Heritage of Arabic Geometry: Al–Samarqandī's Work in Fundamental Geometric Theorems

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ABSTRACT
The manuscripts of Arabic scientific heritage are full of invaluable knowledge which was lost or neglected even by Arabic speaking scholars. Moreover, the authors of the available manuscripts were not studied enough by Arab scholars even though their work had established the basics of different modern sciences, beside the fact that methods and terminologies used in Arabic manuscripts are still used today.

In this paper we shed light on the work of one of the founders of the science of geometry in the Arab heritage, Shams al-Samarqandī , namely Ashkāl al-Tā'sīs, which is translated into English as Fundamental Theorems, that is considered as one of the early writing, in which different basic geometric propositions and their properties are explained. By studying and analysing the information he presented, we try to show the importance of his work, demonstrating some salient examples that present the paramount quality of al–Samarqandī's work for the Arab heritage of geometry. Discussing his methodology and comparing it with the scientific facts established nowadays in geometry, the results show, that he was a pioneer scholar and mathematician of the scientific heritage written in Arabic whose work should be studied more carefully and thoroughly to understand his influence on the science of geometry with a manuscript of less than thirty pages, forming the current used terms nowadays in Arabic geometry. Finally, the conclusions and results suggest some recommendations to preserve and present the rich scientific heritage written in Arabic.

KEYWORDS
Geometry, Arabic Mathematical Heritage, Geometrical Theorems or Propositions, Al–Samarqandī, Qaḍī Zādih.

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INTRODUCTION

The manuscript entitled in Arabic Ashkāl al–Ta'sīs fī al–Handasah was written by Shamsuddīn Muḥammad ibn Ashraf al-Ḥusaynī al–Samarqandī in the second half of the 13th century. If we translate the title literally from Arabic to English the title will be (Basic shapes in geometry), but when examining the work it is clearly means: fundamentals theorems or propositions in geometry, which is widely accepted by the scholars who translated the work into English (De Young, 2001). On the other hand Fazlıoğlu in the paper (The Samarqand Mathematical-Astronomical School: A Basis for Ottoman Philosophy and Science) debate that the actual meaning is basic forms of the existent (Fazlıoğlu, 2008). It is considered one of the early and most writings in the heritage of geometry written in Arabic (Fazlıoğlu, 2007). al- Samarqand had discussed in it the basics for establishing the fundamental theorems of Euclid's plane geometry. The text presented, explained, and discussed 35 of the main important theorems of the plane geometry.

Shamsuddīn al-Samarqandī was born in the middle of 13th century in Samarqand, which is nowadays the capital of Uzbekistan. His death date is between 1302 and 1320 since there is no agreement on his exact date of death. He authored books in different fields, namely, Theology, Logic, Mathematics and Astronomy. His works were the main references taught in the madrasas (schools) in the Arab world for many centuries (al-Zirikli, 1926) and (İhsanoğlu, 2004). After studying the standard curriculum in the basic religious sciences, al–Samarqandī also mastered logic and the science of geometry. One of the clearest characteristics of his works is the idea of understanding the existent due to geometrical forms.

Moreover, al–Samarqandī wrote a commentary (sharḥ) on Naṣīr al–Dīn al–Ṭūsī's (Taḥrīr), a "Recension of Ptolemy's Almagest" in the field of theoretical

astronomy. The Arabic title is Al-Tadhkira fī ʻlm al-Hay'a. Unfortunately, many of al–Samarqandī's astronomical works have not been studied yet.
Al–Samarqandī's most influential work was his different textbooks in which he provided various information about the works of scholars prior to him, thus he greatly impacted future generations, who studied those books.

His geometrical work entitled Ashkāl al-Taʾsīs (hereafter AT) contains 35 theorems or propositions from Euclid's Elements; the first 30 ones are strictly geometrical, while the last five deal with what is called "geometric algebra". It was the textbook used for students of middle level in the Arabic madrasas. Later it was supplemented most often with Qaḍī Zādih al-Rūmī's commentary. For many centuries in Arabic schools both before and after the ottoman era the work was not studied thoroughly by Arabic speaking scholars. Although AT formed the reference for many centuries in Arabic schools, the work was not studied thoroughly, even countries such as Syria and Iraq used to teach and research using Arabic language, especially that Arab academies in Damascus and Cairo claims to study the scientific heritage written in Arabic, it was not well presented in the modern Arabic scientific literature. This paper will shed light on al-Samarqandī's work, demonstrating its contents and methods, showing its importance in shaping the heritage of geometry in the Arab world. We are neither verifying the tract, nor translating it into English since this work have been done previously, our focus is the comparison between his Arabic terminology and nowadays used terms in the science of geometry, and the methods he used to prove his theorems and the implemented methods recently in Arabic school courses, for example the Syrian new curriculum teaching mathematics started to use English terminology despite of the claims of teaching totally in Arabic language.

MANUSCRIPT

The version studied in this paper was obtained as a digital format, scanned from the original manuscript saved in Egyptian book house Cairo, Dār al-

Kutub, riyāḍ 826, under the number 1571 which dates back to 1650-1651 and contains of 20 pages and a hard cover (URL-1). We can notice the comments on this text of Qāḍī Zādeh al-Rūmī (d.1436), director of Ulugh Beg Madrasa in Samarqand, who wrote a very popular commentary on this manuscript later, explaining by his comments on this tract of al–Samarqandī. Nevertheless, we don't know if the original manuscript contained drawings or figures, we assume that those figures on the copy studied in this paper were also added by Qāḍī Zādeh al-Rūmī since they were in the margins with his comments.

METHODOLOGY

The methods we followed in this paper did not aim to edit the manuscript rather than presenting the contents of the manuscript, discussing the author's methodology used in his work, then analysing and checking the correctness of the information the author included in his work. In addition to that, the paper uses a comparative method to study the similarities and differences between the terminologies the author used, and nowadays used terms in geometry in Arabic language.

Finally, the results are related to the influence of the manuscript on the geometric heritage in the Arab world, and the future possible uses and development of the methods and techniques he used to prove his hypothesis.

VERIFICATION OF THE MANUSCRIPT

The commentary of Qāḍī Zādeh on Ashkāl al-Taʾsis was edited by Muḥammad Souissi (1984) in Arabic language. This is the only version studied and tried to recover into modern Arabic scientific language an important and extra ordinary treatise. The objective of this verification and recovery was not to present the value of al–Samarqandī's work but rather than introducing it to Arabic readers using modern Arabic language. Souissi's work did not discuss the importance of a work that established the basis of geometry scientific heritage written in Arabic. Moreover, in his book Souissi did not analyse the methodology and mathematical approach of al–Samarqandī, which he followed in the text of his manuscript. Analysing and discussing the afore-mentioned characteristics of the original treatise may form a part of the scientific heritage itself.

BIBLIOGRAPHICAL WORK

Although the work of al–Samarqandī contributed to the establishment of geometry science in the Arabic written heritage, it did not draw the attention of modern Arab scholars. Looking up the literature, we can find very few articles which deal with this work. Most of the papers are sort of biography of al–Samarqandī, as (Čelebī, 1941), and (al-Sharīf, 1985) for example. Other papers are an inventory of his all manuscripts as (Brockelmann, 1943) and (Dilgan, 1980). We have also found very few papers giving an overview of al–Samarqandī's manuscript: AT and the later scholars who commented on it as (Bingöl, 1991). Only few papers presented and discussed al-Samarqandī's work given in AT, demonstrating the invaluable heritage he left as basis of geometry in the history of Arabic science heritage, for example by (Dilgan, 1960), and (Bağdadlı, 1955) tried to show the importance of his work, but still there were very few papers in English (Dilgan, 1980) and (De Yong, 2001), and even less studies in Arabic discussing and clarifying the importance of his work.

PRESENTATION AND DISCUSSION

In this chapter we present a summary of the manuscript's contents, then we examine the information presented and proved by al–Samarqandī through various examples in the book, and finally we will show some errors and discuss the reasons behind, and suggest possible corrections of them.

SUMMARY OF CONTENTS

The structure of al–Samarqandī's manuscript appears to be in a gradual hierarchical way, distributing the basic geometrical theorems into 35 ones. He began presenting information about lines and angles, then studied triangles, and from that information he moved to explain polygons. Nevertheless, he did not discuss the establishment of circles. On the contrary he used the properties of circles to prove some of his ideas in the afore-mentioned topics. It seems that he considered the theorems on circle and arcs as well established and proven with well-known geometrical properties.

The contents are distributed as following, from theorem 1 to 3 discussed lines and angles, 4 to 8 the triangles and their congruence, then 9 to 12 discussed again lines with angles focusing on the establishing of perpendicular line from a point out of a straight line or inside it. Using the previous information he discussed from theorem 12 to 20 the triangles and their angles, and moved to parallel lines and parallelograms from theorem 21 to theorem 26, while from 27 to 30 he focused on calculating areas of different quadrilaterals. On the other hand he used these areas to prove some what we call nowadays algebraic identities in the theorem from 30 to 35, although the debate in mathematical history was intense about this point (Herz-Fischler, 1987).

PRESENTATION AND METHOD OF PROOF

Al–Samarqandī used different methodologies and approaches to prove his theorems, mainly constructive proof by the geometric properties of the shapes, in addition to proofs by contradiction of hypothesis. We present in the following some examples of both methods:

PROOF USING CONSTRUCTIVE PROPERTIES OF GEOMETRIC SHAPES

In the 17th theorem (correspond to Euclid I,4), page 10 al-Samrqandī proves the congruence of triangles where they had one equal side and two equal angles. He said "If two angles and one side of a triangle were equal to opposite two angles and one side of another triangle, the other angle and two sides will be equal to their opposites, i.e. angle $\hat{A}$ in the triangle $\overset{\Delta}{ABC}$ is equal to angle $\widehat{D}$ in the triangle $\overset{\Delta}{DEF}$, and angle $\hat{B}$ equal to $\hat{E}$, and the side [AB] is equal to [DE]. Imagine plotting [AB] on [DE], then [AC] will coincide with [DF] because $\hat{A}$ is equal to $\widehat{D}$, and [BC] coincides with [EF] because $\hat{B}$ is equal to $\hat{E}$". We notice that the method of al–Samarqandī uses multi-steps: firstly, to imagine plotting one triangle upon the other then depending on the logic that according to geometric characteristics he had proved his hypothesis according to what he has established in the previous theorems. Representing his explanation with geometrical figures following his description will result in Figure 1.

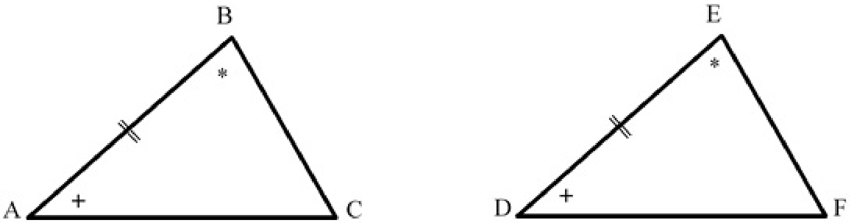

Figure 1. *Constructive proof of two triangle congruence.*

PROOF BY CONTRADICTION

The use of this method was repeatedly applied in many proofs, such as theorems number 5, 6, 7, and 18. For example, in theorem number 7, al–Samarqandī stated that: " If two angles of triangle are equal, the adjacent sides will be equal. For instance Figure 2, the angles $\hat{B}$ and $\hat{C}$ of the triangle

$\overset{\Delta}{ABC}$ are equal, thus [AB]=[AC], otherwise if one of them [AC] is longer than the other, we subtract [DC] from it which is equal to [AB] and we connect D to B the angle $\widehat{DBC}$ will be expectedly equal to $\widehat{DCB}$, but $\widehat{DCB}$ is equal to $\widehat{ABC}$, so compulsory the angle $\widehat{DBC}$ equal to $\widehat{ABC}$ because the part is the whole". Here it is clear that the writer proved the equality of the sides adjacent to equal angles by contradiction, considering them unequal and created equal sides that creates two angles which are supposed to be equal. Figure 2 shows the process of hypothesis contradiction he used to prove in his discussed idea.

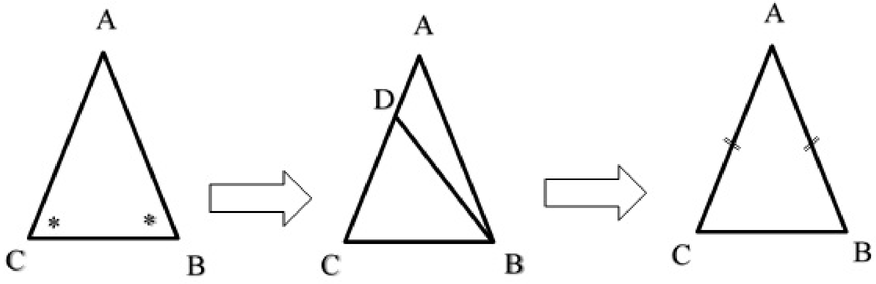

Figure 2. *Proof of two equal sides of triangle by hypothesis contradiction.*

NON MATHEMATICAL PROOF

Although al–Samarqandī used scientific mathematical methods to prove his hypothesis about different geometric theorems, in some cases he just relied on common logic without clear scientific proof. For example, when he tried to discuss theorem number 8 he explained that: "If each side of a triangle is equal to each side of another triangle their opposite angles will be equal and the triangles will be equal. Let a triangle $\overset{\Delta}{ABC}$ and $\overset{\Delta}{DEF}$, if [AB]=[DE], [BC]=[EF] and [AC]=[DF] thus $\widehat{A}$ =$\widehat{D}$, $\widehat{B}$ =$\widehat{E}$, $\widehat{C}$ =$\widehat{F}$. And $\overset{\Delta}{ABC}$= $\overset{\Delta}{DEF}$ because if we imagine plotting [AB] over [DE] it is obligatory that [BC] will plot over

[EF] and [AC] over [DF] otherwise one angle will be smaller than the other and [AC] will not be equal to [DF].” From this paragraph we notice that al–Samarqandī has considered the congruence (used the word *Tasawī* which means equal instead *of Taṭabuq* that means congruence) of two triangles as a logic of their equal areas instead of proving the congruence mathematically. In Figure 3 we demonstrate the previous paragraph explained.

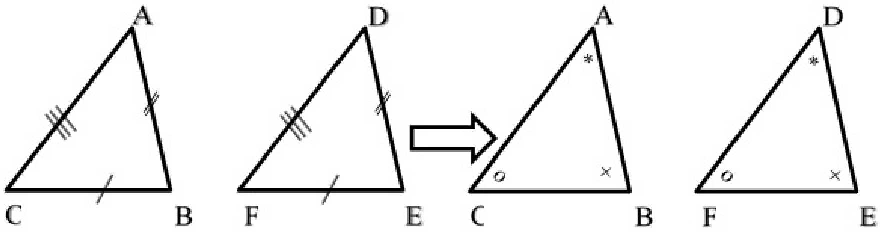

Figure 3. *triangles with equal sides have equal areas and angles.*

Another example where al–Samarqandī used logic to prove a proposition was in the theorem number 13 when he stated: “The longest side in a triangle corresponds to the biggest angle. Let [AB] be longer than [AC] in the triangle $\overset{\Delta}{ABC}$ so the angle $\hat{C}$ is bigger than $\hat{B}$ because if we deduct [AD] from [AB] such that [AD] = [AC], then the angle $\widehat{ADC}$ will be bigger than $\hat{B}$ and equal to $\widehat{ACD}$, and the angle $\widehat{ACB}$ is bigger than $\widehat{ACD}$ so bigger than $\widehat{ADC}$, which is bigger than $\hat{B}$, thus the angle $\widehat{ACB}$ is too much bigger than the angle $\hat{B}$.” Reading the previous text doesn't show the proof why $\widehat{ADC}$ will be bigger than $\hat{B}$, which nowadays is a proven fact that if we have a tringle and one of its vertices is moving out on the strait of its edge, the angle of this vertex is becoming smaller. Moreover, he used some none scientific terminology comparing the angles (too much bigger), and he even did not emphasis the conclusion he was trying to present. Figure 4 is the demonstration of the explained paragraph.

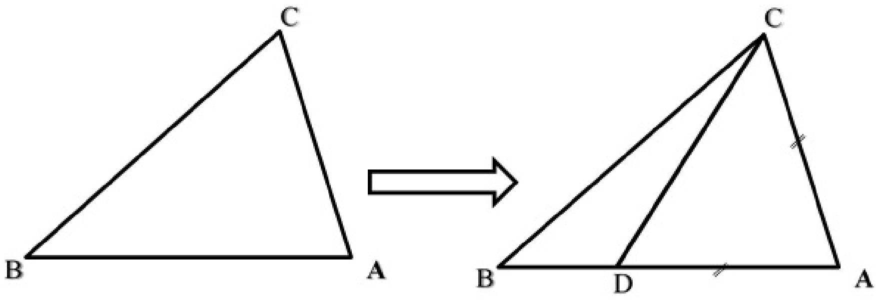

Figure 4. *Proving that if AB>AC then* $\hat{C} > \hat{B}$.

UNPRECEDENTED PROOF

Through his manuscript, the author has proven many geometric properties using an innovative proof unprecedented by other previous mathematicians. For example when he tried to prove the equal areas of two parallelograms without using the formula to calculate geometric areas as in theorem number 23, he stated that "every two parallelogram surfaces sharing one base in one side between two parallel lines are equal. For example ABDC and ABEF, sharing the base [AB] while [AB] is parallel to [CE], are equal because [CD] and [EF] are equal to [AB]. [FD] is shared between the triangle $\overset{\Delta}{ACF}$, $\overset{\Delta}{BDE}$ and the sides of those triangles [CF] and [DE] are equal, and also [AC], [BD] are equal, moreover the angles $\widehat{ACF}$ and $\widehat{BDE}$ are equal internally and externally thus the triangles are congruent. Subtracting and adding the shared triangles $\overset{\Delta}{DGF}$ and $\overset{\Delta}{ABG}$ respectively, the parallelograms result equal." Instead of using the formula to calculate areas, the author divided the shape into triangles and used their geometric properties he proved in previous theorems, in order to confirm the equal areas of the parallelograms he explained in this theorem. Figure 5 presents the methodology he followed in his proof.

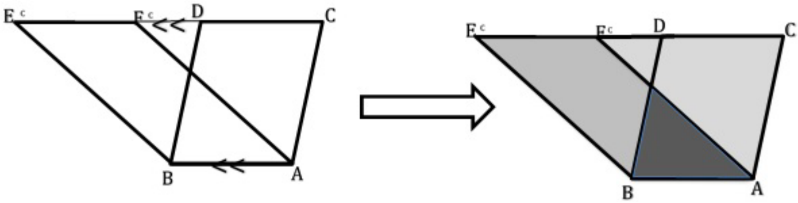

Figure 5. *Dividing the shape into triangles to prove that the areas of two parallelograms sharing the same base and limited by two parallel lines are equal.*

CLASSIFICATION OF INFORMATION PRESENTED

According to what mentioned in the overview of the manuscript contents, we could classify the information presented in the manuscript into different categories as following:

1- Information about the properties of different geometric shapes.

2- Methods for constructing geometric shapes:

3- Proofs of algebraic identities using geometric theorems.

In the following paragraphs we will present and discuss some examples of these categories, and analyse their contents.

INFORMATION ABOUT THE PROPERTIES OF DIFFERENT GEOMETRIC SHAPES

The theorem number 20, al–Samarqandī stated clearly and proved that the sum of the interior angles of a triangle is 180º. He said in page 11 "Every triangle … its angles are equal to two right angles." It was a common knowledge that the measurement of the angles in the Arabic geometric heritage was by parts or multiplications of right angles (90º degrees) so when he said that the interior angles of a triangle are equal to two right angles, he has proved that they are equal to 180º. In the discussion of the theorem, he has proved that fact by using the straight angle rule, which also is equal to 180º, a fact he has presented and proved in the first theorem of his manuscript.

METHODS FOR CONSTRUCTING GEOMETRIC SHAPE

The glorious heritage of Arabic geometric mosaic was constructed using two simple instruments, a straightedge and a compass. Maybe al–Samarqandī was one of the earliest scholars who had established and explained the usage of these two instruments to produce a combination of different geometric shapes. Throughout his manuscript we see many different examples explaining how to construct various shapes using a ruler and a compass, for instance in theorems number 9, 10, 15 the shapes are explained to be drawn using the same tools and methods. In theorem 15, al–Samarqandī explains how to draw a triangle which its sides' lengths are known, he says "We want to make a triangle in which each side is equal to one of three known lines with the condition that the sum of each two is longer than the third. In Figure 6 let the lines A, B, C and let [DE[a straight line we deduct from it [DF] = A and [FG]=B and [GH]=C, we draw from F a circle with a radius equal to [DF], and from G a circle with a radius equal to [GH]. The two circles will intersect in K and L, otherwise [FG] will be equal or longer than the sum [DF] and [GH] together. We connect K with F and G thus the triangle $\overset{\Delta}{KFG}$ is the required triangle because the side [KF] is equal to [DF] equal to A, and [KG] is equal to [GH] equal to C. There is no need for the ruler because the compass is enough to construct."

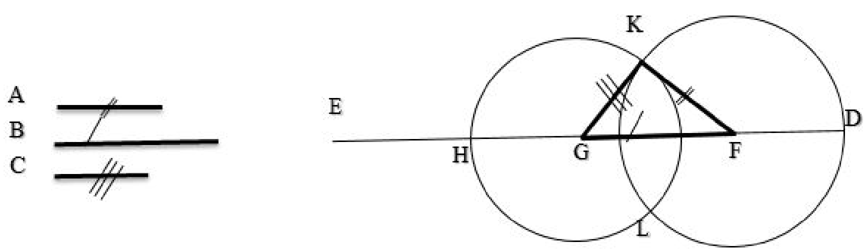

Figure 6. *Drawing a triangle its sides are equal to three known lines, using only a compass and a ruler.*

The author used the features of the circle to construct triangle without the measurement of the lengths of the lines, just by opening the compass directly equal to the predefined lines. This became a common practice in Arabic and Islamic geometric designs production as shown in Figure 6.

ALGEBRAIC IDENTITIES PROVED USING GEOMETRIC THEOREMS

Looking to the theorems 31 to 35, we find that al–Samarqandī had proved some algebraic identities using the theorems of areas calculating of geometric shapes.

For example, in theorem 21 he proved that the multiplication is distributive as in T.K=TX+TY+TZ, where K=X+Y+Z using the geometric characteristics of dividing a line and creating rectangles from the divisions. He explained that "Multiplication of X with Y is equal to the multiplication of X with the parts of Y. For example, multiplying the line A with the line [BC] is equal to multiplying A with the parts of the line [BC], I mean [CE], [ED] and [DB]. Let's consider that [BF] is a column on [BC] equal to A and we complete the shape into a rectangle its area is equal to multiplying A with [BC]. We assume that [DG], [EK] parallel to [BF] thus they are parallel to A, so the sum of the areas of BFGD and DGKE and EKLC is equal to A multiplied with [DB] plus [ED] plus [CE], and all are equal to [BC]." We demonstrate in Figure 7 the description previously translated from the manuscript.

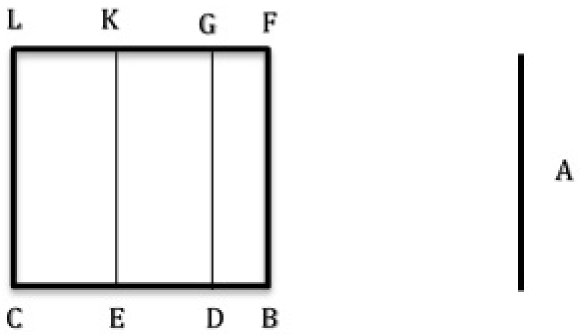

Figure 7. *Proving that T.K=TX+TY+TZ, where K=X+Y+Z using areas of rectangles.*

AMBIGUITY AND ERRORS

Despite the brilliant work of al–Samarqandī, we still can spot some ambiguous ideas and unclear phrases in Arabic, which sometimes lead to misunderstanding of his conclusions, thus we can recognise some very few errors in his discourse compared to our modern knowledge. Following in this paragraph we will present some examples of ambiguous cases and errors committed in his work in the manuscript, although we are not sure if the errors were because of the copying process, the copier of the manuscript, or committed by the author himself.

AMBIGUITY

Although the language used in the manuscript is clear Arabic, and most of the terms are still the same in Arabic science nowadays, hence some of the phrases and terms are ambiguous. For example, the diamond shape named in modern Arabic as Mu'ayan, he used to call it as Mutasaūī Alaḍlā', literally means equal sides geometric shape. Diamond shape is a special form of parallelogram, called in modern Arabic as Mutawazī Alaḍlā, including that each two opposite sides are parallel, he only considers that diamond shape has equal sides length, thus in modern Arabic terminology the Mu'ayan is used to refer for both properties.

On the other hand, apart from the language used, the proposition sometimes or the proof are not clear. An example for this case is seen in the theorem number 33, where he was trying to prove that $(x+y)^2=x^2+2xy+y^{2}$ using geometric theorems by explaining that "The square of the length of a line is equal to the square of its two parts plus the double of the multiplication of their lengths. In Figure 8 let the line [AB] divided into two parts on C we say that the square of [AB] is equal to [AC] square plus [CB] square plus double [AC] times [CB]. That is because if we create [AH] equal to [AB] and [CD] squares, and [CD] parallel to [AF], we connect B to F intersecting [CD] in G. We assume KGL parallel to [AB] the exterior angle of CGB equal to the interior angle AFB, …" The first ambiguous idea is when he said we create [AH], he did not mean the diagonal, he means the triangle where its two opposite corners are A and H.

Another unclear idea is the interior and exterior angles limited by two intersecting lines, they are not explained in the manuscript, despite that in the earliest theorems he discussed and explained the different types of angles and their relations.

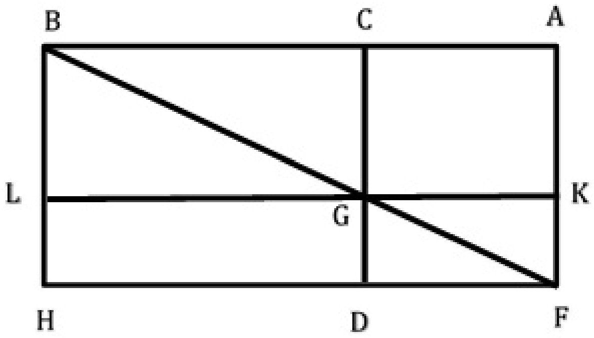

Figure 8. *The ambiguous idea of the interior and exterior angles when trying to prove that (x+y)^2=x^2+2xy+y^(2) using properties of squares and triangles.*

ERRORS

Essentially, we could not find what we can call an error clearly, but still we faced some mixing in using the correct reference of previously explained theorems, especially when he was trying to use it in order to prove the recent theorem, as in the theorem 16 instead of referring to theorem number 15 to use in the proof, he referred to theorem number 8. In the same theorem he committed an error by not defining the correct length of the constructed triangle in order to create an angle from a specific different point. He explained in page 10: "We want to create from a point of a line a given angle. i.e. from point A of the line [AB] an angle as $\hat{C}$, we define on the limited lines of the angle $\hat{C}$ two points D and F respectively, and we connect [DF], then we make on [AB] a triangle where its sides are equal to the sides of $\overset{\Delta}{CDF}$ let it be $\overset{\Delta}{ABG}$ but [AG] should be equal to [CF] and [AB] equal to [CD] and [GB] equal to [DF] thus the created angle on $\hat{A}$, is equal to $\hat{C}$ as seen in theorem 8." Figure 9 shows the process as he explained and demonstrate the error he committed.

It is clear to the reader that the author missed to explain that when choosing D and F from the sides of the angle the length of one of the created sides [CD] or [CF] should be equal to AB, otherwise we would not be able to create the triangle in which [AB] is equal to [CF]. Moreover, he asked to create an equal triangle in spite of that he did not explain yet the method to draw triangles from predefined lines.

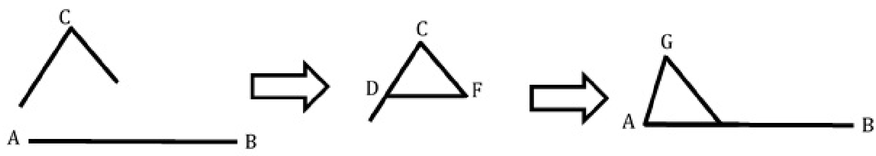

Figure 9. *If the Points D, F are not defined from the beginning that one edge is equal to AB, the triangle will not be constructed as required, and the angle will not be created.*

Finally, as we said he referred to the theorem number 8 in which he explains the congruent triangles instead of the theorem 15 which is explaining more about creating a triangle which its sides' lengths are defined. Nevertheless, even the theorem 8 was not proven mathematically, as we presented previously in this paper, and it relies on common logic discussion.

Another example of the errors is the utility of terminology in different theorems discussion. The author continues to use the term equal triangles meaning the congruent ones as mentioned before. But in one occasion, he mistakenly proved that two parallelograms are equal in areas and described them as equal parallelograms which, as we know from his work about triangles, refers to congruence. This error is seen when explaining in the theorem No. 24 that parallelograms which have equal bases' lengths and limited with parallel line to this base are equal in area. He said in page 13: "Every two parallelograms of equal bases on the same side between two parallel lines are congruent. For example ABCD and EFGH on the equal bases

[BC], [FG] in between the parallel [BG] [AH] lines, that is because connecting [BE], [CH] will make them parallel and equal that is because [BC], [EH] are parallel and equal as seen in theorem 21. Therefore the areas of the two parallelograms ABCD AFGH are equal to the area of the parallelogram EBCH …" He proved that the areas of the parallelograms are equal, but he described them as *mutasāūīān,* congruent in modern terms, at the beginning of his theorem. Figure 10 shows what he means in his discussion.

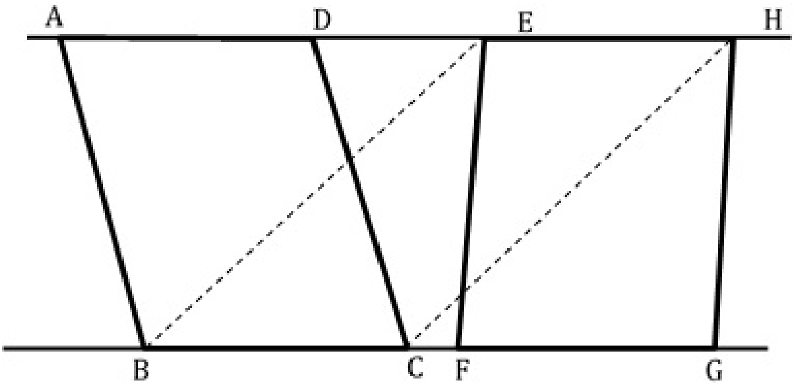

Figure 10. *Proving the equal areas of two parallelograms and considering them congruent.*

Another example about the errors in using correct terms, is when he used the word *Sutū*h, which could mean surfaces or areas to talk about the shapes themselves, when at the same time he used the same word to talk about the areas of these geometric shapes, thus it is difficult to understand which meaning without looking to the context.

Also he used surface of a parallelogram by calling it with the name of its diagonal, as we explained in the theorem 33 previously. For the first case, in the theorem 32 he explained: "The sum of the areas of a line in its segments is equal to its square… " He meant to say that the area of the square created from a line, not the area of the line itself is equal to the sum of the areas of rectangles created from the line segments, where these rectangles depths has the same length of the square. Figure 11 demonstrates the correct meaning of the area of a line he talked about.

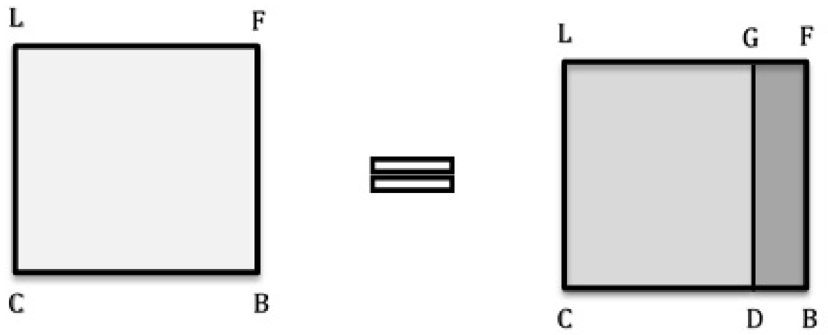

Figure 11. *Area of a line demonstrated by the geometric theorems as al–Samarqandī explained.*

The second case in which he called the area of parallelogram with the area of its diagonal was repeated in many theorems such as 25, 27, 28 and 33…etc. One example is the theorem number 33 where he stated that: "… Thus the area of the parallelogram [DF] is equal sides according to what we have seen in 22…" He means the parallelogram CDEF which its diagonal is [DF] as previously seen in (Fig. 8).

CONCLUSION

This paper shows the importance of the work of early Arabic scholars in establishing the heritage of both scientific research and methodology approaches in the field of geometry. It is of paramount importance to Arab scholars to document, study and analyse this hidden heritage in order to preserve it firstly, then introduce it to the modern scientific society revealing the secrets of the buried manuscripts for more than ten centuries.

One of the important scholars whose work highly impacted the heritage of the Arab science is al–Sasmarqandī, who with a relatively small manuscript had depicted the terminology and theories of geometry in the scientific Arabic history. His work deserves a deep study by Arab historians of science and scientific scholars instead of leaving it to foreigners, who sometimes face difficulties in understanding the language used in this era, in order to realise the role he played in his field.

The text of (Ashkal Al-Ta'asis) is a simple short manuscript of high value for the information it contains, and the methodology it follows. Although it was translated into both English by Gregg de Young, and to modern Arabic by Muhammed Souissi, it is not enough to visualize its influence on the Arabic heritage of geometry.

Despite that the manuscript has minor errors, it still forms a basic reference of geometry in Arabic scientific heritage. We could still use it nowadays to explain to Arab students the terminologies, methodologies and approaches to innovatively prove different theorems using diverse methods.

The information explained and the methods analysed varies from constructive proof to proof by contradiction, and from properties of geometric shapes to constructive methods and instruments. Al–Samarqandī's descriptive discourse needs a figurative interpretation, beside the commentaries by his student Qāḍī Zādeh al-Rūmī, in order to understand and present it better in the modern mathematical language, which we tried to produce, exemplifying by some figures in this paper.

It is also worthy to translate his other works, and similar scholars work into foreign languages to emphasis the cultural communication between different civilisations interacted to build the heritage of science. The future work will be the analysis of the work and comments written by Qāḍī Zādeh, another important scholar, who studied al–Samarqandī's work, analysed, commented and explained the work of his teacher, and then he wrote his own studies contributing to the Arabic heritage of science.

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