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Application of Intuitionistic Fuzzy Topological Operators in Spatial Objects Modeling

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Abstract

The concept of topology is widely used in mathematical modeling of spatial objects and in GIS. One such application of fuzzy topological operators, detecting areas affected by Mikania micrantha, was studied by Shi and Liu. The precision of the results was evaluated by applying the intuitionistic fuzzy pre-interior and pre-clouser operators defined by the author to the data included in this study.

With this application example, it has been shown that the newly defined operators give results closer to the real bounds.

1. Introduction

The most important step for modeling real-world problems is to determine the topological relationships of spatial objects. Topological concepts are used in the analysis of spaces with uncertain boundaries, such as rivers, forests, oceans etc. The boundary determined by the crisp set topology are not suitable for the evaluation of problems involving uncertainty. On the other hand, fuzzy topology allows the definition of uncertainty in these spatial objects. Zadeh (1965) introduced the concept of the fuzzy set as an generalization of crisp set and the theory of fuzzy topology has been developed in following years. Some models have been developed by researchers to modelling the fuzzy relationship between spatial objects. Initially, the interior, boundary and exterior of spatial objects are determined. In the next step, fuzzy membership functions are defined to determine the membership degrees of these topological concepts. Since fuzzy topological concepts depend only on membership degrees, they are not sufficient to rank the hesitation relationships of objects. Intuitionistic fuzzy sets, introduced by Atanassov in 1983, allowed to modelling hesitation relationships of objects that contain uncertainty. Çoker defined the intuitionistic fuzzy topology in 1997[5]. Studies have been conducted by several authors to develop this concept. The topology used in modeling real-world problems is based on identifying and using the common points of different spatial objects. The application of intuitionistic fuzzy topology on spatial objects was examined firstly by M.R. Malek[9]. In 2006, Saadati and Park examined properties of intuitionistic fuzzy metric spaces[12]. Singh and Srivastava studied the separation axioms[14]. In 2020, Marinov and Atanassov defined pre-interior and pre-clouser intuitionistic fuzzy topological operators and showed that these operators can be applied to determine the topological relations of spatial objects[10]. Marinov presented a software implementation of the framework of intuitionistic fuzzy sets[11]. In [16], author introduced $I_{\alpha,\beta}^{\gamma,\omega}$ and $C_{\alpha,\beta}^{\gamma,\omega}$ intuitionistic fuzzy topological operators. In this paper an application of these operators is examined.

Atanassov introduced the concept of Intuitionistic Fuzzy Sets, form an extension of fuzzy sets by expanding the truth value set to the $L^* = \{(x_1, x_2) \in [0,1]^2 \ : \ x_1 + x_2 \leq 1\}$ is a lattice with $(x_1, x_2) \leq (y_1, y_2) :\Leftrightarrow "x_1 \leq y_1 \ \ and \ \ x_2 \geq y_2"$. The operator theory have an important role in intuitionistic fuzzy sets. The concept of intuitionistic fuzzy modal operators was given in 1999 by K. Atanassov and then modal operators have been extensively studied in theoretical and application areas[2, 3, 6, 7]. First

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intuitionistic fuzzy topological operators were defined and new intuitionistic fuzzy topological operators were introduced in subsequent studies[3, 8, 16]. Fuzzy pre-topological/topological operators and intuitionistic fuzzy pre-topological/topological operators were applied to compute of the values of fuzzy relations of spatial objects with uncertainty in determining the boundaries such as forest area, lake, sea, etc [10, 13, 16].

2. Material and Method

Definition[2] An intuitionistic fuzzy set (IFS) on a set X is an object

$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \ : \ x\in X\}$ where $\mu_A(x),(\mu_A \ : \ X\to[0,1])$ is called the degree of membership of x in A, $\nu_A(x),(\nu_A \ : \ X\to[0,1])$ is called the degree of non- membership of x in A,and where μ_A and ν_A satisfy the following condition:

$\mu_A(x)+\nu_A(x)\le 1$, for all $x\in X$.

The class of intuitionistic fuzzy sets on X is denoted by $IFS(X)$. The hesitation degree of x is defined by $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$.

Definition[2] An IFS A is said to be contained in an IFS B (A ô B) if and only if, for all $x\in X \ : \ \mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\ge\nu_B(x)$.

It is clear that $A=B$ if and only if A ô B and B ô A.

Definition[2] Let $A\in IFS$ and let $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \ : \ x\in X\}$ then the above set is callede the complement of A

$A^c=\{\langle x,\nu_A(x),\mu_A(x)\rangle \ : \ x\in X\}$

The intersection and the union of two IFSs A and B on X is defined by

A⊓B
$=\left\{\langle x,\mu_A(x)\wedge\mu_B(x),\nu_A(x)\vee\nu_B(x)\rangle \ : \ x\in X\right\}$

A⊔B
$=\left\{\langle x,\mu_A(x)\vee\mu_B(x),\nu_A(x)\wedge\nu_B(x)\rangle \ : \ x\in X\right\}$

Some special Intuitionistic Fuzzy Sets on X are defined as following;

$O^*=\left\{\langle x,0,1\rangle \ : \ x\in X\right\}$

$X^*=\{<x,1,0>: \ x\in X\}$

Definition[1] An $pre-$closure operator $\mathbf{c} \ : \ \mathbf{X}\to\mathbf{X}$ is a map which associates to each set $A\in\mathbf{X}$ a set $\mathbf{c}(A)$ such that;

1) $\mathbf{c}(\varnothing)=\varnothing$
2) $A\subseteq\mathbf{c}(A)$
3) $\mathbf{c}(A\cup B)=\mathbf{c}(A)\cup\mathbf{c}(B)$, for all $A,B\subset X$.

If in addition to above axioms the operator $\mathbf{c}$ is idempotent, that is $\mathbf{c}(A)=\mathbf{c}(\mathbf{c}(A))$ then $\mathbf{c}$ is called closure operator in $\mathbf{X}$. $\mathbf{X}$ can be $\wp(X),FS(X)$ or $IFS(X)$.

Definition[1] For the $pre-$closure operator $\mathbf{c}$ defined on $\mathbf{X}$ we say that a set $A\in\mathbf{X}$ is closed iff $\mathbf{c}(A)=A$. Also,

$\tau^{\mathbf{c}}=\left\{A \ : \ A\in\mathbf{X} \ \& \ \mathbf{c}(A)=A\right\}$

is the topology generated by the $pre-$closure operator $\mathbf{c}$. If $\mathbf{X}$ is $\wp(X),FS(X)$ or $IFS(X)$ then τ is called crisp topology, fuzzy topology or intuitionistic fuzzy topology, repectively.

Definition[1] An $pre-$ interior operator $\mathbf{i} \ : \ \mathbf{X}\to\mathbf{X}$ is a map which associates to each set $A\in\mathbf{X}$ a set $\mathbf{i}(A)$ such that;

1. $\mathbf{i}(\mathbf{X})=\mathbf{X}$
2. $\mathbf{i}(A)\subseteq A$
3. $\mathbf{i}(A\cap B)=\mathbf{i}(A)\cap\mathbf{i}(B)$, for all $A,B\subset X$.

If in addition to above axioms the operator $\mathbf{i}$ is idempotent, that is $\mathbf{i}(A)=\mathbf{i}(\mathbf{i}(A))$ then $\mathbf{i}$ is called interior operator in $\mathbf{X}$. $\mathbf{X}$ can be $\wp(X),FS(X)$ or $IFS(X)$.

Definition[1] For the $pre-$interior operator $\mathbf{i}$ defined on $\mathbf{X}$ we say that a set $A\in\mathbf{X}$ is open iff

$\mathbf{i}(A)=A$. Also, $\tau_{\mathbf{i}}=\{A\ :\ A\in\mathbf{X}\ \&\ \mathbf{i}(A)=A\}$ is the topology generated by the $pre-$ interior operator $\mathbf{i}$. If $\mathbf{X}$ is $\wp(X), FS(X)$ or $IFS(X)$ then τ is called crisp topology, fuzzy topology or intuitionistic fuzzy topology, repectively.

If $\mathbf{i}$ is $pre-$ interior operator then $\mathbf{c}(A)=\neg\mathbf{i}(\neg A)$ is its corresponding $pre-$ closure. That is $(\mathbf{c}(A),\neg\mathbf{i}(\neg A))$ is a pair of conjugate preclosure-preinterior operators.

Proposition[1] If $\mathbf{i}$ and $\mathbf{c}$ is a conjugate pair of preinterior and preclosure operators in $\mathbf{X}$, then

$\tau^{\mathbf{c}}=\{\neg A\ :\ A\in\tau_{\mathbf{i}}\}$ and $\tau_{\mathbf{i}}=\{\neg B\ :\ B\in\tau^{\mathbf{c}}\}$.

In [10], Marinov and Atanassov generalized the $pre-$ interior and $pre-$ closure operators to intuitionistic fuzzy sets and they introduced new intuitionistic fuzzy topological operators. In the same paper, they examined topological properties of these operators. Then, author defined new generalized intuitionistic fuzzy topological operators by considering the operators defined by Marinov and Atanassov in [16].

Definition[16] Let X be a set and $A\in IFS(X)$. For $\alpha,\beta,\gamma,\omega\in[0,1]$, the topological operator $I_{\alpha,\beta}^{\gamma,\omega}$ is defined as follow;

$$I_{\alpha,\beta}^{\gamma,\omega}\ :\ IFS(X)\to IFS(X)$$

such that

$$\mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x)=\begin{cases}\inf\mu_A(x), & 0\le\mu_A(x)<\alpha\gamma(1-\beta)\\ (1-\beta)\mu_A(x), & \alpha\gamma(1-\beta)\le\mu_A(x)<\alpha\gamma\\ \frac{1}{1-\gamma}(\mu_A(x)-\alpha)+\alpha, & \alpha\gamma\le\mu_A(x)<\alpha\\ \mu_A(x), & \alpha\le\mu_A(x)\le 1\end{cases}$$

and

$$\nu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x)=\begin{cases}\nu_A(x), & 0\le\nu_A(x)<\beta\omega\\ \min\left\{(1-\omega)\nu_A(x)+\beta\omega, 1-\mu_{I_{\alpha,\beta}^{\gamma,\omega}(A)}(x)\right\}, & \beta\omega\le\nu_A(x)<\beta\\ \nu_A(x), & \beta\le\nu_A(x)\le 1\end{cases}$$

Proposition[16] Let X be a set and $A\in IFS(X)$. For $\alpha,\beta,\gamma,\omega\in[0,1]$, the topological operator $I_{\alpha,\beta}^{\gamma,\omega}(A)$ is an intuitionistic fuzzy set.

Proposition[16] Let X be a set and $A\in IFS(X)$. For $\alpha,\beta,\gamma,\omega\in[0,1]$, the operator $I_{\alpha,\beta}^{\gamma,\omega}(A)$ is a $pre-$ interior operator in $IFS(X)$.

Definition[16] Let X be a set and $A\in IFS(X)$. For $\alpha,\beta,\gamma,\omega\in[0,1]$, the topological operator $C_{\alpha,\beta}^{\gamma,\omega}$ is defined as follow;

$$C_{\alpha,\beta}^{\gamma,\omega}\ :\ IFS(X)\to IFS(X)$$

such that

$$\mu_{C_{\alpha,\beta}^{\gamma,\omega}(A)}(x)=\begin{cases}\mu_A(x), & 0\le\mu_A(x)<\beta\omega\\ \min\left\{\begin{matrix}(1-\omega)\mu_A(x)+\beta\omega,\\ 1-\nu_{C_{\alpha,\beta}^{\gamma,\omega}(A)}(x)\end{matrix}\right\}, & \beta\omega\le\mu_A(x)<\beta\\ \mu_A(x), & \beta\le\mu_A(x)\le 1\end{cases}$$

and

$$\nu_{C_{\alpha,\beta}^{\gamma,\omega}(A)}(x)=\begin{cases}\inf\nu_A(x), & 0\le\nu_A(x)<\alpha\gamma(1-\beta)\\ (1-\beta)\nu_A(x), & \alpha\gamma(1-\beta)\le\nu_A(x)<\alpha\gamma\\ \frac{1}{1-\gamma}(\nu_A(x)-\alpha)+\alpha, & \alpha\gamma\le\nu_A(x)<\alpha\\ \nu_A(x), & \alpha\le\nu_A(x)\le 1\end{cases}$$

Proposition[16] Let X be a set and $A\in IFS(X)$. For $\alpha,\beta,\gamma,\omega\in[0,1]$, the topological operator $C_{\alpha,\beta}^{\gamma,\omega}(A)$ is an intuitionistic fuzzy set.

Proposition[16] Let X be a set and $A\in IFS(X)$. For $\alpha,\beta,\gamma,\omega\in[0,1]$, the operator $C_{\alpha,\beta}^{\gamma,\omega}(A)$ is a $pre-$ closure operator in $IFS(X)$.

Proposition[16] The operator $I_{\alpha,\beta}^{\gamma,\omega}$ is generalization of the operator I_μ and the operator $C_{\alpha,\beta}^{\gamma,\omega}$ is generalization of the operator C_ν.

Theorem[16] Let X be a set and $A\in IFS(X)$ then $C_{\alpha,\beta}^{\gamma,\omega}(A)=\neg I_{\alpha,\beta}^{\gamma,\omega}(\neg A)$, i.e $I_{\alpha,\beta}^{\gamma,\omega}$ and $C_{\alpha,\beta}^{\gamma,\omega}$ is a conjugate pair of $pre-$ interior and $pre-$ closure operators. They define the same topology

$$\tau_{I_{\alpha,\beta}^{\gamma,\omega}}=\left\{\neg B\ :\ B\in\tau^{C_{\alpha,\beta}^{\gamma,\omega}}\right\}.$$

The boundary of set A in the intuitionistic fuzzy topology defined by these operators is

$\partial A=C_{\alpha,\beta}^{\gamma,\omega}(A)\cap\left(\neg I_{\alpha,\beta}^{\gamma,\omega}(A)\right)$ according to the IF boundary definition given by Malek[9].

Intuitionistic fuzzy generators are used to construct IFS and they are defined as a function:

Definition[4] A function $\phi : [0,1] \to [0,1]$ will be called intuitionistic fuzzy generator(IFG) if $\phi(x) \le 1 - x$ for all $x \in X$.

3. Results and Discussion

In this study, the problem of detecting areas affected by Mikania micrantha, examined by Shi and Liu[13], is re-examined using intuitionistic fuzzy topological operators, $pre-$ interior and $pre-$closure.

Methodology

(i) Aerial photographs of the studied area are viewed as intuitionistic fuzzy spaces.

(ii) Areas affected by Mikania micrantha in the aerial photo are viewed as intuitionistic fuzzy sets in each of intuitionistic fuzzy spaces.

(iii) The membership and non-membership values of affected areas are calculated.

(iv) The fuzzy value is defined by

$$\mu_A(x) = \begin{cases} \dfrac{\log(\text{Area of certain affected area})}{\log(\text{Total area of affected area})} & if \quad \dfrac{\log(\bullet)}{\log(*)} > 0 \\ 0 & otherwise \end{cases}$$

(v) From Sugeno's generator, the intuitionistic fuzzy non-membership function is given as[15]:

$\nu_A(x) = \dfrac{1-\mu_A(x)}{1+\lambda\mu_A(x)}$, λ is computed using intuitionistic fuzzy entropy(IFE) which is given as:

An important intuitionistic fuzzy generator is defined by Sugeno as following and implemented in the example examined in this study:

$$A_\lambda = \left\{ \left(x, \mu_A(x), \frac{1-\mu_A(x)}{1+\lambda\mu_A(x)} \right) : x \in X \right\}$$

$$IE(A) = \frac{1}{N} \sum_{j=1}^{N} \sum_{i=1}^{N} \pi_A(x_{ij}) e^{1-\pi_A(x_{ij})},$$

and the optimum value of λ is computed as: $\lambda_{opt} = \max_\lambda \left(IE(A, \lambda) \right)$. In this example, $h = 0.5$ was chosen.

Results and Analysis

Each Mikania micrantha area has an identity number and its boundary has been digitized on the aerial photos by Shi and Liu. Table 1 shows the size of each area affected by Mikania micrantha on an aerial photo[13].
In Table2, membership and non-membership degrees of areas were calculated, pre-interior, pre-closure and boundary values obtained: for
This method offers a suitable classification for modeling spatial objects that do not have clear boundaries. While fuzzy topological operators only make a classification over membership degrees, the operators used here provide a classification based on the non-membership degrees and hestigation degrees of spatial objects. Thus, an evaluation much closer to the real bounders is obtained. For different values of provide different values of interior, boundary, and exterior

Table 1. Size of Each Area Affected by Mikania Micrantha

ID	Area	ID	Area	ID	Area
1	7,97	19	60,5	37	195,46
2	8,17	20	61,97	38	265,48
3	8,8	21	63,86	39	293,35
4	10,23	22	73,64	40	312,6
5	10,37	23	73,83	41	315,02
6	15,32	24	76,67	42	343,49
7	17,09	25	77,1	43	349,76
8	17,52	26	82,68	44	388,28
9	24,75	27	85,58	45	401,55
10	25,12	28	87,16	46	403,61
11	28	29	93,8	47	498,05
12	31,69	30	104,38	48	564,57
13	31,92	31	105,35	49	629,68
14	36,75	32	135,05	50	774,58
15	37,83	33	142,95	51	786,1
16	42,46	34	155,6	52	855,94
17	53,36	35	184,86	53	1014,44
18	57,1	36	192,21		195,46
				Total Average	10713,58 202,14

Table 2. $\alpha = 0.6, \beta = 0.5, \gamma = 0.7$ and $\omega = 0,3$

ID	Area	μ_{Area}	ν_{Area}	$\mu_{I_{\alpha,\beta}^{\gamma,\omega}}$	$\nu_{I_{\alpha,\beta}^{\gamma,\omega}}$	$\mu_{C_{\alpha,\beta}^{\gamma,\omega}}$	$\nu_{C_{\alpha,\beta}^{\gamma,\omega}}$	μ_{ϑ}	ν_{ϑ}
1	7,97	0,223691	0,698217	0,111845	0,698217	0,301783	0,698217	0,301782849	0,698217151
2	8,17	0,226362	0,69498	0,113181	0,69498	0,30502	0,69498	0,305019876	0,694980124
3	8,8	0,234367	0,685325	0,117183	0,685325	0,314057	0,685325	0,314056724	0,685324602
4	10,23	0,250594	0,665963	0,125297	0,665963	0,325415	0,665963	0,325415487	0,665963382
5	10,37	0,252058	0,664229	0,126029	0,664229	0,326441	0,664229	0,32644086	0,664229345
6	15,32	0,294114	0,615389	0,147057	0,615389	0,35588	0,615389	0,355879553	0,615389176
7	17,09	0,305896	0,602025	0,152948	0,602025	0,364127	0,602025	0,364127406	0,602025085
8	17,52	0,308574	0,599007	0,154287	0,599007	0,366002	0,596689	0,366001984	0,596689036
9	24,75	0,345806	0,557756	0,172903	0,557756	0,392064	0,459188	0,392064136	0,459187891
10	25,12	0,347405	0,556014	0,173703	0,556014	0,393184	0,45338	0,393183535	0,453379758
11	28	0,359102	0,543341	0,179551	0,543341	0,401371	0,411135	0,401371478	0,411135078
12	31,69	0,372443	0,529038	0,186222	0,529038	0,41071	0,363461	0,410710326	0,363460961
13	31,92	0,373223	0,528208	0,186611	0,528208	0,411256	0,360693	0,411255856	0,360692641
14	36,75	0,388408	0,512134	0,194204	0,512134	0,421885	0,307113	0,421885339	0,307113324
15	37,83	0,391529	0,508855	0,195765	0,508855	0,42407	0,296184	0,424070312	0,296183984
16	42,46	0,403972	0,49587	0,201986	0,497109	0,43278	0,252898	0,432780268	0,252898438
17	53,36	0,428596	0,470563	0,028655	0,479394	0,450018	0,168542	0,450017541	0,168542478
18	57,1	0,435897	0,463158	0,05299	0,474211	0,455128	0,143861	0,455127848	0,143861371
19	60,5	0,44213	0,456872	0,073767	0,46981	0,459491	0,122905	0,459491066	0,122905233
20	61,97	0,444717	0,454272	0,082391	0,46799	0,461302	0,114238	0,461302085	0,114238461
21	63,86	0,447955	0,451026	0,093183	0,465718	0,463568	0,103419	0,463568422	0,10341853
22	73,64	0,463311	0,435746	0,144371	0,455022	0,474318	0,052486	0,455022097	0,144370614
23	73,83	0,463589	0,435471	0,145296	0,45483	0,474512	0,051571	0,454829908	0,145296261
24	76,67	0,467657	0,431457	0,158855	0,45202	0,47736	0,038189	0,452019651	0,158855285
25	77,1	0,468259	0,430863	0,160864	0,451604	0,477782	0,03621	0,451604039	0,160864346
26	82,68	0,475789	0,423469	0,185965	0,446429	0,483053	0,011565	0,446428572	0,18596491
27	85,58	0,479505	0,419838	0,198349	0,443887	0,485653	0,209919	0,443886738	0,209919099
28	87,16	0,481476	0,417916	0,20492	0,442541	0,487033	0,208958	0,442540981	0,208957843
29	93,8	0,489388	0,410231	0,231294	0,437161	0,492572	0,205115	0,437161463	0,231294323
30	104,38	0,500906	0,399131	0,269686	0,429392	0,500906	0,199565	0,429391578	0,269685726
31	105,35	0,501903	0,398175	0,273009	0,428722	0,501903	0,199087	0,428722445	0,273008566
32	135,05	0,528667	0,372791	0,362224	0,410954	0,528667	0,186396	0,410953984	0,36222428
33	142,95	0,534794	0,367056	0,382646	0,406939	0,534794	0,183528	0,406939493	0,382646093
34	155,6	0,543932	0,358554	0,413106	0,400988	0,543932	0,179277	0,400987656	0,413106002
35	184,86	0,562501	0,341462	0,475004	0,389024	0,562501	0,170731	0,38902364	0,47500391
36	192,21	0,566703	0,337629	0,48901	0,38634	0,566703	0,168815	0,386340482	0,489009952
37	195,46	0,56851	0,335985	0,495033	0,385189	0,56851	0,167992	0,38518931	0,495033139
38	265,48	0,601506	0,306356	0,601506	0,364449	0,601506	0,153178	0,364449175	0,601506486
39	293,35	0,612265	0,296858	0,612265	0,3578	0,612265	0,148429	0,357800406	0,612264547
40	312,6	0,619114	0,290851	0,619114	0,353596	0,619114	0,145426	0,353595715	0,619114008
41	315,02	0,619945	0,290124	0,619945	0,353087	0,619945	0,145062	0,35308704	0,619945077
42	343,49	0,629269	0,282003	0,629269	0,347402	0,629269	0,141001	0,347401991	0,629269291
43	349,76	0,631219	0,280312	0,631219	0,346219	0,631219	0,140156	0,346218505	0,631218712
44	388,28	0,642478	0,270596	0,642478	0,339417	0,642478	0,135298	0,339417103	0,642478171
45	401,55	0,6461	0,267488	0,6461	0,337242	0,6461	0,133744	0,337241768	0,646099721
46	403,61	0,646651	0,267016	0,646651	0,336911	0,646651	0,133508	0,336911057	0,646651166
47	498,05	0,669309	0,247772	0,669309	0,323441	0,669309	0,123886	0,323440707	0,669309356
48	564,57	0,682819	0,236453	0,682819	0,315517	0,682819	0,118227	0,315517194	0,682819464
49	629,68	0,694582	0,22669	0,694582	0,305418	0,694582	0,113345	0,305418036	0,694581964
50	774,58	0,716902	0,208398	0,716902	0,283098	0,716902	0,185017	0,283098473	0,716901527
51	786,1	0,718492	0,207106	0,718492	0,281508	0,718492	0,185017	0,2815075	0,7184925
52	855,94	0,727665	0,199683	0,727665	0,272335	0,727665	0,185017	0,272334764	0,727665236
53	1014,44	0,745974	0,185017	0,745974	0,254026	0,745974	0,185017	0,25402602	0,74597398

Table 3. $\alpha = 0.8, \beta = 0.2, \gamma = 0.3$ and $\omega = 0,6$

ID	Area	μ_{Area}	ν_{Area}	$\mu_{I^{\gamma,\omega}_{\alpha,\beta}}$	$\nu_{I^{\gamma,\omega}_{\alpha,\beta}}$	$\mu_{C^{\gamma,\omega}_{\alpha,\beta}}$	$\nu_{C^{\gamma,\omega}_{\alpha,\beta}}$	μ_{∂}	ν_{∂}
1	7,97	0,223691	0,698217	0,178952	0,698217	0,223691	0,654596	0,223690558	0,654595929
2	8,17	0,226362	0,69498	0,181089	0,69498	0,226362	0,649972	0,226361503	0,649971606
3	8,8	0,234367	0,685325	0,187493	0,685325	0,234367	0,636178	0,234366748	0,636178003
4	10,23	0,250594	0,665963	0,015134	0,665963	0,250594	0,608519	0,250593553	0,608519117
5	10,37	0,252058	0,664229	0,017226	0,664229	0,252058	0,606042	0,252058371	0,606041922
6	15,32	0,294114	0,615389	0,077305	0,615389	0,294114	0,53627	0,294113646	0,536270252
7	17,09	0,305896	0,602025	0,094138	0,602025	0,305896	0,517179	0,305896294	0,517178693
8	17,52	0,308574	0,599007	0,097963	0,599007	0,308574	0,512867	0,308574262	0,51286673
9	24,75	0,345806	0,557756	0,151151	0,557756	0,345806	0,453938	0,345805909	0,453937668
10	25,12	0,347405	0,556014	0,153436	0,556014	0,347405	0,451448	0,347405049	0,451448468
11	28	0,359102	0,543341	0,170146	0,543341	0,359102	0,433344	0,359102112	0,433343605
12	31,69	0,372443	0,529038	0,189205	0,529038	0,372443	0,412912	0,372443323	0,41291184
13	31,92	0,373223	0,528208	0,190318	0,528208	0,373223	0,411725	0,373222651	0,411725417
14	36,75	0,388408	0,512134	0,212011	0,512134	0,388408	0,388763	0,388407627	0,388762853
15	37,83	0,391529	0,508855	0,21647	0,508855	0,391529	0,384079	0,391529018	0,38407885
16	42,46	0,403972	0,49587	0,234245	0,49587	0,403972	0,365528	0,403971812	0,365527902
17	53,36	0,428596	0,470563	0,269424	0,470563	0,428596	0,329375	0,428596487	0,329375348
18	57,1	0,435897	0,463158	0,279853	0,463158	0,435897	0,318798	0,435896925	0,31879773
19	60,5	0,44213	0,456872	0,288757	0,456872	0,44213	0,309817	0,442130095	0,309816528
20	61,97	0,444717	0,454272	0,292453	0,454272	0,444717	0,306102	0,444717264	0,306102198
21	63,86	0,447955	0,451026	0,297078	0,451026	0,447955	0,301465	0,447954889	0,301465084
22	73,64	0,463311	0,435746	0,319016	0,435746	0,463311	0,279637	0,435745852	0,319015978
23	73,83	0,463589	0,435471	0,319413	0,435471	0,463589	0,279245	0,435471297	0,319412683
24	76,67	0,467657	0,431457	0,325224	0,431457	0,467657	0,273509	0,431456644	0,325223694
25	77,1	0,468259	0,430863	0,326085	0,430863	0,468259	0,272661	0,430862912	0,32608472
26	82,68	0,475789	0,423469	0,336842	0,423469	0,475789	0,262099	0,423469388	0,336842104
27	85,58	0,479505	0,419838	0,342149	0,419838	0,479505	0,256912	0,419838198	0,342149462
28	87,16	0,481476	0,417916	0,344966	0,417916	0,481476	0,254165	0,417915687	0,344965864
29	93,8	0,489388	0,410231	0,356269	0,410231	0,489388	0,243187	0,410230661	0,356268995
30	104,38	0,500906	0,399131	0,372722	0,399131	0,500906	0,22733	0,399130826	0,372722454
31	105,35	0,501903	0,398175	0,374147	0,398175	0,501903	0,225964	0,398174922	0,374146528
32	135,05	0,528667	0,372791	0,412382	0,372791	0,528667	0,189702	0,372791406	0,412381834
33	142,95	0,534794	0,367056	0,421134	0,367056	0,534794	0,181509	0,367056419	0,42113404
34	155,6	0,543932	0,358554	0,434188	0,358554	0,543932	0,169363	0,358553794	0,434188287
35	184,86	0,562501	0,341462	0,460716	0,341462	0,562501	0,144946	0,341462343	0,460715961
36	192,21	0,566703	0,337629	0,466719	0,337629	0,566703	0,13947	0,33762926	0,466718551
37	195,46	0,56851	0,335985	0,4693	0,335985	0,56851	0,137121	0,335984729	0,469299917
38	265,48	0,601506	0,306356	0,516438	0,306356	0,601506	0,094794	0,306355965	0,516437837
39	293,35	0,612265	0,296858	0,531806	0,296858	0,612265	0,081225	0,296857723	0,531806496
40	312,6	0,619114	0,290851	0,541591	0,290851	0,619114	0,072644	0,290851021	0,54159144
41	315,02	0,619945	0,290124	0,542779	0,290124	0,619945	0,071606	0,290124344	0,542778682
42	343,49	0,629269	0,282003	0,556099	0,282003	0,629269	0,060004	0,282002844	0,556098987
43	349,76	0,631219	0,280312	0,558884	0,280312	0,631219	0,057589	0,28031215	0,558883875
44	388,28	0,642478	0,270596	0,574969	0,270596	0,642478	0,043708	0,270595862	0,574968815
45	401,55	0,6461	0,267488	0,580142	0,267488	0,6461	0,039269	0,267488241	0,580142458
46	403,61	0,646651	0,267016	0,58093	0,267016	0,646651	0,038594	0,267015796	0,580930237
47	498,05	0,669309	0,247772	0,613299	0,247772	0,669309	0,011103	0,247772438	0,61329908
48	564,57	0,682819	0,236453	0,632599	0,236453	0,682819	0,189163	0,236453134	0,632599235
49	629,68	0,694582	0,22669	0,649403	0,22669	0,694582	0,181352	0,226690478	0,649402805
50	774,58	0,716902	0,208398	0,681288	0,208398	0,716902	0,166718	0,208398038	0,681287895
51	786,1	0,718492	0,207106	0,683561	0,207106	0,718492	0,165684	0,207105593	0,683560714
52	855,94	0,727665	0,199683	0,696665	0,199873	0,727665	0,159747	0,199873369	0,696664622
53	1014,44	0,745974	0,185017	0,72282	0,194007	0,745974	0,185017	0,194006825	0,722819971

Table 3. $\alpha = 0.8, \beta = 0.2, \gamma = 0.3$ and $\omega = 0,6$

The optimal $\alpha, \beta, \gamma, \omega$ values can be obtained for the spatial object examined. Also operators studied by Shi and Liu give more zero values for chosen alpha values. In the intuitinistic fuzzy topological operators examined in this study, the number of non-zero values obtained is larger. This allows us to obtain more realistic results in modeling objects with uncertain boundaries.

4. Conclusion and Suggestions

In this paper, $I_{\alpha,\beta}^{\gamma,\omega}$ and $C_{\alpha,\beta}^{\gamma,\omega}$ intuitionistic fuzzy topological operators studied on a spatial object example. For optimal $\alpha,\beta,\gamma,\omega$ values topological interior, closure and boundary values were obtained. Table 2 and Table 3 shows that the topological values obtained here are more sensitive than those obtained with fuzzy topological operators and crisp topology. This classification of topological relations offers a new way of modeling spatial object problems with uncertain boundaries.

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