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Çoklu Yansıma Ortamlarında Geniş Menzilli Hedeflerin Uyarlanabilir Radar Tespiti

Harun Taha HAYVACI

Öne Çıkanlar:
- Karmaşık ve çoklu yansımaların bulunduğu ortamlarda geniş hedefler için sağlam ve güvenilir radar tespit algoritması
- Kontrol gözetim, güvenlik ve uzaktan algılama uygulamaları

Anahtar Kelimeler:
- Adaptif Radar Tespiti
- Çoklu Yol Kullanımı
- Geniş Menzilli Hedefler
- Kısıtlı Optimizasyon
- Genelleştirilmiş Olasılık Oranı Testi (GLRT)

ÖZET:

Bu makalede, Gauss gürültüsü altında gömülü geniş menzilli radar hedeflerinin uyumlu algılama problemi, dağınık çoklu yol ortamı varsayımı altında ele alınmıştır. Her aralık hücresinden hedef yankısı, doğrudan yol bileşenini hesaba katan bilinmeyen bir ölçek faktörüyle belirlenen belirli bir veri vektörü ve bir parlak yüzeyin çoklu yankılarını temsil eden bilinmeyen bir kovaryans matrisiyle Gaussian dağılımlı rastgele veri vektörünün toplamı olarak modellemiştir. Tasarım aşamasında, her birincil veri kovaryans matrisi, ikincil veri seti ile elde edilen örnek bir kovaryans matrisi çevresinde yer aldığı varsayılır. Adaptif tespit problemi için bir kısıttlı Genelleştirilmiş Olasılık Oranı Testi (GLRT) ele alınmıştır. Geliştirilen algoritmanın, literatürdeki iyi bilinen adaptif dedektörlerle performans analizi yapılmıştır. Sunulan sonuçlar ve performans analizi, önerilen yaklaşımın yayılmış çoklu yol varlığı olan ortamlarda geniş menzilli radar hedeflerinin tespit performansını artırdığını vurgulamaktadır.

Adaptive Radar Detection of Extended Targets in Diffuse Multipath Environments

Highlights:
- Robust and reliable radar detection algorithm for extended targets in cluttered and multipath environments.
- Surveillance, security, and remote sensing applications.

Keywords:
- Adaptive Radar Detection
- Multipath Exploitation
- Range Extended Targets
- Constrained Optimization
- Generalized Likelihood Ratio Test (GLRT)

ABSTRACT:

This paper discusses the adaptive detection of extended radar targets buried in Gaussian clutter, assuming a diffuse multipath environment. The target return signal from each range cell is modeled as the sum of a deterministic data vector, which includes an unknown scaling factor representing the direct path component, and a randomly distributed data vector in a Gaussian distribution with unknown covariance matrix representing multipath echoes. During the design phase, it is assumed that the primary data covariance matrix falls within the vicinity of a sample covariance matrix that is devised from the secondary data set. The paper proposes a constraint Generalized Likelihood Ratio Test (GLRT) for the adaptive detection problem of extended radar targets in diffuse multipath environments, and conducts a performance analysis comparing the developed algorithm with well-known adaptive detectors in the literature. The results and performance analysis demonstrate that the proposed approach enhances the detection performance of extended radar targets in environments with diffuse multipath. Overall, this article provides valuable insights for improving the adaptive detection of extended targets in challenging environments, with potential applications in radar and sensing technologies.

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INTRODUCTION

Radar systems are commonly used for remote sensing and detection of targets in various environments. However, in cluttered and multipath environments, the performance of radar detection algorithms can be significantly degraded due to interference and scattering from surrounding objects (Pamela et al, 1995; Richards et al, 2010; Fertig et al, 2012). In particular, detecting extended targets, such as vehicles or buildings, in diffuse multipath environments remains a challenging problem (Gerlach et al, 1999). To address this issue, adaptive radar detection algorithms have been developed to mitigate the effects of multipath interference and clutter (Fante et al, 1991; Fante et al, 1995). These algorithms employ adaptive signal processing techniques to estimate the interference statistics and adaptively suppress them, while preserving the target signals of interest. Adaptive detectors, which are proposed to detect extended (range spread) radar targets embedded in Gaussian clutter, experience performance degradation for several reasons (Aubry et al, 2016; Tang et al, 2017). Modern High-Resolution Radars (HRR's) may have range resolutions, which are smaller than the target to be identified, thus backscattered signals from the target can be received through not just one but more than one isolated point, which are considered as range cells (Bachman, 1965). Therefore, various techniques have been developed for adaptively detecting extended radar targets during the last decades.

Many studies in literature considering adaptive detectors for distributed targets have modeled target returns as known signals with unknown scaling factors (Conte et al, 2001; Conte at al, 2001; Aubry et al, 2013). It can be regarded as convenient when the target echoes consist only of a line-of-sight signal return. However, the received signals often contain multipath echoes along with line-of-sight echo, which distort the direct path returns and cause steering vector mismatch between the actual and nominal one (Hayvaci et al, 2013, Kumbul et al, 2019, Yilmaz et al, 2021). In this content, adaptive subspace detectors for both point-like and extended targets are devised to overcome steering vector mismatch related challenges by exploiting prior knowledge about the environment (Kraut et al, 2001; Bandiera et all, 2007). Though, it is not always easy or practical to predict target subspace particularly in glistening surfaces such as sea surface, which causes diffuse multipath (Fante et al, 1991). In such scenarios, the backscattered target echoes are reached to the radar receiver through many propagation paths. It is exceedingly difficult to predict the signal parameters of multipath components, such as direction of arrival, due to unpredictable dynamic behavior of the scene. A novel adaptive detection algorithm is developed for point-like targets, which models the multipath echoes via random variables with an unknown covariance matrix to tackle these challenges. The devised detector, referred to as T-AMF (Tunable-Adaptive Matched Filter), considers the radar signal return as the sum of direct and multipath returns from the target. The signal return is assumed to be buried in Gaussian noise with unknown covariance matrix (Aubry et al, 2015).

In this article, the author presents a novel adaptive radar detection algorithm for extended targets in diffuse multipath environments where geometry of the problem is depicted in Figure 1. The algorithm is based on a constrained Generalized Likelihood Ratio Test that models the target and interference signals using a joint probability density function. The author investigates the detection of radar signal returns from extended targets, which are buried in Gaussian clutter under the assumption of a radar-target environment with diffuse multipath. The target echo from each range cell is modeled as the sum of a deterministic data, which is direct path component, and a zero-mean complex Gaussian distributed random vector, which is diffuse multipath component, as in (Aubry et al, 2015). A suitable disturbance model involving the characteristics of multipath environment is critical for robust adaptive detection. Author designs Generalized Likelihood Ratio Test with the assumption of each data coming from different range cell has a covariance matrix, which is in the restrained vicinity of the sample covariance matrix that is devised from the secondary data set. Thus, the author devises a set of constrained optimization problems with Maximum Likelihood (ML) estimate of the primary data covariance matrices by exploiting knowledge about the environment. The author derives the optimal detection criterion based on this model and proposes an efficient algorithm for its implementation. The performance of the proposed algorithm is evaluated using simulated data through Monte Carlo simulations, demonstrating its effectiveness in detecting extended targets in challenging environments.

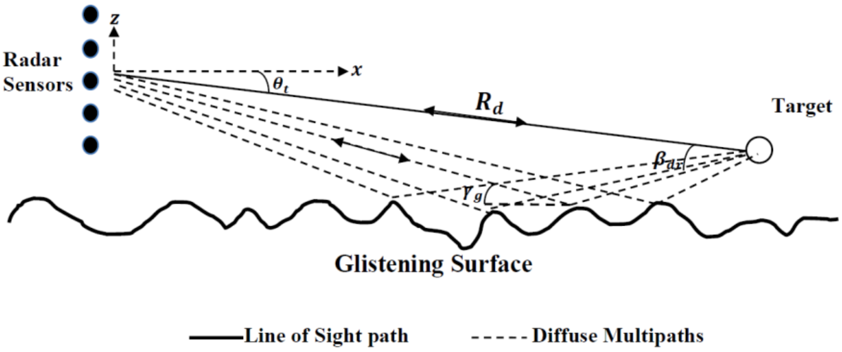

Figure 1. Geometry of the problem

Overall, this work contributes to the development of robust and reliable radar detection algorithms for extended targets in cluttered and multipath environments. The proposed algorithm has potential applications in various fields, including surveillance, security, and remote sensing.

The organization of the paper is as follows. In Problem Formulation section, the author describes the hypothesis-testing problem, and defines the parameters of target echo and disturbance model. In Detector Design section, the author devises the proposed detector to enhance detection of extended radar targets in diffuse multipath environments. In Performance Assessment section, the author presents the performance analysis and evaluation of the proposed detector along with well-known adaptive detectors, namely extended version of Kelly's receiver and Adaptive Matched Filter (AMF). Finally, the author addresses the concluding remarks in Conclusions.

MATERIALS AND METHODS

In this study, the author employs certain mathematical notation to denote vectors, matrices, and operations. Vectors are represented with boldface lowercase letters, such as $\boldsymbol{a}$, while boldface uppercase letters, such as $\boldsymbol{A}$, denotes matrices. The notation $(\ )^{\dagger}$ represents the conjugate transpose, and $det(\ )$ denotes the determinant of a square matrix argument. The symbol $\boldsymbol{I}$ represents the identity matrix, with its size determined by the context. The author uses C^N, $C^{N,K}$, and H^N respectively to represent the sets of N-dimensional vectors of complex numbers, $N \times K$ matrices of complex numbers, and $N \times N$ Hermitian matrices. The notation $\lambda(X) = [\lambda_1(X), \lambda_2(X), \ldots, \lambda_N(X)]^T$ is used to represent the vector of eigenvalues of a Hermitian matrix $X \in H^N$, arranged in descending order, with $\lambda_i(X)$ representing the i-th ordered eigenvalue. The symbol $\succeq$ (and its strict form $\succ$) is used to denote generalized matrix inequality. For any $\boldsymbol{A} \in H^N$, $\boldsymbol{A} \succeq 0$ denotes that $\boldsymbol{A}$ is a positive semi-definite matrix (with $\boldsymbol{A} \succ 0$ representing positive definiteness). The author uses $\parallel \boldsymbol{x} \parallel$ to represent the Euclidean norm of $\boldsymbol{x}$. For any complex number x, $|x|$ represents the modulus of x. The symbol $\parallel A \parallel_2$ denotes the spectral norm of the matrix $\boldsymbol{A} \in C^{N,M}$. Finally, the notation $E\,[\cdot]$ is used to denote statistical expectation.

Problem Formulation

The author explores a set of radar sensors that gather data from N channels. These channels can be temporal, spatial, or a combination of the two (spatial-temporal). The author tackles the problem of detecting radar return signals from an extended radar target across H range cells. The primary data set, in other words data under test, is assumed to be a set of N-dimensional vectors of complex numbers, denoted by $\boldsymbol{r}_t \in C^N$. The secondary data, denoted by $\boldsymbol{r}_l$, $l = 1,\ldots,K$ ($K \geq N$), consists of vectors that are free of signal of interest. All data are obtained at the radar receiver. The radar receiver is operating in a diffuse multipath environment, which is caused by a glistening surface (Yilmaz et al, 2021). Thus, the detection problem is depicted as a binary hypothesis testing problem, where the goal is to determine whether a signal of interest is present or not. The relevant binary hypothesis testing problem is depicted as the following.

$$\begin{cases} H_0: \begin{cases} \boldsymbol{r}_t = \boldsymbol{n}_t, & t = 1,\ldots,H \\ \boldsymbol{r}_l = \boldsymbol{n}_l, & l = 1,\ldots,K \end{cases} \\ H_1: \begin{cases} \boldsymbol{r}_t = \alpha_t\, \boldsymbol{p} + \boldsymbol{s}_t + \boldsymbol{n}_t & t = 1,\ldots,H \\ \boldsymbol{r}_l = \boldsymbol{n}_l & l = 1,\ldots,K \end{cases} \end{cases} \tag{1}$$

where

- $\boldsymbol{p} \in C^N, \| \boldsymbol{p} \|^2 = 1$, The target steering vector is denoted by the target steering $\boldsymbol{p} \in C^N$, where $\boldsymbol{p}$ has a Euclidean norm of 1, $\| \boldsymbol{p} \|^2 = 1$. This target's steering vector takes into account the line of sight component, which represents the direct path between the transmitter and the target.
- The parameter $\alpha_t \in C$ is a deterministic quantity that encompasses the target reflectivity as well as the channel propagation effects for the line of sight component associated with each range cell.
- The author uses the notations $\boldsymbol{n}_t \in C^N$, $t = 1,\ldots,H$, and $\boldsymbol{n}_l \in C^N$, $l = 1,\ldots,K$, to represent the clutter interference and noise contributions for each range cell and the secondary data set, respectively. It is assumed that all $\boldsymbol{n}_t$ and $\boldsymbol{n}_l$ are independent and identically distributed (iid) complex normal random vectors, with a mean of zero and a positive definite covariance matrix $\overline{\boldsymbol{M}} \succ \boldsymbol{0}$.
$$E\left[\boldsymbol{n}_t \boldsymbol{n}_t^\dagger\right] = E\left[\boldsymbol{n}_l \boldsymbol{n}_l^\dagger\right] = \overline{\boldsymbol{M}}, \quad t = 1,\ldots,H; \quad l = 1,\ldots,K;$$
- The data vector, $\boldsymbol{s}_t$, with $= 1,\ldots,H$, represents the diffuse multipath phenomena for each range cell in an extended target scenario (Fante et al, 1991). In the current context, the signal of interest radiating from the target is received through many diverse propagation paths over a glistening surface. The data vector, therefore, models the sum of echoes from multiple spatially distributed reflectors. In this study, we assume that the data vector, $\boldsymbol{s}_t$, follows a complex, zero-mean, circularly symmetric Gaussian process with a covariance matrix, $\boldsymbol{\Sigma}_t$, which is unknown. This assumption is based on the Central Limit Theorem.

In the current context, the covariance matrix of the primary data is a key factor in testing the H_1 and H_0 hypotheses. Under the H_1 hypothesis, the covariance matrix is denoted as $\boldsymbol{M}_t = \boldsymbol{M} + \boldsymbol{\Sigma}_t$, where $t = 1,\ldots,H$. In contrast, under the H_0 hypothesis, the covariance matrix is $\boldsymbol{M}_t = \boldsymbol{M}$, without any additional terms. Additionally, the term $\boldsymbol{s}_t$ in this formulation is included to account for potential mismatches that may arise due to propagation effects in the environment.

In order to compare and assess performance, the author examines the widely-known Kelly's receiver and AMF among the available adaptive detection strategies, as given in (Kelly, 1986; Robey et al, 1992). However, in this study author use the extended target version of these receivers. Kelly's receiver is presented. The extended target version of Kelly's receiver is presented below.

$$t_{Kelly} = \sum_{t=1}^{H} \frac{\left|\boldsymbol{p}^{\dagger}\boldsymbol{S}^{-1}\boldsymbol{r}_t\right|^2}{(\boldsymbol{p}^{\dagger}\boldsymbol{S}^{-1}\boldsymbol{p})\sum_{t=1}^{H}\boldsymbol{r}_t^{\dagger}\boldsymbol{S}^{-1}\boldsymbol{r}_t} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_1, \tag{2}$$

and the extended target version of AMF is given as,

$$t_{AMF} = \sum_{t=1}^{H} \frac{\left|\boldsymbol{p}^{\dagger}\boldsymbol{S}^{-1}\boldsymbol{r}_t\right|^2}{\boldsymbol{p}^{\dagger}\boldsymbol{S}^{-1}\boldsymbol{p}} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_2. \tag{3}$$

Detector Design

The author introduces a new adaptive detector, named extended T-AMF, by developing a constrained generalized likelihood ratio test (GLRT) in this section. The hypothesis testing that leads to the constrained GLRT is outlined below.

$$\frac{\underset{[\alpha_1,\ldots,\alpha_H \in C, \boldsymbol{M}_1,\ldots,\boldsymbol{M}_H \in \Omega_t^{\epsilon}]}{max} f_{1,H}}{\underset{\overline{\boldsymbol{M}}=\boldsymbol{S}}{max} f_{0,H}} \underset{H_2}{\overset{H_1}{\gtrless}} \eta_3 \tag{4}$$

where $f_{1,H}$ and $f_{0,H}$ are defined as

$$f_{1,H} = f_1(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_H; \alpha_1, \ldots, \alpha_H, \boldsymbol{M}_1, \ldots, \boldsymbol{M}_H) = \frac{1}{\prod_{t=1}^{H}\det(\pi \boldsymbol{M}_t)} e^{-\sum_{t=1}^{H}(\boldsymbol{r}_t - \alpha_t \boldsymbol{p})^{\dagger}\boldsymbol{M}_t^{-1}(\boldsymbol{r}_t - \alpha_t \boldsymbol{p})} \tag{5}$$

and

$$f_{0,H} = f_0\left(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_H;\ \overline{\boldsymbol{M}}\right) = \frac{1}{\prod_{t=1}^{H}\det(\pi \overline{\boldsymbol{M}})} e^{-\sum_{t=1}^{H}\boldsymbol{r}_t^{\dagger}\overline{\boldsymbol{M}}^{-1}\boldsymbol{r}_t} \tag{6}$$

respectively. In addition, the constrained set for $\boldsymbol{M}_t$ is defined as

$$\Omega_t^{\epsilon} = \left\{\boldsymbol{M}_t \succ \boldsymbol{0}: \left\|\boldsymbol{S}^{\frac{1}{2}}\boldsymbol{M}_t^{-1}\boldsymbol{S}^{\frac{1}{2}} - \boldsymbol{I}\right\|_2 \leq \epsilon_t\right\} \quad \epsilon_t \geq 0, t = 1, \ldots, H \tag{7}$$

The signal detection threshold, denoted as η_3, is determined with respect to the desired probability of false alarm rate P_{fa}. In equation (7), a collection of positive-definite matrices $\boldsymbol{M}_t \succ \boldsymbol{0}$ is defined such that $\boldsymbol{S}^{1/2}\boldsymbol{M}_t^{-1}\boldsymbol{S}^{1/2}$ is ϵ_t-similar to the identity matrix $\boldsymbol{I}$, following the method described in (Aubry et al, 2015). In other words, the suitable neighborhood of $\boldsymbol{S}$ for locating $\boldsymbol{M}_t$ is defined as Ω_t^{ϵ}, with ϵ representing a parameter that controls the size of uncertainty region of the covariance matrix. Higher values of ϵ are recommended for environments with strong reflection contributions, while lower values of ϵ are more suitable for weak multipath returns.

In order to derive the extended T-AMF equation (4), the following substitution is required as a preliminary step.

$$\boldsymbol{X}_t = \boldsymbol{S}^{\frac{1}{2}}\boldsymbol{M}_t^{-1}\boldsymbol{S}^{\frac{1}{2}}, \qquad t = 1, \ldots, H$$

Therefore, equation (4) can be reformulated as follows:

$$\max_{\bar{\alpha}_t \in \mathbb{C}, \boldsymbol{X}_t \in \Omega_1^{\epsilon_t}} \left(\prod_{t=1}^{H} \det(\boldsymbol{X}_t) \right) e^{-\sum_{t=1}^{H} \bar{f}_t(\bar{\alpha}_t, \boldsymbol{X}_t; \bar{\boldsymbol{r}}_t)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_4 \tag{8}$$

where $\bar{f}_t(\bar{\alpha}_t, \boldsymbol{X}_t; \bar{\boldsymbol{r}}_t)$ and $\bar{\boldsymbol{r}}_t$ are defined respectively as

$$\bar{f}_t(\bar{\alpha}_t, \boldsymbol{X}_t; \bar{\boldsymbol{r}}_t) = (\bar{\boldsymbol{r}}_t - \bar{\alpha}_t \bar{\boldsymbol{p}})^\dagger \boldsymbol{X}_t (\bar{\boldsymbol{r}}_t - \bar{\alpha}_t \bar{\boldsymbol{p}}) - \|\bar{\boldsymbol{r}}_t\|^2$$
$$\Omega_1^{\epsilon_t} = \left\{ \boldsymbol{X}_t \succ \boldsymbol{0} \colon \|\boldsymbol{X}_t - \boldsymbol{I}\|_2 \leq \epsilon_t \right\},$$

and

$$\bar{\boldsymbol{r}}_t = \boldsymbol{S}^{-1/2} \boldsymbol{r}_t,\ \bar{\boldsymbol{p}} = \frac{\boldsymbol{S}^{-1/2} \boldsymbol{p}}{\|\boldsymbol{S}^{-1/2} \boldsymbol{p}\|},\ \bar{\alpha}_t = \alpha_t \|\boldsymbol{S}^{-1/2} \boldsymbol{p}\|$$

where $t = 1, \ldots, H$.

Once one takes the logarithm of (8), the original decision test depicted (4) can be formulated as

$$t_{\text{T-AMFI}}(\epsilon) = \max_{\bar{\alpha}_t \in \mathbb{C} \boldsymbol{X}_t \in \Omega_1^{\epsilon_t}} \boldsymbol{A} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{9}$$

where

$$\boldsymbol{A} = \sum_{t=1}^{H} \left(\log \det(\boldsymbol{X}_t) - \bar{f}_t(\bar{\alpha}_t, \boldsymbol{X}_t; \bar{\boldsymbol{r}}_t) \right)$$

with η the modification of η_4 in (9). Later, one can observe that it is to maximize $\left[\sum_{t=1}^{H} \left(\log \det(\boldsymbol{X}_t) - (\bar{\boldsymbol{r}}_t - \bar{\alpha}_t \bar{\boldsymbol{p}})^\dagger \boldsymbol{X}_t (\bar{\boldsymbol{r}}_t - \bar{\alpha}_t \bar{\boldsymbol{p}}) \right) \right]$ with respect to $\bar{\alpha}_1, \ldots \bar{\alpha}_H \in \mathbb{C}$, and $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_H \in \Omega_1^{\epsilon_t}$, $\left(\|\bar{\boldsymbol{r}}_t\|^2 \right.$ are constant values). Constraint and objective functions are separable functions of $\boldsymbol{X}_t$, thus one can obtain the optimal solution by solving the optimization problem as the following.

$$\mathcal{P}_{\boldsymbol{y}_t}^{\boldsymbol{y}_t} \begin{cases} \max\limits_{\boldsymbol{X}_t} \log \det(\boldsymbol{X}_t) - \boldsymbol{y}_t^\dagger \boldsymbol{X}_t \boldsymbol{y}_t \\ \text{s.t. } \boldsymbol{X}_t \in \Omega_1^{\epsilon_t} \end{cases},\ t = 1, \ldots, H \tag{10}$$

where $\boldsymbol{y}_t \in \mathbb{C}^N$ is an N-dimensional complex vector and it stands for $(\bar{\boldsymbol{r}}_t - \bar{\alpha}_t \bar{\boldsymbol{p}})$.
Thus, based on Proposition III.1 in (Aubry et al, 2015), the decision test denoted as $t_{\text{T-AMF}}(\epsilon)$ in equation (10) can be expressed as following:

$$t_{\text{T-AMF}}(\epsilon) = \sum_{t=1}^{H} f_{\epsilon_t}(\gamma_t) + \sum_{t=1}^{H} \gamma_t^o \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{11}$$

where

$$f_{\epsilon_t}(\gamma_t) = \log(\lambda_t^\star) - \lambda_t^\star \gamma_t + (N-1) \log(1 + \epsilon_t),$$

and $\gamma_t^o = \|\bar{\boldsymbol{r}}_t\|^2$ and $\gamma_t = \|\bar{\boldsymbol{r}}_t\|^2 - \left| \bar{\boldsymbol{p}}^\dagger \bar{\boldsymbol{r}}_t \right|^2$.

In this context, every $\lambda_t^\star$, where $t = 1, \ldots, H$, represents a distinct and optimal solution to a strictly concave optimization problem stated as follows:

$$\mathcal{P}_{\gamma_t} \begin{cases} \max\limits_{\lambda_t} \log(\lambda_t) - \lambda_t \gamma_t \\ \text{s.t. } \|\lambda_t - 1\| \leq \epsilon_t \lambda_t \geq 0 \end{cases},\ t = 1, \ldots, H \tag{12}$$

Thus, the optimal solution for each optimization problem $\mathcal{P}_{\gamma_t}$ is obtained through

1) $0 \leq \epsilon < 1$.

$$\lambda_t^\star = \begin{cases} 1+\epsilon_t & \text{if } \gamma_t \leq \dfrac{1}{1+\epsilon_t} \\ \dfrac{1}{\gamma_t} & \text{if } \dfrac{1}{1+\epsilon_t} < \gamma_t \leq \dfrac{1}{1-\epsilon_t} \\ 1-\epsilon_t & \text{if } \gamma_t > \dfrac{1}{1-\epsilon_t} \end{cases} \tag{13}$$

2) $\epsilon \geq 1$.

$$\lambda_t^\star = \begin{cases} 1+\epsilon_t & \text{if } \quad \gamma_t \leq \dfrac{1}{1+\epsilon_t} \\ \dfrac{1}{\gamma_t} & \text{if } \quad \gamma_t > \dfrac{1}{1+\epsilon_t} \end{cases}$$

Finally, the algorithm for computing decision statistics of the devised T-AMF for the extended target model is given as the following.

Algorithm 1 : T-AMF1 decision statistic computation for extended target model.

```
Input: r_1, ..., r_L, S, ε_1 ..., ε_L.
Output: the decision test t_T-AMF1(ε).
set t_T-AMF1(ε) = 0
for t=1:L do
    compute r̄_t = S^{-1/2} r_t, p̄ = S^{-1/2} p / ||S^{-1/2} p||, γ_t^o = ||r̄_t||^2 and γ̄_t = ||r̄_t||^2 − |p̄† r̄_t|^2.
    if 0 ≤ ε_t < 1 then
        case γ̄_t ≤ 1/(1+ε_t):
            f_{ε_t}(γ̄_t) = N log(1 + ε_t) − γ̄_t(1 + ε_t)
        case 1/(1+ε_t) < γ̄_t ≤ 1/(1−ε_t) :
            f_{ε_t}(γ̄_t) = − log(γ̄_t) − 1 + (N − 1) log(1 + ε_t)
        case γ̄_t > 1/(1−ε_t) :
            f_{ε_t}(γ̄_t) = log(1 − ε_t) − γ̄_t(1 − ε_t) + (N − 1) log(1 + ε_t)
    else
        case γ̄_t ≤ 1/(1+ε_t) :
            f_{ε_t}(γ̄_t) = N log(1 + ε_t) − γ̄_t(1 + ε_t)
        case γ̄_t > 1/(1+ε_t) :
            f_{ε_t}(γ̄_t) = − log(γ̄_t) − 1 + (N − 1) log(1 + ε_t)
    end
    compute t_T-AMF1(ε) = t_T-AMF1(ε) + γ_t^o + f_{ε_t}(γ̄_t)
end for
output t_T-AMF1(ε)
```

RESULTS AND DISCUSSION

In this section, the author evaluates the performance of a detector that has been developed by measuring its probability of detection P_d at a predetermined false alarm rate P_{fa}. To compare the performance of the developed detector with existing ones, extended versions of Kelly's receiver and the AMF, as shown in equations (2) and (3), respectively, are also evaluated.

Conventional Monte Carlo simulation methods are utilized to obtain the performance metrics. The false alarm rate P_{fa} is set to a nominal value of 10^{-4} to facilitate the computational process. To ensure that the assigned false alarm rate is achieved, the threshold value is determined through $100/P_{fa}$

independent trials. Furthermore, each value of P_d is estimated through 10^4 independent realizations of the decision statistics. The performance analysis simulations are carried out for $N = 16$ space channels, $K = 32$ secondary data, and $H = 10$ range cells.

In order to simulate the multipath environment, the author assumes $N_{ML} = 4$ mainlobe and $N_{SL} = 4$ sidelobe scatterers from glistening surface points for each range cell. The directions of arrival for both main lobe and side lobe scatterers are modeled as uniformly distributed independent random variables within the ranges of $[-2, 2]$ and $[8.5, 11.5]$ degrees, respectively. The complex amplitudes of the reflected multipaths, denoted as α_i for $i = 1, \dots, N_{ML} + N_{SL}$, are modeled as dependent on the useful signal power with a factor of path loss L associated with multipath.

$$\alpha_i = \alpha \frac{x_i}{\sqrt{L}}, \qquad i = 1, \dots, N_{ML} + N_{SL} \tag{16}$$

In this context, the random variable x_i is an independent, circularly symmetric complex normal distribution with zero mean and a unit variance. The factor L, which is associated with the multipath, represents the path loss and is measured in decibels. In summary, the formulation of the primary data covariance matrix can be expressed as follows.

$$\boldsymbol{M}_t = \sigma_c^2 \overline{\boldsymbol{M}} + \sigma_n^2 \boldsymbol{I} + \boldsymbol{\Sigma}_t(\alpha_t, L), \tag{17}$$

where $t = 1, \dots, H$ and the entry located at row n and column m of $\overline{\boldsymbol{M}}$ is (Barbarossa et al, 1994)

$$\overline{\boldsymbol{M}}(n, m) = e^{-(n-m)^2/(2\sigma_d^2)} \tag{18}$$

Here, the parameters $\sigma_d = 0.995$, $\sigma_n^2 > 0$, and $\sigma_c^2 > 0$ are defined as follows: σ_d represents the one-lag correlation coefficient, σ_n^2 denotes the thermal noise power, and σ_c^2 is the clutter power. Moreover, the covariance matrix is defined as the following.

$$\Sigma_t(\alpha_t, L) = \sum_{i=1}^{N_{ML}+N_{SL}} \frac{|\alpha_i|^2}{L} v(\theta_i) v(\theta_i)^\dagger \tag{19}$$

where θ_i, for $i = 1, \dots, N_{ML} + N_{SL}$, are the angles of arrival from the glistening surface, and they are defined as the following.

$$v(\theta\_i) = \frac{1}{\sqrt{N}} \left[1, e^{j2\pi\frac{d}{\lambda}sin(\theta_i)}, \dots, e^{j2\pi(N-1)\frac{d}{\lambda}sin(\theta_i)}\right]^T \tag{20}$$

It is important to underscore that when considering the H_0 hypothesis, where $\alpha_i = 0$ for $i = 1, 2, \dots, N_{ML} + N_{SL}$, the multipath returns as described in (19) are absent. In addition, the Signal-to-Interference-Plus-Noise Ratio (SINR) can be defined as follows.

$$\text{JSINR} = |\alpha|^2 \boldsymbol{p}^\dagger \boldsymbol{M}^{-1} \boldsymbol{p} \tag{21}$$

Figure 1 depicts a scenario where there is an absence of multipath, and it showcases the probability of detection plot, P_d versus the Signal-to-Interference-Plus-Noise Ratio (SINR), where the loss parameter L equals 40 dB, and the parameter ϵ is equal to 0. The results from the figure validate the effectiveness and accuracy of the devised detector, as it matches the performance of the AMF in the absence of multipath and when ϵ is equal to 0. This verification is essential in establishing the reliability of the devised detector.

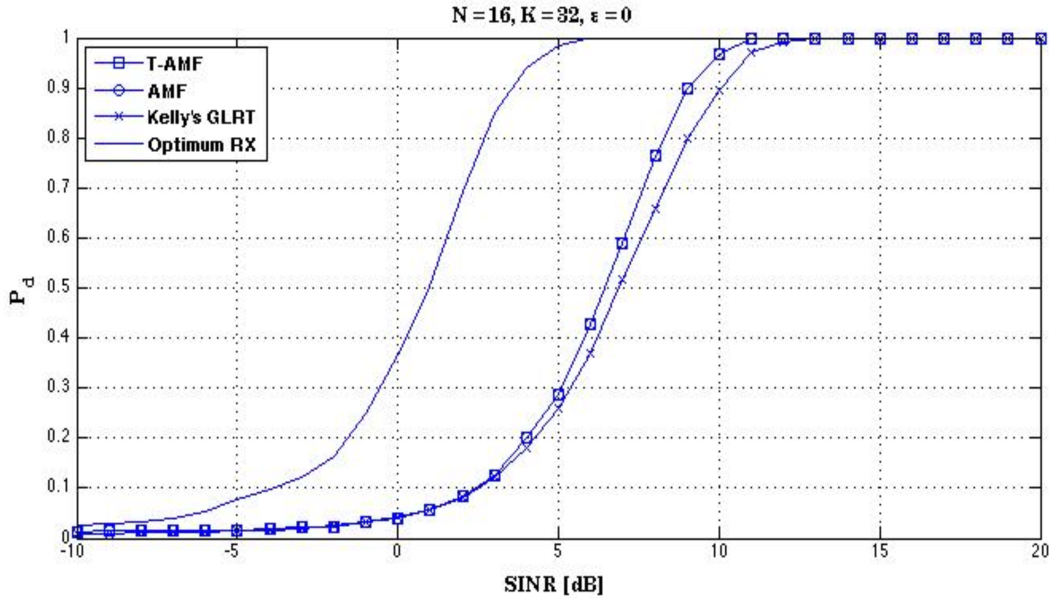

Figure 1. P_d versus SINR for the extended T-AMF, the extended AMF and the extended version of Kelly's receiver; $N = 16$, $K = 32$, $H = 10$, $P_{fa} = 10^{-2}$, $\epsilon = 0$, and $L = 40\ dB$.

In Figure 2, the absence of multipath is assumed, and the probability of detection plot, P_d versus the Signal-to-Interference-Plus-Noise Ratio (SINR), is presented for a loss parameter L equal to 40 dB, while the parameter ϵ is equal to 0.75. The results from the figure indicate that when the value of ϵ is high, such as 0.75, the detection performance of the extended T-AMF is inferior to that of the AMF and Kelly's GLRT. This outcome is not unexpected since the sample data covariance matrix $\boldsymbol{M}$ is expected to be similar to the actual primary data covariance matrix $\boldsymbol{S}$ when there is no multipath (i.e., $\boldsymbol{M} \simeq \boldsymbol{S}$). Higher values of ϵ imply a higher probability of mismatch between the sample data covariance matrix and the actual primary data covariance matrix, which leads to inferior detection performance.

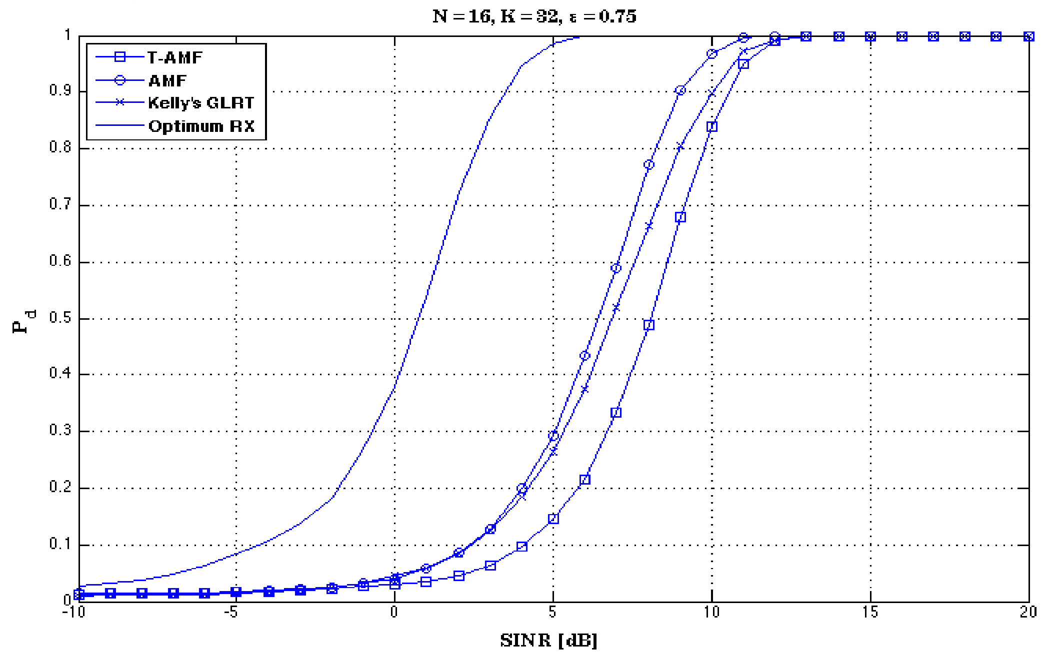

Figure 2. P_d versus SINR for the extended T-AMF, the extended AMF and the extended version of Kelly's receiver; $N = 16$, $K = 32$, $H = 10$, $P_{fa} = 10^{-2}$, $\epsilon = 0.75$, and $L = 40\ dB$.

Figure 3 explores a scenario where there is a significant multipath effect, with a loss parameter L of $20\ dB$. In this scenario, Kelly's GLRT experiences severe performance degradation due to its strong selectivity. On the other hand, the AMF exhibits better detection performance in a multipath environment than Kelly's receiver, but it still has a notable performance loss compared to the extended T-AMF. For this particular scenario, the parameter ϵ is chosen as 0.75, allowing the extended T-AMF to exploit the multipath environment and achieve better performance.

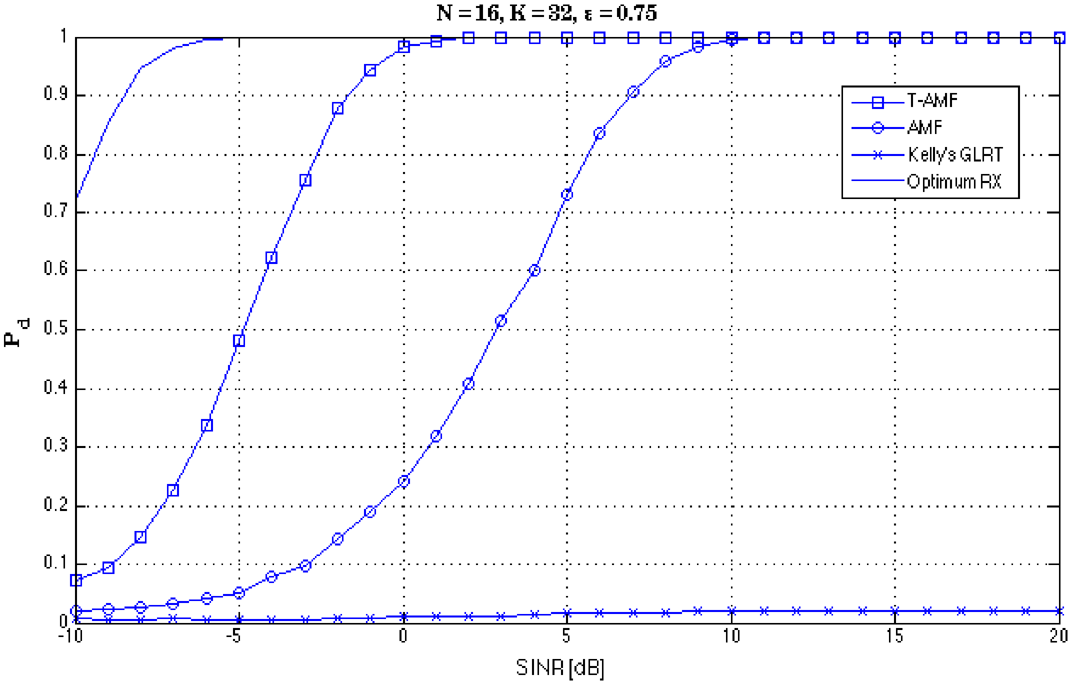

Figure 3. P_d versus SINR for the extended T-AMF, the extended AMF and the extended version of Kelly's receiver; $N = 16$, $K = 32$, $H = 10$, $P_{fa} = 10^{-2}$, $\epsilon = 0.75$, and $L = 20\ dB$.

CONCLUSION

The paper discusses the adaptive detection of extended targets in a clutter rich environment with diffuse multipath. The author models the target return signal from each range cell as a combination of a deterministic signal and a Gaussian-distributed random vector, with the deterministic data vector representing the direct path component and the random data vector representing multipath echoes. For the respective hypothesis testing, the author implements a constraint GLRT in Section II, assuming that the primary data covariance matrix of each range bin is within a certain vicinity of a sample covariance matrix that is devised from the secondary data set. The similarity between primary covariance and sample covariance matrix of the secondary data is set by an adjustable parameter, which depends on the power of multipath components. Thus, the mismatch between the sample data covariance matrix and the actual primary data covariance matrix is adjustable. The performance analysis demonstrates that the devised extended T-AMF performs better than the extended versions of conventional adaptive receivers in diffuse multipath environments.

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Conflict of Interest

The article authors declare that there is no conflict of interest between them.

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