



POLİTEKNİK DERGİSİ

JOURNAL of POLYTECHNIC

ISSN: 1302-0900 (PRINT), ISSN: 2147-9429 (ONLINE)

URL: <http://dergipark.org.tr/politeknik>



Simetrik v- şekilli plakalardaki gerilme yığılma faktörünün yapay sinir ağı kullanılarak modellenmesi

Modeling of stress concentration factor using artificial neural networks for a flat tension bar with opposite v-shaped notches

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To cite to this article: Eren M., Toktas I. ve Ozkan M.T. ,“Modeling of stress concentration factor using artificial neural networks for a flat tension bar with opposite v-shaped notches”, *Journal of Polytechnic*, 26(3): 1199-1205, (2023).

Bu makaleye su şekilde atıfta bulunabilirsiniz: Eren M., Toktas I. ve Ozkan M.T. ,“Modeling of stress concentration factor using artificial neural networks for a flat tension bar with opposite v-shaped notches”, *Politeknik Dergisi*, 26(3): 1199-1205, (2023).

Erişim linki (To link to this article): <http://dergipark.org.tr/politeknik/archive>

DOI: 10.2339/politeknik.1275466

Modeling of Stress Concentration Factor Using Artificial Neural Networks for a Flat Tension Bar with Opposite V-Shaped Notches

Highlights

- ❖ Plate with V-Shaped Notched Stress Concentration Faktör
- ❖ Artificial neural Networks
- ❖ Stress Concentration Factor

Graphical Abstract

In this study, Stress Concentration Factors (SCF) formed on symmetrical v-shaped channel plates were remodeled using Artificial Neural Network technique.

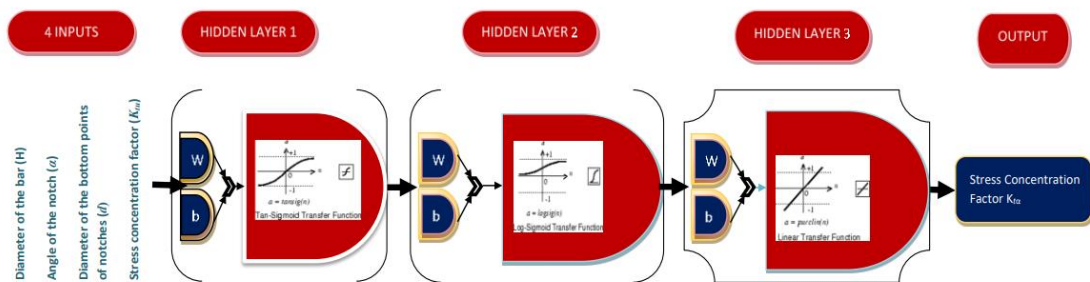


Fig. Basic artificial neural network model

Aim

This study aims to determination stress concentration factor (SCF) for V-Shaped Notch in a plate.

Design & Methodology

In the study, ANN and converting into numerical techniques have been used.

Originality

This study presents a model for the development of ANN methodology for determination of Stress Concentration factor for V-Shaped Notches. It is thought that the study will make a significant contribution to the literature in this respect.

Findings

A new methodology has been developed for calculating the stress concentration factor in V-shaped ribbed plates according to parametric measurements and presented to the use of designers.

Conclusion

According to parametric dimensional measurements, the stress concentration factor in V-shaped channel plates was determined with a sensitivity of $R^2=0.99999$.

Declaration of Ethical Standards

The authors of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

Modeling of Stress Concentration Factor Using Artificial Neural Networks for a Flat Tension Bar with Opposite V-Shaped Notches

Araştırma Makalesi / Research Article

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(Geliş/Received : 02.04.2023 ; Kabul/Accepted : 23.05.2023 ; Erken Görünüm/Early View : 02.06.2023)

ABSTRACT

Machine parts are exposed to stress accumulation due to geometric differences. Determining the stress accumulation locations is crucial to the design procedures. Studies on stress concentrations have been conducted in the past using a variety of theoretical and experimental methodologies, and distinct interpretations have been offered depending on the geometry of the machine part to be produced. The ability to complete activities with the least amount of effort and in the shortest amount of time has emerged as a result of the new computer technologies and software that have impacted many aspects of our everyday lives. One of these methods is the artificial neural networks (ANN) model, which is a branch of artificial intelligence. It is argued as a thesis in this study that fast and low-cost solutions can be found to problems in the field of solid mechanics by using the ANN model. For this purpose, a model has been developed to determine the SCF value with the ANN model of a plate with symmetrical V-shaped notch. The graphs obtained from previous experimental studies were converted to digital format and the K_t values obtained for the V-shaped notch problem with different parameters were converted into a data file. In this file, the SCF values to be obtained according to the strength upper limit safety factor value of the machine part, depending on the dimensional dimensions and material type required for the design, are calculated numerically in the form of an Excel file. An ANN-based code was created in MATLAB software and a new solution method was presented for parts containing a V-shaped notch.

Keywords : Stress concentration factor (scf), artificial neural network (ann), opposite v-shaped notches, stress-strain analysis, computational methods

1. INTRODUCTION

Designing mechanical parts are an important subject since worldwide industry has evolved from the beginning, and adjusting the shapes of these parts is the most challenging concern.

Considering tension is applied to homogeneous shaped machine parts, this results in equally distributed stresses. However, stress is concentrated on the edge or non-uniform shapes of parts, instead of distributed equally. An object can be considered strong if there is no reduction in the area. In case of reduction such as a crack inside the object causes localized stress and leads to deformation. The mentioned is generally called stress concentration. The ANN model is used to predict the stress concentration on a flat tension bar with opposing V-shaped notches in this paper.

Keeping the material below the specified elasticity strength is a crucial design principle. There are different methods of determining the SCF. These methods can be named FEA, experimental, numerical, ANN, etc

Basic stresses and their effects were investigated in the literature; especially stress concentration for a flat tension bar with opposite V-shaped notches.

For all varieties of notches on a circular bar, Nao-Aki Noda and Yasushi Takase created a formula for stress concentration [1]. Using the body force technique, Hironobu Nisitani and Hironobu Nisitani investigated the stress concentration of a cylindrical bar with V-shaped notches [2]. Results were determined by superpositioning the stress fields. F. J. Ortega et al. created a mathematical model to predict SCF on a notched flat bar in axial tension. They introduced the polynomial equation of the second degree by carrying out one hundred simulations on finite element software [3]. C J Gomes et al. worked on theoretical SCFs for short flat tension bars with opposite U-shaped notches. The finite element method and uniform tension have been applied [4]. N. A. Noda et al formula of SCFs for notched round and flat bars has been determined.

They investigated the mechanical properties of the bars under various loading conditions [5]. Ozkan et al. studied solid mechanics and performed to FEA solution and improved its ANN models. Good results were obtained using ANN [6-13].

SCFs, K_{ta} have been obtained for the mentioned bar as a function of the V angle, α [14]. Leven-Frocht's method of relating K_{ta} to the K_{tu} was utilized [15]. In the case of V notched flat bar, while $H/d=1,66$ it is seen in Figure 1 that V angle zero is similar to the V angle 90. But $H/d=1,66$ and angle equal 120°. K_{ta} has big differences. When the angle becomes 150° K_{ta} has very big differences. When

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$H/d=3$ similar decreases to the statements above can be seen for K_{ta} . While the V angle increases, the value of K_{ta} decreases.

Peterson's studies have been updated with this study. On a flat tension bar, as shown in Figure 1, experiments were carried out by Peterson's studies, and results were pointed to numerical values [16]. These values have been collected in an Excel database file. New technologies in software and hardware have been used for converting the data. By using computer technology, a highly reliable database has been obtained. Obtaining a value from the chart is a very hard job and this job can differ according to the person's abilities. With the usage of computer technology, these jobs bring reliability. This study is about the determination of a flat tension bar with opposite V-shaped notch effects. Charts had been obtained from experimental studies. But there are not any mathematical models or engineering equations to determine the SCF. There is only a statistical model obtained before [16]. An engineering approach is needed for experimental techniques. Without any engineering approach, SCF can be determined using the ANN method. This method's results are compatible with the experimental results.

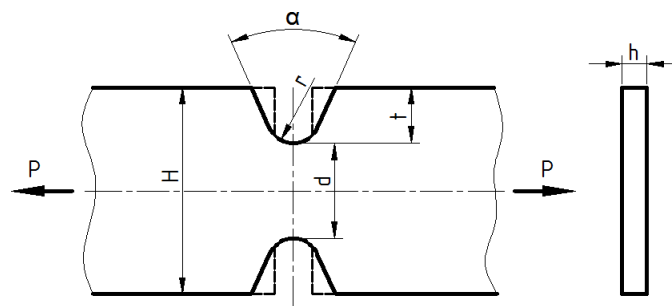


Figure 1. Model of tension applied flat bar with opposite V-shaped notches

2. APPLIED METHOD

The research investigates machine component stability levels based on stress concentration. There are three main factors that affect machine parts namely product strength, design, and material properties. Various operational environments could affect machine components. The stress level of the component depends on shape irregularities, channels, and notches. Stress conditions were previously analyzed using methods such as FEM, photoelastic, experimental, numerical, statistical, ANN, etc.) [6-13, 17-21].

A verified application data table has been used in the design to examine the mentioned criteria. However, this table lacks any mathematical representation, and values are derived from experimental reading. Such a reading process is often open to errors and might differ from reader to reader. As users desire to obtain the same numerical values, the data has been converted to graphics with the help of high-tech software.

The data has been recorded and separated into sheets of a .xlsx file. The resultant ANN model's overall sensitivity

was higher than that of conventional regression models, where the sensitivity should be raised in accordance with the degree of the equation. It is challenging to compute outcomes using such equations due to the degree of the equation. The user could not achieve direct results without any ready-made MATLAB formula [22].

3. ARTIFICIAL NEURAL NETWORK (ANN) MODEL

ANN is a sub-method of artificial intelligence, and it is a method that has a wide range of applications in recent years, including mathematical approaches. The main component of ANN is humans.

The tasks performed by neurons and dendrites in the brain are modeled using computer and mathematical approaches. Properties of neurons, such as their function, shape, and size, can vary. According to the number of neurons and the differences in their functions, a model can be created in which real results can be obtained. ANN can be considered as a system in which neurons (process elements) are arranged in a hierarchical order to produce

outputs for certain inputs. Each neuron is responsible for collecting data, processing and transmitting results [23-29]. Neuronal activities can be categorized as inputs, weights, summation functions, activation functions, and outputs (Fig. 2) [6-13].

Hidden layers are the parts where the main learning function is performed. In the creation of a new ANN model, there must be a minimum of 2 hidden layers. According to the number of parameters, the number of rows and columns of the data, these hidden layer numbers can be increased according to a certain algorithm. Learning can be performed by increasing the number of hidden layers and using different numbers of neurons. While the learning process is being implemented, the process is completed by determining a real model by using different learning functions. The final values obtained derive the output results produced in response to the inputs to the user in the output tab. This study is interested in SCF determination for a flat tension bar with opposite V-shaped notches. Peterson's SCF charts have been examined. Machine parts are affected the different types of stresses. Shape and size variation has changed

the stress on the parts. The highest stress is comprised of the notch area. The maximum stress point is changed according to the magnitude of the load and shape differences. Stress Concentration is the measure of when the most tension was present. The information contained in these curves must be interpreted in order to determine the stress concentration for a given issue. Obtaining SCF from a chart value according to notch size and reliability of the value can be varied by reader abilities. To minimize the reading error, computer technology is used. So, more reliability values have been obtained for converting the chart data.

results were obtained with an ANN model with three hidden layers and [9+11+1] neurons in each layer (Figure 2). Following this, the ANN models shown in Figure 3 were obtained with the input layer with four inputs, the 1st hidden layer with nine neurons, the 2nd hidden layer with 11 neurons, and the 3rd hidden layer with 1 neuron, and as a result, an output hop model with a single output. Information can be represented using weights between layers. Levenberg-Marquardt (LM) learning algorithm methodology was used to generate the outputs.

r , α , H , d , K_{tu} , and K_{ta} (Table 1) are the determination of the SCF K_{ta} for a flat tension bar in V-shaped notches

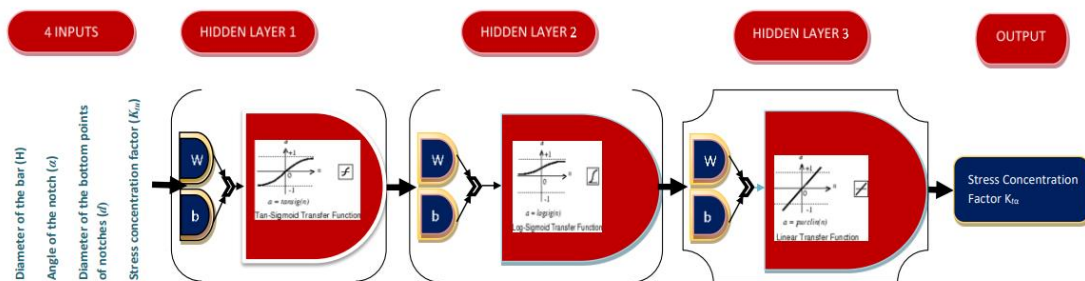


Figure 2. Basic artificial neural network model

An xlsx file containing data has been compiled. Charts that have been transformed have provided data. An updated ANN model was enhanced. The data are 3371 lines by 5 columns, as per the study's specifications. Of these, 15% of the data were randomly chosen and used as test data, 15% were used as validation data, and the remaining 70% were used as training data [6-13].

4. RESULTS AND DISCUSSION

The network was estimated in this study using the dependent parameters (r , H , d and K_{tu}) to generate K_{ta} results for the stress concentration factor parameters and statistical error analysis techniques. Statistical error levels for the training and test datasets are shown in Table 2. Different learning methods and models with different numbers of hidden layers and neurons are tested. The best

Table 1. Stress concentration factors K_{tu} and K_{α} for a flat tension bar in V-shaped notches.

r	α	H	d	K_{tu}	K_{ta}
0.1-5	0-60 90 120 150 180	0.5-40	0.16-13.3	1-4	1-4

Desing the parameters, α , H , d , and K_{tu} , the SCF K_{ta} for a flat tension bar with opposing V-shaped notches has been calculated. These parameters were employed as input layers, and the ANN model utilized the LM method and

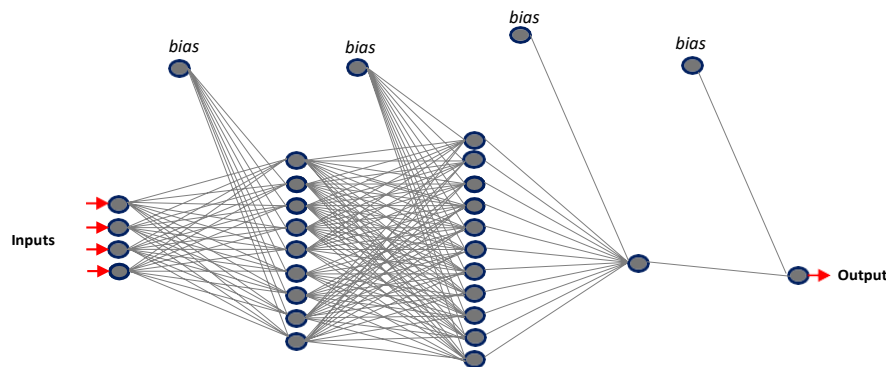


Figure 3. ANN architecture with [4+9+11+1+1] processing elements at three hidden layers

MLP (Multi-Layer Perception). α , H, d, and K_{tu} have been utilized as the input layer while K_{ta} has been used as the output layer of the ANNs. The following are the expressions for the tansig, logsig, and purelin transfer functions (f) utilized in the ANN model (Eqs 1-4) [23-29]:

$$NET_i = \sum w_{ij} \cdot x_j + w_{bi} \tag{1}$$

$$a = \text{tansig}(n) = \frac{2}{(1+e^{-2n})} - 1 \tag{2}$$

$$a = \text{logsig}(n) = \frac{1}{(1+e^{-n})} \tag{3}$$

$$a = \text{purelin}(n) \tag{4}$$

n: Number of processing elements in the previous layer where NET is the weighted sum of the input.

The ANN model has been enhanced using Matlab NN Toolbox. The data file was loaded into the Matlab NN Tool editor. ANN code has been created in Matlab Toolbox. Tests have been done on various models and their effectiveness. Different model means training technique, the number of the hidden layer, and employed function type's iteration. Among the iteration models, the one that performed the best has been picked. The model comprises 4 inputs, 3 hidden layers (9+11+1), and 1 output. For the first hidden layer, which contains nine elements, the Tansig function was used, for the second, which contains eleven, the Logsig function, and for the third, which contains one, the Purelin function. The training method has employed the Levenberg-Marquardt (LM) and Multi-Layer Perception (MLP) training algorithms. Figure 4 illustrates the ANN model (4+9+11+1). In this section; 4 is input parameters (α , H, d, and K_{tu}); 9 is hidden layer 1 (tansig) neuron number, 11 is hidden layer 2 (logsig) neuron numbers, 1 is hidden layer 3 (purelin) neutron number and last 1 is the Output SCF K_{ta} . The training efficiency of the ANN model is displayed in Figure 5. R= 1 has been calculated. Figure 6 displays the ANN's the best training performance. In this figure; the model's best training performance has been obtained at 1.5252 E-06. ANN 30-Bins error histogram has been presented in Figure 7. In the figure Error histogram 20-Bins; addition of training, test and validation percentage has been more than 85% zero error. The remaining 15% data contains negligible minor errors. The proof of this value is $R^2=0.999$. Figure 8 shows the model training, test and validation top performance. The training sensitivity has 10E-8, Validation sensitivity has 10E-5 and test sensitivity has 10E-4. In a good ANN model, the most sensitive value should be the training step. Figure 9 displays the Performance for training, testing, and validating the ANN predictions. In the model, training, test, validation and overall performance has more than 0.999. These results have been obtained from Matlab code that was prepared in NN toolbox [22-29].

Table 2. Statistical Performance of Training processes the ANN model.

Absolute Fraction of Variance (R ²)	Root Mean Square Error (RMSE)	Mean Error Percentage (MEP %)
0.999999212	0.001939947	0.069474395

Results from the ANN model were contrasted with information obtained statistically. The statistical performance of the training step is displayed in Table 2

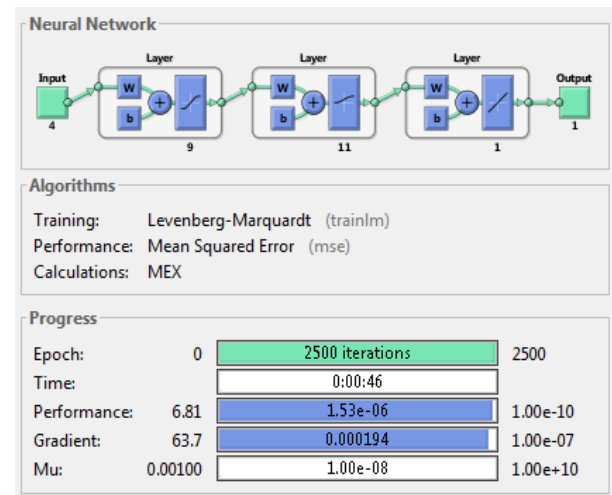


Figure 4. Improved an ANN model using MATLAB

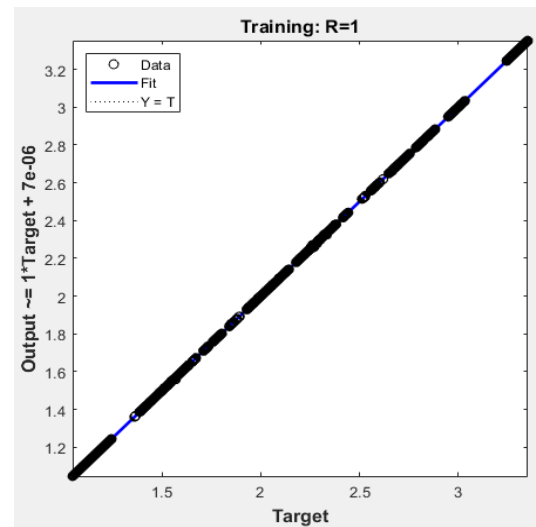


Figure 5. Training effectiveness of the ANN model

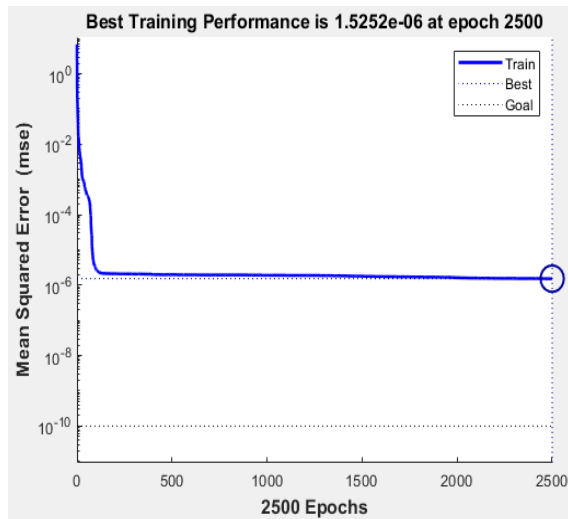


Figure 6. The ANN model's best training effectiveness

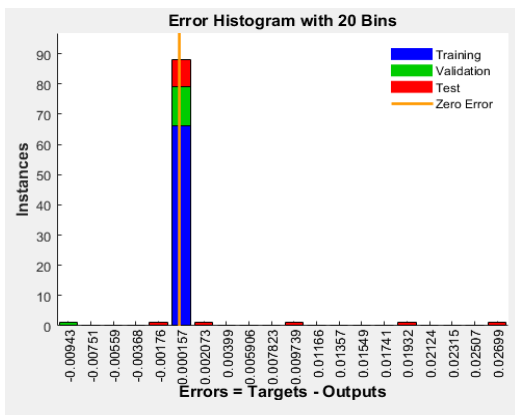


Figure 7. The ANN's error histogram

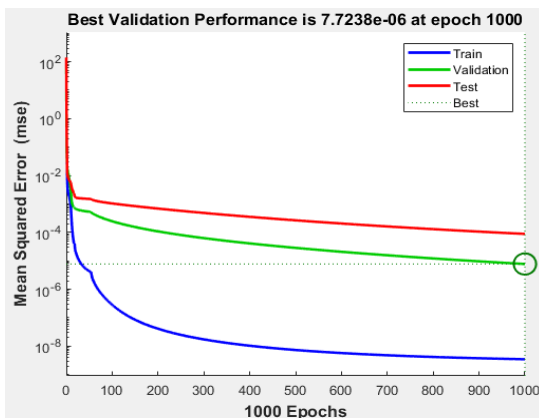


Figure 8. The ANN's error histogram

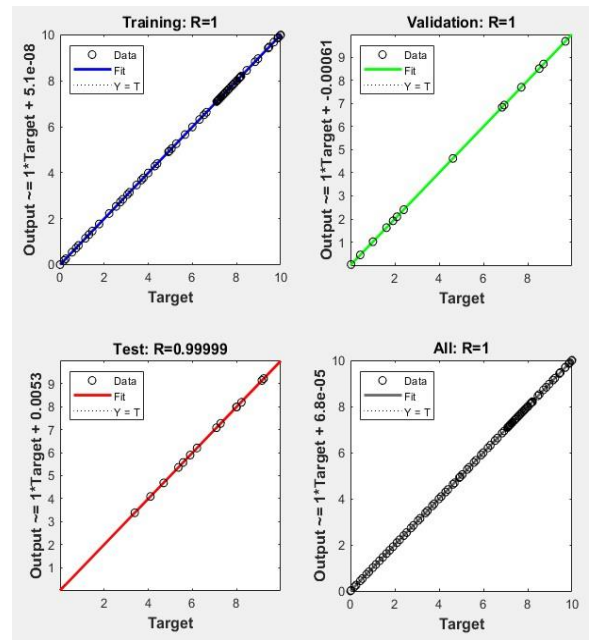


Figure 9. Performance for training, testing, and validating the ANN predictions

The prediction level of an ANN model could be evaluated by applying unknown data to the network and analyzing the results. The results could be compared using statistical methods such as R^2 , RMSE, and MEP [19-25]. Additionally, regression analysis results could be used for calculating these data. These values are determined by the following Eqs (5-7) [30-32]:

$$RMSE = \left(\frac{1}{p} \sum_j |t_j - o_j|^2 \right)^{1/2} \tag{5}$$

$$R^2 = 1 - \left(\frac{\sum_j (t_j - o_j)^2}{\sum_j (o_j)^2} \right) \tag{6}$$

$$MEP = \frac{\sum_j \left(\frac{t_j - o_j}{t_j} \times 100 \right)}{p} \tag{7}$$

The trial-error method was selected to change the network structure (i.e. the number of neurons and hidden layers). Then, the network was trained. Three hidden layers have been selected to obtain desired and accurate results. Additionally, the number of neurons were changed on each of the hidden layer (e.g. from 5 to 150). This alteration was done to obtain the network with the best statistical error results.

Figure 10 shows the comparison of converted data and ANN model results. ANN result has a good agreement with the chart data. In this model has been very good performance by statistical methods. ($R^2=0,999999212$,

RMSE=0,00193994761083, and
MEP%=0,0694743950331501.)

operations. Performed the experiments and analyse the results.

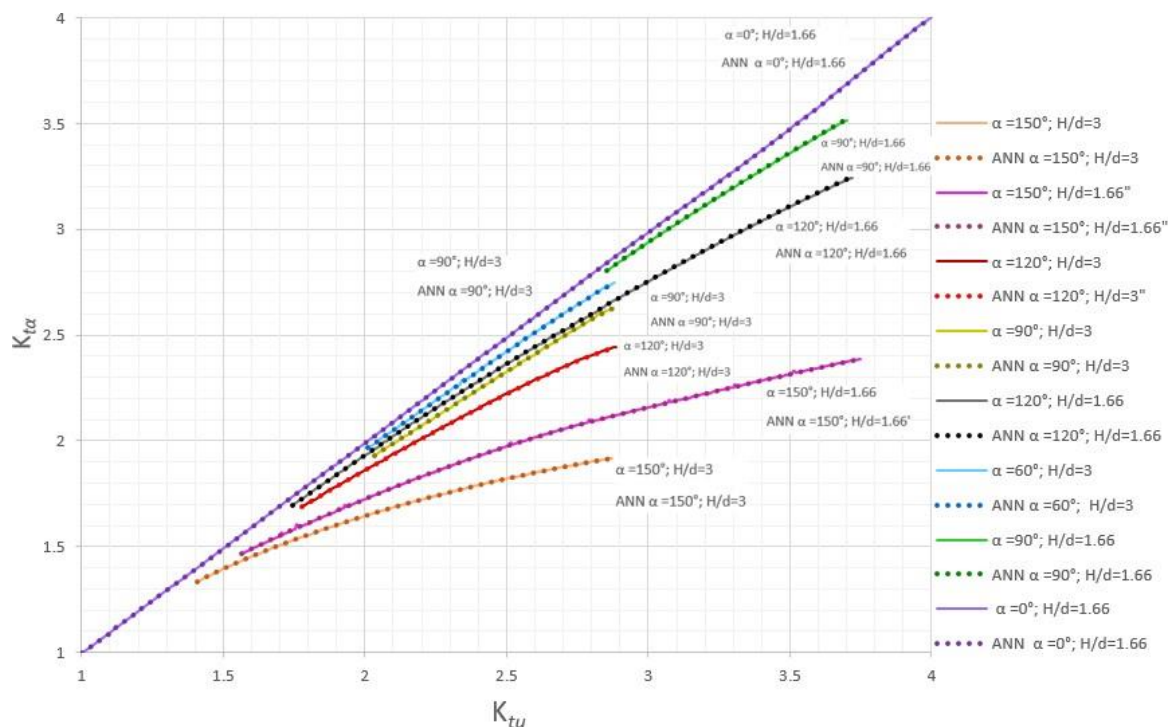


Figure 10. SCF $K_{t\alpha}$ for a flat tension bar with opposite V-shaped notches

6. CONCLUSION

This research uses ANN, an application of computer technology. SCF has been calculated using the ANN approach for a flat tension bar with opposing V-shaped notches. The charts from Peterson have been transformed into a numeric value. The graphical data from Peterson was used to establish a data bank. An excel file with the data has been assembled. The Matlab NN tool editor has received the ANN Excel file. Several ANN models have been evaluated. The most effective performance model was selected. The ANN model has been used to calculate the stress concentration factor for a flat tension bar with opposing V-shaped notches. Statistical techniques have been used to evaluate ANN model performance. Calculated values include $R^2=0,999999212$, RMSE=0,00193994761083, and MEP%=0,0694743950331501. The ANN model test was used to create samples. Peterson's data and the results of the test have been compared. Findings showed that the original data and ANN Test data are complementary to one another. An ANN model was used to improve the simple and affordable technique. This model was a reliable and practical technique that may be utilized to increase accuracy.

AUTHORS' CONTRIBUTIONS

Mehmet EREN: Performed the engineering design and analyse the results.

İhsan TOKTAŞ: He posed the problem, determined the analysis methods, and performed the writing

Murat Tolga ÖZKAN: He posed the problem, determined the analysis methods, and performed the writing operations. Performed the experiments and analyse the results.

CONFLICT OF INTEREST

There is no conflict of interest in this study.

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