ANALYTICAL SOLUTION OF THE TRUNCATED INTERARRIVAL ERLANGIAN QUEUE : $E_R/M/C/K$ WITH BALKING AND RENEGING

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ABSTRACT

This paper aimed to treat the analytical solution of the truncated interarrival Erlangian queue: $E_r/M/c/k$ with balking and reneging for general values of r, c and k. The discipline considered here is FIFO. Some previously published results are shown to be special cases of the present results.

Key Words: *Balking, Reneging, Steady-state Probability, Truncated Interarrival Erlangian Queue.*

1. INTRODUCTION

Gupta [4] has emphasized that the solution of the steady - state probabilities of the truncated Erlangian queues without balking and reneging concepts should be numerically and computer oriented. White et al. [18] have solved some special cases numerically for k= 1, 2 and r = 2 only but without any concept. Morse [8] treated the non truncated case without balking and reneging. Shawky [11] treated the system of machine interference model: M/M/1/k/k with balking, reneging and an additional server for longer queues, in [12] he studied the system: M/M/c/k/N with balking, reneging and spares, and also in [13] he discussed the system: $H_r/M/c/k/N$ with balking and reneging. Al-Seedy and Al-Ibraheem [1] studied the system: $H_2/M/1/k+Y/k+Y$ with the concepts of balking, reneging, state – dependent, spares and an additional server for longer queues. Shawky and El-Paoumy [15] treated the queue: $H_k/M/c/N$ with balking and reneging, in [16] they treated the queue: $H_k/M/2/N$ with balking, reneging and two heterogeneous servers, and also in [17] they discussed the queue: $M/H_k/1/N$ with balking and reneging. Jain and Rakhee [5] discussed the system of N-policy for a machine repair system with spares and reneging, Pearn and Chang [9] dealt with optimal management problem of the N-policy $M/E_k/1$ queueing system with a removable service station under steady-state condition, Sadlam and Torum [10] studied the queueing system: M/M/2 with Poisson arrivals, two heterogeneous parallel servers and no waiting line and Shawky [14] discussed the service Erlangian machine interference model: $M/E_r/1/k/N$ with balking and reneging.

In this paper, the truncated multi - channel queue : $E_r/M/c/k$ is treated with both balking and reneging concepts. The steady - state probabilities of the model herein are

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In this paper, the truncated multi - channel queue : $E_r/M/c/k$ is treated with both balking and reneging concepts. The steady - state probabilities of the model herein are derived together with some measures of effectiveness where these measures are analytically deduced. Finally, some previously published works are shown to be special cases of the model herein.

2. DESCRIPTION OF THE SYSTEM

Consider the multi - servers truncated interarrival Erlangian queue having r - stages each with interarrival rate $r\lambda$ and the service time is exponential with rate μ. The capacity of the system is k and the concepts of balking and reneging are assumed. The queue discipline is assumed to be first come, first served.

Assume the balk concept with probability :

$$\beta = \text{prob. \{a unit joins the queue\}},$$

where $0 \le \beta < 1$ if $c \le n \le k$ and $\beta = 1$ if $1 \le n < c$.

It is also assumed that the units may renege according to an exponential distribution, $f(t) = \alpha e^{-\alpha t}$, $t > 0$, with parameter α. The probability of reneging in a short period of time Δt is given by $r_n = (n-c)\,\alpha\,\Delta t$, for $c < n \le k$ and $r_n = 0$, for $0 \le n \le c$.

3. THE STEADY-STATE EQUATIONS AND THEIR SOLUTION

Let us define $P_{n,j}$, the equilibrium probability that there are n units in the system and j^{th} arrival stage occupied the next arriving unit, where n= 0, 1, ..., k and j= 1, 2, ...r.

Then, the steady–state probability difference equations, in the presence of balking and reneging are :

$$\left.\begin{aligned} r\lambda P_{0,j} &= r\lambda P_{0,j+1} + \mu P_{1,j}, & 1 \le j < r \\ r\lambda P_{0,r} &= \mu P_{1,r}, & j = r \end{aligned}\right\}, \qquad n = 0 \tag{1}$$

$$\left.\begin{aligned} (r\lambda + n\mu)P_{n,j} &= r\lambda P_{n,j+1} + (n+1)\mu P_{n+1,j}, & 1 \le j < r \\ (r\lambda + n\mu)P_{n,r} &= r\lambda P_{n-1,1} + (n+1)\mu P_{n+1,r}, & j = r \end{aligned}\right\}, \qquad 1 \le n < c \tag{2}$$

$$\left.\begin{aligned} (\beta r\lambda + c\mu)P_{c,j} &= \beta r\lambda P_{c,j+1} + (c\mu + \alpha)P_{c+1,j}, & 1 \le j < r \\ (\beta r\lambda + c\mu)P_{c,r} &= r\lambda P_{c-1,1} + (c\mu + \alpha)P_{c+1,r}, & j = r \end{aligned}\right\}, \qquad n = c \tag{3}$$

$$\left.\begin{aligned} [\beta r\lambda + c\mu + (n-c)\alpha]P_{n,j} &= \beta r\lambda P_{n,j+1} + [c\mu + (n-c+1)\alpha]P_{n+1,j}, & 1 \le j < r \\ [\beta r\lambda + c\mu + (n-c)\alpha]P_{n,r} &= \beta r\lambda P_{n-1,1} + [c\mu + (n-c+1)\alpha]P_{n+1,r}, & j = r \end{aligned}\right\}, \qquad c+1 \le n < k \tag{4}$$

$$\left.\begin{aligned} [\beta r\lambda + c\mu + (k-c)\alpha]P_{k,j} &= \beta r\lambda P_{k,j+1}, & 1 \le j < r \\ [\beta r\lambda + c\mu + (k-c)\alpha]P_{k,r} &= \beta r\lambda P_{k-1,1} + \beta r\lambda P_{k,1}, & j = r \end{aligned}\right\}, \qquad n = k. \tag{5}$$

Now, to solve the probability - difference equations use the iterative (recursive) method as follows.

From first equation of (5),

$$P_{k,j+1} = \phi_k P_{k,j}, \qquad 1 \le j < r \tag{6}$$

where

$$\phi_n = 1 + \theta_n, \qquad \theta_n = \begin{cases} \dfrac{n\mu}{r\lambda}, & 0 \le n < c \\ \dfrac{c\mu + (n-c)\alpha}{r\beta\lambda}, & c \le n \le k. \end{cases} \tag{7}$$

Put j = 1, 2, …, r-1 in (6), we get

$$P_{k,j} = \phi_k^{j-1} u_k P_{k,1}, \quad u_k = 1, \quad 1 \le j \le r. \tag{8}$$

Using second equation of (5) and (8) we have

$$P_{k-1,1} = u_{k-1} P_{k,1}, \qquad u_{k-1} = \phi_k^r - 1. \tag{9}$$

From first equation of (4) at n = k-1,

$$P_{k-1,j+1} = \phi_{k-1} P_{k-1,j} - \theta_k P_{k,j}, \qquad 1 \le j < r. \tag{10}$$

For j = 1, 2, …, r-1 in (10) and using equations (8) and (9),

$$P_{k-1,j} = \left[\phi_{k-1}^{j-1} u_{k-1} - \theta_k u_k \sum_{a_1=1}^{j-1} \phi_k^{a_1-1} \phi_{k-1}^{j-1-a_1} \right] P_{k,1}, \qquad 1 \le j \le r. \tag{11}$$

Using second equation of (4), (8) and (11) for n = k-1, j = r we obtain

$$P_{k-2,1} = u_{k-2} P_{k,1}, \qquad u_{k-2} = \phi_{k-1}^r u_{k-1} - \theta_k u_k \sum_{a_1=1}^{r} \phi_k^{a_1-1} \phi_{k-1}^{r-a_1}. \tag{12}$$

Also, from first equation of (4) at n = k - 2,

$$P_{k-2,j+1} = \phi_{k-2} P_{k-2,j} - \theta_{k-1} P_{k-1,j}, \qquad 1 \le j < r. \tag{13}$$

Put j = 1, 2, …, r-1 in (13) and using the equations (9) and (12) we get

$$P_{k-2,j} = \left[\phi_{k-2}^{j-1} u_{k-2} - \theta_{k-1} u_{k-1} \sum_{a_1=a_0}^{j-1} \phi_{k-1}^{a_1-1} \phi_{k-2}^{j-1-a_1} + \theta_k \theta_{k-1} u_k \sum_{a_1=a_0}^{j-2} \phi_{k-1}^{a_1-1} \sum_{a_2=a_1}^{j-2} \phi_k^{a_2-a_1} \phi_{k-2}^{j-a_2-2} \right] P_{k,1},$$

$$1 \le j \le r, \; a_0 = 1. \tag{14}$$

Similarly, from equations (4), (11) and (12) at n = k-2, j = r, we have

$$P_{k-3,1} = u_{k-3} P_{k,1}, \tag{15}$$

where

$$u_{k-3} = \phi_{k-2}^r u_{k-2} - \theta_{k-1} u_{k-1} \sum_{a_1=a_0}^{r} \phi_{k-1}^{a_1-1} \phi_{k-2}^{r-a_1} + \theta_k \theta_{k-1} u_k \sum_{a_1=a_0}^{r-1} \phi_{k-1}^{a_1-1} \sum_{a_2=a_1}^{r-1} \phi_k^{a_2-a_1} \phi_{k-2}^{r-1-a_2}. \tag{16}$$

Using first equation of (4) at n = k-3, j = 1, 2, …, r-1,

$$P_{k-3,j} = \Big[\phi_{k-3}^{j-1}u_{k-3} - \theta_{k-2}u_{k-2}\sum_{a_1=a_0}^{j-1}\phi_{k-2}^{a_1-1}\phi_{k-3}^{j-1-a_1} + \theta_{k-2}\theta_{k-1}u_{k-1}\sum_{a_1=a_0}^{j-2}\phi_{k-2}^{a_1-1}\sum_{a_2=a_1}^{j-2}\phi_{k-1}^{a_2-a_1}\phi_{k-3}^{j-2-a_2}$$

$$-\theta_{k-2}\theta_{k-1}\theta_k u_k \sum_{a_1=a_0}^{j-3}\phi_{k-2}^{a_1-1}\sum_{a_2=a_1}^{j-3}\phi_{k-1}^{a_2-a_1}\sum_{a_3=a_2}^{j-3}\phi_k^{a_3-a_2}\phi_{k-3}^{j-3-a_3}\Big]P_{k,1}. \qquad (17)$$

In general, for n = k, k-1, k-2, …, c and j = 1, 2, …, r, we have

$$P_{n,j} = \eta_{n,j}P_{k,1}, \qquad c \le n \le k, \quad 1 \le j \le r, \qquad (18)$$

where

$$\eta_{n,j} = \phi_n^{j-1}u_n + \sum_{\ell=0}^{k-n-1}\Big\{(-1)^{\ell+1}u_{n+\ell+1}\Big[\prod_{i=0}^{\ell}\Big(\theta_{n+i+1}\sum_{a_{i+1}=a_i}^{j-\ell-1}\phi_{n+i+1}^{a_{i+1}-a_i}\Big)\Big]\phi_n^{j-\ell-1-a_{\ell+1}}\Big\},$$

$$0 \le n \le k, \quad 1 \le j \le r, \qquad (19)$$

$$u_n = \begin{cases} 1, & n = k \\ \phi_k^r - 1, & n = k-1 \\ \phi_{n+1}^r u_{n+1} + \sum_{\ell=0}^{k-n-2}(-1)^{\ell+1}u_{n+\ell+2}\Big[\prod_{i=0}^{\ell}\Big(\theta_{n+i+2}\sum_{a_{i+1}=a_i}^{r-\ell}\phi_{n+i+2}^{a_{i+1}-a_i}\Big)\Big]\phi_{n+1}^{r-\ell-a_{\ell+1}}, & 0 \le n \le k-2. \end{cases}$$

Similarly, from equations (2), (3), (18) and (19) we obtain

$$P_{n,j} = \beta\eta_{n,j}P_{k,1}, \qquad 0 \le n < c, \quad 1 \le j \le r. \qquad (20)$$

Equations (18) and (20) give all the probabilities in terms of $P_{k,1}$, which itself may now be determined by using the normalizing condition:

$$\sum_{n=0}^{k}\sum_{j=1}^{r}P_{n,j} = 1. \qquad (21)$$

Then,

$$P_{k,1} = 1/\Big(\beta\sum_{n=0}^{c-1}\sum_{j=1}^{r}\eta_{n,j} + \sum_{n=c}^{k}\sum_{j=1}^{r}\eta_{n,j}\Big), \qquad (22)$$

hence all the probabilities are completely known in terms of the queue parameters.

Therefore, the expected number of units in the system and in the queue are, respectively,

$$L = \sum_{n=1}^{k}\sum_{j=1}^{r}nP_{n,j} = \Big[\beta\sum_{n=1}^{c-1}\sum_{j=1}^{r}n\eta_{n,j} + \sum_{n=c}^{k}\sum_{j=1}^{r}n\eta_{n,j}\Big]P_{k,1},$$

$$L_q = \sum_{n=c}^{k}\sum_{j=1}^{r}(n-c)P_{n,j} = \sum_{n=c+1}^{k}\sum_{j=1}^{r}(n-c)\eta_{n,j}P_{k,1},$$

and the expected waiting time in both the system and the queue are obtained by:

$$W = \frac{L}{\lambda'} \quad , \qquad W_q = \frac{L_q}{\lambda'}, \qquad \lambda' = \mu(L - L_q) .$$

EXAMPLE

In the above system : E_r/M/c/k with balking and reneging, letting r = 3, c=5 and k = 8, i.e., the queue : E_3/M/5/8 with balking and reneging, the results are :

$P_{8,2} = \phi_8 P_{8,1}$, $P_{8,3} = \phi_8^2 P_{8,1}$, $P_{7,j} = \eta_{7,j} P_{8,1}$, $P_{6,j} = \eta_{6,j} P_{8,1}$, $P_{5,j} = \eta_{5,j} P_{8,1}$,

$P_{4,j} = \beta\eta_{4,j} P_{8,1}$, $P_{3,j} = \beta\eta_{3,j} P_{8,1}$, $P_{2,j} = \beta\eta_{2,j} P_{8,1}$, $P_{1,j} = \beta\eta_{1,j} P_{8,1}$, $P_{0,j} = \beta\eta_{0,j} P_{8,1}$, $j = 1, 2, 3,$

where

$$\eta_{7,1} = u_7 = \phi_8^3 - 1 , \quad \eta_{7,2} = \phi_7 u_7 - \theta_8 , \quad \eta_{7,3} = \phi_7^2 u_7 - \theta_8(\phi_7 + \phi_8),$$

$$\eta_{6,1} = u_6 = \phi_7^3 u_7 - \theta_8(\phi_7^2 + \phi_7\phi_8 + \phi_8^2) , \qquad \eta_{6,2} = \phi_6 u_6 - \theta_7 u_7 ,$$

$$\eta_{6,3} = \phi_6^2 u_6 - \theta_7 u_7(\phi_6 + \phi_7) + \theta_7\theta_8 , \quad \eta_{5,1} = u_5 = \phi_6^3 u_6 - \theta_7 u_7(\phi_6^2 + \phi_6\phi_7 + \phi_7^2) + \theta_7\theta_8(\phi_6 + \phi_7 + \phi_8) ,$$

$$\eta_{5,2} = \phi_5 u_5 - \theta_6 u_6 , \quad \eta_{5,3} = \phi_5^2 u_5 - \theta_6 u_6(\phi_5 + \phi_6) + \theta_6\theta_7 u_7 ,$$

$$\eta_{4,1} = u_4 = \phi_5^3 u_5 - \theta_6 u_6(\phi_5^2 + \phi_5\phi_6 + \phi_6^2) + \theta_6\theta_7 u_7(\phi_5 + \phi_6 + \phi_7) - \theta_6\theta_7\theta_8$$

$$\eta_{4,2} = \phi_4 u_4 - \theta_5 u_5 , \quad \eta_{4,3} = \phi_4^2 u_4 - \theta_5 u_5(\phi_4 + \phi_5) + \theta_5\theta_6 u_6 ,$$

$$\eta_{3,1} = u_3 = \phi_4^3 u_4 - \theta_5 u_5(\phi_4^2 + \phi_4\phi_5 + \phi_5^2) + \theta_5\theta_6 u_6(\phi_4 + \phi_5 + \phi_6) - \theta_5\theta_6\theta_7 u_7$$

$$\eta_{3,2} = \phi_3 u_3 - \theta_4 u_4 , \quad \eta_{3,3} = \phi_3^2 u_3 - \theta_4 u_4(\phi_3 + \phi_4) + \theta_4\theta_5 u_5 ,$$

$$\eta_{2,1} = u_2 = \phi_3^3 u_3 - \theta_4 u_4(\phi_3^2 + \phi_3\phi_4 + \phi_4^2) + \theta_4\theta_5 u_5(\phi_3 + \phi_4 + \phi_5) - \theta_4\theta_5\theta_6 u_6$$

$$\eta_{2,2} = \phi_2 u_2 - \theta_3 u_3 , \quad \eta_{2,3} = \phi_2^2 u_2 - \theta_3 u_3(\phi_2 + \phi_3) + \theta_3\theta_4 u_4 ,$$

$$\eta_{1,1} = u_1 = \phi_2^3 u_2 - \theta_3 u_3(\phi_2^2 + \phi_2\phi_3 + \phi_3^2) + \theta_3\theta_4 u_4(\phi_2 + \phi_3 + \phi_4) - \theta_3\theta_4\theta_5 u_5$$

$$\eta_{1,,2} = \phi_1 u_1 - \theta_2 u_2 , \quad \eta_{1,3} = \phi_1^2 u_1 - \theta_2 u_2(\phi_1 + \phi_2) + \theta_2\theta_3 u_3 ,$$

$$\eta_{0,1} = u_0 = \phi_1^3 u_1 - \theta_2 u_2(\phi_1^2 + \phi_1\phi_2 + \phi_2^2) + \theta_2\theta_3 u_3(\phi_1 + \phi_2 + \phi_3) - \theta_2\theta_3\theta_4 u_4$$

$$\eta_{0,,2} = u_0 - \theta_1 u_1 , \quad \eta_{0,3} = u_0 - \theta_1 u_1(1 + \phi_1) + \theta_1\theta_2 u_2 ,$$

$$\theta_1 = \frac{\mu}{3\lambda}, \quad \theta_2 = \frac{2\mu}{\lambda}, \quad \theta_3 = \frac{\mu}{\lambda} , \quad \theta_4 = \frac{4\mu}{3\lambda}, \quad \theta_5 = \frac{5\mu}{3\beta\lambda}, \quad \theta_6 = \frac{5\mu + \alpha}{3\beta\lambda},$$

$$\theta_7 = \frac{5\mu + 2\alpha}{3\beta\lambda}, \quad \theta_8 = \frac{5\mu + 3\alpha}{3\beta\lambda}, \quad \phi_n = 1 + \theta_n, \quad n = 0(1)8, \ \theta_0 = 0.$$

Then,

$$P_{8,1} = 1/\left[\beta\sum_{n=0}^{4}\sum_{j=1}^{3}\eta_{n,j} + \sum_{n=5}^{7}\sum_{j=1}^{3}\eta_{n,j} + \phi_8^2 + \phi_8 + 1\right].$$

Moreover, if we put $\beta = 0.6$, $\alpha = 0.3$, $\lambda = 3$ and $\mu = 2$, we get

$P_{8,1} = 5.839039E-9$, $P_{8,2} = 1.762525E-8$, $P_{8,3} = 5.320213E-8$, $P_{7,1} = 1.547526E-7$
$P_{7,2} = 4.4674E-7$, $P_{7,3} = 1.288097E-6$, $P_{6,1} = 3.709194E-6$, $P_{6,2} = 1.048036E-5$
$P_{6,3} = 2.959376E-5$, $P_{5,1} = 8.351262E-5$, $P_{5,2} = 2.310907E-4$, $P_{5,3} = 6.390461E-4$
$P_{4,1} = 1.059611E-3$, $P_{4,2} = 1.908695E-3$, $P_{4,3} = 3.348545E-3$, $P_{3,1} = 5.614978E-3$,
$P_{3,2} = 8.416421E-3$, $P_{3,3} = 1.233075E-2$, $P_{2,1} = .01757477$, $P_{2,2} = .03726447$,
$P_{2,3} = .08133951$, $P_{1,1} = .1815717$, $P_{1,2} = .1984879$, $P_{1,3} = .1929103$,
$P_{0,1} = .1273266$, $P_{0,2} = .08697737$, $P_{0,3} = .4286894$,

$L = .9547255$, $L_q = 4.756249E-5$, $W = .5000249$, *and* $W_q = 2.491023E-5$,

4. SPECIAL CASES

Some queueing systems can be obtained as special cases of this model.

1) If we put $\beta = 1$ and $\alpha = 0$, then we get the queue : $E_r/M/c/k$ without balking and reneging, and the results are :

$$P_{n,j} = \eta_{n,j} P_{k,1}, \qquad 0 \le n \le k,\ 1 \le j \le r,$$

$$P_{k,1} = 1/\sum_{n=0}^{k}\sum_{j=1}^{r}\eta_{n,j},$$

where

$$\eta_{n,j} = \begin{cases} \phi_c^{j-1}u_n + \sum_{i=1}^{k-n}(-1)^i\binom{j-1}{i}\theta_c^i\phi_c^{j-i-1}u_{n+i}, & c \le n \le k \\ \phi_n^{j-1}u_n + \sum_{\ell=0}^{k-n-1}\{(-1)^{\ell+1}u_{n+\ell+1}\left[\prod_{i=0}^{\ell}\left(\theta_{n+i+1}\sum_{a_{i+1}=a_i}^{j-\ell-1}\phi_{n+i+1}^{a_{i+1}-a_i}\right)\right].\phi_n^{j-\ell-1-a_{\ell+1}}\}, & 0 \le n < c \end{cases}$$

$$1 \le j \le r, \quad a_0 = 1,$$

$$u_n = \begin{cases} 1, & n = k \\ \phi_c^r - 1, & n = k-1 \\ \sum_{i=1}^{k-n-1}(-1)^i\binom{r}{i}\theta_c^i\phi_c^{r-i}u_{n+1+i}, & c \le n < k-1, \\ \phi_{n+1}^r u_{n+1} + \sum_{\ell=0}^{k-n-2}(-1)^{\ell+1}u_{n+\ell+2}\left[\prod_{i=0}^{\ell}\left(\theta_{n+i+2}\sum_{A_{i+1}=A_i}^{r-\ell}\phi_{n+i+2}^{A_{i+1}-A_i}\right)\right]\phi_{n+1}^{r-\ell-A_{\ell+1}}, & 0 \le n < c, \end{cases}$$

$$\phi_n = 1+\theta_n\ , \qquad \theta_n = \begin{cases} \dfrac{n\mu}{r\lambda}, & 0 \le n < c \\ \dfrac{c\mu}{r\lambda}, & c \le n \le k. \end{cases}$$

2) Let j = r = 1, then we get the queue : M/M/c/k with balking and reneging, and the results are :

$$P_n = \begin{cases} (\prod_{i=n+1}^{k}\theta_i)P_k, & c \le n < k \\ \beta(\prod_{i=n+1}^{k}\theta_i)P_k, & 0 \le n < c, \end{cases}$$

$$P_k = 1/(1+\beta\sum_{n=0}^{c-1}\prod_{i=n+1}^{k}\theta_i + \sum_{i=c}^{k-1}\prod_{i=n+1}^{k}\theta_i),$$

where

$$\theta_n = \begin{cases} \dfrac{n\mu}{\lambda}, & 1 \le n < c \\ \dfrac{c\mu+(n-c)\alpha}{\beta\lambda}, & c \le n \le k. \end{cases}$$

Moreover, let $\beta = 1$ and $\alpha = 0$, we have the system : M/M/c/k without balking and reneging, and the results are :

$$P_n = \begin{cases} \left(\dfrac{c}{\rho}\right)^{k-n} P_k, & c \le n \le k \\ \dfrac{(c)_{c-n}c^{k-c}}{\rho^{k-n}} P_k, & 0 \le n < c, \end{cases}$$

$$P_k = 1/\left[\sum_{n=0}^{c-1}\frac{(c)_{c-n}c^{k-c}}{\rho^{k-n}} + \sum_{i=c}^{k}\left(\frac{c}{\rho}\right)^{k-n}\right]$$

where

$$\rho = \frac{\lambda}{\mu}\ , \quad (k)_n = k(k-1)(k-2)\ldots(k-n+1), \quad n \ge 1\ ,\ (k)_0 = 1\ ,$$

which studied by Kleinrock [6], White et al. [18], Medhi [7], Gross and Harris [3], Bunday [2], Morse [8] and others.

5. CONCLUSIONS

In this paper, the queue: E_r/M/c/k is studied with balking and reneging. The steady-state probabilities and some measures of effectiveness are derived in explicit form. We discussed the numerical example and deduced the expected number of units in the system and in the queue, and the expected waiting time in both the system and the queue.

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KUYRUĞA DAHİL OLMAMA VE KUYRUĞU TERK ETME İLE BUDANMIŞ GELİŞLERARASI ERLANG KUYRUK MODELİ E_R/M/C/K NIN ANALİTİK ÇÖZÜMÜ

ÖZET

Bu makalede kuruğa dahil olmama ve kuyruğu terk etme ile budanmış gelişlerarası Erlang kuyruk modeli E_r /M/c/k nin r,c ve k nın genel değerleri için analitik çözümü ele alınmıştır. Çalışmada dikkate alınmış olan kuyruk disiplini ilk gelen ilk hizmet alınır (FIFO) şeklindedir. Daha evvelce yayınlanmış sonuçların, mevcut çalışmanın özel durumları olduğu da ayrıca gösterilmiştir.

Anahtar Kelimeler: *Budanmış gelişlerarası Erlang kuyruk modeli, Durağan durum olasılığı, Kuyruğa dahil olmama, Kuyruğu terk etme.*