step to be heated in the ovens to strengthen this bond with high temperature and pressure. The "Lid Attach" is the last step that places the lid cover on the top of the die to encapsulate the substrate.

Our problem is observed at the UC step of the production line. We looked at the previous step, the underfill process, to determine if it affected the UC step. Three machines are available for this process that can handle all types of products. The only machine-related constraint is that mixed products cannot be simultaneously processed on the same machine. The daily capacity for underfill machines is 80K/day regardless of product type; thus, this production line step does not constitute a bottleneck. The magazines can be directly placed in the oven as soon as the underfill step is completed; no minimum waiting time is required between these steps. On the other hand, the materials can be kept for a maximum of 3 hours in a dry box before the UC step. If they cannot be placed in the ovens before 3 hours, they are mainly scrapped through the quality department, causing a high cost to the company. For this reason, all magazines must be taken to the ovens within the specified time limit to avoid high scrap costs.

The ovens run cyclically during the workday; the UC process takes 3 hours once the ovens are started. The ovens constitute the bottleneck of the production line due to non-interrupted long processing time, eligibility restrictions regarding the product types, random placement of magazines into eligible ovens, and low capacity compared to other processes in the production line. The other important reasons why the ovens could not be used efficiently are as follows:

- Daily production quantity is determined for each product without considering the capacity of UC ovens.
- The capacity of ovens is the same in terms of the number of magazines; however, the quantities each magazine can carry are different based on product type, making the capacity control of ovens difficult.
- The maximum time between the underfill and underfill cure steps must be respected.

For this reason, providing effective scheduling for the UC process is an important management concern. We identified the problem as the scheduling problem of parallel ovens with batch processing and eligibility constraints. We developed a mathematical programming-based solution approach that helps schedule the daily required production quantities of each product on the ovens. The proposed solution approach will determine the size of the batches processed in the ovens and their sequencing according to which cycle they are assigned. In this manner, operators will place magazines depending on the scheduling decisions. In addition, the ovens will only operate for a specified number of cycles during the day instead of running all day to be available to process the completed production orders from the underfill step whenever sent.

Besides, we propose to determine the production plan of the underfill step according to the schedules of ovens since the underfill process already has enough capacity to actualize this plan. With this backward scheduling, the completed production orders from the underfill step are processed within the specified maximum time in the UC step, thus eliminating quality defects and scrap costs. As the company will have a detailed production plan for the UC bottleneck step and the previous underfill step using the proposed models in this study, the company will use the available resources efficiently and successfully manage the production plan and order commitment.

4. Proposed Mathematical Model Formulations

We proposed two mixed-integer programming (MIP) models that will help improve the utilization rate of ovens by minimizing the maximum cycle number among the ovens, forming batches of magazines and assigning them to the appropriate cycles of the ovens to be treated. The main constraints that must be considered in the scheduling of ovens are the oven–product eligibility restrictions and monthly product demand. The first model (Model I) forms batches of magazines from only a single product type to load into an oven, whereas the second model (Model II) forms batches from different product types to place in the same oven. The assumptions common for both models are given as follows:

- The maximum number of cycles each oven can operate is known and is the same for all ovens.
- Each oven is run without interruption during each cycle.
- The number of magazines that can be placed in each oven is the same.
- The number of boats placed in the magazines changes according to the size of the product, making the oven capacity different for each product type.
- Eligibility requirements exist for the type of product that each oven can process.
- The maximum waiting time between the underfill and underfill cure steps is not considered because the production plan for the underfill process is made regarding the scheduling decisions of ovens obtained by the proposed model.

4.1 Integer Linear Programming Formulation of Model I

The notation used to develop Model I, the objective function, and the constraints are as follows:

Indices:

- i Index for the product type, $i = 1, 2, 3, \dots, I$
 o Index for the oven, $o = 1, 2, 3, \dots, O$
 c Index for the cycle number, $c = 1, 2, 3, \dots, C$ (max # of cycles: 6 in our case)

Parameters:

- K_i The capacity of the magazine based on product type i
 IQ_i Daily ordering quantity of product type i

| | |
|-----------|--|
| OT_{io} | 1, if oven o is eligible for processing product type i ; 0, otherwise. |
| Cap | Magazine capacity of each oven (max: nine magazines) |
| M | Big number |

Decision Variables:

| | |
|------------|---|
| X_{ioc} | 1, if product type i is assigned to oven o for cycle c , and 0 otherwise |
| P_i | The number of magazines required for product type i in line with daily planned production |
| Q_{ioc} | The number of magazines for product type i assigned to oven o in cycle c |
| C_{max} | The maximum cycle number scheduled among the ovens |
| CPO_{io} | The maximum cycle number scheduled for product type i assigned to oven o |

Objective Function:

$$\text{Min } M * C_{max} + \sum_{o=1}^O \sum_{i=1}^I CPO_{io} \quad (1)$$

Subject to:

$$x_{ioc} * c \leq C_{max} \quad \forall o, \forall c, \forall i \quad (2)$$

$$\sum_{i=1}^I x_{ioc} \leq 1 \quad \forall o, \forall c \quad (3)$$

$$\sum_{c=1}^C x_{ioc} \leq M * OT_{io} \quad \forall i, \forall o \quad (4)$$

$$K_i * P_i \geq IQ_i \quad \forall i \quad (5)$$

$$\sum_{o=1}^O \sum_{c=1}^C x_{ioc} * Cap \geq P_i \quad \forall i \quad (6)$$

$$\sum_{i=1}^I x_{i,o,c+1} \leq \sum_{i=1}^I x_{ioc} \quad \forall o, \forall c, c \leq C \quad (7)$$

$$Q_{ioc} \leq Cap * x_{ioc} \quad \forall o, \forall c, \forall i \quad (8)$$

$$\sum_{o=1}^O \sum_{c=1}^C Q_{ioc} = P_i \quad \forall i \quad (9)$$

$$x_{ioc} * c \leq CPO_{io} \quad \forall o, \forall c, \forall i \quad (10)$$

$$x_{ioc} \in \{0, 1\} \quad \forall i, \forall o, \forall c \quad (11)$$

$$P_i \geq 0 \text{ and integer} \quad \forall i \quad (12)$$

$$Q_{ioc} \geq 0 \text{ and integer} \quad \forall i, \forall o, \forall c \quad (13)$$

By multiplying the C_{max} decision variable with a big number ($M = 10000$), the objective function (1) first minimizes the maximum number of cycles (C_{max}) scheduled considering all ovens operating, providing a preemptive optimal solution. Then, the sum of the maximum number of cycles executed for each product and oven is minimized. The first term aims to minimize the makespan, while the second term allocates products to ovens as evenly as possible. Constraint (2) ensures that the C_{max} decision variable is either equal to or greater than the cycle number of each product for each oven, thus determining the maximum cycle number scheduled among all ovens. Constraint (3) guarantees that, at most, one product type is allocated to each oven for each cycle. According to Constraint (4), a product can only be assigned to an oven if it is eligible to be processed in that oven. The big-M value is taken as six in this constraint because the number of cycles scheduled for a product in an oven cannot exceed the maximum cycle number allowed. Constraint (5) determines the number of magazines required for product type i in line with the planned production quantity. Constraint (6) ensures that the required number of magazines allocated to the ovens during the workday satisfies the planned daily production quantities for the products. The scheduled cycles of each oven must be sequential when Constraint (7) is enforced. Constraint (8) guarantees that the maximum number of magazines for each cycle does not exceed the capacity of each oven if the related product is assigned. The total number of magazines assigned to the ovens for product type i is equal to the required number of magazines for that product by Constraint (9). Constraint (10) finds the maximum cycle number run for each product type on

each oven during the workday. Constraint (11) indicates the binary variables, whereas constraints (12 -13) indicate the integer variables.

4.2 Integer Linear Programming Formulation of Model II

The same notation used for Model I is used for developing Model II. The differences between Model I and Model II are as follows:

Constraint (3) is omitted since more than one product type can be processed in an oven for a given cycle, forming batches of different product types. The rest of the constraints are the same. In addition, the following Constraint (14) is added to the model.

$$\sum_{i=1}^I Q_{ioc} \leq Cap \quad \forall o, \forall c \quad (14)$$

Constraint (14) guarantees that the total number of magazines comprised of different product types assigned to each oven and cycle does not exceed the oven capacity.

5. Case Study

In the current production, five ovens can be used for the underfill cure process, and the capacity of each oven is nine, which can have only one product type. The ovens can work 24 hours a day, but according to time studies, the Overall Equipment Efficiency (OEE) calculated for the ovens is 90%. Therefore, the ovens can work 90% of the daily working time, and the remaining 10% is allocated for engineering, maintenance, break time, loading, and unloading. Eventually, the ovens can run for 21.6 hours during the workday. The duration of required activities that need to be done by operators is listed below:

- The duration of loading magazines to the oven: 6 mins
- The duration of unloading magazines to the oven: 3 mins
- The duration of the curing process: 3 hrs

The ovens are run cyclically during the workday. Therefore, considering the times mentioned above, each oven can be scheduled to run a maximum of 7 cycles daily. Not all ovens can treat all types of products because of the different temperature, pressure, and atmospheric profiles. Each product must be allocated to an eligible oven to meet the product reliability requirements. The eligibility constraints for the ovens are listed below:

- Oven 1: This oven can process Product 1 and Product 2.
- Oven 2: This oven can process Product 1, Product 2, and Product 3.
- Oven 3: This oven can process Product 4 and Product 5.
- Oven 4: This oven can process only Product 5.
- Oven 5: This oven can process all product types.

Each boat accommodates different numbers of products due to varying product sizes, whereas each magazine can carry only five boats. Table 1 shows us how many products each magazine can carry based on product type.

Table 1. The number of units in a magazine based on product type

| | Product Size | # Units/Boat | # Units/Magazine |
|-----------|--------------|--------------|------------------|
| Product 1 | 13x13 | 60 | 300 |
| Product 2 | 15x15 | 50 | 250 |
| Product 3 | 23x23 | 32 | 160 |
| Product 4 | 23x23 | 32 | 160 |
| Product 5 | 24x24 | 30 | 150 |

The production data for the past six months of 2022 were initially extracted from the SAP system; the daily production quantities of each product are illustrated in Table 2. According to the planning calendar of the company, every quarterly period is considered three months and 13 weeks as a standard. For this reason, the first and second months of each quarter have assumed four weeks, and the third month has five weeks, which is equivalent to 28 working days for the four-week months and 35 working days for the five-week months. If there is any shutdown

due to public holidays or scheduled maintenance, it is deduced from the working days. For instance, there are three days of public holiday in October 2022 (Month 10'22), so all product orders must be completed in 25 days.

Models I and II were run using LINGO 20.0 Optimization software for these six months based on the required daily production quantities. The recommended daily production schedules for the ovens were developed for each month, respectively. Models I and II are composed of 381 variables and 631 constraints, and it took less than one second to run the models.

Table 2. The daily product order quantity for the past six months

| Months | # Working Days/Month | Daily Product Order Quantity | | | | |
|-------------|----------------------|------------------------------|-----------|-----------|-----------|-----------|
| | | Product 1 | Product 2 | Product 3 | Product 4 | Product 5 |
| Month 7'22 | 28 | 7250 | 5846 | 9907 | 11800 | 0 |
| Month 8'22 | 28 | 6953 | 4450 | 10380 | 12392 | 0 |
| Month 9'22 | 35 | 8750 | 6500 | 10908 | 9430 | 0 |
| Month 10'22 | 25 | 6750 | 6100 | 11150 | 12402 | 0 |
| Month 11'22 | 28 | 8901 | 6250 | 13519 | 13643 | 0 |
| Month 12'22 | 35 | 7692 | 4286 | 13948 | 13940 | 0 |

5.1 Implementation of Model I

The maximum cycle number scheduled considering all the ovens operating daily (C_{max}) and the maximum cycle number scheduled for each product processed in the assigned ovens (CPO_{oi}) are listed in Table 3. The maximum cycle number is six. Accordingly, product 1 is last processed in Oven 1 in cycle 6 ($CPO_{11} = 6$), and product 2 is last processed in Oven 1 in cycle 3 ($CPO_{21} = 3$).

Table 3. The maximum cycle number scheduled for the products on the assigned ovens for Month 7'22

| Max. Cycle Numbers | |
|--------------------|---|
| C_{max} | 6 |
| CPO_{11} | 6 |
| CPO_{21} | 3 |
| CPO_{32} | 6 |
| CPO_{43} | 4 |
| CPO_{35} | 1 |
| CPO_{45} | 6 |

Table 4 summarizes the results for the total number of magazines for each product and the number of magazines assigned to ovens and cycles. The number of products that can be placed in a magazine varies depending on the product type. The total number of magazines processed for each product (P_i) is calculated by dividing the product's daily production quantity by the magazine's capacity for the related product type and rounding it up to a higher integer when necessary to meet the daily production requirements.

Table 4. The number of magazines allocated to ovens and cycles for each product in Month 7'22

| Product | # Magazines/Product | # Assigned Magazines |
|---------|---------------------|----------------------|
|---------|---------------------|----------------------|

| | | |
|-------|----|---|
| P_1 | 25 | $Q_{114} = 9, Q_{115} = 9, Q_{116} = 7$ |
| P_2 | 24 | $Q_{211} = 9, Q_{212} = 9, Q_{213} = 6$ |
| P_3 | 62 | $Q_{321} = 9, Q_{322} = 9, Q_{323} = 9, Q_{324} = 9$ $Q_{325} = 9, Q_{326} = 8, Q_{351} = 9$ |
| P_4 | 74 | $Q_{431} = 9, Q_{432} = 9, Q_{433} = 9, Q_{434} = 9$ $Q_{452} = 9, Q_{453} = 9, Q_{454} = 9, Q_{455} = 9, Q_{456} = 2$ |
| Total | | 185 |

The Q_{ioc} values show the assignment of magazines to ovens and cycles for each product. For instance, the daily production quantity of product 1 is 7250 units in Month 7'22, and the magazine's capacity for this product is 300; the P_1 value is found to be 25 magazines. These 25 magazines are allocated as nine magazines to Oven 1 in cycle 4, 9 magazines to Oven 1 in cycle 5, and 7 magazines to Oven 1 in cycle 6. Considering all product order quantities, 185 magazines have been scheduled daily.

From the results in Table 4, the proposed schedules of ovens by Model I for Month 7'22 are illustrated in Figure 4. This Gantt chart displays the proposed daily production schedule for ovens.

| Month 7 '22 | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 | Cycle 7 |
|-------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|---------|
| RESOURCES | | | | | | | |
| Oven 1 | Order 2 9 magazines | Order 2 9 magazines | Order 2 6 magazines | Order 1 9 magazines | Order 1 9 magazines | Order 1 7 magazines | |
| Oven 2 | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 8 magazines | |
| Oven 3 | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | | | |
| Oven 4 | | | | | | | |
| Oven 5 | Order 3 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 2 magazines | |

Figure 4. The Gantt chart of the proposed schedules of ovens for Month 7'22 using Model I

The schedules suggested for the ovens obtained from Model 1 for the rest of the months are illustrated in Figure 5. Oven 4 is not scheduled since it is only eligible to produce Product 5, and this product is new and will be produced after Month 3'23. If the product orders had been produced according to the obtained schedules, the ovens could have been planned more efficiently according to the needs of the production. For instance, in Month 7'22 (Figure 4), while Ovens 1, 2, and 5 would have been scheduled to work for six cycles, Oven 3 would have been scheduled for four cycles. In this way, the company would have more capacity to confirm more product orders. A similar situation may be observed for other ovens.

| Month 8 '22 | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 | Cycle 7 |
|--------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| Oven 1 | Order 2 9 magazines | Order 2 9 magazines | Order 1 9 magazines | Order 1 9 magazines | Order 1 6 magazines | | |
| Oven 2 | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 2 magazines | |
| Oven 3 | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | | |
| Oven 4 | | | | | | | |
| Oven 5 | Order 3 9 magazines | Order 3 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 6 magazines | |
| Month 9 '22 | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 | Cycle 7 |
| Oven 1 | Order 2 9 magazines | Order 1 9 magazines | Order 1 9 magazines | Order 1 9 magazines | Order 1 3 magazines | | |
| Oven 2 | Order 2 9 magazines | Order 2 8 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | |
| Oven 3 | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 5 magazines | | |
| Oven 4 | | | | | | | |
| Oven 5 | Order 4 9 magazines | Order 4 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 6 magazines | |
| Month 10 '22 | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 | Cycle 7 |
| Oven 1 | Order 2 9 magazines | Order 2 9 magazines | Order 2 7 magazines | Order 1 9 magazines | Order 1 9 magazines | Order 1 5 magazines | |
| Oven 2 | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 7 magazines | |
| Oven 3 | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | | |
| Oven 4 | | | | | | | |
| Oven 5 | Order 3 9 magazines | Order 3 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 6 magazines | |
| Month 11 '22 | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 | Cycle 7 |
| Oven 1 | Order 2 9 magazines | Order 2 9 magazines | Order 2 7 magazines | Order 1 9 magazines | Order 1 9 magazines | Order 1 9 magazines | Order 1 3 magazines |
| Oven 2 | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines |
| Oven 3 | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 5 magazines |
| Oven 4 | | | | | | | |
| Oven 5 | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 4 magazines | |
| Month 12 '22 | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 | Cycle 7 |
| Oven 1 | Order 2 9 magazines | Order 2 9 magazines | Order 1 9 magazines | Order 1 9 magazines | Order 1 8 magazines | | |
| Oven 2 | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 7 magazines |
| Oven 3 | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | |
| Oven 4 | | | | | | | |
| Oven 5 | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 7 magazines |

Figure 5. The Gantt chart of the proposed schedules of ovens for the past months using Model I

The results of Model I are summarized in Table 5 regarding the total number of magazines processed daily, the maximum cycle number of each oven run during the day, and the occupancy rates of ovens for each month. The occupancy rate of each oven is calculated by dividing the number of magazines processed by each oven by the total magazine capacity of the oven, considering the scheduled cycles during the workday. Let us calculate the occupancy rate of Oven 1 for Month 7'22. The ovens can process a maximum of 9 magazines in a cycle, so when Oven 1 works for six cycles as scheduled, it can process a maximum of 54 magazines daily. We have allocated 49 magazines to this oven for processing during the day. Thus, the occupancy rate is calculated as 91% (49/54).

Table 5. Summarized results for the past six months with Model I

| | | Oven 1 | Oven 2 | Oven 3 | Oven 5 |
|----------------|---|--------|--------|--------|--------|
| Month 7'22 | Total number of processed magazines daily | 49 | 53 | 36 | 47 |
| | Occupancy rate | 91% | 98% | 100% | 87% |
| | Maximum cycle number | 6 | 6 | 4 | 6 |
| Month 8'22 | Total number of processed magazines daily | 42 | 47 | 45 | 51 |
| | Occupancy rate | 93% | 87% | 100% | 94% |
| | Maximum cycle number | 5 | 6 | 5 | 6 |
| Month 9'22 | Total number of processed magazines daily | 39 | 53 | 41 | 51 |
| | Occupancy rate | 87% | 98% | 91% | 94% |
| | Maximum cycle number | 5 | 6 | 5 | 6 |
| Month 10'22 | Total number of processed magazines daily | 48 | 52 | 45 | 51 |
| | Occupancy rate | 89% | 96% | 100% | 94% |
| | Maximum cycle number | 6 | 6 | 5 | 6 |
| Month 11'22 | Total number of processed magazines daily | 55 | 63 | 59 | 49 |
| | Occupancy rate | 87% | 100% | 94% | 91% |
| | Maximum cycle number | 7 | 7 | 7 | 6 |
| Month 12'22 | Total number of processed magazines daily | 44 | 61 | 54 | 61 |
| | Occupancy rate | 98% | 97% | 100% | 86% |
| | Maximum cycle number | 5 | 7 | 6 | 7 |

5.2 Implementation of Model II

Model II is run for the past six months' data, and the proposed schedules for each month for the ovens are displayed in Figure 6. Compared to the proposed schedule for Oven 1 for month 7'22 in Figure 4, the number of cycles scheduled is reduced to 5 from 6. This reduction has been achieved by scheduling a batch of products 1 and 4 for the second cycle of Oven 5, marked in yellow in Figure 6. The other yellow marks represent similar situations for other months and ovens. The results of Model II are summarized in Table 6. The improved cycle numbers and occupancy rates compared to those of Model I are given in bold.

| Month 7 '22 | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 | Cycle 7 |
|--------------|--|--|--|------------------------|------------------------|------------------------|------------------------|
| Oven 1 | Order 2 9 magazines | Order 2 9 magazines | Order 2 6 magazines | Order 1 9 magazines | Order 1 9 magazines | | |
| Oven 2 | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 8 magazines | |
| Oven 3 | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | | | |
| Oven 4 | | | | | | | |
| Oven 5 | Order 3 9 magazines | Order 4 2 magazines Order 1 7 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | |
| Month 8 '22 | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 | Cycle 7 |
| Oven 1 | Order 2 9 magazines | Order 2 9 magazines | Order 1 9 magazines | Order 1 9 magazines | Order 1 6 magazines | | |
| Oven 2 | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | | |
| Oven 3 | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | | |
| Oven 4 | | | | | | | |
| Oven 5 | Order 3 2 magazines Order 4 6 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | |
| Month 9 '22 | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 | Cycle 7 |
| Oven 1 | Order 2 9 magazines | Order 2 9 magazines | Order 2 8 magazines | Order 1 9 magazines | Order 1 9 magazines | | |
| Oven 2 | Order 1 3 magazines Order 3 6 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | | |
| Oven 3 | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 5 magazines | | |
| Oven 4 | | | | | | | |
| Oven 5 | Order 4 9 magazines | Order 4 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | | |
| Month 10 '22 | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 | Cycle 7 |
| Oven 1 | Order 2 9 magazines | Order 2 9 magazines | Order 2 7 magazines | Order 1 9 magazines | Order 1 9 magazines | | |
| Oven 2 | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | |
| Oven 3 | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | | |
| Oven 4 | | | | | | | |
| Oven 5 | Order 3 4 magazines Order 1 5 magazines | Order 3 9 magazines | Order 3 3 magazines Order 4 6 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | |
| Month 11 '22 | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 | Cycle 7 |
| Oven 1 | Order 2 9 magazines | Order 2 9 magazines | Order 2 7 magazines | Order 1 9 magazines | Order 1 9 magazines | Order 1 9 magazines | Order 1 3 magazines |
| Oven 2 | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines |
| Oven 3 | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines |
| Oven 4 | | | | | | | |
| Oven 5 | Order 4 5 magazines Order 3 4 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 3 9 magazines | Order 3 9 magazines | | |
| Month 12 '22 | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 | Cycle 7 |
| Oven 1 | Order 2 9 magazines | Order 2 9 magazines | Order 1 9 magazines | Order 1 9 magazines | Order 1 8 magazines | | |
| Oven 2 | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 3 7 magazines |
| Oven 3 | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | |
| Oven 4 | | | | | | | |
| Oven 5 | Order 3 9 magazines | Order 3 9 magazines | Order 3 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 9 magazines | Order 4 7 magazines |

Figure 6. The Gantt chart of the proposed schedules for ovens for the past six months with Model II

Table 6. Summarized results for the past six months with Model II

| | | Oven 1 | Oven 2 | Oven 3 | Oven 5 |
|-------------|---|------------|-------------|-------------|-------------|
| Month 7'22 | Total number of processed magazines daily | 42 | 53 | 36 | 54 |
| | Occupancy Rate | 93% | 98% | 100% | 100% |
| | Maximum cycle number | 5 | 6 | 4 | 6 |
| Month 8'22 | Total number of processed magazines daily | 42 | 45 | 45 | 53 |
| | Occupancy Rate | 93% | 100% | 100% | 98% |
| | Maximum cycle number | 5 | 5 | 5 | 6 |
| Month 9'22 | Total number of processed magazines daily | 53 | 45 | 41 | 45 |
| | Occupancy Rate | 98% | 100% | 91% | 100% |
| | Maximum cycle number | 6 | 5 | 5 | 5 |
| Month 10'22 | Total number of processed magazines daily | 43 | 54 | 45 | 54 |
| | Occupancy Rate | 96% | 100% | 100% | 100% |
| | Maximum cycle number | 5 | 6 | 5 | 6 |
| Month 11'22 | Total number of processed magazines daily | 55 | 63 | 63 | 45 |
| | Occupancy Rate | 87% | 100% | 100% | 100% |
| | Maximum cycle number | 7 | 7 | 7 | 5 |
| Month 12'22 | Total number of processed magazines daily | 44 | 61 | 54 | 61 |
| | Occupancy Rate | 98% | 97% | 100% | 97% |
| | Maximum cycle number | 5 | 7 | 6 | 7 |

5.3 Comparison of Model I, Model II, and Past Production Decisions

The current average occupancy rate for each oven is calculated by dividing the number of magazines processed by each oven monthly by the total magazine capacity of ovens for 7-cycle daily work during the month, as given in Table 7. In the current situation, since the production plan is unclear at the beginning of the month, the ovens must run continuously throughout the day since they require long heating times for the coming product order when stopped. For this reason, the ovens work full day even if they are not loaded with magazines. An oven can process a maximum of nine magazines in a cycle, so when it works for 7 cycles, it can process 63 magazines in a day. The monthly total magazine capacity is calculated when the daily magazine capacity is multiplied by the number of working days each month. When the monthly magazine quantity processed is divided by the monthly magazine capacity, e.g., for Month 8'22, the occupancy rate is calculated as 73% (5180/7056).

Table 7. The current average occupancy rate for the past six months

| Month | #Working Days | # Daily Magazine Quantity Produced | # Monthly Magazine Quantity Produced | Monthly Magazine Capacity | Current Average Occupancy Rate |
|-------------|---------------|------------------------------------|--------------------------------------|---------------------------|--------------------------------|
| Month 8'22 | 28 | 185 | 5180 | 7056 | 73% |
| Month 8'22 | 28 | 185 | 5180 | 7056 | 73% |
| Month 9'22 | 35 | 184 | 6440 | 8820 | 73% |
| Month 10'22 | 25 | 196 | 4900 | 6300 | 78% |
| Month 11'22 | 28 | 226 | 6328 | 7056 | 90% |
| Month 12'22 | 35 | 220 | 7700 | 8820 | 87% |

Figure 7 shows the results of Model I and Model II compared with the current situation as a summary. Using Model I, the average occupancy rate increases while the average number of cycles decreases. For instance, the product orders would have been completed on an average of 5.5 cycles instead of 7 cycles for Month 7'22, and

the occupancy rate would have been 94% instead of %73. With Model II, we can further improve the capacity usage. For instance, if it were possible to batch different products, the orders of Month 7'22 would have been completed on an average of 5.25 cycles, and the occupancy rate would have been 98%. This way, the company can produce more orders using capacity more efficiently.

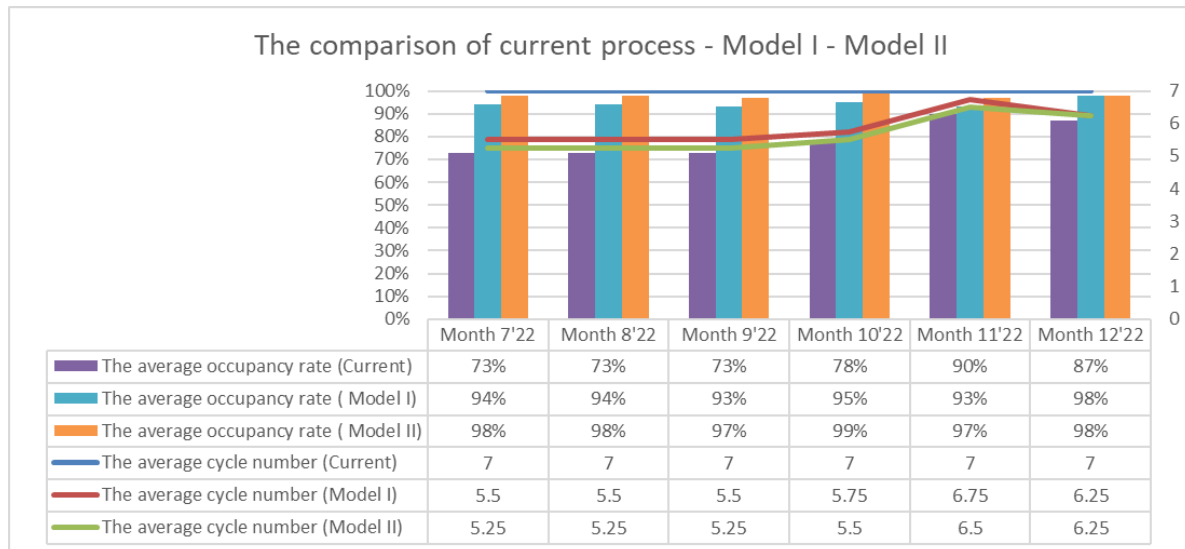


Figure 7. The comparison between the current situation - Model I – Model II

6. Conclusion

In today's economy, rapid growth and competitiveness are on the cutting edge of the industry. Therefore, it is critical to schedule manufacturing resources efficiently. We can apply different scheduling methods for scheduling these resources for different cases in different areas of the industry. Semiconductor manufacturers need a good scheduling strategy to compete in a growing industry. Otherwise, they cannot use resources efficiently and cannot quickly respond to these demands. This study proposed new MIP models to schedule batch-processing parallel ovens with eligibility constraints for a semiconductor manufacturing company. Two MIP models have been formulated concerning the real case of this company. The first MIP model (Model I) was developed based on the restriction that only one product-type batch can be placed in the same oven for the underfill cure operation to avoid mixing between the product types. The second MIP model (Model II) was developed based on batching of different product types for increased flexibility in scheduling parallel batch processing ovens.

This paper has the following advantages for the company:

- The company can prepare the daily production schedule for ovens at the beginning of each month.
- The company can extend the production plan to the previous production step to prevent the scrap cost due to expired staging time between underfill and underfill cure operations.
- The company can manage the oven capacities more efficiently and commit to more product orders or pull-in requests.
- Model II facilitates a scheduling plan with batches of different types of products. In this way, the usage of the ovens can be increased relatively more than that of Model I.

In future research, uncertainties about daily production requirements and oven capacities can be modeled and solved using simulation-based optimization or stochastic programming.

Contributions of Authors

Şeyda Topaloğlu Yıldız: Conceptualization, Supervision, Methodology, Software, Validation, Writing – original draft, **Ezgi Güleç:** Methodology, Software, Validation, Data curation, Writing - review and editing.

Conflicts of Interest

No potential conflicts of interest were reported by the authors.

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