ESKİŞEHİR TEKNİK ÜNİVERSİTESİ BİLİM VE TEKNOLOJİ DERGİSİ B- TEORİK BİLİMLER

Eskişehir Technical University Journal of Science and Technology B- Theoretical Sciences

2023, 11(2), pp. 116-124, DOI: 10.20290/estubtdb.1269417

RESEARCH ARTICLE

NUCLEAR ASYMPTOTIC NORMALIZATION COEFFICIENT FOR ^{27}Al $\rightarrow$ ^{26}Mg+p REACTION

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ABSTRACT

The ^{26}Mg(p,ɣ)^{27}Al reaction is important in nuclear astrophysics as it play a crucial role in understanding the nucleosynthesis processes in red giants and Wolf-Rayet stars. The ^{26}Mg(p,ɣ)^{27}Al reaction is responsible for the production of ^{27}Al in these stars, while the ^{26}Mg(^{3}He,d)^{27}Al reaction provides information on the asymptotic normalization coefficient for the ground state of ^{27}Al.The asymptotic normalization coefficient (ANC) method is an indirect method that provides information on the normalization of the overlap functions for a given reaction. This information is crucial for nuclear astrophysics as it allows for the calculation of the direct component of the reaction rate at astrophysical relevant energies.In this work, the angular distribution of the ^{26}Mg(^{3}He,d)^{27}Al reaction have been analyzed using separate sets of optical potentials via the Distorted Wave Born Approximation which allows for a better understanding of the reaction mechanism and the determination of the ANC. Consequently,thecross section and Astrophysical S factor for ^{27}Al→^{26}Mg + p have been calculated for the direct capture.

Keywords: Direct reaction, DWBA analysis, Asymptotic normalization coefficient, Nuclear astrophysics

1. INTRODUCTION

Nuclear astrophysics is a field that seeks to understand the processes involved in the production, evolution, and distribution of chemical elements in the universe. One of the challenges in this field is to explain the origin and abundance of heavier elements, such as those beyond iron, which are formed through processes involving fusion, neutron capture, and explosive events such as supernovae[1].

One important element in this regard is Aluminum-26 (^{26}Al), half-life of^{26}Al is determined as a($t_{1/2}$=7.2*10^5 y) [2]. This isotope is produced primarily through the Mg-Al cycle [2], a series of nuclear reactions that occur in the interiors of massive stars. Understanding the production and distribution of ^{26}Al is important because its decay produces gamma rays that are observable in the galaxy.The abundance of stable aluminum-27 (^{27}Al) is also an important consideration in nuclear astrophysics. This isotope is not produced in significant quantities through nuclear fusion, but rather through the slow capture of neutrons in the s-process [3], a type of nucleosynthesis that occurs in the orduring to later stages of stellar evolution. The ratio of ^{27}Al to ^{26}Al in the galaxy can provide insights into the correspondingadditions of the s-process and the Mg-Al cycle to the production of these isotopes [2].

Another challenge in nuclear astrophysics is to realizethe galactic distribution of^{26}Al. Observations have shown that the proportion of the ground level to the isomeric level in ^{26}Al varies acrossdifferent regions of the galaxy[4]. This ratio may be influenced by the destruction of ^{26}Si [4], which can decay into ^{26}Al. This process may occur in novae, which are explosive events that can produce significant amounts of ^{26}Al. Understanding the details of these processes is important for accurately modelling the production and distribution of ^{26}Al in the galaxy.

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Received: 22.03.2023 Published: 28.08.2023

2. ASYMPTOTIC NORMALIZATION COEFFICIENT

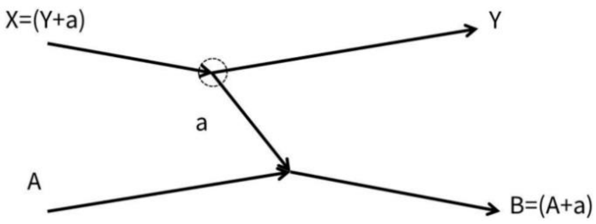


Figure 1: Sketch of a general transfer function

The ANC method, together with the DWBA formalism, permit to extract information on the nuclear structure of the initial and final states involved in the transfer reaction, and on the strength and energy dependence of the direct capture process A+a→B+γ at astrophysical energies. This information is important to understand the nucleosynthesis of heavy elements in stars and other astrophysical environments.It should be noted that the ANC method has some limitations and assumptions, such as the validity of the DWBA approximation, the neglect of higher-order effects in the transfer reaction, the assumption of a single-particle model for the bound state wave functions, and the dependence of the results on the choice of the potential model used to describe the nuclear interaction. Therefore, it is important to compare the results obtained with the ANC method with other experimental and theoretical approaches, and to carefully assess the uncertainties and systematic errors in the data analysis. In particular, the reactionA+a→B+γ can be studied via ANC in term of the radial overlap integral of a suitable one-particle transfer reaction A(X,Y)B, in which X=Y+A and B=A+a depicted in (Fig.1).This method has been carried out to a number of transfer reactions involvingα-particles [6], protons [4], neutrons [5], and indirect methods can be used to investigate nuclear reactions for astrophysics at the Gamow energies [7,8, 9].

One nucleon nuclear transfer reaction could be parameterized employing the distorted wave Born approximation (DWBA)[10]. This can be made as the spectroscopic factors -S relative to the initial and final states related to a particular bound state:

$$\frac{d\sigma}{d\Omega} = \sum_{j_B,j_X} S_{Aa,l_B,j_B}\, S_{Ya,l_X,j_X}\, \sigma^{DWBA}_{l_B,j_B,l_X,j_X}. \tag{1}$$

the differential cross section can be expressed as

$$\frac{d\sigma}{d\Omega} = \sum_{Aa,l_B,j_B} \left(C^B_{Aa,l_B,j_B} C^X_{Ya,l_X j_X}\right)^2 \times\times \frac{\sigma^{DWBA}_{l_B,j_B,l_X,j_X}}{b^2_{Aa,l_B,j_B} b^2_{Ya,l_X,j_X}}. \tag{5}$$

In Equation 5, the function $\sigma^{DWBA}_{l_B,j_B,l_X,j_X}$is the cross-section and it could be used to reproduce the angular DWBA.The ANC theory, as stated in the introduction, has an extension that allows determining the ANC's for the mirror nuclei. In the case of a proton transfer, in fact, the coefficients for the process A+p→B can be extracted from its appropriatemirror partner reaction D+n→E,D and having inverted number of protons and neutrons with respect to *A* and *B* (the vice versa is also valid) [11-12]. Therefore possible to extract the ANC's for the direct capture into bound states and the and the $\Gamma_p$for the resonant ones of ^{26}Mg(p,γ)^{27}Al, applying the ANC method on data for the ^{26}Mg(^{3}He,d)^{27}Al reaction.

3. DWBA METHOD

The ^{26}Mg (^{3}He, d) ^{27}Al low energy nuclear reaction has been interpreted with the help of the DWBA, In the DWBA formalism, the scattering amplitude is obtained by multiplying the distorted wave functions of the incoming and outgoing particles with the transition matrix element, which describes the probability of transferring a nucleon from the projectile to the target nucleus. The distorted wave functions are obtained by solving the Schrödinger equation with the appropriate optical potentials[10].The DWBA calculation also requires knowledge of the spectroscopic factor, which describes the overlap between the wave function of the initial state of the projectile and the final state of the residual nucleus[10].The full finite-range approximation takes into account the finite range of the nuclear interaction and improves the accuracy of the DWBA calculation. The DWUCK-5 and FRESCO codes are widely used for DWBA calculations in nuclear physics.In this paper, I applied full finite-range approximation using DWUCK-5[13], and FRESCO code [14] within the DWBA. The DWBA calculation with the full finite-range approximation and the appropriate optical potentials and spectroscopic factors provides a powerful tool for understanding the structure of nuclei and the mechanisms of the ^{26}Mg (^{3}He, d) ^{27}Al nuclear reactions.Here, the entrance channel optical potential parameters were taken from the experimental [15],and exit channel optical potential parameters were obtained from the experimental d+^{27}Al[15]. The optical potential model parameters could be shown in the real and imaginary part oftotal potential

$$U = V_C(r_C) - V_0\big(f(x_0)\big) + \frac{\hbar^2}{m_\pi c} V_{LS}(LS)\frac{1}{r}\frac{d}{dr}f(x_{LS}) - i\left[W_v(x_v) - 4w_D\frac{d}{dx_D}\,f(x_D)\right] \quad (6)$$

In the optical potential formula of (6), real potential is responsible for scattering and imaginer potential is responsible for absorbing. Here, V_o, $V_{LS}$and V_C (r_C) representsthe real part of potential, the spin-orbit term and the Coulomb potential, respectively. W_V and W_D are the depth of the volume term and the depth of surface term for the imaginary part of the potential, respectively. In order to calculate DWBA, the radial dependence of form of the Woods – Saxon volume potential was used. Obtained parameters for Wood- Saxon are that Real potential-V_0 = - 47.56, Coulomb radius-r_C= 1.25 and diffuseness parameter- a_C=0.65

Table 1. Optical potential parameters for of ^{26}Mg (^{3}He,d)^{27}Al reaction and corresponding to SF and ANC coefficient .

Parameter	^{3}He	D-I	D-II	D-III
V_r (MeV)	217.6	76.75	89.17	85.81
r_r	1.15	1.25	1.13	1.13
a_r	0.636	0.737	0.8	0.75
W_W	32.5	--------	-------	------
W_S	---------	13.5	12.35	12.35
r_i	1.4	1.25	1.4	1.325
a_i	0.936	0.738	0.6	0.55
V_{so}	6.2	6.2	6.2	6.2
r_{so}	1.01	1.01	1.01	1.01
a_{so}	0.75	0.75	0.75	0.75
r_c	1.25	1.25	1.25	1.25
SF	--------	0.37	0.34	0.3

Optical potential parameters for exit and entrance channel are presented in Table.1. The results of DWBA calculation compared to the different experimental data are presented in Figures 2. a,b and c. Thetheoretical astrophysical S- factor quantity in MeV has been calculated inoperating the RADCAP code [17], employing a potential model for the Woods-Saxon well of the ^{26}Mg + p compound system. The potential is adjusted to match the ANC value. This calculation shows that the S(E) is nearly constant between 0 and 2 MeV. This range was selectedtaking into account the Gamow windows and Gamow energies (E_G) for the process at the temperatures reported in [18].

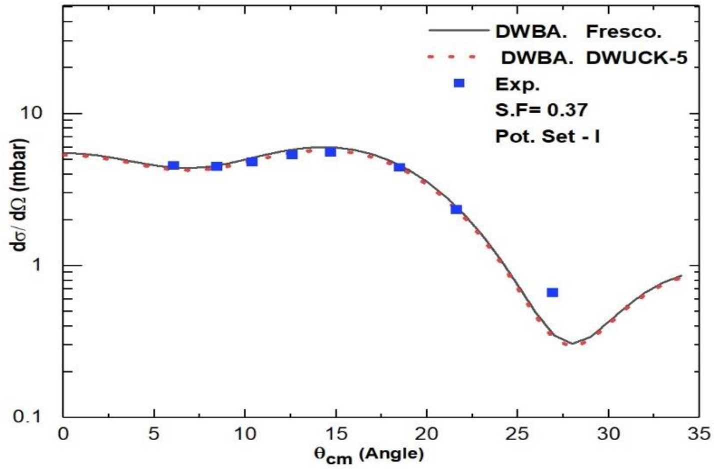


Figure 2 (a)

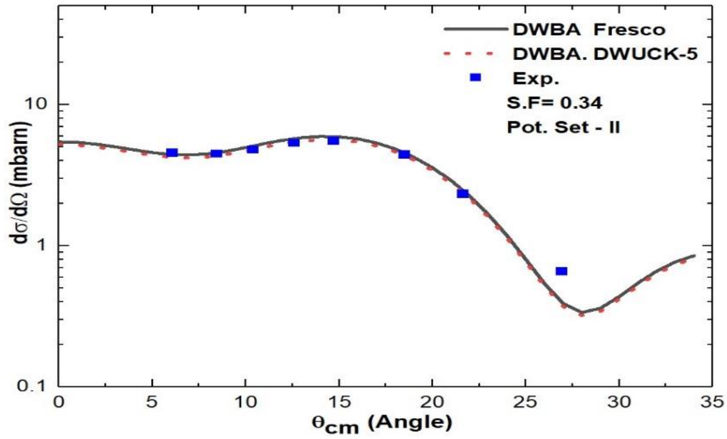


Figure 2. (b)

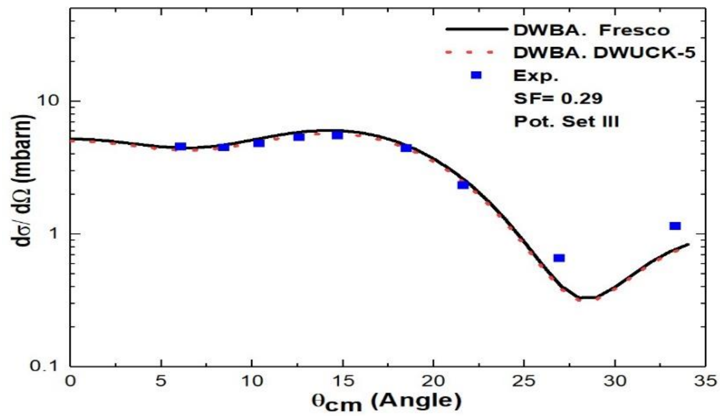


Figure 2. (c)

Figure 2. The experimental and theoretical ^{26}Mg (^{3}He,d)^{27}Al reaction differential cross sections for the transitions leading to the ground state in^{27}Al with the incident energy of 25 MeV. Blue square dots refer to experimental angular distributions from Vernotte[15]. FRESCO and DWUCK-5 codes were compared black curve and red dashed line. D-I, D-II and D-III optical potential used for producing theoretical angular distributions depicted in Figure 2. (a) and Figure 2. (b), and Figure 2. (c), respectively.

4. SUMMARY AND CONCLUSION

This paper describes a study on the one proton transfer reaction $^{26}Mg(^{3}He,d)^{27}Al$, which is important for determining nuclear asymptotic normalization coefficients (ANCs) and investigating astrophysical S-factors and cross sections. The paper reanalyses the experimental angular distribution for this reaction, leading to the ground state of ^{27}Alin $1d_{5/2}$ were shown in Figures 2. (a,b-c).

Using the extracted ANC, the $^{26}Mg(p,\gamma)$ S-factors and cross section for capture to the ground state were calculated depicted in Figune 4. And Figure 5. without the need for additional normalization constants. In order to get ANC coefficient, angular momentum transfer at low angle is crucial, one can understand that DWBA theory explain very well experimental data at low angles. DWBA method and experimental data consisted at very low angles. The squares of proton asymptotic normalization coefficient (ANC) for $^{27}Al \rightarrow {}^{26}Mg + p$ is extracted to be 8.2 ± 2 $fm^{-1}$from the angular distributions of the $^{26}Mg(^{3}He,d)^{27}Al$ reaction leading to ground state of ^{27}Al based on DWBA theory. The ANC coefficient of projectile ^{4}He interrelated to the vertex $^{3}He \rightarrow d+p$ in the channel is recognised with the high certainty and its value is also determined as a $(C_{He}{}^{4})^{2}= 3.90 \pm 0.06$ fm^{-1}. [20].This parameter used for analysis of determining ANC of ^{27}Al.

The paper also discusses the extraction of the proton ANC of $^{27}Al \rightarrow {}^{26}Mg + p$ using the wave function of ^{27}Al presented in Figure3. The calculations show that applying the FR-DWBA with the suitable optical potential angular distributions for the ground state of ^{27}Al are able to theoretically reproduce the experimental data in the locality of the first peak, which was sufficient for determining an ANC of ^{27}Al from the reaction. The theoretical angular distributions were obtained using Dwuck-5 and Fresco codes, which showed the same behaviour and results which are depicted in Figures 2. (a-b-c). We also calculated spectroscopic factor depending on different optical potential sets which is crucial for determination of ANC presented in Table-1.

However, I also note that the contribution of the reaction rate $^{27}Al \rightarrow {}^{26}Mg + p$ mainly comes not only from direct contribution but also from resonance contributions such as $J^{\Pi}=5/2^{+}$, $J^{\Pi}=1/2^{+}$, and $J^{\Pi}=3/2^{-}$. Therefore, additional computations were needed to include these other resonance contributions in the total S-factor and reaction rate calculation.

In summary, the ANC method is a useful indirect technique for studying nuclear reactions, especially direct capture, and has been successfully applied to transfer reactions involving protons, neutrons, and α-particles. The method involves determining the radial overlap integral of a suitable one-particle transfer reaction, which can be parameterized using the distorted wave Born approximation and spectroscopic factors. The radial overlap function can also be described in the asymptotic limit using the Wittaker function and Sommerfeld parameter[21]. The ANC method can be extended to determine ANC's for mirror nuclei, and can be used to extract ANC's for direct capture and resonant states in nuclear reactions which is $^{26}Mg(p,\gamma)^{27}Al$, using data from the $^{26}Mg(^{3}He,d)^{27}Al$ reaction. Because of lacking experimental data for ground state transition of $^{26}Mg(p,\gamma)^{27}Al$ reaction, i presented here just theoretical S-factor and Cross section data. Furthermore, this reaction has special interest for nuclear astrophysics and we plan to investigate experimental investigation of $^{26}Mg(p,\gamma)^{27}Al$ reaction in near future to get experimental S-factor and cross section for ground state contribution.

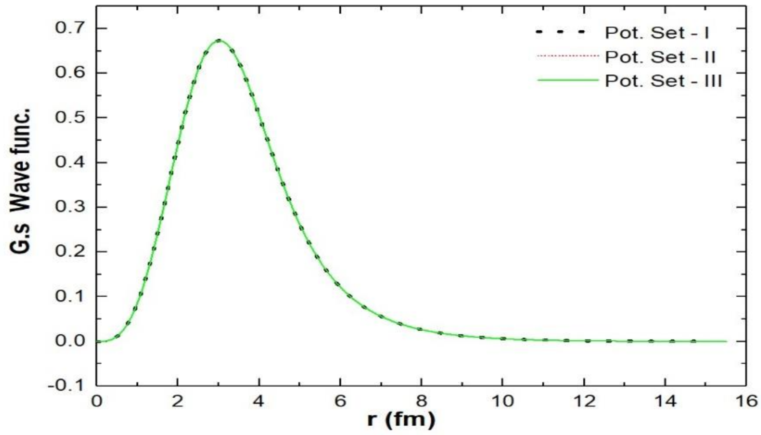


Figure 3. Extracted wave functions in terms of different optical potential parameters of Set –I, II and III.

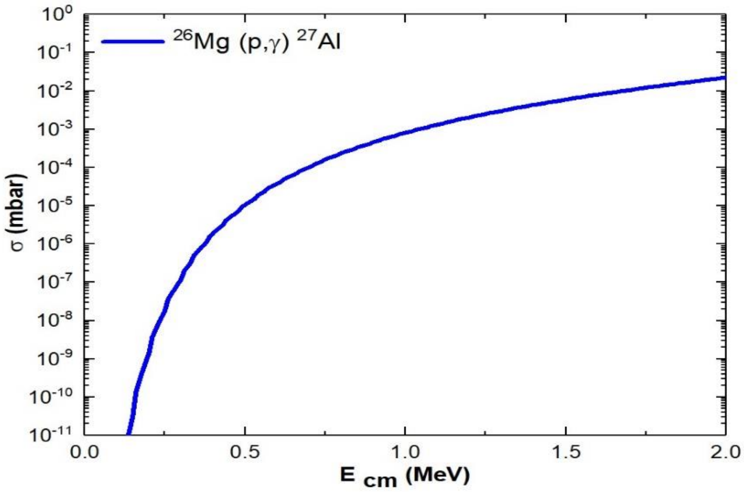
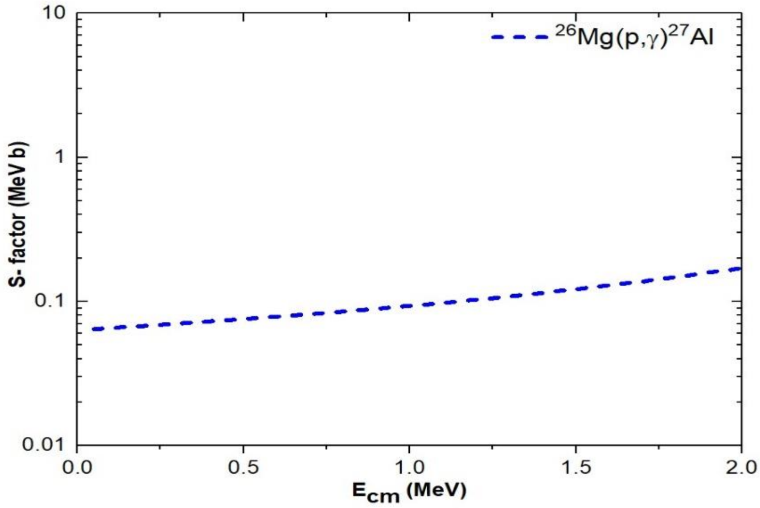


Figure 4. Direct contribution cross-section of ^{26}Mg (p, g) ^{27} Al reaction calculated using ANC method.



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CONFLICT OF INTEREST

The author stated that there are no conflicts of interest regarding the publication of this article.

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