H.Ü. İktisadi ve İdari Bilimler Fakültesi Dergisi, Cilt 26, Sayı 1, 2008, s. 99-119

MEASURING EFFICIENCY OF TURKISH AUTOMOTIVE FIRMS WITH THE FUZZY DEA MODEL

Tuba YAKICI AYAN*
Selçuk PERÇİN**

Abstract

The aim of this paper is to compare the efficiency of automotive firms in the context of standard DEA, bounded (crisp) DEA, and fuzzy DEA approaches and to apply a bounded fuzzy DEA model by imposing bounds on input and output factors. Actual data on 37 automotive firms recorded in Istanbul Chamber of Industry (ISO) were obtained for illustration purposes of fuzzy-DEA and compared the efficiency results with those obtained with standard DEA and bounded (crisp) approaches. According to the analysis results, average efficiencies differ significantly across methods. Besides, fuzzy-DEA model results have outlined that real evaluation of one problem in the context of DEA is generally applicable, and in many situations is likely to result in more realistic estimates of efficiency than standard DEA and bounded (crisp) approaches.

Keywords: Efficiency, data envelopment analysis (DEA), Fuzzy DEA, Fuzzy linear programming.

Öz

Bulanık Veri Zarflama Analizi İle Türk Otomotiv Firmalarının Etkinlik Ölçümü

Bu çalışmanın amacı, Türk otomotiv firmalarının standart VZA (veri zarflama analizi), sınırlı VZA ve bulanık VZA yöntemleri ile hesaplanan etkinliklerini karşılaştırmak ve girdi-çıktı faktör ağırlıklarını sınırlandırarak bir sınırlı bulanık VZA modeli uygulamaktır. Sınırlı bulanık VZA nın gösterimi amacı ile İstanbul Sanayi Odasına (ISO) kayıtlı 37 otomotiv firmasının gerçek verileri elde edilmiş ve hesaplanan etkinlik sonuçları standart VZA ve sınırlı

* Yrd.Doç.Dr., Karadeniz Technical University, The Faculty of Economics and Administrative Sciences, TRABZON, ayan@ktu.edu.tr

** Yrd.Doç.Dr., Karadeniz Technical University, The Faculty of Economics and Administrative Sciences, TRABZON, spercin@ktu.edu.tr

yaklaşımlardan elde edilen sonuçlarla karşılaştırılmıştır. Analiz sonuçlarına göre sözkonusu metodlar, birbirlerinden önemli ölçüde farklı etkinlik puanları üretmişlerdir. Bunun yanı sıra, Bulanık VZA modelinin diğer VZA modellerinden daha gerçekçi sonuçlar verdiği sonucuna varılmıştır.

Anahtar Sözcükler: Etkinlik, veri zarflama analizi (VZA), Bulanık VZA, Bulanık doğrusal programlama.

INTRODUCTION AND BACKGROUND

DEA, widely known as a non-parametric approach, is basically a mathematical programming technique developed by Charnes, Cooper and Rhodes (CCR) (1978) to evaluate the relative efficiency of a set of homogenous ‘‘decision making units’’ (DMUs) (Emel *et al*., 2003). Conceptually, the relative efficiency of DMUs that is constrained to be no more than 1 is compared by using a ratio of the weighted sum of outputs to the weighted sum of inputs. Specifically, DEA determines a set of weights such that the efficiency of one DMU relative to the other DMUs is maximized and identifies the source of inefficiency in each input relative to each output for the DMUs (Lertworasirikul *et al*., 2003).

In the business sector, DMUs can be companies, firms, service centers, management or employees (Dia, 2004). Although, the real evaluation of DMUs often implies strong imprecision and great uncertainty, evaluating the traditional DEA with these entities requires that data be accurate (crisp) in its analysis. Indeed, as the system's complexity increases, accurate evaluation of data becomes extremely difficult (Dia, 2004:268). In such situations, in order to compare the performance of DMUs, it is thus important to incorporate fuzzy modeling with traditional DEA especially since the former is reflecting a general feeling or experience of experts, whereas later particularly sensitive to the quality of the data (Guo and Tanaka, 2001; Dia, 2004).

DEA has been extensively used to measure efficiency of entities such as schools in performing their education (Sarrico and Dyson, 2000), hospitals, hotels, banks and libraries in providing their services (Al-Shammari, 1999; Ramanathan, 2005; Barros and Alves, 2004; Asish and Ravisankar, 2000; Weber 2002; Chen, 1997) in which outputs and inputs are always multiple forms. An advantage of DEA in these examples, it is a powerful tool for efficiency measurement. However, in some situations, such as in a manufacturing system, and in production process or a service system, inputs and outputs are easy to change. It is difficult to measure them in an accurate way to obtain precise data (Lertworasirikul *et al*., 2003). Some researchers, however, when they evaluating the problems, used stochastic efficient frontier techniques to deal with inaccurate and fluctuating inputs and outputs in DEA models (Aigner *et al*., 1977). In addition, this method can not be used for the multiple-input, multiple-output form (Charnes *et al*., 1978; Lertworasirikul, 2002). In the light of above findings, one has to say that unbounded version of the DEA model can give rise to undesirable consequences (Roll *et al.,* 1991*)*. Because, traditional DEA requires precise input and output data, it is very difficult to evaluate the efficiency of DMUs with unbounded inputs and outputs by traditional DEA models (Guo and Tanaka, 2001). For this reason, linguistic or qualitative data presented by fuzzy numbers should be used to deal with the fluctuating or imprecise data in evaluating the efficiency of entities.

In recent years, a number of different approaches have been carried out as a way to quantify imprecise and vague data in DEA models by Lertworasirikul, (2002), Lertworasirikul *et al*., (2003), Guo and Tanaka (2001), Triantis and Girod (1998), Maeda *et al*., (1998) and Entani *et al*., (2002). Lertworasirikul (2002) was, for example, interested in modeling some small examples in a fuzzy context using a DEA approach while Lertworasirikul *et al*., (2003) developed a possibility approach that transforms fuzzy DEA models into possibility DEA models by using possibility measures of fuzzy constraints. Similarly, Guo and Tanaka (2001) evaluated the fuzzy DEA and also proposed a model with considering the relationship between DEA and RA (regression analysis). Besides, Triantis and Girod (1998) developed a mathematical programming approach to measure technical efficiency in a preprint and packaging manufacturing line in the context of fuzzy DEA environment. Entani *et al*., (2002) proposed a DEA model with an interval efficiency consisting efficiencies obtained from the pessimistic and the optimistic viewpoints to deal with fuzzy data. Meanwhile, mentioned above researchers (Zimmerman, 1996; Guo and Tanaka 2001; Maeda *et al*., 1998) have been used tolerance, ranking and parametric programming approaches when comparing of fuzzy DEA to other efficiency measurement models. In these researches, it is shown that fuzzy DEA models can more realistically represent real-world problems than the traditional DEA models. Fuzzy set theory also allows linguistic or qualitative data to be used directly within the DEA models (Lertworasirikul et al., 2003:380).

In this paper, firstly, by adding upper and lower bounds on the weights of inputs and outputs, bounded (crisp) version of the standard DEA is obtained. Secondly, to model the uncertainty of the bound values, crisp bounds are replaced by the fuzzy numerical values. Finally, to determine the fuzzy numerical values with triangular membership functions, upper and lower limits of numerical data merging into parametric programming approaches are transformed to linear programming models, and solved by a Lindo 6.1 program.

This approach uses the traditional DEA framework with bounded constraints and then merges this concept developed in fuzzy parametric programming (Carlsson and Korhonen, 1986; Triantis and Girod, 1998).

The paper is organized as follows. In the first section of the study, the fuzzy model was used to develop and to solve the uncertainty in the bounded DEA (crisp) model. In Section 2, the entire procedure was presented, and possible interpretations to differences in efficiency ratings, obtained with the standard DEA, bounded (crisp) model and Fuzzy DEA, were discussed. Section 3 closed with final remarks and future directions.

1. FUZZY MODEL FOR THE BOUNDED DEA

In this section, it was examined to impose bounds on factor weights and to combine this with the fuzzy DEA. Fuzzy DEA aims at evaluating the performance of organizations in an uncertain and vague context (Dia, 2004). Incorporating fuzzy data in bounded DEA (crisp) models not only allows for a greater realism in modeling, it also represents the upper and lower bounds on factor weights of data in the model so that none of the factors are ignored or over emphasized with high weights (Kabnurkar, 2001; Dia, 2004; Allen *et al*., 1997).

The relative efficiency of DMUs with in the bounded DEA framework can be maximized as the ratio of weighted outputs to weighted inputs for each DMU (Roll *et al.*, 1991; Pedraja-C. *et al*., 1997). This fractional programming problem is equivalent to a linear programming model developed by Charnes *et al*., (1962, 1978). In order to determine appropriate values for the bounds of each DMU, some methods were proposed by Roll *et al*., (1991) and Roll and Golany (1993). With respect to those methods, appropriate bounds can only be set after examining the results from an unbounded DEA, and may have to be varied from one DMU to another (Roll *et al*., 1991; Pedraja-C. *et al*., 1997). In implementing this method, firstly, an unbounded DEA model is run, and a weights matrix is compiled with eliminating extreme values leading to anomalies in the results for calculating the bounds. Secondly, specified bounds are added as upper and lower bound constraints to the original DEA (CCR) model to obtain the bounded DEA (crisp) model (Allen *et al*., 1997; Kabnurkar, 2001; Roll *et al*., 1991). The absolute bounded DEA (crisp) model as introduced by Roll *et al.* (1991) is represented as follows:

$$\max \frac{u^T y_0}{v^T x_0}$$
$$s.t$$
$$\frac{u^T Y}{v^T X} \leq 1$$
$$LB_r \leq u_r \leq UB_r \qquad \forall r,$$
$$LB_i \leq v_i \leq UB_i \qquad \forall i,$$
$$u, v \geq 0 \tag{1}$$

y_0 is the known as column vector of outputs produced by considered DMU, and Y is the matrix of outputs of all DMUs. x_0 is the column vector of inputs consumed by the DMU, and X is the matrix of inputs of all DMUs. The u and v are column vector of variable multipliers or most favorable factor weights to maximize the efficiency of DMUs. UB_r and LB_r define upper and lower bound on weight of output r ($r = 1,\ldots,s$), whereas UB_i and LB_i upper and lower bound on weight of input i ($i = 1,\ldots,m$). In Eq. (1), the efficiency ratio of each DMU should no exceed unity and all factor weights should be positive. To model the uncertainty of the bound values in (1), crisp bounds of UB_r, LB_r, UB_i, LB_i are replaced by the fuzzy numerical values of $UB_r^f, LB_r^f, UB_i^f, LB_i^f$ respectively. Fuzzy numerical values express the concept close to the original crisp bounds (Kabnurkar, 2001:93). To incorporate fuzzy numerical values in bounded DEA (crisp) models, coefficients in fuzzy DEA models are to be defined by triangular membership functions. Otherwise, fuzzy DEA models can not be solved like a crisp model because coefficients in this model are imprecise and vague data (Lertworasirikul *et al*., 2003:383). Fractional fuzzy DEA model with fuzzy coefficients are re-formulated as follows (Werners, 1987):

$$\max \frac{u^T y_0}{v^T x_0}$$
$$s.t$$
$$\frac{u^T Y}{v^T X} \leq 1$$
$$LB_r^f \leq u_r \leq UB_r^f \qquad \forall r,$$
$$LB_i^f \leq v_i \leq UB_i^f \qquad \forall i,$$
$$u, v \geq 0 \tag{2}$$

In Eq. (2), the superscript f signifies a fuzzy numerical value. Also, parameters of fuzzy DEA model are fuzzy sets, $u^T y_0$ is approximately equal to one which indicates that $u^T y_0 / v^T x_0$ is less than or equal to one. However, such models are called unsymmetrical, because there is not "symmetry" between the constraints and the objective function in (2). These models can be solved if the crisp objective function can be represented as a maximizing set (Kabnurkar, 2001). Therefore, to solve Eq. (2), it is needed to determine the most desirable value, the least desirable values and the form of membership functions of the fuzzy weight bounds. The most desirable bound values get a membership grade of 1 specified by decision maker. Also, to determine the least desirable bounds two methods are proposed. There are many techniques or different approaches can be taken to set the least desirable bounds of this model. In this study, setting the bounds is of four steps: (1) compiling a weights matrix by running the unbounded DEA model (2) eliminating the topmost and bottommost extreme values from all columns, (3) taking the average of the remaining values as average after truncation, and (4) choosing the desirable ratio between the largest and smallest weight values as proposed by Roll and Golany (1993). Determining the values of the upper and lower bounds by using a value of d=2 and the formulas $LB = 2\overline{u}_r / (1+d)$ and $UB = 2d\overline{u}_r / (1+d)$, most desirable bounds can be calculated. Also, for determining the least desirable bounds required by the fuzzy model, a value of d=3 are chosen instead of 2 (Roll and Golany, 1993). However, as the second method, the highest and lowest values of optimal multipliers obtained for efficient DMUs in the unbounded weight matrix, are used as the least desirable bounds.

The half of linear triangular membership function for the fuzzy bounds can be graphically depicted as follows:

Figure 1: Membership function of *UB*

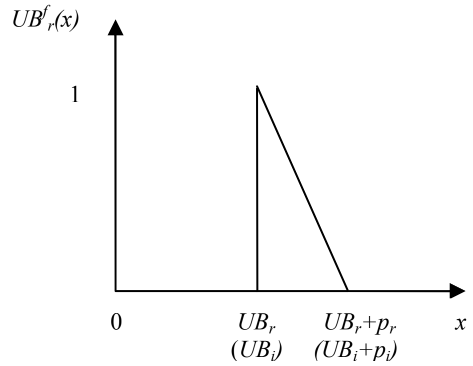
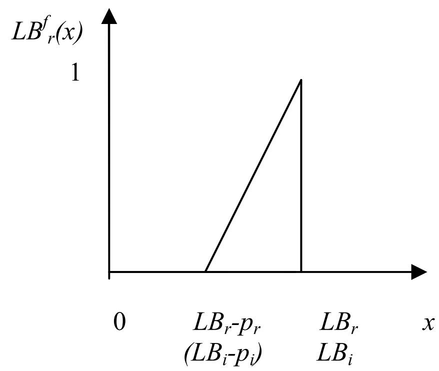


Figure 2: Membership function of *LB*



As shown in figure 1 and 2, UB_r+p_r or UB_i+p_i and LB_r-p_r or LB_i-p_i has the effect of relaxing the weight restriction constraints. The value of the objective function of a linear programming model is optimized when the constraints are most relaxed. As seen figure 1 and 2, neither of bounds values is tightening the constraints. Therefore, the membership function of objective function will favor the bounds values which relax the constraints. On the other hand, the membership function of the fuzzy constraints will favor the bounds specified by the decision-maker. Thus, the maximizing solution of objective function of fuzzy DEA model will be a compromising solution between the relaxed and specified (most desirable) bounds (Kabnurkar, 2001: 94).

By the membership function of the bound of UB_r^f, the degree of satisfaction of $x \le UB_r^f$ can be obtained as:

$$UB_r^f(x) = \begin{cases} 1 & \text{if } x \le UB_r \\ \dfrac{UB_r + p_r - x}{p_r} & \text{if } UB_r \le x \le UB_r + p_r \\ 0 & \text{if } UB_r + p_r \le x \end{cases} \quad (3)$$

The satisfaction degrees of other fuzzy constraints such as UB_i^f, LB_r^f and LB_i^f can be formulated with small changes as in Eq. (3) (Werners, 1987).

Equating the denominator of the objective function to 1, adding as a constraint and multiplying the objective function and all the constraints by the transformation factor $T_0 = (v^T x_0)^{-1}$, fractional programming model in (2) can be converted a linear programming model (Werners, 1987):

$$\begin{aligned} & f = \max \mu^T y_0 \\ & s.t \\ & \eta^T x_0 = 1 \\ & -\eta^T X + \mu^T Y \le 0 \\ & T_0 LB_r^f \le \mu_r \le T_0 UB_r^f \qquad \forall r, \\ & T_0 LB_i^f \le \eta_i \le T_0 UB_i^f \qquad \forall i, \\ & \mu, \eta \ge 0 \qquad \text{where } \mu_r = T_0 u_r, \ \eta_i = T_0 v_i. \end{aligned} \quad (4)$$

To solve Eq. (4), crisp objective function should be represented as a maximizing set. The maximizing set is formed by determining the upper and lower bounds of the crisp function over the fuzzy domain. To determine the maximizing set for the objective function, the two values of the objective function (f_o and f_1) should be specified by solving two linear programming problems . f_o is the value of the objective function when the weight bound constraints are most relaxed. To determine f_0, left –hand and right-hand sides of constraints of μ_r and η_i in (4) can be rearranged in the form of LB_r-p_r, LB_i-p_i, UBr+p_r,and UB_i+p_i respectively. Contrast to f_0, f_1 is the value of the objective function when the weight bound constraints are tightest. In other words, to determine f_1 specified bounds on constraints of μ_r and η_i in (4) are arranged in the form of UB_r, LB_r, UB_i and LB_i respectively.

The fuzzy set of optimal values or the membership function of the objective function can be presented as follows (Werners, 1987):

$$G(w)=\begin{cases} 1 & \text{if } f_0 \le \mu^T y_0 \\ \dfrac{\mu^T y_0 - f_1}{f_0 - f_1} & \text{if } f_1 < \mu^T y_0 < f_0 \\ 0 & \text{if } \mu^T y_0 \le f_1 \end{cases} \quad (5)$$

where w defines the set of all factor weights (μ , η).

Objective function is of maximization type, f_0 will be its upper bound (most desirable) and f_1 will be its lower bound (least desirable).

Figure 3: Membership function of the goal

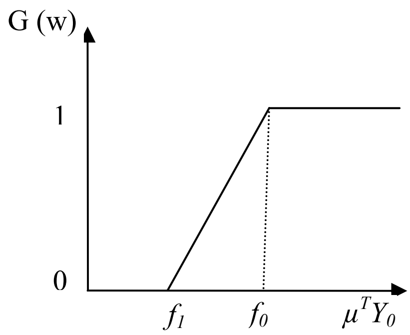


Figure 3 graphically shows the variation of the membership degree of the objective function between 0 and 1 as the objective function varies between f_0 and f_1.

As seen in figure 3, there is a problem of finding a point which satisfies the constraints and goal with the maximum degree. Mathematically, this problem can be explained to find a set of weights by introducing an additional variable λ. Crisp variable λ which represents intersection between constraints and objectives of the crisp model and called as satisfaction degree must be maximized subject to as follows:

$$\max \lambda = \min\left\{ \frac{\mu^T y_0 - f_1}{f_0 - f_1}, \frac{T_0(UB_r + p_r) - \mu_r}{T_0 p_r}, \frac{\mu_r - T_0(LB_r - p_r)}{T_0 p_r}, \right.$$
$$\left. \frac{T_0(UB_i + p_i) - \eta_i}{T_0 p_i}, \frac{\eta_i - T_0(LB_i - p_i)}{T_0 p_i} \right\} \quad (6)$$

In other words, the following crisp model must be solved by rearranging the Eq.(6):

$$\begin{aligned}
&\max \lambda \\
&\lambda(f_0 - f_1) - \mu^T Y_0 \leq -f_1 \\
&\eta^T X_0 = 1 \\
&-\eta^T X + \mu^T Y \leq 0 \\
&\lambda T_0 p_r + \mu_r \leq T_0(UB_r + p_r) \\
&\lambda T_0 p_i + \eta_i \leq T_0(UB_i + p_i) \\
&-\lambda T_0 p_r + \mu_r \geq T_0(LB_r - p_r) \\
&-\lambda T_0 p_i + \eta_i \geq T_0(LB_i - p_i) \\
&\lambda \leq 1 \\
&\mu, \eta, \lambda \geq 0
\end{aligned} \quad (7)$$

To solve Eq. (7), parametric algorithm can be used to reach the maximum feasible value in determining the different values of λ.

The solution of the parametric algorithm provides simultaneously both λ values and optimal efficiency scores of each DMUs.

2. THE SAMPLE AND RESULTS

Turkish industrial firms listed in the yearbook of Istanbul Chamber of Industry (ISO 500), a sample of 37 private sector companies, were used for our study. The input and output data for the year 2003 were obtained from the ISO 500 (2003) guide. The population size was 43 firms. However, due to unavailability of data on some input and output measures six companies were eliminated from the study. The final sample size was reduced to 37 firms.

Data on input and output measures for the year 2003 were obtained from the financial statements of companies as given in the ISO 500's guide and listed in Table-1. As seen Table 1, a set of 37 DMUs with the numerical values of two outputs and three inputs are documented.

Table 1: Input-Output Data for Automotive Firms*

No.	DMU	Net Sales (1,000 $)	Profit After taxes (1,000 $)	Net Assets (1,000 $)	Employee Number	Equity (1,000 $)
1	Aka	48.529	2.84	24.85	415	7.144
2	Anadolu Isuzu	128.566	17.25	94.39	495	55.227
3	Autolive	114.585	12.79	39.42	467	21.343
4	Bmc	316.721	73.82	990.31	2.354	256.874
5	Bosch	37.382	399	21.33	184	12.992
6	Cms	94.880	6.64	57.02	757	21.190
7	Coskunoz	54.979	2.19	40.90	803	17.582
8	Delphi	114.862	1.48	41.90	1.514	19.366
9	Federal Mogul	120.056	9.07	79.48	1.000	62.894
10	Fiat	69.196	9.23	126.21	379	117.004
11	Fnss	288.728	89.71	180.57	395	69.554
12	Ford	2.073,893	175.72	1.74,719	5.153	346.108
13	Hayes	48.782	5.29	48.45	536	22.213
14	Hema	103.220	15.92	130.81	1.250	67.987
15	Honda	268.627	27.34	73.17	444	53.936
16	Kale	55.588	2.91	36.80	612	11.523
17	Karsan	118.64	486	86.45	835	46.486
18	Mako	93.30	7.62	46.74	946	19.829
19	Man	364.28	16.84	203.32	2.497	71.225
20	Matay	60.62	7.95	18.86	237	10.205
21	Mercedes	949.98	51.83	390.27	3.645	155.656
22	Nursan	52.48	3.34	29.26	602	9.535
23	Otokar	140.28	16.20	109.62	707	41.231
24	Oyak	1.81,312	127.27	519.79	3.810	326.361
25	Stprofil	79.80	3.89	112.50	1.357	41.439
26	Sywiring	39.08	5.70	21.31	235	10.359
27	Temsa	205.63	17.23	102.22	718	8.333
28	Termo	55.06	7.35	37.52	300	2.084
29	Tirsan	89.19	6.10	105.99	257	43.137
30	Teknik Malz.	133.23	9.98	56.92	535	16.783
31	Tofas	1.60,844	28.65	1.54,418	4.138	410.223
32	Turk Traktor	277.04	62.14	200.81	804	93.913
33	Tusas Hava	70.44	6.14	255.03	1.717	148.096
34	Tusas Motor	67.10	4.56	103.654	630	71.591
35	Tudemsas	37.69	4.94	27.532	1.861	8.063
36	Uzel	192.41	14.87	111.506	1.187	22.451
37	Yazaki	90.95	6.47	57.837	1.300	36.891

*Indicative exchange rates on banknotes announced on 12/31/2003 by the Central Bank of Turkey were used in this study.

The outputs, namely, net sales, profit after taxes, while inputs; net assets, employee number and equity were chosen based on literature (Al-Shammari, 1999; Yeh, 1996; Zhu, 2000).

The efficiency score is a numerical value that describes a system's efficiency in terms of inputs and outputs. The optimal input/output weights and efficiency scores for all DMUs calculated by the CCR model are presented in Table 2.

According to the standard DEA analysis results, out of the sample of 37 companies, 5 (13.52%) were found to be relatively efficient (h_0=1), and 32 (86.48%) were found to be relatively inefficient (h_0<1). The interpretation for the inefficiency in some units is that some inputs are not fully utilized. In contrast, efficient units are getting more output per unit of input for these resources (Al-Shammari, 1999). The DEA relative efficiency score of the 32 inefficient DMUs ranged from 0.084 to 0.931. The mean efficiency score was 0.6216 with a standard deviation of 0.2636. The majority of the relatively inefficient DMUs (64.86%) fall within the 0.084-0.69 band. The same also applies to the DMUs with scores of 0.74-0.93 (21.62%). It is interesting to note that eight companies within this band may be more efficient than other companies as using a given level of inputs to produce more outputs or producing the same amount of outputs with less input.

In Table-2, *UB* and *LB* are the most desirable bounds computed with d= 2. The least desirable bounds, ^1UB+p and ^1LB-p are computed with d=3 whereas ^2UB+p and ^2LB-p are the highest and the lowest weight values respectively. As seen Table 2, weights of some inputs and outputs are assigned zero as well as the weights of some others are assigned high values. By assigning zero weights to some of the inputs and outputs means that these factors are ignored when solving standard DEA model. To eliminate the extreme weight values and to minimize variation between the weights assigned to different inputs and outputs, the decision maker sets bounds on the weight values (Kabnurkar, 2001: 112).

Table 2: Bound Values and Standard DEA Model Results

DMU	U1	U2	V1	V2	V3	Efficiency
1	1.14E-04*	0*	2.99E-04	2.07E-04	0*	0.7790266
2	1.7E-05	6.8E-05	6.3E-05	3.4E-05	1.23E-04	0.4644968
3	0*	0	0*	0*	0	0.9318318
4	5E-06	1E-06	1.5E-05	0	1.88E-04	0.242919
5	1E-04	0	2.53E-04	1.77E-04	1E-04	0.5253701
6	4.5E-05	0	1.17E-04	8.1E-05	0	0.5977605
7	6E-05	0	1.56E-04	1.08E-04	0	0.4607394
8	5.7E-05	0	1.49E-04	1.03E-04	0	0.9210506
9	2.4E-05	0	0	8.9E-05	0	0.411442
10	2.8E-05	0	0	1.2E-05	2.08E-03	0.2695988
11	0	7.9E-05	0	3.9E-05	0	1
12	3E-06	0	9E-06	0	1.1E-04	0.8990018
13	4.9E-05	0	1.23E-04	8.6E-05	4.9E-05	0.3338927
14	7E-06	8.6E-05	9E-06	5E-05	0	0.2901954
15	1E-05	1.63E-04	0	9.7E-05	0	1
16	7.4E-05	0	1.94E-04	1.34E-04	0	0.5795723
17	2.6E-05	0	6.5E-05	4.6E-05	2.6E-05	0.4305343
18	5.3E-05	0	1.37E-04	9.5E-05	0	0.691626
19	1.3E-05	0	3.4E-05	2.3E-05	0	0.6603497
20	4.7E-05	5.95E-04	6.3E-05	3.47E-04*	0	1
21	6E-06	0	1.6E-05	1.1E-05	6E-06	0.8575824
22	9.1E-05	0	2.37E-04	1.64E-04	0	0.6678254
23	1.7E-05	7E-05	6.5E-05	3.5E-05	1.26E-04	0.49797
24	4E-06	0	1E-05	7E-06	4E-06	0.8388885
25	2.3E-05	0	6E-05	4.1E-05	0	0.2559487
26	6.2E-05	4.05E-04	3.45E-04	1.68E-04	0	0.6576718
27	3.5E-05	0	8.7E-05	6.1E-05	3.5E-05	1
28	4.7E-05	6.29E-04*	6.06E-04*	1.55E-04	0	1
29	3.9E-05	0	0	1.7E-05	2.923E-03	0.4902082
30	4.9E-05	0	1.24E-04	8.6E-05	4.9E-05*	0.9155787
31	3E-06	0	9E-06	0	1.13E-04	0.6534063
32	4E-06	6E-05	0	3.6E-05	0	0.6642991
33	9E-06	0	2.2E-05	1.5E-05	9E-06	0.08403032
34	1.9E-05	0	5.6E-05	0	6.9E-04	0.1837541
35	6E-05	3.9E-04	3.33E-04	1.62E-04	0	0.5456448
36	2.7E-05	0	7.2E-05	5E-05	0	0.7421695
37	3.6E-05	0	8.2E-05	7.1E-05	0	0.4555013
Aver.	3.41351E-	6.881E-05	1.02973E-	7.5865E-05	1.79216E-04	0.62161857
Av.tr.	3.28286E-	5.477E-05	9.15429E-	7.0286E-05	1.05943E-04	0.64313367
UB	4.37714E-	7.303E-05	1.22057E-	9.3714E-05	1.41257E-04	1.33*Avg
LB	2.18857E-	3.651E-05	6.10286E-	4.6857E-05	7.06286E-05	0.67*Avg
[1]*UB+p*	4.92429E-	8.216E-05	1.37314E-	1.0543E-04	1.58914E-04	1.5*Avg
[1]*LB-p*	1.64143E-	2.739E-05	4.57714E-	3.5143E-05	5.29714E-05	0.5*Avg
[2]*UB+p*	1.14E-04	0.000629	6.06E-04	3.47E-04	2.923E-03	
[2]*LB-p*	0	0	0	0	0	

*Eliminated values from the analysis.

It signifies the degree to which the decision makers consider as acceptable differences in the impact of the various factors when assessing performance of different DMUs (Roll *et al.*, 1991:6).

The efficiency scores obtained using the crisp model and fuzzy DEA models and λ values that represent the degree to which the bounds specified by the decision maker were satisfied in the final solution is shown in Table-3.

In Table-3, results from bounded (crisp) and fuzzy models are compared. In the second column, using values of *UB* and *LB* from Table-2, the efficiency results of bounded DEA (crisp) model are shown. In Table-3, two different sets of f_0 values are used with f_1 values to determine different values of least desirable bounds with respect to proposed methods on fuzzy constraints.

For this purpose, to obtain f_1 values with using values of *UB* and *LB*, and to obtain one set of f_0 with using ^1UB+p and ^1LB-p, they are solved by Eq. (3). Founded results were inserted to the Eq. (7) with using the parametric algorithm to obtain λ values and efficiency scores of each DMUs. Similarly, the other set of f_0 values with using ^2UB+p and ^2LB-p and f_1 values from the Table-2, are solved twice by Eq. (3). Founded results were plugged into the Eq. (7), which is solved using the parametric algorithm. Fifth and sixth columns of Table 3 as well as third and fourth columns, shows the efficiency scores and λ values of fuzzy models.

To determine the difference between the scores obtained by using the crisp model and the fuzzy models, two-tail t test was performed. For the columns 2 and 3 of Table-2, a p- value of 0.001 while a p-value 0.000 for the columns 2 and 5 were found which mean that the null hypotheses that there is not a significant difference in the average efficiency calculated by the two models can be rejected. This means that there is a significant difference between the efficiency scores calculated by crisp and fuzzy DEA models.

Considered closely to the efficiency scores of Table-3, it is noticed that there is a significant difference between the crisp and fuzzy scores of one of the DMUs (DMU20).

Table 3: Comparison of Results of Crisp and Fuzzy DEA models

No.	**Crisp bounds** **(*UB*, *LB*)**	**Efficiency** **(*UB, LB*) &** **(^1UB+*p*, ^1LB-*p*)**	**λ**	**(*UB, LB*) &** **(^2UB+*p*, ^2LB-*p*)**	**λ**
1	0.3212192	0.7307448	0.5	0.7503836	0.7
2	0.4644968	0.4644968	0.5	0.4644968	0.7
3	0.8933367	0.89359	0.5	0.8959661	0.7
4	0.234268	0.238284	0.5	0.2447004	0.7
5	0.2335077	0.4945752	0.5	0.5061183	0.7
6	0.5688285	0.5750709	0.5	0.5843771	0.7
7	0.3642388	0.4302752	0.5	0.4426312	0.7
8	0.724878	0.8097222	0.5	0.8502344	0.7
9	0.3807087	0.411442	0.5	0.4476158	0.7
10	0.1846664	0.1847309	0.5	0.1856405	0.7
11	1	1	1	1	1
12	0.8990018	0.9044405	0.5	0.9117981	0.7
13	0.3330266	0.3331667	0.5	0.3334557	0.7
14	0.2753012	0.2891293	0.5	0.2901954	0.7
15	1	1	1	1	1
16	0.3625636	0.5411434	0.5	0.5567905	0.7
17	0.3665392	0.4020882	0.5	0.4293054	0.7
18	0.6520246	0.6607191	0.5	0.6732408	0.7
19	0.6125587	0.6549283	0.5	0.6587164	0.7
20	0.4441214	0.9893346	0.7	1	0.9
21	0.8575824	0.8585613	0.5	0.865972	0.7
22	0.3539693	0.6257685	0.5	0.6427786	0.7
23	0.49797	0.49797	0.5	0.49797	0.7
24	0.8388885	0.8391605	0.4	0.8398885	0.7
25	0.2162036	0.2444529	0.5	0.2554636	0.7
26	0.2982774	0.6556051	0.5	0.6566083	0.7
27	1	1	1	1	1
28	0.4116658	0.9092576	0.5	0.9543204	0.7
29	0.3078403	0.4895507	0.4	0.578267	0.7
30	0.8823206	0.898387	0.5	0.9052491	0.7
31	0.6534063	0.6835069	0.6	0.6967553	0.7
32	0.6642991	0.6642991	0.5	0.6658247	0.7
33	0.08403032	0.08583803	0.5	0.08744721	0.7
34	0.172932	0.1833745	0.5	0.1840102	0.7
35	0.2714745	0.4394442	0.5	0.4944271	0.7
36	0.6853849	0.7421695	0.5	0.7698522	0.7
37	0.4142932	0.432593	0.5	0.451984	0.7
Aver.	0.51150876	0.601591541		0.61667833	

This implies that DMU20 move from the inefficient set to the efficient set when the bounds are changed from crisp to fuzzy (Kabnurkar, 2001). Since DMU20 satisfied the bounds to a degree as high as 90%, 10% relaxation of the bounds could change efficient DMU set.

The relaxation is acceptable because the bounds were already specified subjectively by decision maker. In Table 4, the new bounds set which were computed by using the 90% satisfaction level of the original bounds are presented.

Table 4: Modified Set of Bounds
(90% Satisfaction Level of Original Bounds)

Factor	Upper bound	Lower bound
U1	5.07943E-05	1.96971E-05
U2	1.286E-04	3.286E-05
V1	1.70451E-04	5.49257E-05
V2	1.1904E-04	4.2171E-05
V3	4.19431E-04	6.35657E-05

Visually comparing the results in the Table-5, it is also seen that DMU20 entered the efficient set, when the bounds were re-adjusted. For this reason, DMU 20 is referred to as borderline. In Table-5, comparison of efficiency scores obtained by using original and modified sets of bounds is shown.

In table 5, efficiency score of DMU20 was found to be 1. In this case, the decision maker obtained a new chance to modify the original set of bounds, and to make the bounds favorable to the borderline DMUs.

Table 5: Comparison of Efficiency Scores Obtained Using Original and Modified Sets of Bounds

	Efficiency Scores	
No.	**With Original set of bounds**	**With Modified set of bounds**
1	0.3212192	0.7412798
2	0.4644968	0.4644968
3	0.8933367	0.8952061
4	0.234268	0.242919
5	0.2335077	0.4953264
6	0.5688285	0.5775081
7	0.3642388	0.4399073
8	0.724878	0.8334948
9	0.3807087	0.4299287
10	0.1846664	0.1850441
11	1	1
12	0.8990018	0.9117981
13	0.3330266	0.3332864
14	0.2753012	0.2891293
15	1	1
16	0.3625636	0.5555881
17	0.3665392	0.4278737
18	0.6520246	0.667527
19	0.6125587	0.6558819
20	0.4441214	1
21	0.8575824	0.8615824
22	0.3539693	0.6372841
23	0.49797	0.49797
24	0.8388885	0.8398885
25	0.2162036	0.2506915
26	0.2982774	0.6566083
27	1	1
28	0.4116658	0.9543204
29	0.3078403	0.5487935
30	0.8823206	0.9013495
31	0.6534063	0.6867553
32	0.6642991	0.6642991
33	0.08403032	0.08603032
34	0.172932	0.1839875
35	0.2714745	0.4917342
36	0.6853849	0.7503122
37	0.4142932	0.4466556
Aver	0.51150876	0.610931298

CONCLUSION

The study has indicated how to compare the efficiency of automotive firms in the context of standard DEA, bounded (crisp) DEA, and fuzzy DEA approaches by imposing bounds on input and output factors. For this reason firstly, the paper proposes general guidelines for setting bounds on factor weights. Then, it presents bounded (crisp) and fuzzy DEA approaches to the same data set to make a reasonable efficiency comparison of DMUs. Finally, the implication of these approaches is discussed. The results can be interpreted in the following way:

DEA approaches were formulated in terms of output factors representing, net sales, and profit after taxes relative to input factors representing the net assets, employee number and equity. According to the standard DEA results, a sample of 37 companies which its 5 (13.514%) were found to be relatively efficient. However, when the analysis is performed by fuzzy DEA approach, the efficient firms become 4 (10.81%) and they form the reference set for the inefficient firms.

Evaluating the performance of many activities by a traditional DEA approach requires precise input and output data. In order to make a reasonable efficiency comparison of DMUs, inputs and outputs should be linguistic or qualitative data characterized by fuzzy numbers. In implementing fuzzy DEA approach and determining fuzzy coefficients, standard DEA model should be transformed into the bounded (crisp) DEA approach. To determine upper and lower bounds on factor weights of bounded (crisp) DEA approach, an unbounded DEA model is run and a weights matrix is compiled eliminating extremely high weights. In this study, for example, the average weight, for each factor is calculated and a certain amount of allowable variation about each mean is decided upon subjectively, giving an upper and lower bound for each factor weight (Allen *et al*., 1997). Then, to model the uncertainty in a bounded (crisp) DEA approach, a fuzzy mathematical programming approaches proposed by Sengupta (1992) and Werners (1987) was preferred. For this purpose, the determined crisp weight bounds were replaced by fuzzy numbers. When fuzzy numbers were used instead of crisp numbers, the decision maker can specify a range of values instead of one value. This means that setting the flexibility into the fuzzy DEA approach to determine upper and lower bounds of fuzzy numerical values.

Application part of this study aims at comparing various approaches presented throughout the paper. This comparing with t- test for means displayed that the efficiency scores calculated by the fuzzy model are significantly different from those by standard DEA and crisp model. The efficiency scores calculated by the fuzzy model represent a compromise between maximization of the efficiency scores and satisfaction of the decision maker with the bounds. In other words, the fuzzy approach produces maximum possible efficiency value at which the satisfaction of the decision maker with the bounds is maximized. For example, some DMUs can move from the inefficient set to efficient set, when the bounds are changed from crisp to fuzzy by just 10% relaxation of bounds. Thus, these DMUs can be referred to as 'borderline'. If DEA analysis is perform to make important decisions, borderline DMUs become important. In such cases, fuzzy model gives the decision maker an opportunity to revise the bounds and reassessment the DMUs on the borderline. Besides, fuzzy-DEA model results have outlined that real evaluation of one problem in the context of DEA is generally applicable, and in many situations is likely to result in more realistic estimates of efficiency than standard DEA and bounded (crisp) approaches.

In this study, the linear membership function which can affect efficiency results was used. But linear membership function may not be suitable in every case. For this reason, other form of membership functions like hyperbolic, logistic, s-shaped, etc. should be considered. Also, apart from the fuzzy CCR approach, other approaches could be applied to the DEA models.

REFERENCES

Aigner, D.J, P. Levell Schmidt (1977) "Formulation and Estimation of Stochastic Frontier Production Models",. **Journal of Econometrics**, 6, 21-37.

Allen, R., A. Athanassopoulos, R.G. Dyson and E. Thanassoulis (1997) "Weights Restrictions and Value Judgements in Data Envelopment Analysis: Evolution, Development, and Future Directions", **Annals of Operations Research**, 73, 13-34.

Al-Shammari, M. (1999) "A Multi-Criteria Data Envelopment Analysis Model for Measuring the Productive Efficiency of Hospitals", **International Journal of Operations & Production Management**, 19(9), 879-890.

Asish, S., T.S. Ravisankar (2000) "Rating of Indian Commercial Banks: A DEA Approach", **European Journal of Operational Research**, 124, 187-203.

Barros, C.P., F.P. Alves (2004) "Productivity in the Tourism Industry", **International Advances in Economic Research**, 10(3), 215-225.

Carlsson, C., P. Korhonen (1986) "A Parametric Approach to Fuzzy Linear Programming", **Fuzzy Sets and Systems**, 20, 17–30.

Charnes,, A, W.W Cooper (1962) “Programming with Linear Fractional Functions”, **Naval Research Logistics Quarterly**, 9(3-4), 181-186.

Charnes, A, W.W Cooper, E. Rhodes (1978) “Measuring the Efficiency of Decision Making Units”, **European Journal of Operational Research**, 2, 429–44.

Chen, T. (1997) “A Measurement of the Resource Utilization Efficiency of University Libraries”, **International Journal of Production Economics**, 53, 71-80.

Dia M. (2004) “A Model of Fuzzy Data Envelopment Analysis”, **Information Systems & Operational Research**, 42(4), 267-279.

Emel, A.B., M. Oral, A. Reisman, R. Yolalan (2003) “A Credit Scoring Approach for the Commercial Banking Sector”, **Socio-Economic Planning Sciences**, 37, 103–123.

Entani, T., Y. Maeda, H. Tanaka (2002) “Dual Models of Interval DEA and its Extension to Interval data, **European Journal of Operational Research**, 136, 32–45.

Guo, P., H. Tanaka (2001) “Fuzzy DEA: a Perceptual Evaluation Method”, **Fuzzy Sets and Systems**, 119, 149-160.

Kabnurkar, A. (2001) **Mathematical Modeling for Data Envelopment Analysis with Fuzzy Restrictions on Weights**, Unpublished Master’s Thesis, Virginia Polytechnic Institute and State University, Department of Industrial and Systems Engineering, Virginia.

Lertworasirikul, S. (2002) “Fuzzy Data Envelopment Analysis for Supply Chain Modeling and Analysis”, **Dissertation Proposal in Industrial Engineering**, North Carolina State University.

Lertworasirikul, S., S.C Fang, J.A. Joines, H.L.W. Nuttle (2003) “Fuzzy Data Envelopment Analysis (DEA): a Possibility Approach”, **Fuzzy Sets and Systems**, 139, 379-394.

Maeda, Y., T. Entani, H. Tanaka (1998) “Fuzzy DEA with Interval Efficiency”, **6th European Congress on Intelligent Technics and Soft Computing**, 2, 1067-1071.

Pedraja, C.F., J.J. Salinas, P. Smith (1997) “On the Role of Weight Restrictions in Data Envelopment Analysis”, **Journal of Productivity Analysis**, 8, 215-230.

Ramanathan, R. (2005) “Operations Assessment of Hospitals in the Sultanate of Oman”, **International Journal of Operations & Production Management**, 25(1), 39-54.

Roll, Y., W.D. Cook, B. Golany (1991) “Controlling Factor Weights in Data Envelopment Analysis”, **IIE Transactions**, 23(1), 2 - 9.

Roll Y., B. Golany (1993) "Alternative Methods of Treating Factor Weights in DEA", **OMEGA The International Journal of Management Science**, 21(1), 99-109.

Sarrico, C.S., R.G Dyson (2000) "Using DEA for Planning in UK Universities: An Institutional Perspective", **Journal of the Operational Research Society**, 51, 789-800.

Sengupta, J.K. (1992) "A Fuzzy Systems Approach in Data Envelopment Analysis", **Computers and Mathematics with Applications**, 24(8-9), 259-266.

Triantis, K., O. Girod (1998) "A Mathematical Programming Approach for Measuring Technical Efficiency in a Fuzzy Environment", **Journal of Productivity Analysis**, 10, 85-102.

Weber, W.L. (2002) "The Global Economy and Japanese Bank Performance", **Managerial Finance**, 28(10), 33-46.

Werners, B. (1987) "An Interactive Fuzzy Programming System". **Fuzzy Sets and Systems**, 23(1), 131-147.

Yeh, Q.J. (1996) "The Application of Data Envelopment Analysis in Conjunction with Financial Ratios for Bank Performance Evaluation", **Journal of the Operational Research Society**, 47, 980–988.

Zimmerman, H.J. (1996) **Fuzzy Set Theory and Its Implications**, London: Kluwer Academic Publishers.

Zhu, J. (2000) "Multi-Factor Performance Measure Model with an Application to Fortune 500 Companies", **European Journal of Operational Research**, 123(1), 105–124.