Buca Eğitim Fakültesi Dergisi, 2023, sayı 56, ss. 949-973

Araştırma Makalesi

The Journal of Buca Faculty of Education, 2023, issue 56, pp. 949-973

Research Article

Knowledge Levels of Pre-Service Mathematics Teachers on the Basic Concepts of Algebra*

Matematik Öğretmen Adaylarının Cebirin Temel Kavramlarına Yönelik Bilgi Düzeyleri

Mahir Biber[1]

[1]*Dr. Öğr. Üyesi, İstanbul Üniversitesi, mahir.biber@iuc.edu.tr, (https://orcid.org/0000-0003-4044-6966)*

Geliş Tarihi: *15.03.2023* ***Kabul Tarihi:*** *08.06.2023*

ABSTRACT

This study aimed to assess the knowledge levels of pre-service teachers specializing in secondary school mathematics regarding the fundamental concepts of algebra and analyze them based on various variables. The research was conducted with 151 pre-service teachers enrolled in a Primary Education Mathematics Teaching program at a state university, using the convenience sampling method. A case study approach was employed to gather data, involving nine open-ended questions designed by the researcher. Both qualitative and quantitative analysis techniques were applied to analyze the data. The findings revealed that pre-service teachers specializing in secondary school mathematics exhibited inadequate knowledge of the basic concepts of algebra and their interrelationships. It was observed that their understanding of algebra improved as they progressed to higher grade levels, influenced by the courses taken during their undergraduate education, although it did not reach a high level of proficiency. Furthermore, although the gender difference did not significantly impact the pre-service teachers' algebra knowledge levels, the mean algebra scores of female participants were found to be higher than those of their male counterparts.

Keywords: Mathematics, pre-service teacher, algebra, knowledge level.

ÖZ

Bu araştırmada, ortaokul matematik öğretmen adaylarının cebir öğrenme alanının temel kavramlarına yönelik bilgi düzeylerini belirlemek ve çeşitli değişkenlere göre incelemek amaçlanmıştır. Araştırma bir devlet üniversitesinin İlköğretim Matematik Öğretmenliği programına devam eden ve kolay örnekleme yöntemine göre belirlenen 151 öğretmen adayı ile yürütülmüştür. Araştırmada durum çalışması deseninden yararlanılmıştır. Araştırmanın verileri araştırmacı tarafından hazırlanan 9 açık uçlu soru ile toplanmıştır. Verilerin analizinde hem nicel hem de nitel analiz tekniklerinden yararlanılmıştır. Elde edilen sonuçlar, ortaokul matematik öğretmen adaylarının cebirin temel kavramlarına ve kavramlar arasındaki ilişkilere yönelik bilgi düzeylerinin yetersiz olduğunu göstermektedir. Öğretmen adaylarının cebir bilgilerinin üst sınıflara doğru geliştiği, buna karşın üst düzey bir cebir bilgisinin oluşturulamadığı anlaşılmaktadır. Ayrıca,

*This research is an extended version of the study presented by the author at the 5th International Symposium on Computer and Mathematics Education in Turkey.

cebir bilgi düzeylerinin cinsiyete göre farklılaşmadığı, buna karşın kızların ortalamalarının erkeklerden daha yüksek olduğu sonucuna ulaşılmıştır.

Anahtar Kelimeler: Matematik, öğretmen adayı, cebir öğrenme alanı, bilgi düzeyi.

INTRODUCTION

The Secondary School Mathematics Curriculum encompasses five key learning areas: numbers and operations, algebra, geometry and measurement, data processing, and probability (Ministry of National Education-MoNE, 2018). Among these areas, algebra plays a crucial role in the development of thinking skills and enables individuals to make sense of everyday life. It is widely acknowledged by mathematics educators and teachers that the algebra learning area is one of the most essential components of school mathematics. Algebra nurtures skills such as reasoning, which are particularly vital for individuals to comprehend the events occurring around them and achieve success in their lives (Birgin & Demirören, 2020). Moreover, algebra significantly contributes to perceiving the symbolic structure of mathematics and fostering the use of mathematical language. Consequently, the teaching of algebra assumes a prominent position.

The introduction to algebra instruction starts at an early age by focusing on pattern recognition and establishing connections with arithmetic concepts (Mason et al., 2009). The National Council of Teachers of Mathematics-NCTM (1989) emphasized the importance of working with patterns to foster algebraic reasoning skills in primary school children. Similarly, Ahuja (2008) noted that children's algebraic thinking begins with establishing relationships between patterns, number sequences, and arithmetic operations. In Turkey, the teaching of algebra in secondary school mathematics courses commences in 6th grade with the exploration of number patterns to identify desired terms (MoNE, 2019). Dougherty et al. (2015) highlighted the significance of algebraic topics in preparing secondary school students for advanced mathematics and emphasized the crucial role of early algebra instruction in their career planning.

Algebra plays a pivotal role in fostering students' abstract thinking, making connections, and logical reasoning abilities. However, it is concerning that student performance in algebra subjects remains relatively low in Turkey. This observation is particularly evident in exams such as TIMSS and PISA, which assess students' mathematical knowledge and skills and facilitate international comparisons. Algebra often proves to be one of the most challenging sections for students in these exams. Among the 79 countries participating in PISA 2018, Turkey ranked 42nd in mathematics performance overall and 33rd among the 37 OECD countries (MoNE, 2019). This underperformance primarily stems from struggles in the areas of algebra and geometry. According to the TIMSS (2019) results, Turkey's algebra achievement significantly lags behind the average mathematics performance (MoNE, 2020). Consequently, this situation necessitates a separate evaluation of algebra instruction.

It is believed that one of the primary factors contributing to the failure in algebra education in Turkey may be the inadequate content knowledge of mathematics teachers. Demir & Tuğrul (2021) supported this notion in their study, revealing that pre-service teachers, who represent the future educators, struggle significantly in this area. Even (1990) emphasized the crucial role of teachers' content knowledge in students' comprehension of the subject, stressing the importance of teachers possessing a high-level understanding of the subject matter. Subject matter knowledge entails teachers' competencies and perceptions, such as the ability to define, associate, and provide examples of concepts within a field (Ball, 1988; Shulman, 1986). Ahuja (2008) specifically applied this notion to algebra and asserted that teachers should exhibit a high level of algebraic thinking skills to foster young children's development in this area. Toheri & Winarso (2017) also underscored the significance of teachers' knowledge and skills in shaping and nurturing students' algebraic thinking abilities.

Given the shortcomings in algebra education, it is essential to evaluate the role of teachers. It is of great importance to examine the deficiencies in the subject matter knowledge of secondary school mathematics teachers and pre-service teachers receiving teacher education, as they form the foundation of students' algebraic knowledge. For effective algebra instruction, secondary school mathematics teachers and pre-service teachers are expected to have a clear understanding of the basic concepts taught at the secondary school level, provide appropriate examples, and comprehend the relationships between these concepts. Previous studies have focused on assessing the conceptual and procedural knowledge levels of pre-service mathematics teachers regarding algebra, as well as their understanding of algebraic expressions, equations, and equalities (Dede et al., 2010; Huang & Kulm, 2012; Serbin, 2021; Sitrava, 2017; Zuya, 2017). There have also been studies examining the algebraic thinking levels of pre-service teachers (Çelik, 2007). However, no comprehensive research has been identified that evaluates the fundamental principles of algebra in the context of secondary education and assesses the proficiency of pre-service mathematics teachers in these concepts.

With this study, it is anticipated that the readiness levels of pre-service mathematics teachers specializing in secondary school mathematics regarding algebra, as well as their progress in this field during their undergraduate education, can be determined. The study encompasses not only the limited concepts taught at the secondary school level but also high school level concepts such as relations, functions, and polynomials. This approach is intended to enable pre-service teachers to enhance their content knowledge not only for their future teaching careers but also for a broader understanding of algebra concepts. Consequently, this research aims to evaluate the knowledge levels of pre-service mathematics teachers specializing in secondary school mathematics concerning the basic concepts of algebra and analyze them based on various variables. The study is significant in terms of revealing the current knowledge levels of pre-service mathematics teachers, who will become future educators and questioning the algebra education provided at the secondary, high school, and undergraduate levels. The sub-problems addressed within the research scope are as follows:

1) What is the knowledge level of the pre-service teachers participating in the research about the basic concepts of learning algebra?

2) Do the pre-service teachers' knowledge levels in the research on the basic concepts of learning algebra differ according to their grade levels?

3) Do the knowledge level of the pre-service teachers participating in the research on the basic concepts of learning algebra differ according to their gender?

METHOD

2.1. Model of the Research

In this study, the case study, which is one of the qualitative research designs, was used. According to Creswell (2013), a case study involves systematically collecting data using various data collection tools and thoroughly examining one or more specific cases over an extended period. Given the objective of this study, which is to investigate the knowledge levels of mathematics pre-service teachers regarding the basic concepts of algebra through open-ended questions, the case study method was considered appropriate.

2.2. Participants

The research was conducted with 151 pre-service teachers studying in the primary school mathematics teaching department at a university in Istanbul. The study involved 151 pre-service teachers enrolled in the primary school mathematics teaching department at a university in Istanbul. Convenience case sampling was used to select the participants for this research. While this sampling method may have lower levels of validity and reliability compared to other sampling methods (Creswell, 2013), it was considered appropriate in this case because the study focused on pre-service mathematics teachers and the researcher was a lecturer in the same department. By including participants from different grade levels, the research aimed to assess the impact of the courses they had taken at previous educational levels as well as at the university level on their knowledge of algebra. This approach aimed to increase the diversity of the sample and obtain a more reliable representation. Table 1 provides descriptive information regarding the demographic characteristics of the pre-service teachers.

Table 1

Descriptive Information on Demographic Characteristics of Pre-service Teachers

Variable		Participants	Frequency (f)
Gender	Female	S_3, S_{6-13}, S_{18-45}, S_{47-52}, S_{57}, S_{59-62}, S_{64-66}, S_{69-75}, S_{78-83}, S_{87-96}, $S_{100-103}$, $S_{109-114}$, $S_{116-120}$, $S_{122-129}$, $S_{131-140}$, $S_{142-151}$	117
	Male	S_1, S_2, S_4, S_5, S_{14-17}, S_{46}, S_{53-56}, S_{58}, S_{63}, S_{67}, S_{68}, S_{76}, S_{77}, S_{84-86}, S_{97-99}, $S_{104-108}$, S_{115}, S_{121}, S_{130}, S_{141}	34
Grade Level	1	S_2, S_3, S_6, S_8, S_9, S_{15}, S_{16}, S_{19}, S_{20}, S_{23}, S_{25}, S_{26}, S_{29}, S_{32}, S_{33}, S_{35}, S_{37-39}, S_{42}, S_{47}, S_{51}, S_{54}, S_{58}, S_{64}, S_{67-70}, S_{72}, S_{74}, S_{76}, S_{79}, S_{81}, S_{82}, S_{89}, S_{94-97}, S_{99}, S_{103}, S_{105}, S_{108}, S_{111}, S_{119}, S_{121}, S_{126}, $S_{128-130}$, S_{132}, S_{137}, S_{142}, S_{144}, S_{147}, S_{148}, S_{150}	58
	2	S_7, S_{11}, S_{12}, S_{17}, S_{21}, S_{22}, S_{24}, S_{28}, S_{34}, S_{41}, S_{45}, S_{46}, S_{48}, S_{49}, S_{50}, S_{53}, S_{61}, S_{65}, S_{66}, S_{71}, S_{78}, S_{80}, S_{84}, S_{86}, S_{87}, S_{88}, S_{91}, S_{92}, S_{100}, S_{102}, S_{104}, S_{109}, S_{110}, S_{112}, S_{113}, S_{115}, S_{116}, S_{117}, S_{118}, S_{122}, S_{123}, S_{125}, S_{127}, S_{136}, S_{140}, S_{149}, S_{151}	47
	3	S_1, S_4, S_{10}, S_{13}, S_{14}, S_{18}, S_{27}, S_{30}, S_{31}, S_{36}, S_{40}, S_{43}, S_{44}, S_{56}, S_{57}, S_{59}, S_{60}, S_{75}, S_{77}, S_{83}, S_{85}, S_{90}, S_{93}, S_{98}, S_{101}, S_{106}, S_{107}, S_{114}, S_{120}, S_{124}, S_{131}, S_{133}, S_{134}, S_{135}, S_{138}, S_{139}, S_{143}, S_{145}, S_{146}	39
	4	S_5, S_{52}, S_{55}, S_{62}, S_{63}, S_{73}, S_{141}	7

2.3. Materials (Data Collection Tools)

In this research, the "Algebra Learning Field Basic Concepts Scale" was utilized to assess the participants' knowledge levels regarding the fundamental concepts of algebra. The scale was developed by the researcher and consisted of nine open-ended questions designed to evaluate pre-service teachers' abilities in defining, exemplifying, and establishing connections with algebraic concepts. The scale items were carefully constructed to align with the expected learning outcomes of basic algebraic concepts within the secondary and high school curriculum, allowing for data collection in two main categories. The first category focused on the pre-service teachers' definitions of key algebraic concepts such as proposition, explicit proposition, relation, function, equation, inequality, identity, and polynomial. The second category involved requesting examples of these concepts or explanations of the relationships between them. The "Algebra Learning Field Basic Concepts Scale" was subjected to review by eight experts, including four mathematics educators and two secondary school/high school mathematics teachers, whose feedback was incorporated to refine the scale's questions.

2.4. Data Collection and Permissions

The data were obtained from the written explanations of the pre-service teachers by the researcher. The scale was applied to pre-service teachers in a face-to-face environment and 30-45 minutes were given to each pre-service teacher. Before the scale was applied Ethics Committee approval numbered 2021/237 was taken from Istanbul University-Cerrahpasa Social and Human Sciences Research Ethics Committee.

2.5. Data Analysis

The analysis of the data was carried out using descriptive and predictive statistics according to the scores obtained with the help of a rubric prepared by the researcher. The rubric presented in Table 2 has been prepared by considering the studies of Kartal & Çinar (2017) and Karakuş (2018).

Table 2

Rubric Used in Quantitative Analysis of Data

Category	Explanation
No answer or wrong answer (0-point)	Answers that are left blank or contain Wrong statements
No explanation (1-point)	Answers without any explanation despite containing correct statements
Answer with error (2-points)	Wrong statements as well as correct explanations
Partially correct (3-points)	Incomplete explanations without Wrong statements
Completely correct (4-points)	Complete answers that are completely correct

Based on the rubric used for scoring, the "Algebra Learning Field Basic Concepts Scale" allowed pre-service teachers to obtain a minimum of 0 points and a maximum of 64 points. When scoring with rubrics, Moskal & Leydens (2000) highlighted the significance of considering inter-rater reliability. Consequently, both intra-rater and inter-rater reliability were calculated. For intra-rater reliability, a randomly selected student's responses were scored twice by the researcher, with a four-week interval, and the agreement percentage between the scores was examined following the recommendations of Miles & Huberman (1994). To ensure the reliability of the scale's coding, the consistency between the evaluations of the researcher was examined, employing the formula proposed by Miles & Huberman (1994).

Reliability Percentage = Number of Consensuses / (Number of Consensus+Number of Disagreements)

The percentage of agreement for the scoring was 85%. It was decided that the scoring of the scale was appropriate since the percentage of agreement was 70% and above (Tavşancıl & Aslan, 2001).

In terms of inter-rater reliability, Cohen (1960) Kappa statistics were employed to assess the agreement between scores given by two raters. In this regard, a faculty member, independent of the researcher, was asked to score ten randomly selected scales. The results revealed an 82% agreement between the scores. According to Landis & Koch (1977), an agreement of .80 and above signifies a perfect agreement. Therefore, these findings indicate a very high level of agreement between the raters.

Descriptive and predictive statistics were employed in this research. Descriptive statistics were utilized to assess the knowledge levels of pre-service teachers regarding algebra concepts. Prior to examining the status of these levels based on grade level and gender, the normality assumption was first evaluated. Since the data did not meet the normality assumption, Kruskal-Wallis H and Mann-Whitney U tests were employed.

FINDINGS

The first sub-problem of this research addresses the question, "What is the knowledge level of the pre-service teachers participating in the study regarding the basic concepts of algebra learning?" Descriptive analysis was conducted on the scores obtained by the pre-service teachers using the scale. These scores were then evaluated based on the criteria presented in Table 3.

Table 3

Evaluation Criteria for Scores Obtained from the Scale

Score Range	Level
0 - 21,33	Low
21,34 - 42,67	Middle
42,68 - 64,00	High

The results of the descriptive statistics for the scores obtained by the pre-service teachers from the scale are presented in Table 4.

Table 4

Descriptive Statistics Results of Scores Obtained from the Scale

N	Mean	SD	Minimum	Maximum
151	21,37	7,38	2,00	39,00

Table 4 shows that the algebra scores of the pre-service teachers are at a moderate level. The relatively low average suggests that pre-service teachers have a limited understanding of algebra concepts and their proficiency in this area is insufficient.

Additionally, the findings obtained in this context were analyzed separately for each concept. Firstly, the results of the descriptive analysis regarding the definitions provided by the pre-service teachers for the concept of proposition are presented in Table 5.

Table 5

Descriptive Analysis Results of Definitions for the Concept of Proposition

Category	Frequency (f)	Percentage (%)
Correct	95	62,9
Partially Correct	23	15,2
Wrong	21	13,9
Empty	12	8
Total	151	100

In Table 5, it is evident that the majority of pre-service teachers (f=95; 62.9%) correctly defined the concept of proposition. The responses of two pre-service teachers who provided accurate definitions are provided below:

***S_{21}:** They are statements that state true or false definite judgments.*

***S_{141}:** If the value of an expression put forward in logic is true or false, this statement is called a proposition.*

On the other hand, there are some pre-service teachers who defined the concept of proposition partially correctly (f=23; 13.9%) or incorrectly (f=21; 13.9%). It is evident that these pre-service teachers either overlooked the requirement for a proposition to be true or false or used expressions that lacked meaning when defining the concept. Below are some examples of incorrect answers provided by the pre-service teachers:

***S_8:** Discourses whose truth and falsity are not certain.*

***S_{16}:** Expressing something with its certainty.*

Additionally, it was observed that pre-service teachers who provided partially correct definitions often presented incomplete explanations. These definitions lacked mention of the truth value, contained incorrect generalizations, or solely focused on expressing the truth value. Here are a few examples of partially correct definitions provided by the pre-service teachers:

***S_5*: They are expressions expressing judgment.**

***S_{41}*: Transaction presented as true or false.**

***S_{10}*: They are expressions with a truth value of 1 or 0.**

The research also investigated the concept of the open proposition. The descriptive analysis results of the definitions provided by the pre-service teachers are presented in Table 6.

Table 6

Descriptive Analysis Results of Definitions for the Concept of Explicit Proposition

Category	Frequency (f)	Percentage (%)
Correct	58	38,4
Partially Correct	6	4
Wrong	28	18,5
Empty	59	39,1
Total	151	100

The findings indicate that a significant number of pre-service teachers either left the concept of open proposition blank (f=59; 39.1%) or provided correct definitions (f=58; 38.4%). Here are some examples of the answers provided by pre-service teachers who defined the concept correctly:

S_2*: *It is a proposition whose truth value changes depending on the variables.

S_{17}*: *These are the expressions that are true or false according to the value of the unknown.

A significant number of pre-service teachers (f=28; 18.5%) misidentified the concept of explicit proposition. It is evident that these pre-service teachers often mistakenly associate an explicit proposition with being a true proposition, or they believe that logical conjunctions or symbols must be used in its formulation. Here are some examples of incorrect definitions provided by the pre-service teachers:

S_{42}*: *Explicit Propositions are the true propositions.

S_{143}*: *Concepts such as ⊥, ∥, ⇒, ∧ are used in explicit propositions.

It was observed that pre-service teachers who partially correctly defined the concept of explicit proposition often overlooked the requirement of having a variable in it. An example of such a definition is as follows:

S_{84}*: *It is a proposition whose truth value changes with certain conditions.

The concept of "relation" was another concept that pre-service teachers were asked to define within the scope of the research. The results of the descriptive analysis of the definitions are presented in Table 7.

Table 7

Descriptive Analysis Results of Definitions for the Concept of Relation

Category	Frequency (f)	Percentage (%)
Correct	32	21,2
Partially Correct	52	34,4
Wrong	41	27,2
Empty	26	17,2
Total	151	100

In Table 7, it is observed that more than half of the pre-service teachers define the concept of relation either completely (f=32; 21.1%) or partially correctly (f=52; 34.4%). However, it is also noteworthy that a considerable number of pre-service teachers misidentified the concept (f=41; 27.2%) or left it blank (f=26; 17.2%). Examples of answers provided by pre-service teachers who correctly define the concept are as follows:

S$_{48}$: *Let A and B be two sets that are not empty. Each subset of the AXB Cartesian product is a relation from A to B.*

S$_{79}$: *Any subset of the Cartesian product of two sets.*

It was observed that a significant number of pre-service teachers (f=41; 27.2%) who misidentified the concept of relation provided explanations that were meaningless or confused it with concepts such as groups or equations. Additionally, some pre-service teachers defined the concept of relation as the relationship between functions or operations. Examples of definitions reflecting these situations are as follows:

S$_{26}$: *Concept formed by associating two or more relations.*

S$_{118}$: *Equation established by mathematical operations between two or more numbers, dependency.*

S$_{4}$: *It is the assumed relationship between two or more functions.*

It is evident that a considerable number of pre-service teachers (f=52; 34.4%) who provided deficient definitions of the concept of relation primarily described it as a mathematical relationship. An example definition illustrating this situation is as follows:

S$_{20}$: *The relationship established between two or more attributes through mathematical operations.*

On the other hand, there are pre-service teachers who define the concept of relation as a simple relationship between sets, disregarding the Cartesian product set or its subsets. Here are some example definitions:

S$_{21}$: *The matching of an element in any set A to an element in a set B is called a relation from A to B.*

S$_{141}$*: A and B are a set and pairs (x, y) formed as $\forall x \in A$ ve $\forall y \in B$, are called a relation from A to B.*

It was observed that some pre-service teachers defined the concept of relation as the rule of function. An example of this situation is as follows;

S$_{120}$: ***f****: In the* $\boldsymbol{R} \rightarrow \boldsymbol{R}$ *function, f is called a relation.*

The concept of function was also examined within the scope of the research. The results of the descriptive analysis of the definitions made by the pre-service teachers are presented in Table 8.

Table 8

Descriptive Analysis Results of the Definitions Made for the Concept of Function

Category	Frequency (f)	Percentage (%)
Correct	18	11,9
Partially correct	87	57,6
Wrong	31	20,5
Empty	15	10
Total	151	100

In Table 8, it is observed that a very small number of pre-service teachers (f=18; 11.9%) define the concept of function correctly, while the majority (f=87; 57.6%) provide partially correct definitions. Additionally, there are pre-service teachers who define the concept wrongly (f=31; 20.5%) or are unable to define it at all (f=15; 10%). Examples of correct answers provided by the pre-service teachers are as follows:

***S_{25}:** They are special relations defined from one set to another set in which no element in the domain is left idle and matches only one element in the value set.*

***S_{47}:** Let A and B be two sets that are not empty. The relation f from A to B is called a function if each element of A matches only one element of B.*

It can be observed that the definitions provided by the pre-service teachers regarding the concept of function mostly consist of meaningless explanations that are not relevant to the concept. Examples of such explanations can be provided as follows:

***S_{3}:** The result found by replacing an x variable in any equation.*

***S_{105}:** It is the entering and exiting of numbers in a transaction.*

On the other hand, some pre-service teachers define the concept of function only as a quantity or confuse it with concepts such as set, equation, and polynomial. Example definitions for these situations are as follows;

***S_{69}:** The quantity changes depending on one or more variables.*

***S_{71}:** It is the set of expressions obtained by processing the elements of any set.*

***S_{129}:** They are equations with a certain definition and value set.*

It is evident that the definitions considered partially correct are primarily based on situations where the function is defined solely as a transformation, perceiving it merely as a relation while disregarding specific rules, or neglecting the requirement of matching elements of the domain set with only one element in the range set. Examples of such definitions are provided below:

***S_{5}:** Expressions that give certain inputs as outputs by processing them.*

***S_{88}:** The relation defined from the domain to the value set is called a function.*

***S_{35}:** A relation from A to B can specify a function defined from A to B. The basic condition here is that all elements in the domain must be matched.*

Another concept examined in the research was the equation. The results of the descriptive analysis of the definitions made by the pre-service teachers regarding the concept of the equation are presented in Table 9.

Table 9

Descriptive Analysis Results of Definitions for the Concept of Equation

Category	Frequency (f)	Percentage (%)
Correct	49	32,5
Partially Correct	69	45,7
Wrong	25	16,6
Empty	8	5,2
Total	151	100

In Table 9, it is evident that the concept of the equation is mostly defined correctly (f=49; 32.5%) or partially correctly (f=69; 45.7%) by the pre-service teachers. However, some pre-service teachers provided incorrect definitions (f=25; 16.6%). The pre-service teachers who defined the concept correctly provided the following expressions as examples:

S_{13}*: *Equations containing at least one unknown.

S_{120}*: *They are the equations expressed with one or more unknowns.

It is observed that the majority of pre-service teachers who misidentified the concept of the equation provided meaningless explanations, while some defined the equation as a tool for problem-solving. Examples of these definitions are as follows:

S_{78}: *A new proposition is created by putting "=" between two propositions.*

S_{119}: *It is a mathematical representation with symbols on both sides.*

S_{3}: *It is a system we use to solve any problem.*

In addition, definitions with incorrect examples were also considered wrong. An example definition for this situation is as follows;

S_{89}: *Equations are expressions such as $x^2 + 2x + 3$.*

It is observed that pre-service teachers who partially correctly define the concept of an equation often neglect either the condition of the unknown quantity or the condition of equality. Examples of these situations can be given as follows:

S_{5}: *Expressions showing the equality of two quantities.*

S_{40}: *They are expressions that contain at least one unknown.*

Within the scope of the research, pre-service teachers were also asked to provide an example of the concept of an equation in a separate question. The findings obtained are presented in Table 10.

Table 10

Descriptive Analysis Results of Examples Given Regarding the Concept of Equation

Category	Frequency (f)	Percentage (%)	Sample Data
Correct	135	89,4	Let a, b, c and z be constants and x,y variables, $ax + by + c = z$ (S_{11})
Wrong	8	5,3	$\boldsymbol{3x^2} + \boldsymbol{4x} + \boldsymbol{1}$ (S_1) $3+2=1+4$ (S_{145})
Empty	8	5,3	

In Table 10, it can be observed that the majority of pre-service teachers had no difficulties in providing examples of equations (f=135; 89.4%). However, there were also students who gave incorrect examples (f=8; 5.3%) or could not provide any examples (f=8; 5.3%).

Another concept examined in the research is inequality. The results of the descriptive analysis of the definitions made by the pre-service teachers regarding the concept of inequality are presented in Table 11.

Table 11

Descriptive Analysis Results of Definitions for the Concept of Inequality

Category	Frequency (f)	Percentage (%)
Correct	36	23,8
Partially Correct	30	19,9
Wrong	68	45
Empty	17	11,3
Total	151	100

In Table 11, it can be observed that a significant portion of the pre-service teachers faced challenges in comprehending the concept of inequality (f=68; 45%). However, it is also evident that a substantial number of pre-service teachers defined the concept correctly (f=36; 23.8%) or partially correctly (f=30; 19.9%). One of the pre-service teachers who defined the concept correctly provided the following expression as an example:

S_{62}: *They are relations formed with symbols $<, >, \leq, \geq$ and mathematical expressions.*

It is evident that a considerable number of pre-service teachers who misidentified the concept of inequality provided meaningless explanations or defined inequality as an equation. Examples of these definitions are as follows:

S_7: *Expressing two propositions by putting the signs $<, >, \leq, \geq$ is called.*

S_{24}: *It is the relationship between two equations given with symbols such as $<, >, \leq, \geq, \neq$.*

S_{39}: *The fact that an equation is less than, greater, or equal to a number value.*

On the other hand, the pre-service teachers who partially correctly defined the concept of inequality either defined it without considering symbols, defined it solely based on symbols without generalizing, or defined it solely as a relationship of magnitude. Examples of these definitions are as follows:

S_3: *The two expressions are not equal to each other.*

S_{144}: *They are the expressions in which the symbols $<, >, \leq, \geq$ are used.*

S_{63}: *Specifying greatness-smallness between expressions.*

In addition, the definitions made by including the symbol "=" when defining the concept of inequality were accepted as partially correct. An example definition for this situation is as follows;

***S_{25}:** They are comparison operations using symbols such as =, >, <.*

Within the scope of the research, pre-service teachers were also asked to give examples of the concept of inequality. The obtained findings are presented in Table 12.

Table 12

Descriptive Analysis Results of Examples Given Regarding the Concept of Inequality

Category	Frequency (f)	Percentage (%)	Sample Data
Correct	135	89,4	$2x - 10 < 0$ ($Ö_{29}$)
Wrong	7	4,6	$3x+8=4x+4$ ($Ö_4$)
Empty	9	6	

In Table 12, it can be observed that the majority of pre-service teachers did not encounter any difficulties in providing examples of the concept of inequality (f=135; 89.4%). However, there were a few instances where pre-service teachers provided incorrect examples (f=7; 4.6%) or did not provide any examples at all (f=9; 6%).

Another significant algebra concept examined within the scope of this research is identity. The descriptive analysis results of the definitions provided by pre-service teachers regarding the concept of identity are presented in Table 13.

Table 13

Descriptive Analysis Results of Definitions for the Concept of Identity

Category	Frequency (f)	Percentage (%)
Correct	20	13,2
Partially Correct	56	37,1
Wrong	51	33,8
Empty	24	15,9
Total	151	100

In Table 13, it can be observed that the pre-service teachers faced challenges in providing correct definitions of the concept of identity (f=20; 13.2%), with the majority offering partially correct definitions (f=56; 37.1%) or incorrect definitions (f=51; 33.8%). Examples of responses from pre-service teachers who provided correct definitions are presented below.

***S_{10}:** Types of equations provided for all real numbers*

***S_{18}:** Expressions that contain variables and provide equality for all values that the variable will take*

It is observed that the misidentifications made by the pre-service teachers regarding the concept of identity mostly consist of meaningless explanations or are related to factorization. Examples of such situations are provided below:

***S_{82}:** It helps in solving equations. It is solved by establishing an equation with one unknown, and the identity status is checked.*

***S_{68}:** Representation of expansions such as square and cube.*

In addition, some pre-service teachers generalize the concept of identity to perfect square expressions or define them with wrong examples. Examples of these situations are as follows;

S_{57}*: *Identities are perfect square expressions that contain an unknown quantity.

S_{121}*: *Expressions such as $(a + b)$, $(a - b)^2$, $(a + b)^2$, … are called identity.

It is evident that the partially correct definitions mainly stem from disregarding the fact that the solution set of the identity comprises real numbers or perceiving the identity as a simple algebraic expression without considering its specific properties. Examples illustrating these situations are provided below:

S_{25}*: *Equalities that are not the same as an equation but are generally explained together.

S_{13}*: *They are expressions that contain at least one unknown quantity.

In addition, some definitions neglect the case of identity being a special case of the equation. An example statement for this situation is as follows;

S_{101}*: *It is an open proposition that is true for every value of the unknown quantity.

Within the scope of the research, pre-service teachers were also asked to give examples of the concept of identity. The obtained findings are presented in Table 14.

Table 14

Descriptive Analysis Results of Examples Given Regarding the Concept of Identity

Category	Frequency (f)	Percentage (%)	Sample Data
Correct	88	58,3	$3x + 6x = 9x$ (**S_{16}**) $a^2 - b^2 = (a - b)(a + b)$ (**S_{21}**)
Wrong	32	21,2	$(x - 4)(x - 5)$ (**S_3**) $x^2 - 9 = 0$ (**S_6**)
Empty	31	20,5	

In Table 14, it is evident that the pre-service teachers mostly provided correct examples of the concept of identity (f=88; 58.3%). However, a significant number of pre-service teachers also provided incorrect examples (f=32; 21.2%) or did not provide any examples at all (f=31; 20.5%).

The final concept examined within the scope of the research is the polynomial. The descriptive analysis results of the definitions provided by the pre-service teachers concerning the concept of the polynomial are presented in Table 15.

Table 15

Descriptive Analysis Results of Definitions for the Concept of Polynomial

Category	Frequency (f)	Percentage (%)
Correct	11	7,3
Partially Correct	57	37,7
Wrong	58	38,4
Empty	25	16,6
Total	151	100

In Table 15, it is observed that a significant portion of the pre-service teachers (f=57; 37.7%) provided partial or incorrect definitions of the concept of polynomial, while a notable number (f=58; 38.4%) gave wrong definitions. Additionally, there were a considerable number of pre-service teachers (f=25; 16.6%) who could not define the concept at all, while only a small number (f=11; 7.3%) were able to provide a correct definition. An example of a correct definition given by one of the pre-service teachers is as follows:

S_{65}*: *They are functions whose exponents are natural numbers and whose solution set is real numbers.

It was observed that a significant number of incorrect answers provided by the pre-service teachers regarding the concept of polynomials involved neglecting the requirement that the exponents of the unknowns in the polynomial should be natural numbers or failing to consider that the domain set consists of real numbers. Examples of these situations are as follows:

S_2*: *They are expressions consisting of a certain number of independent variables and constant numbers.

S_{21}*: *They are algebraic expressions formed as a result of the multiplication of numbers and unknowns.

Apart from this, a significant number of pre-service teachers define the polynomial as an equation. An example definition for this situation is as follows;

S_{12}*: *They are m-degree equations with n unknowns.

It was observed that the pre-service teachers who provided partially correct definitions of the concept often defined it as a function without specifying any special conditions or simply presented an appropriate algebraic expression that exemplified a polynomial without actually providing a comprehensive definition. Examples of these situations are as follows:

S_{10}*: *The special case of the function.

S_{106}*: *They are the expressions in the form of $P(x) = a_1x + a_2x^2 + \cdots + a_nx^n$.

In addition, some pre-service teachers provided definitions without expressing the domain set and the coefficients of the variables, or they misidentified the powers of the unknowns. Examples of these situations are as follows:

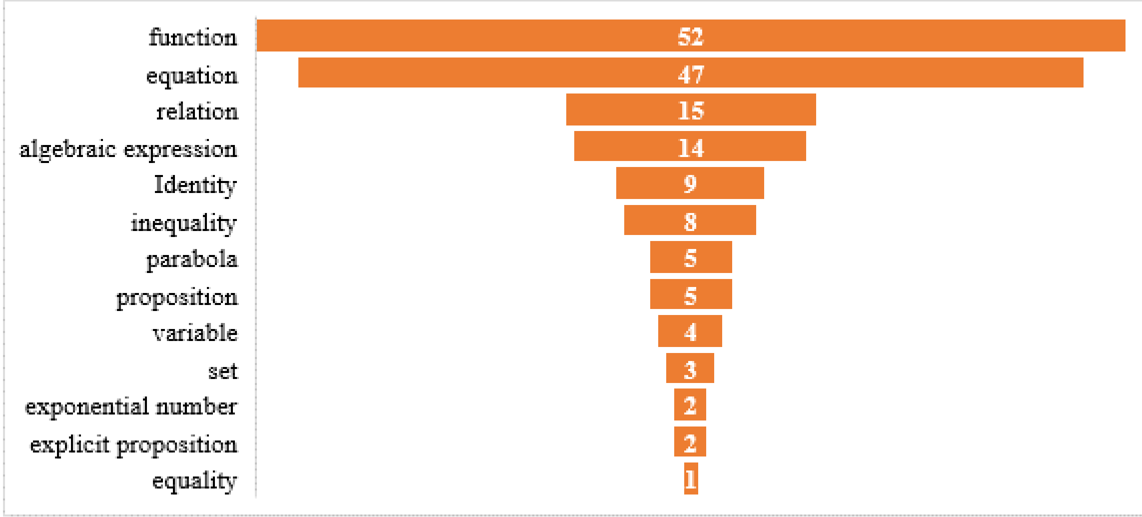
S_{112}*: *An algebraic expression with a variable and the degree of the terms are natural numbers.

S_{48}*: *They are functions where $a \in R$ and $n \geq 0$ as $ax^n = 0$.

Within the scope of the research, pre-service teachers were also asked about the algebraic concepts they associate with the concept of a polynomial. The data obtained are presented in Figure 1.

Figure 1

Associations with the Concept of Polynomial and Other Concepts of Algebra

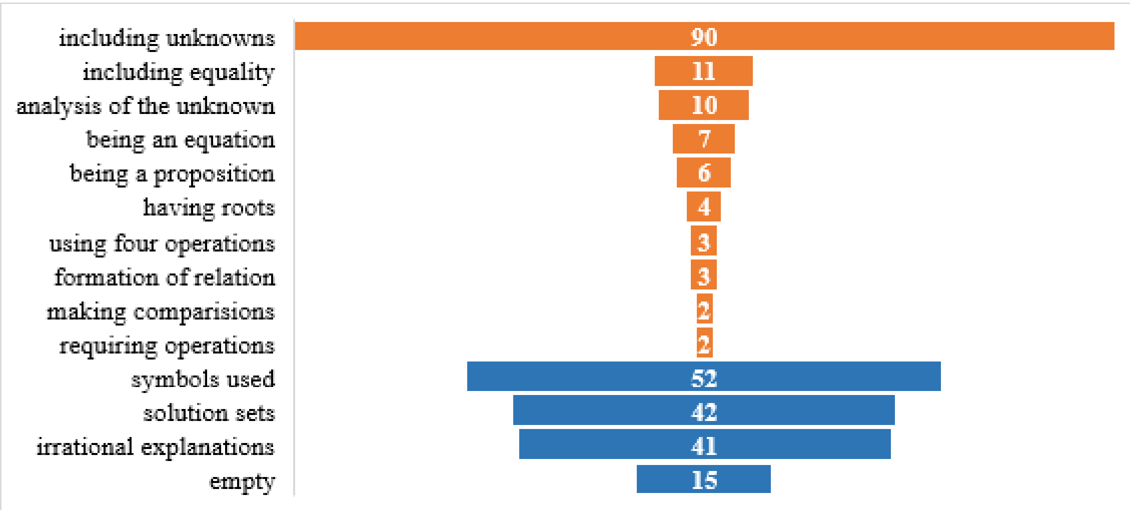


When examining Figure 1, it is evident that pre-service teachers predominantly associate the concept of a polynomial with the concepts of function (f=52; 34.4%) and equation (f=47; 31.1%). Additionally, a notable number of pre-service teachers associate it with concepts such as relation (f=15; 10%) and algebraic expressions (f=14; 9.3%).

Within the scope of the research, pre-service teachers were also asked to identify the common and distinct aspects of the concepts of equation, inequality, and identity. The findings are presented in Figure 2.

Figure 2

Associations on the Concepts of Equation, Inequality, and Identity



When Figure 2 is examined, a significant number of pre-service teachers (f=90; 59.6%) stated that all three concepts of the equation, identity, and inequality, contain unknown quantities. Some of these views are as follows;

S$_{18}$*: *There is at least one unknown term in all three of them.

S$_{25}$*: *There is a case of finding the unknown and concluding by using an equation for all three concepts.

Additionally, it is noteworthy that some mistaken explanations include statements such as all three concepts contain equality (f=11; 7%) and are created to solve for the unknown (f=10; 6.6%), or that they are simply equations (f=7; 4.6%). Here are some examples of these explanations:

S$_{69}$*: *All of them show at least one equality.

S$_{23}$*: *It is tried to find the unknown by performing various operations in all of them.

S$_{36}$*: *All three are equations.

Regarding different aspects of the concepts, they emphasized the symbols used (f=52; 34.4%) and the solution sets (f=42; 27.8%). Here are some examples of these explanations:

S$_{18}$*: *Equation and identity express equality, but there is no equality in inequality. Expressions such as $<$, $>$, $\leq$, $\geq$ are used in inequality.

S$_{10}$*: *Different aspects of equation, inequality, and identity are that the equation has a certain number of truth values. Identity is true for all real numbers, and inequality is true within a certain range of values.

Furthermore, there are illogical explanations provided for different aspects of the concepts (f=41; 27.2%). Here are some examples of these explanations:

S$_{12}$*: *The result is numerical values in the equation. The results are non-numeric expressions in identity and numerical values that unknowns will not take in inequality.

S$_{35}$*: *If we consider the equation as the basis of inequality and identity, inequality is an interval, and identity is the equation in which equality expressions are used.

Within the scope of the research, pre-service teachers were also asked to relate the concepts of relation and function. The obtained findings are presented in Table 16.

Table 16

Content Analysis Results of Answers Regarding Function-Relation Relationship

Category	**Frequency (f)**	**Percentage (%)**
Correct	43	28,5
Partially Correct	27	17,9
Wrong	41	27,2
Empty	40	26,4
TOTAL	151	100

Upon examining Table 16, it is evident that a significant portion of the pre-service teachers provided correct answers (f=43; 28.5%), while another significant portion provided incorrect answers (f=41; 27.2%). Additionally, it is noteworthy that a considerable number of pre-service teachers did not provide any explanations (f=40; 26.4%). Here are some examples of the correct answers provided by the pre-service teachers:

S$_{8}$*: *The function is the special case of the relation. Every function is a relation, but not every relation is a function.

***S_{25}*: *Relation is a situation that establishes a relationship between sets. On the other hand, the function mentions certain rules while establishing this relationship. Relations with these rules are called functions.*

Upon examining the explanations provided by the pre-service teachers that were considered incorrect, it was observed that they made mistakes such as regarding correlation as a special case of the function or misinterpreting the rule of the function when explaining the relationship between relation and function. Here are some examples illustrating this situation:

***S_{32}*: *For an expression to be a relation, it must have reflection, transition, and symmetry properties. Every relation is a function, but not every function is a relation. Because there is no requirement for the function to provide these three properties.*

***S_{106}*: *Thanks to the relation, the function takes the domain to the value set.*

In addition, some pre-service teachers make meaningless explanations. One of these explanations is as follows;

***S_{13}*: *The main difference in relation and function is one-to-oneness. If the relation is one-to-one, it is a function.*

Upon examining the explanations that were accepted as partially correct, it was observed that the pre-service teachers often provided superficial explanations without establishing a clear relationship between the concepts. Additionally, they sometimes explained a one-way relationship without expressing the reciprocal relationship between the concepts. Here are some examples illustrating these situations:

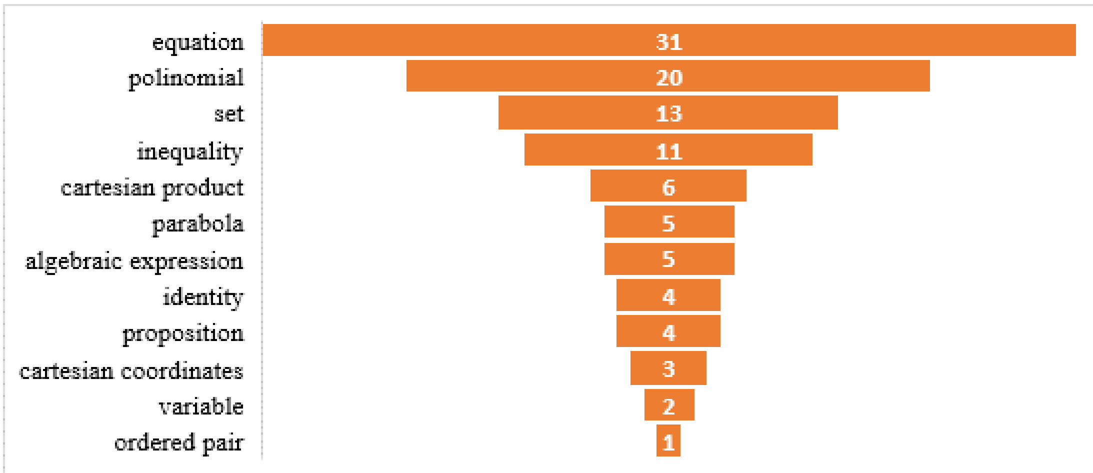
***S_{18}*: *Both determine a relationship between the given elements.*

***S_{48}**: *Let A and B be two sets that are not empty. Let it be $a \in A$ ve $b \in B$. Let f be a function from A to B. In this case, since f(a)=b $(a, b) \in AXB$, it is seen that a forms an ordered pair with b in B in both the function and the relation. Every function is a relation.*

Finally, pre-service teachers were asked to associate the concepts of relation and function with other concepts of algebra. The obtained findings are presented in Figure 3.

Figure 3

Pre-service teachers' Associations of Relation and Function Concepts with Other Concepts of Algebra



When examining Figure 3, it becomes evident that pre-service teachers associate the concepts of relation and function with various other concepts. The most common association is with the concept of equations (f=31; 20.5%), followed by polynomials (f=20; 13.2%), sets (f=13; 8.6%), and inequalities (f=11; 7.3%).

The second sub-problem of the research is "Does the knowledge levels of the pre-service teachers participating in the research on the basic concepts of algebra learning differ according to their grade levels?". In light of this sub-problem, non-parametric tests were selected as the number of data points did not meet the condition of having more than thirty observations in each group, which is necessary for using parametric tests. Therefore, non-parametric tests were chosen, and the findings obtained from the Kruskal-Wallis H test are presented in Table 17.

Table 17

Kruskal Wallis H Test Results of Algebra Knowledge According to Grade Levels

Grade	N	Average Rank	df	χ^2	P	Sig. Difference
1	58	56,25	3	21,191	.000*	1-2
2	47	83,74				1-3
3	39	90,40				1-4
4	7	107,43				

*p < .05

In Table 17, it is evident that the knowledge levels of pre-service teachers regarding the basic concepts of algebra learning differ significantly based on their grade levels ($\chi2 = 21.191$; $p < .05$). Upon examining the average ranks, it becomes apparent that the algebra knowledge levels of pre-service teachers increase as they progress to higher grade levels. The multiple comparison results obtained from the Kruskal-Wallis H test indicate that the algebra scores of pre-service teachers in the 2nd, 3rd, and 4th-grade levels significantly differ from those in the 1st grade.

The last sub-problem of the study was, "Does the knowledge levels of the pre-service teachers participating in the research on the basic concepts of algebra learning differ according to their genders?". To determine the appropriate analysis technique for this sub-problem, the normal distribution of the data was first examined. As the number of observations exceeded 29 (Kalaycı, 2008), the Kolmogorov-Smirnov test was employed to assess the data's conformity to a normal distribution. The findings of this analysis are presented in Table 18.

Table 18

Normality Test Results of Algebra Scores by Gender

Gender	Statistic	df	p
Female	.097	117	.009
Male	.076	34	.200*

*p > .05

The findings indicate that the data does not conform to a normal distribution. As a result, non-parametric tests were deemed necessary for analyzing the data. Consequently, the results of the Mann-Whitney U test, which aimed to investigate whether the knowledge levels of pre-service teachers regarding the basic concepts of algebra learning field differ based on their gender, are presented in Table 19.

Table 19

Mann Whitney U Test Results of Algebra Knowledge Levels by Gender

Gender	N	Average Rank	Order Total	U	p
Female	117	78,25	9155,00	-1,173	.241
Male	34	68,26	2321,00		
Total	151				

In Table 19, it can be observed that there is no statistically significant difference in the algebra knowledge levels of the pre-service teachers based on their genders (U = -1,173; p > .05). When considering the mean rank, it is evident that the average algebra scores of females (78.25) are higher than those of males (68.26).

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

In this research, the knowledge levels of pre-service secondary school mathematics teachers regarding the basic concepts of algebra were examined with respect to various variables. The findings indicate that the pre-service mathematics teachers possess a moderate level of understanding in the basic concepts of algebra. However, the proximity of the averages to the low level suggests the insufficiency of their algebraic knowledge. This finding highlights the inadequacy of algebra instruction provided in both secondary education institutions and undergraduate programs. Previous studies conducted at the middle school level (Didiş Kabar & Amaç, 2018) and university level (Dede et al., 2010; Sitrava, 2017) also support this result.

It was observed that, in general, pre-service teachers had a basic understanding of the concept of propositions, but they struggled to grasp the mathematical significance of a proposition requiring a judgment. This lack of understanding led to their inability to recognize the role of symbols in mathematical expressions and to perceive mathematical judgments. Consequently, they had difficulty recognizing concepts such as equations, inequalities, and identities as propositions. Furthermore, it was evident that pre-service teachers had insufficient comprehension of the concept of variables. This finding aligns with the results of a study conducted by Asquith et al. (2007) with secondary school teachers. The concept of variables serves as the foundation for the development of algebraic concepts (Akgün, 2009). Graham & Thomas (2000) emphasized the crucial role of perceiving the concept of variables in establishing an algebraic thinking structure and understanding algebraic concepts. The lack of sufficient understanding of the concept of variables indicates a deficiency in skills such as abstract thinking and generalization. Moreover, the insufficient formation of the concept of variables in pre-service teachers reveals their lack of knowledge in concepts such as open propositions, equations, and identities, as well as their limited ability to make associations.

Another significant finding is that pre-service teachers demonstrated proficiency in providing examples of equations, inequalities, and identities. However, they struggled with adequately defining and associating these concepts. This result is consistent with the findings of studies conducted by Lima & Tall (2006) and Aydin & Köğce (2008). It became apparent that pre-service teachers had difficulty establishing connections between all three concepts and instead relied on pairwise associations to express their common and distinct aspects. For instance, pre-service teachers acknowledged variables as the most critical shared feature among the three concepts. However, they failed to recognize or articulate that equations and identities must necessarily involve variables, whereas inequalities may not. Another oversight was observed when they identified symbols as a distinguishing characteristic, overlooking that equations and identities are founded upon the same symbols. These concepts hold significant importance in preparing students cognitively for advanced mathematical concepts and enabling practical problem-solving in daily life. The lack of understanding among pre-service teachers regarding these concepts will likely have a detrimental impact on students' algebraic development.

It was understood that pre-service teachers also had difficulties defining the concepts of relation and function and were insufficient to associate the concepts with each other. This result coincides with the results of Even (1990) and Norman (1992) on the concept of function, Dede et al. (2010) on the concept of relation, and Aydin & Köğce (2008) on the association of both concepts. The fact that the pre-service teachers especially see the concepts of relation and function as a type of equation shows that they do not make sense of the logical foundations of the concepts and they experience conceptual confusion. Dubinsky & Harel (1992) stated that especially the concept of function carries students from the basic level to the advanced mathematical level. This situation reveals the importance of well-perceiving students’ concepts of relation and function. On the other hand, Kabael (2010) dealt with the difficulties and misconceptions of students in the perception of the concept of function in three classes: definition of the concept, representation, and relations between them, and mathematical language. The fact that pre-service teachers have difficulties defining the concepts of relation and function indicates that they cannot internalize the concepts.

One of the important results of the research is that the concept of polynomials comes first among the concepts that pre-service teachers have the most difficulty in perceiving. In general, it has been observed that they see the concept of the polynomial as a special case of the function, but they do not know enough about the conditions of being a polynomial. It is thought that the fact that pre-service teachers especially associate polynomials with the equation, that is, they see polynomials as a type of equation, shows that their conceptual knowledge is not sufficiently developed. It is thought that this misperception is because the concept of the polynomial has a representation as "P(x)=…". The use of the "=" symbol here means that, unlike the equation, each value of the variable or variables in the polynomial will correspond to a result. In other words, while the polynomial has a dynamic structure, the equation, as it is stated by Dede et al. (2010) represents a more static structure provided for one or more values that the unknowns in it will take. It is thought that pre-service teachers do not particularly understand this situation.

Within the scope of the research, the knowledge levels of pre-service teachers on the basic concepts of algebra were also compared according to their grade level and gender. According to the results obtained, the knowledge levels of mathematics pre-service teachers on the basic concepts of algebra differ according to their grade levels. Accordingly, it was understood that the pre-service teachers' algebra knowledge improved as the grade level increased, and it differed significantly, especially compared to the first graders. This can be attributed to the influence of the algebra courses they took at the undergraduate level. In the first year of undergraduate education, mathematics pre-service teachers generally repeat their mathematics knowledge at secondary and high school levels and increase their readiness for upper grades. Therefore, the knowledge level of pre-service teachers at the first-grade level is limited to the knowledge they brought from high school. Starting from the second grade, pre-service teachers' knowledge of algebra develops with the influence of linear algebra, analytical geometry, and abstract algebra courses. Therefore, the result obtained within the scope of the research is an expected result. However, the fact that the general algebra levels of the pre-service teachers were insufficient suggests that these courses do not have enough impact on the conceptual development of the pre-service teachers.

The final result of the research is that the knowledge levels of pre-service mathematics teachers about the basic concepts of algebra do not differ according to their gender. Despite this result, when the group averages are taken into account, it was seen that the females' knowledge of algebra was better than the males. This result is quite remarkable in terms of the perception that men are more prone to mathematical thinking. This result, which was obtained especially in the field of algebra learning, where mathematical thinking is most intense, suggests that this perception can be refuted.

The results of the research show that the concepts of algebra in the secondary and high school curricula are not adequately taught, especially at the conceptual level. The mathematical language skills of students for these concepts are also insufficient. This situation requires the use of learning methods that support classroom dialogues and discussion environments in algebra teaching. In this context, it is suggested that contemporary teaching methods and techniques such as cooperative learning, problem-solving, discussion, brainstorming, and group work should be included in learning environments. The inadequacy of algebra teaching at secondary and high school levels can be associated with teachers' field knowledge. Students' cognitive development, especially towards mathematics, seems to be closely related to teachers' field knowledge. Teachers shape a large part of this field knowledge at the undergraduate level. This situation requires a review of the content and efficiency of algebra courses given in education faculties.

According to the results, another point that should be noted is that teachers generally give more importance to practice in learning environments and do not spare enough time for conceptual development, such as definition and association. It is very difficult to comprehend an area based on abstract concepts such as algebra with a practice-based teaching method. Therefore, it is recommended that teachers focus on conceptual development practices for algebra teaching to be more effective. In this context, especially concept definition, explaining and discussing the relations between concepts, etc., are considered very important to include such applications.

Ethical Approval: This research was conducted with the permission of the Istanbul University-Cerrahpasa Social and Human Sciences Research Ethics Committee with decision no 2021/237 dated 05.10.2021.

REFERENCES

Ahuja, O. P. (1998). Importance of algebraic thinking for preservice primary teachers. *The Mathematics Educator, 3*(1), 72-92.

Akgün, L. (2009). Eight-grade students’ connection skills between word problems and the concept of variable. *Mersin University Journal of the Faculty of Education*, *5*(2), 275–284. https://doi.org/10.17860/efd.00303

Asante, K. O. (2010). Sex differences in mathematics performance among senior high students in Ghana. *Gender and Behaviour*, *8*(2), 3279-3289. https://doi.org/10.4314/gab.v8i2.61947

Asquith, P., Stephens, A. C., Knuth, E. J., & Alibali, M. W. (2007). Middle school mathematics teachers’ knowledge of students’ understanding of core algebraic concepts: Equal sign and variable. *Mathematical Thinking and Learning*, *9*(3), 249–272. https://doi.org/10.1080/10986060701360910

Aydin, M., & Köğce, D. (2008). Preservice teachers’ perceptions of “equation and function” conceptions. *Journal of Yüzüncü Yıl University Education Faculty*, *5*(1), 46-71.

Ball, D. L. (1988). *Research on teaching mathematics: Making subject matter knowledge part of the equation.* National Center for Research on Teacher Education, Michigan State University.

Birgin, O., & Demirören, K. (2020). Investigation of 7th and 8th grade students' performance about algebraic expressions. *Pamukkale University Journal of Education*, *50*, 99-117. https://doi.org/10.9779/pauefd.567616

Çelik, D. (2007). *Analytical examination of the preservice teachers' algebraic thinking skills* [Unpublished doctoral thesis]. Karadeniz Technical University.

Cohen, J. (1960). A coefficient of agreement for nominal scales. *Educational and Psychological Measurement*, *20*(1), 37–46. https://doi.org/10.1177/001316446002000104

Creswell, J. W. (2013). *Research design: Qualitative, quantitative, and mixed methods approaches* (4th ed.). Sage Publications.

Dede, Y., Bayazit, İ., & Soybaş, D. (2010). Prospective teachers' understanding of equation, function, and polynomial concepts. *Kastamonu Education Journal*, *18*(1), 67-88.

Demir, H., & Tuğrul, K. (2021). The reasons for failure in the teaching content knowledge test of pre-service elementary mathematics teachers. *Social Sciences Studies Journal*, *7*(84), 2672–2687. https://doi.org/10.26449/sssj.3243

Didiş Kabar, M. G., & Amaç, R. (2018). Investigating pre-service middle-school mathematics teachers' knowledge of student and instructional strategies: An algebra case. *Journal of Abant İzzet Baysal University Education Faculty*, *18*(1), 157–185.

Dougherty, B., Bryant, D. P., Bryant, B. R., Darrough, R. L., & Pfannenstiel, K. H. (2015). Developing concepts and generalizations to build algebraic thinking: The reversibility, flexibility, and generalization approach. *Intervention in School and Clinic*, *50*(5), 273–281. https://doi.org/10.1177/1053451214560892

Dubinsky, E., & Harel, G. (1992). *The concept of function: Aspects of epistemology and pedagogy*. Mathematical Association of America.

Duru, A., & Savaş, E. (2005). Gender difference in mathematics teaching. *Erzincan University Journal of Education Faculty*, *7*(1), 35-46.

Even, R. (1990). Subject matter knowledge for teaching and the case of functions. *Educational Studies in Mathematics*, *21*(6), 521–544.

Graham, A. T., & Thomas, M. O. J. (2000). Building a versatile understanding of algebraic variables with a graphic calculator. *Educational Studies in Mathematics*, *41*(3), 265–282.

Huang, R., & Kulm, G. (2012). Prospective middle grade mathematics teachers' knowledge of algebra for teaching. *Journal of Mathematical Behavior*, *31*(4), 417–430. https://doi.org/10.1016/j.jmathb.2012.06.001

Kabael, T. U. (2010). Concept of the function: Historical development, understanding process, misconceptions and the teaching strategies. *TÜBAV Science Journal*, *3*(1), 128-136.

Kalaycı, Ş. (2008). *Spss uygulamalı çok değişkenli istatistik teknikleri [Multivariate statistical techniques with spss applications]*. Asil Publication.

Karakuş, F. (2018). Investigation of primary pre-service teachers' concept images on cylinder and cone. *Elementary Education Online*, *17*(2), 1033–1050.

Kartal, B., & Çinar, C. (2017). Examining pre-service mathematics teachers' geometry knowledge of polygons. *Journal of Ahi Evran University Kırşehir Education Faculty*, *18*(2), 375-399.

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33*(1), 159–174. https://doi.org/10.2307/2529310

Lima, R. N., & Tall, D. (2006). The concept of equations: What have students met before? *PME, 4*, 233–241.

Mason, J., Stephens, M., & Watson, A. (2009). Appreciating mathematical structure for all. *Mathematics Education Research Journal*, *21*(2), 10–32. https://doi.org/10.1007/BF03217543

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook, 2nd ed* (pp. xiv, 338). Sage Publications, Inc.

MoNE. (2018). *İlkokul ve ortaokul matematik dersi öğretim programı [Primary and secondary school mathematics curriculum]*. Ministry of National Education.

MoNE. (2019). *PISA 2018 Türkiye ön raporu [PISA 2018 Turkey preliminary report]* (No. 10; Eğitim Analiz ve Değerlendirme Raporları Serisi [Training Analysis and Evaluation Reports Series]).

MoNE. (2020). *TIMMS 2020 Türkiye ön raporu [TIMSS 2020 Turkey preliminary report]* (No. 15; Eğitim Analiz ve Değerlendirme Raporları Serisi).

Moskal, B. M., & Leydens, J. A. (2000). Scoring Rubric Development: Validity and Reliability. *Practical Assessment, Research & Evaluation*, *7*(10).

NCTM. (1989). *Curriculum and evaluation standards for school mathematics*. National Council of Teachers of Mathematics.

Norman, D. A. (1992). Design principles for cognitive artifacts. *Research in Engineering Design*, *4*(1), 43–50. https://doi.org/10.1007/BF02032391

Serbin, K. S. (2021). *Prospective teachers' knowledge of secondary and abstract algebra and their use of this knowledge while noticing students' mathematical thinking*.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching -. *Educational Researcher*, *15*(2), 4–14. https://doi.org/10.2307/1175860

Sitrava, R. T. (2017). Prospective mathematics teachers' concept images of algebraic expressions and equations. *Cumhuriyet International Journal of Education*, *6*(2), 1123-1143. https://doi.org/10.30703/cije.331098

Tavşancıl, E., & Aslan, A. E. (2001). *Sözel, yazılı ve diğer materyaller için içerik analizi ve uygulama örnekleri [Content analysis and application examples for oral, written and other materials]*. Epsilon Publishing.

Toheri, T., & Winarso, W. (2017, April 17). *Improving algebraic thinking skill, beliefs and attitude for mathematics throught learning cycle based on beliefs* [MPRA Paper].

Zuya, H. E. (2017). Prospective teachers' conceptual and procedural knowledge in mathematics: The case of algebra. *American Journal of Educational Research*, *5*(3), 310-315. https://doi.org/10.12691/education-5-3-12

GENİŞLETİLMİŞ ÖZ

Matematik eğitimcileri ve öğretmenlere göre okul matematiğinin en önemli alanlarının başında cebir gelmektedir. Özellikle bireylerin çevrelerindeki olayları anlamlandırabilmeleri ve hayatlarında başarılı olabilmelerinde önemli olan akıl yürütme ve muhakeme etme gibi becerilerin gelişiminde cebirin önemli bir rolü vardır (Birgin & Demirören, 2020). Cebir konularının öğrencilerin soyut düşünme, ilişkilendirme, muhakeme etme gibi becerileri üzerinde ciddi etkileri olmaktadır. Buna karşın, ülkemizde öğrencilerin cebir konularına yönelik performanslarının düşük olması dikkat çekicidir (Dede vd., 2010). Bu durum, özellikle öğrencilerin matematik bilgi ve becerilerini ölçen ve uluslararası karşılaştırmalar yapan TIMMS ve PISA gibi sınavlarda açıkça görülmektedir. Toheri ve Winarso (2017) öğretmenlerin bilgi ve becerilerinin öğrencilerdeki cebirsel düşünme becerisinin oluşturulması ve gelişmesi açısından önemine vurgu yapmışlardır. Cebir öğretimindeki başarısızlık dikkate alındığında öğretmen faktörünün değerlendirilmesi ve özellikle öğrencilerdeki cebir bilgisinin temelini oluşturan ortaokul matematik öğretmenlerinin ve öğretmen adaylarının alan bilgilerindeki eksikliklerin incelenmesi oldukça önemli görülmektedir. Dolayısıyla bu araştırmada, ortaokul matematik öğretmen adaylarının cebirin temel kavramlarına yönelik bilgi düzeylerini belirlemek ve cinsiyet ve sınıf düzeyi değişkenlerine göre incelemek amaçlanmıştır. Araştırmanın, geleceğin matematik öğretmenleri olacak öğretmen adaylarının cebire yönelik mevcut bilgi düzeylerini ortaya koyması ve ortaokul, lise ve lisans seviyelerinde verilen cebir eğitiminin sorgulanması açısından önemli olduğu düşünülmektedir.

Bu araştırmada, matematik öğretmen adaylarının cebirin temel kavramlarına yönelik bilgi düzeylerinin açık uçlu sorular aracılığıyla derinlemesine incelenmesi amaçlandığından durum çalışması yönteminin kullanılması uygun görülmüştür. Araştırma İstanbul ilinde yer alan bir üniversitenin ilköğretim matematik öğretmenliği bölümünde öğrenim gören 151 öğretmen adayı ile yürütülmüştür. Araştırmanın katılımcıları kolay ulaşılabilir durum örneklemesi ile belirlenmiştir. Araştırmada katılımcıların cebirin temel kavramlarına yönelik bilgi düzeylerini belirlemek için araştırmacı tarafından geliştirilen "Cebir Öğrenme Alanı Temel Kavramları Ölçeği" kullanılmıştır. Hazırlanan ölçek 9 adet açık uçlu sorudan oluşmaktadır. Ölçekte öğretmen adaylarının cebir öğrenme alanı kavramlarına yönelik tanımlama, örnekleme ve ilişki kurma becerilerini ölçecek nitelikte sorular yer almaktadır. Ölçek maddeleri ortaokul ve lise öğretim programlarında yer alan cebirin temel kavramlarına yönelik kazanımlar dikkate alınarak ve öğretmen adaylarından iki kategoride bilgi toplanmasına imkân tanıyacak biçimde hazırlanmıştır. İlk maddede öğretmen adaylarından araştırmacı tarafından belirlenen cebirin temel kavramlarını (önerme, açık önerme, bağıntı, fonksiyon, denklem, eşitsizlik, özdeşlik, polinom) tanımlamaları diğer maddelerde ise bu kavramlara yönelik örnekler vermeleri veya kavramlar arası ilişkileri açıklamaları istenmiştir. Bu çalışma kapsamında sadece ortaokul düzeyi kavramlarla sınırlı kalınmamış, bağıntı, fonksiyon ve polinom gibi lise düzeyi önemli kavramlara da yer verilmiştir. Bu sayede öğretmen adaylarının sadece öğretmenlik hayatlarında anlatacakları içeriğe değil cebirin geneline ilişkin kavramsal düzeylerine ulaşılabileceği düşünülmüştür. Araştırma kapsamında veriler öğretmen adaylarının yazılı yanıtlarından elde edilmiştir. Ölçek öğretmen adaylarına yüz-yüze ortamda uygulanmış ve her bir öğretmen adayına 30-45 dk arası süre verilmiştir. Verilerin analizi araştırmacı tarafından hazırlanan bir rubrik yardımıyla elde edilen puanlara göre betimsel ve kestirimsel istatistikler kullanılarak yapılmıştır. Öğretmen adaylarının cebir kavramlarına yönelik bilgi düzeylerinin incelenmesinde betimsel istatistikler, bu düzeylerinin sınıf düzeyi ve cinsiyete göre durumu incelenirken ise veriler normallik varsayımını karşılamadığı için Kruskal Wallis H ve Mann Whitney U testlerinden yararlanılmıştır.

Araştırmanın bulguları matematik öğretmen adaylarının cebirin temel kavramlarına yönelik bilgi düzeylerinin orta düzeyde olduğunu göstermektedir. Buna karşın, ortalamaların alt düzeye çok yakın olması öğretmen adaylarının cebir kavramlarına yönelik bilgilerinin yetersiz olduğu anlamına gelmektedir. Genel olarak öğretmen adaylarının cebir kavramlarına yönelik yetersiz bilgilerinin temelinin değişken kavramına yönelik düşük algıya dayandığı düşünülmektedir. Değişken kavramı, cebirsel kavramların oluşumunun temelidir (Akgün, 2009). Graham ve Thomas (2000) değişken kavramının algılanmasının cebirsel düşünme yapısının oluşması ve cebir kavramlarının anlaşılması açısından önemine vurgu yapmışlardır. Değişken kavramının algılanmaması özellikle soyut düşünme ve genelleme yapma gibi becerilerin bireyde yeterince oluşmadığını göstermektedir. Öğretmen adaylarında değişken kavramının yeterince oluşmaması, açık önerme, denklem, özdeşlik gibi kavramlardaki bilgi eksikliklerini ve ilişkilendirme yetersizliklerini de ortaya çıkarmaktadır.

Araştırma kapsamında elde edilen bir diğer sonuç, matematik öğretmen adaylarının cebirin temel kavramlarına yönelik bilgi düzeylerinin sınıf seviyelerine göre farklılaştığıdır. Buna göre, öğretmen adaylarının cebir bilgilerinin sınıf seviyesi arttıkça geliştiği ve özellikle birinci sınıflara göre anlamlı düzeyde farklılaştığı gözlemlenmiştir. Bu durum, lisans düzeyinde aldıkları cebir derslerinin etkisine dayandırılabilir. Araştırmada elde edilen son sonuç ise matematik öğretmen adaylarının cebirin temel kavramlarına yönelik bilgi düzeylerinin cinsiyetlerine göre farklılaşmadığıdır. Elde edilen bu sonuca karşın, grup ortalamaları dikkate alındığında anlamlı düzeyde bir farklılık oluşmasa da kızların cebir ortalamalarının erkeklere göre daha yüksek olduğu görülmüştür.

Araştırmanın sonuçları ortaokul ve lise müfredatlarında yer alan cebir kavramlarının özellikle kavramsal boyutta yeterli düzeyde öğretilemediğini ve matematiksel dil becerilerinin de yetersiz kaldığını göstermektedir. Bu durum, cebir öğretiminde sınıf içi diyalogları ve tartışma ortamlarını destekleyen öğrenme yöntemlerinin kullanılmasını gerektirmektedir. Özellikle işbirlikli öğrenme, problem çözme, tartışma, beyin fırtınası ve grup çalışması gibi çağdaş öğretim yöntem ve tekniklerinin öğrenme ortamlarına dahil edilmesi önerilmektedir. Öğrencilerin özellikle matematiğe yönelik bilişsel gelişimleri öğretmenlerin alan bilgileri ile yakından ilişkili görülmektedir. Öğretmenler bu alan bilgilerinin büyük bir kısmını lisans düzeyinde şekillendirmektedirler. Bu durum, eğitim fakültelerinde verilen cebire yönelik derslerin içeriklerinin ve verimliliğinin yeniden gözden geçirilmesini gerektirmektedir.