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AN EMPIRICAL INVESTIGATION OF PRICE CHANGES IN ISTANBUL STOCK EXCHANGE (ISE)

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Abstract

The assumption that stock price changes have Normal distribution has become foundation of many of the main-stream models in financial literature. The validity of these models in financial theory depends critically on this assumption. Today it is well known empirical fact that the distribution of stock market returns are usually not normal but *leptokurtic*, i.e. the empirical distribution has fat tails and a high degree of peakedness as compared to the normal distribution. Thus, the usual statistical tools may be badly misleading from the point of financial decision making. The main concern of this study is to investigate whether the stock price changes of an emerging market (Istanbul Stock Exchange) have a Normal or Gaussian Distribution or can price changes be characterized by stable Paretian distribution. It has been concluded from the findings of this study that stock price changes do not fit to Normal or Gaussian distribution but can be characterized by stable Paretian distribution.

Keywords: Istanbul Stock Exchange, stable Paretian, normal distribution.

Öz

İMKB'de Fiyat Değişimleri Üzerine Ampirik Bir Araştırma

Hisse senedi fiyat değişimlerinin normal dağılım gösterdiği kabulü finansal literatürde bir çok model için temel oluşturur. Finansal teorideki bu modellerin geçerliliği büyük ölçüde bu kabulün doğruluğuna bağlıdır. Günümüzde hisse senedi getirilerinin dağılımının normal değil *leptokurtic* olduğu ampirik çalışmalarla tespit edilmiştir. Bir başka ifadeyle, hisse senedi fiyat değişimlerinin dağılımı normal dağılım ile mukayese edildiğinde, kuyruklarda beklenenden daha fazla gözlem sayısı bulunmaktadır. Bu nedenle, finansal karar verme açısından geleneksel istatistiksel araçlar yanıltıcı sonuçlar doğurabilir. Bu çalışmanın ana

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amacı gelişmekte olan bir piyasada (İMKB'de) hisse senedi fiyat değişimlerinin normal dağılımla tanımlanıp tanımlanamayacağına yöneliktir. Çalışmada elde edilen bulgulardan, hisse senedi fiyat değişimlerinin normal dağılıma uymadığı fakat stable Paretian dağılımı ile tanımlanabileceği sonucuna varılmıştır.

Anahtar Sözcükler: İstanbul Menkul Kıymetler Borsası, stable Paretian, normal dağılım.

INTRODUCTION

Academic analyses of the unconditional distribution of stock returns date back to the beginning of the twentieth century and have been an important research area in financial economics. Major part of the speculative market researches focused on the question of forcasting stock market prices from the past history of the prices themselves. Earlier studies defined price behaviors as random walk[1] or brownian motion.The theory of random walks is based on two hypothesis: i) successive price changes in an individual security are independent, and ii) the price changes conform to some probability distribution. The first hypothesis implies that past series of price changes cannot be used to predict the future. The hypothesis concerning the distribution of the price changes implies that price changes of an individual security can be characterized by any distribution. In terms of Efficient Market Hypothesis (EMH), the first one is the most important because if successive price changes are not independent then the theory of random walks is not valid. This implies that if successive price changes are independent then a market is efficient regardless of distribution of price changes. However, if successive price changes are not independent then a market is said to be inefficient regardless of distribution of price changes. Thus, the hypothesis concerning the distribution of the price changes does not provide any information about EMH.

Bachelier (1900: 17-78), in his original work "Theory of Speculation", applied statistical methods to stocks, bonds, futures, and options . In his paper, Osborne (1964: 100-128) claimed that stock price changes follow a random walk on brownian motion. Kendall (1964: 85-99) found successive arithmetic first differences in British stock price averages uncorrelated and concluded that these changes were independent.

Analyses of the ISE about the theory of random walks, mostly, focused on the independence hypothesis in order to test the efficiency of the stock market. For example, Bekçioglu & Ada (1985: 1-17), using monthly closing prices of forty two stocks beginning 1975 and ending 1981, appllied serial correlation and run test to analyze the efficiency of the ISE. They have concluded that ISE does not even exhibit "weak-form" efficiency. A similar study was done by Balaban (1995: 3-15) using daily and weekly data of ISE Composite Index over the period of 1988-1994. Kıyalar (1997: 75-137) found out that daily closing price changes of forty five stocks are highly correlated over the period of 1988-1994. Both Balaban's (1995: 3-15) and Kıyılar's (1997: 75-137) findings are consistent with the results of Bekçioğlu and Ada (1985: 1-17).

Even though the general theory of random walks does not require to specify the form or shape of the distribution of price changes, the second hypothesis of the theory is also as significant as the first one. It is important because, from the point of view of the investors, determining the form or shape of the distribution is directly related to riskiness of their investments.

The prevalent view was that stock returns could be edequately characterised by the Gaussian curve. The assumption that stock price changes have Normal or Gaussian distribution has become foundation of many of the main-stream models such as mean-variance approach to portfolio selection (Markowitz, 1952: 77-91), The Capital Asset Pricing Model (Sharpe, 1964: 425-442), Black-Scholes (1973: 637-659) option pricing formula. The validity of these models in financial theory depends critically on the normality assumption. However, today it is well known empirical fact that the distribution of stock market returns are usually not normal but *leptokurtic*, i.e. the empirical distribution has fat tails and a high degree of peakedness as compared to the normal distribution. Thus, the usual statistical tools may be badly misleading from the point of financial decision making.

Mandlebrot (1963: 307-332) made an important contribution by pronouncing leptokurtic features of the stock returns. He argued that stock returns are usually more peaked at the centre with thicker tails (leptokurtic) than the normal distribution curve. Mandlebrot, in his analysis, fitted the "infinite"[2] variance Stable Paretian class of distributions with characteristic exponent (α)[3] between one and two. Later, the most comprehensive study of the distribution of stock returns (log price changes) has been made by Fama (1965: 34-105). Fama showed Mandlebrot's (1963: 307-332) hypothesis that a stable Paretian distribution with a characteristic exponent less than two describes stock returns better than a Normal distribution. Granger & Morgenstern (1970: 177-192) stated that β parameter[4] is greater than zero for the distribution of stock price changes.

More recently, Peters (1991: 27-38) found that the distribution of the S&P500 stock returns exhibit negative skewness, fat tails, and a high peakedness. Praetz (1972: 49-55), Officer (1972: 807-811), Aparicio & Estrada (1997: 1-14), and Broca (2002: 129-140) considered alternative distributions in order to find out the best fit to the data. Dillen & Stoltz (1999: 49-53) found out that the empirical distribution of stock returns and the residuals are leptokurtic for twenty stocks quoted on the Stockholm Stock Exchange.

The first hypothesis of the general theory of random walks is out of the scope of this paper. The main purpose of this paper is to analyse the behavior of price changes in Istanbul Stock Exchange. This study investigates the evidence from the Istanbul Stock Exchange (ISE) in terms of distribution of price changes which has recieved less attention so far.

Another stimulus behind this study is that ISE is an emerging market and it should be useful to examine whether the behavior of stock price changes can be characterized by the unconditional distributions.

The structure of the study is as follows. The first section briefly describes the data used in the analyses. The second section explains the design of the research and the methodological approach. The third section gives some information about Istanbul Stock Exchange as an emerging market. The fourth, fifth and sixth section presents statistical analysis of the empirical evidence and the results of these analyses. The final section is the conclusion remarks of the study.

I. DATA

Daily observations[5] of 87 stocks traded in ISE and the Istanbul Stock Exchange Composite Index (ISECI-100[6]) are used to investigate behavior of ISE. The stocks that are under consideration in the analysis are selected randomly. Some of these randomly selected stocks are in the ISECI-100 and some are not. Thus, stocks in the analysis can be considered as a good representation of the whole market.

Daily observations of the ISECI-100 range between July 3, 1987 and March 3, 2001. The time period for 87 stocks vary stock to stock, but the final date, March 30, 2001, is the same for all stocks, however, initial date differs. Thus, number of observations varies between 1474 and 3377 for individual stocks, the number of observations for ISECI-100 is 3411. In this study logarithmic price changes are considered as the main data, therefore this transformation results in one less observation for all samples. The logarithmic price changes are calculated as follows:

$$R_t = \log(P_t) - \log(P_{t-1}) \qquad (1)$$

Where, P_t is the price of individual stock at the time t, P_{t-1} is the price of individual stock at the time t-1 and R_t is the log price changes of individual stocks, i.e., returns to individual stocks. The above equation can be reformulated for ISECI-100 as follows:

$$R_t = \log(I_t) - \log(I_{t-1}) \qquad (2)$$

Where, I_t and R_t refer to ISECI-100 index number and return to ISECI-100 at the time t, respectively.

II. RESEARCH DESIGN

This study provides an investigation to detect if the empirical evidence from Istanbul Stock Exchange is consistent with the assumption that stock price changes have Normal (Gaussian) distribution, or can it be characterized by stable Paretian distribution. In this study comparison of several statistical methods for the analysis of the behavior of stock price changes are used. Generally, the normality assumption of stock price changes can be checked easily by statistical tests such as Kolmogorov-Simirnov (K-S), Jarque-Bera, Shapiro-Wilk (Kadılar, 2005: 97-100). As in all statistical hypothesis tests, the results of the test can be changed by the selection of significance level (α) which is selected at the beginning of the analysis. For instance, in conducting K-S test at 0,01 significance level (α=0,01) a set of data may turn out to be not normally distributed, whereas at higher significance levels (α=0,05 or α=0,10) the same set of data may exhibit normal distribution behavior. Thus, selection of significance level is exteremely important for critical data sets such that it may cause missing some important observations at the extereme tails of the distribution. Mostly, 99% and 95% confidence levels are often considered in statistical analyses. If the set of data conform to normal distribution, 99% and 95% confidence intervals of mean are $\mu \pm 2{,}575\sigma$, $\mu \pm 1{,}96\sigma$, respectively. Thus, selection of significance levels directly influences the confidence intervals.

From the point of view of a statistician 99% and 95% confidence intervals of mean may be sufficient for an analysis. However, investors want to know all possibilities that may influence their wealths. In this sense, detailed analysis should be conducted about the distribution of price changes. As stated before, selection of significance levels must be extremely important from the point of view of investors because there is a possibility that investors would not take all the potential excess gains and losses into account with higher significance levels.

Another critical point is that when the results of the normality tests (such as K-S) do not conform to normal distribution, generally a statistician, if there are sufficient number of data, may choose to exclude extreme observations to fit the data to normal distribution. Hence, the result of exclusion of extreme observations causes information losses. However, for investors these extreme observations are substantially important in terms of their investments and imply excess gains or losses.

In this study data are analyzed for different confidence intervals for different significance levels and the results confirm that extreme tails (beyond $\mu \pm 5\sigma$) contain more observations than would be expected if the distribution were normal.

In the first step histograms with normal curves of the empirical data are drawn to see whether they have similar behaviors with the normal curves. Further for a detailed analysis empirical frequency distributions are computed in order to make comparisons with the unit normal frequency distributions[7] within given standart deviations of mean changes for each sample. As the third step, normal probability graphs are constructed for detecting departures from normality.

The last part consists of analysis of determining the characteristic exponent (α) and invariance under addition for each sample. Characteristic exponent (α) is determined by the following equation:

$$R_n = n^{1/\alpha} R_1, \tag{3}$$

$$\alpha = \frac{\log n}{\log R_n - \log R_1} \tag{4}$$

The test of invariance under addition for each sample is done by the process of taking sums of the daily logarithmic price changes for different intervals (4-day, 9-day, 16-day and 25-day logarithmic price changes are calculated). This process changes the scale of the distribution. We know that after adjustment for the scale stable Paretian distribution should retain its statistical properties if they are added together. For instance, if the daily distribution is stable Paretian, then 4-day logarithmic price changes will have a mean of 4μ, but the variance will be unstable. Here, μ is the mean of daily logarithmic price changes.

It is also known that if the distribution of observations of each sample were Normal or Gaussian, the expected mean and variance for 4-day logarithmic price changes would be 4μ and $4\sigma^2$ (μ is the mean and σ^2 is the variance of daily logarithmic price changes), respectively. Then a comparison can be done for each sample with the expected values by computing means and variances of different sums of the daily logarithmic price changes to see wheter the empirical evidence has stable or unstable variances.

III. THE ISTANBUL STOCK EXCHANGE AND A GLANCE AT ISECI-100

1980s were the new era for Turkish economy, with the support of IMF stabilization program in 1980 Turkish economy switched from an inward-development to an outward-oriented one. Financial liberilization and integration of financial markets were the two major components of the IMF Stabilization program. As one of the first responses to that program, in 1986 The Istanbul Securities Exchange (ISE) began its operations with 42 listed companies. In 1989, the Turkish financial system was further liberalized and foriegn investors were permitted to hold stock portofolios at ISE. As of 2003, 285 corporatiıons are traded by a computer-assisted system with an average daily trading volume of $372 million relative to $3 million average trading volume in 1989. In terms of trading volume ISE is ranked 23rd in the world surpassing several European stock exchanges such as Kopenhagen, Oslo, Athens, Warsaw,Vienna, Budapest, and Luxembourg (Istanbul Stock Exchange Journal, 2002). Similar to other emerging markets, ISE's return volatilities have been high throughout its history.

It will be illuminating to have some idea about the behavior of ISE before getting into a detailed analysis of individual stocks. It is well-known that in most financial data, there are high volatile and low volatile phases, and that they alternate periodically. In practice, stock markets often have long periods of relative activity, followed by long periods of relative inactivity i.e.large changes tend to be followed by large changes, and small changes tend to be followed by small changes. This is observed in ISECI-100 data and this empirical observation is known as volatility clustering. Based on the volatility clustering, it is expected that data are excessively peaked around zero in their unconditional distribution with heavy tails.

Figure 1a shows the logarithm of ISECI-100 for the period of July 3, 1987 until March 3, 2001. Figure 1b shows returns of the ISECI-100 and volatility clustering. Figure 1c shows the empirical density of the returns which is leptokurtic and heavy tailed distributed.

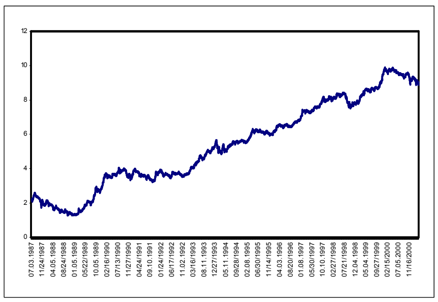


Figure 1a. Logarithm of the ISECI-100, 100 July 3, 1987 – March 3, 2001

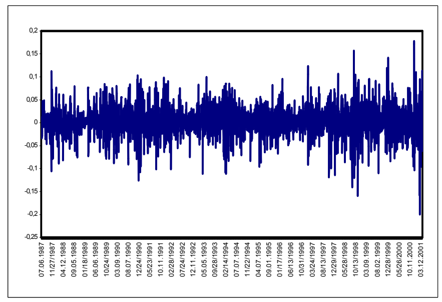


Figure 1b. Daily Returns of the ISECI- July 3, 1987 – March 3, 2001

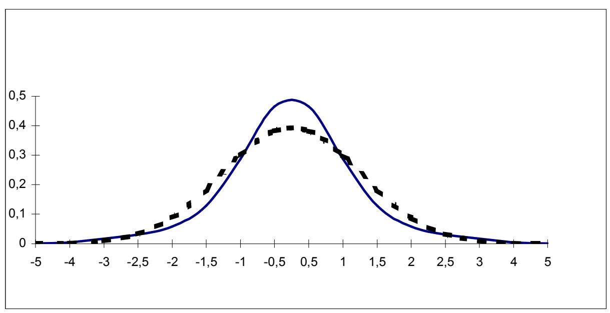


Figure 1c. Solid curve represents the empirical density of ISECI-100 daily returns
Figure 1. Logarithmic prices, returns , and empirical density of the ISECI-100

IV. HISTOGRAMS WITH NORMAL CURVE AND FREQUENCY DISTRIBUTIONS

A common first step and simple way of analyzing the distribution of logarithmic price changes is to construct histograms and the frequency distributions for each stock and ISECI-100. Figure 2 shows an histogram illustrating departure from normality, the bell shaped curve represents the Gaussian curve. Figure 3. shows the empirical histograms of individual stocks and ISECI-100 with normal curves. As can be seen from figure 3 almost all of histograms with normal curves have similar shapes with Figure2 implying departures from normality.

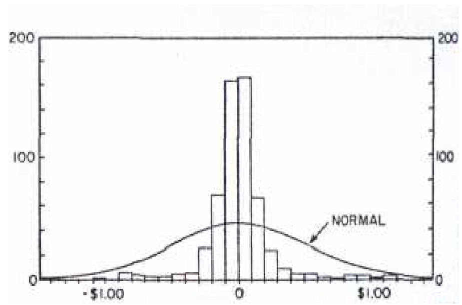
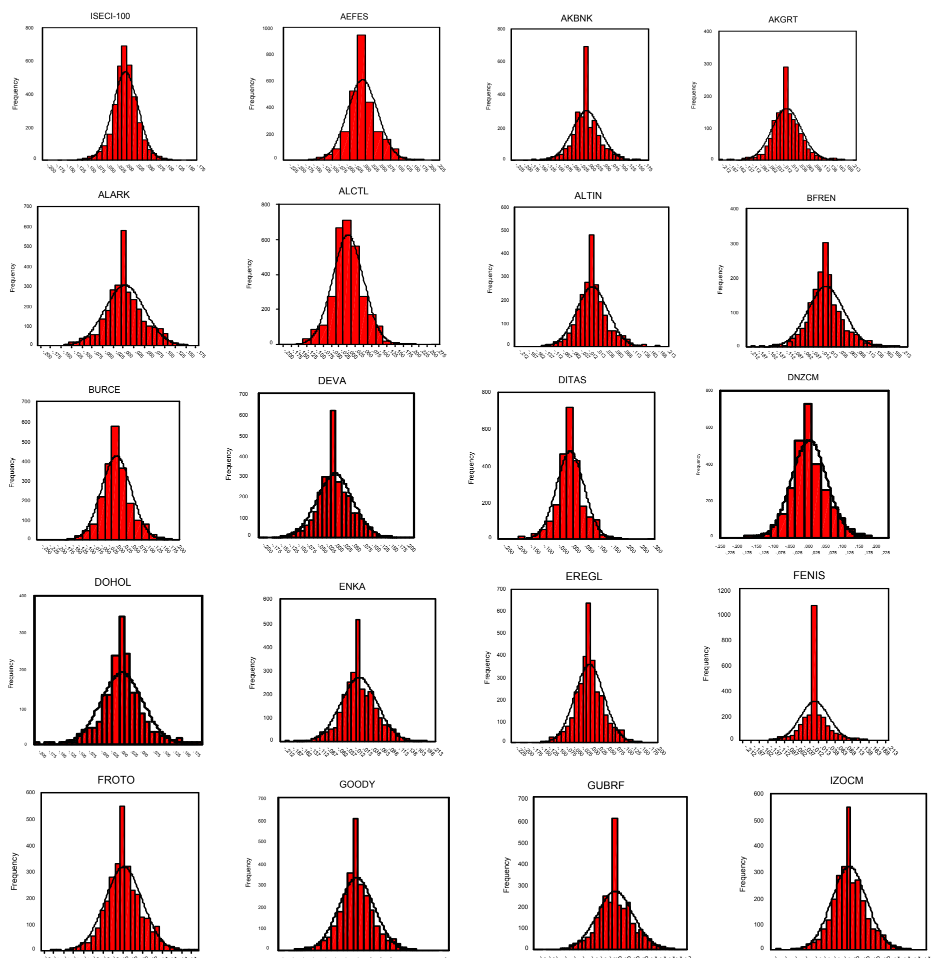


Figure-2: Histogram illustrating departure from normality of monthly wool prices, 1890-1937. The continous bell-shaped curve represents Gaussian curve. Taken from Mandlebrot (p. 308)



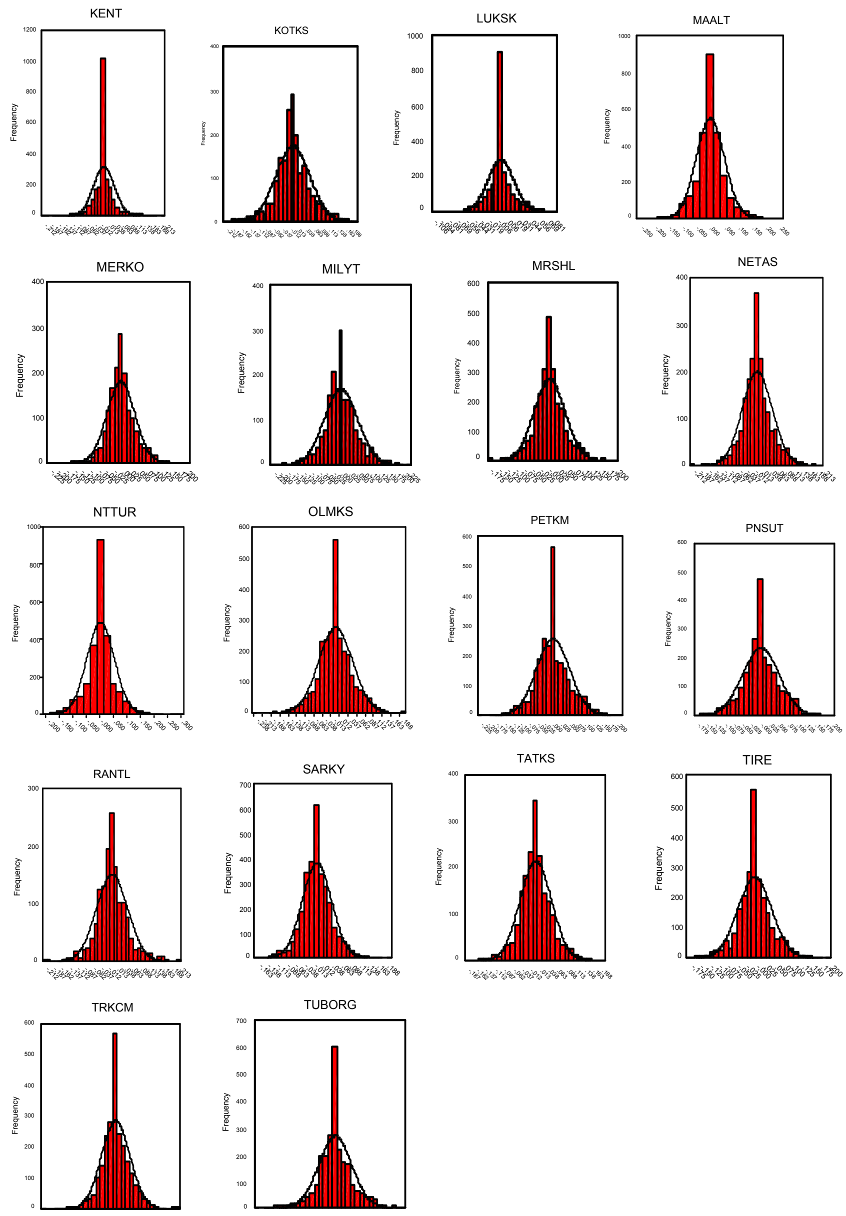


Figure-3: Selected Histograms of Empirical Evidence Illustrating Departure From Normality. (Continued)

In order to be more confident, another crucial analysis is to construct empirical frequency distributions for each sample. This can be done by computing empirical proportions of logarithmic price changes within given standard deviations of mean changes for each sample. Further, empirical frequency distributions can be compared with the expected frequency distributions. Table 1 gives empirical distributions of observations within 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0, and 5.0 standart deviations. Unit normal distributions in the first row would be the expected distributions if the empirical distributions of observations were normal. Table 2 is the comparison of empirical distributions with the unit normal distributions within the given standart deviations. Numbers in Table 2 were calculated by subtracting unit normal distribution from empirical frequency distributions. A positive (negative) number implies an excess (a deficiency) of relative frequency within the given interval. For instance, the expected distribution for the unit normal within 0.5 standard deviation of mean change is 0,3830 whereas it is 0,4648 for ISECI-100. This implies that ISECI-100 contains 8,18 per cent more of the total frequency within the one-half standard deviation of the mean than would be expected if the distribution were normal. Individual stocks such as FENIS, KENT, LUKSK, NTTUR have more than 20 per cent excess frequency within the one-half of the standard deviation of the mean change when compared with the unit normal distribution. To be more illuminating, this implies that within the one-half standard deviation of mean change FENIS, KENT, LUKSK, NTTUR have 573 (0,2209*2595), 544 (0,2315*2350), 519 (0,2132*2432), 506 (0,2002*2531) more observations than the expected number of observations. On the average of all individual stocks there are 290 more observations than the expected number.

A detailed analysis of Table 2 presents an excess of relative frequencies within the intervals of 0.5-1.5 standart deviations of mean change and deficiency of relative frequency within the intervals 2.0-5.0 standart deviation of the mean change on the average. Standard deviations of the mean change greater than 5.0 standart deviations exhibits excess of relative frequencies on the average. Row 1 of Table 2 is the comparison of empirical frequency distribution of ISECI-100 with unit normal distribution exhibiting an excess of relative frequency within the interval of 0.5-1.5 standart deviations of mean change and deficiency of relative frequency within the 2.0-5.0 standart deviation of mean change and excess relative frequency 5.0 standart deviations of mean change. This is consistent with individual stock behaviors. The striking part here is that the excess of relative frequency of ISECI-100 is 2443 times greater than the expected frequency if it would be a normal distribution. This implies that the observation that should be realized once every 6745 years, realized once every 3 years. The results of Table 2 confirms peakedness around zero and fat tails of empirical distribution and hollow in between.

Table 3 is the analysis of the extreme tail areas in terms of actual number of observations beyond 2.0, 3.0, 4.0, and 5.0 standard deviations from their means. It implies that empirical evidence shows the number of observations is greater than expected number of observations beyond the given standard deviations implying excess frequencies in the extreme tails. For instances, on the average, empirical evidence is 666 times greater than the expected number of observations beyond five standard deviations.

Table-1: Empirical Frequency Distributions

STOCKS	FREQUENCY DISTRIBUTION								
	INTERVALS								
	0.5S	1.0S	1.5S	2.0S	2.5S	3.0S	4.0S	5.0S	>5.0S
UNIT NORMAL	0,3830	0,6826	0,8664	0,9545	0,9876	0,9973	0,999938	0,9999994	0,0000006
ISECI-100	0,4648	0,7528	0,8818	0,9405	0,9713	0,9886	0,997361	0,9985337	0,0014663
AEFES	0,5097	0,7415	0,8675	0,9359	0,9730	0,9888	0,998200	0,9996400	0,0003600
AKBNK	0,5210	0,7573	0,8707	0,9329	0,9735	0,9863	0,998104	0,9996208	0,0003792
AKGRT	0,4795	0,7531	0,8919	0,9459	0,9713	0,9850	0,996091	1,0000000	0,0000000
AKSA	0,5075	0,7636	0,8852	0,9446	0,9800	0,9927	0,997814	0,9992714	0,0007286
ALARK	0,4764	0,7273	0,8520	0,9376	0,9792	0,9941	0,998960	1,0000000	0,0000000
ALCAR	0,4832	0,7425	0,8693	0,9408	0,9757	0,9920	0,998233	0,9995583	0,0004417
ALCTL	0,4809	0,7256	0,8602	0,9416	0,9805	0,9937	0,998681	0,9996702	0,0003298
ALTIN	0,5052	0,7581	0,8736	0,9339	0,9738	0,9873	0,995626	1,0000000	0,0000000
ANACM	0,4845	0,7345	0,8684	0,9449	0,9763	0,9881	0,998681	0,9996702	0,0003298
ARCLK	0,4636	0,7433	0,8771	0,9412	0,9771	0,9900	0,997011	0,9993358	0,0006642
ASELS	0,4906	0,7545	0,8826	0,9583	0,9870	0,9927	0,998853	0,9992352	0,0007648
AYGAZ	0,5306	0,7765	0,8931	0,9528	0,9827	0,9906	0,998920	0,9992801	0,0007199
BAGFS	0,4538	0,7347	0,8723	0,9471	0,9797	0,9907	0,997906	0,9997009	0,0002991
BFREN	0,4986	0,7593	0,8828	0,9411	0,9691	0,9857	0,997713	1,0000000	0,0000000
BRISA	0,4733	0,7347	0,8653	0,9403	0,9769	0,9927	0,998680	0,9996700	0,0003300
BSPRO	0,5018	0,7533	0,8702	0,9433	0,9762	0,9868	0,998538	0,9996345	0,0003655
BTCIM	0,5051	0,7624	0,8921	0,9457	0,9722	0,9837	0,994569	0,9986422	0,0013578
BURCE	0,4979	0,7451	0,8645	0,9375	0,9773	0,9905	0,998105	1,0000000	0,0000000
DEVA	0,4721	0,7318	0,8637	0,9409	0,9772	0,9911	0,999010	1,0000000	0,0000000
DITAS	0,5288	0,7599	0,8635	0,9350	0,9757	0,9873	0,997533	0,9995888	0,0004112
DNZCM	0,4601	0,7313	0,8613	0,9434	0,9807	0,9907	0,998925	1,0000000	0,0000000
DOHOL	0,5117	0,7561	0,8656	0,9333	0,9682	0,9891	0,998437	1,0000000	0,0000000
ECILC	0,4529	0,8546	0,8704	0,9393	0,9774	0,9928	0,998493	1,0000000	0,0000000
ECZYT	0,4586	0,7253	0,8668	0,9412	0,9779	0,9902	0,998549	1,0000000	0,0000000
EGGUB	0,4919	0,7692	0,8873	0,9510	0,9813	0,9910	0,998920	0,9992798	0,0007202
EGPRO	0,5017	0,7453	0,8682	0,9374	0,9737	0,9916	0,997207	0,9988827	0,0011173
ENKA	0,4684	0,7333	0,8628	0,9401	0,9800	0,9895	0,998186	1,0000000	0,0000000
EPLAS	0,4843	0,7549	0,8893	0,9420	0,9722	0,9876	0,996448	1,0000000	0,0000000
ERBOS	0,4893	0,7619	0,8878	0,9415	0,9704	0,9862	0,995871	0,9986235	0,0013765
EREGL	0,4963	0,7473	0,8672	0,9357	0,9752	0,9907	0,998071	0,9996786	0,0003214
FENIS	0,6039	0,7900	0,8794	0,9283	0,9595	0,9807	0,993449	0,9992293	0,0007707
FRIGO	0,5052	0,7626	0,8844	0,9436	0,9670	0,9931	0,995871	0,9993118	0,0006882
FROTO	0,4676	0,7273	0,8668	0,9452	0,9754	0,9900	0,999336	1,0000000	0,0000000
GARAN	0,5080	0,7798	0,9028	0,9589	0,9843	0,9933	0,999252	0,9996262	0,0003738
GLBYO	0,5452	0,7983	0,9164	0,9700	0,9873	0,9950	0,999546	0,9995457	0,0004543
GOODY	0,4947	0,7528	0,8751	0,9358	0,9699	0,9869	0,997459	0,9996370	0,0003630
GUBRF	0,4558	0,7396	0,8602	0,9361	0,9776	0,9913	0,998917	1,0000000	0,0000000
GUSGR	0,5013	0,7619	0,8909	0,9422	0,9718	0,9801	0,997433	0,9987163	0,0012837
HURGZ	0,5089	0,7504	0,8685	0,9407	0,9728	0,9848	0,998663	0,9995544	0,0004456
INTEM	0,4668	0,7240	0,8570	0,9425	0,9812	0,9919	0,998895	0,9996315	0,0003685
ISCTR	0,4830	0,7645	0,8901	0,9559	0,9828	0,9933	0,998131	0,9992523	0,0007477
IZOCM	0,4656	0,7357	0,8686	0,9399	0,9788	0,9910	0,998560	1,0000000	0,0000000

Table-1: Empirical Frequency Distributions (Continued)

STOCKS	FREQUENCY DISTRIBUTION								
	INTERVALS								
	0.5S	**1.0S**	**1.5S**	**2.0S**	**2.5S**	**3.0S**	**4.0S**	**5.0S**	**>5.0S**
KARTN	0,5036	0,7559	0,8717	0,9369	0,9707	0,9890	0,999408	0,9994076	0,0005924
KCHOL	0,4875	0,7592	0,8881	0,9512	0,9801	0,9937	0,999004	0,9993358	0,0006642
KENT	0,6145	0,8034	0,9009	0,9387	0,9600	0,9757	0,990213	0,9982979	0,0017021
KONYA	0,4986	0,7641	0,8752	0,9401	0,9740	0,9856	0,995336	0,9996113	0,0003887
KORDS	0,4960	0,7469	0,8720	0,9418	0,9777	0,9894	0,997805	0,9992685	0,0007315
KOTKS	0,4829	0,7537	0,8686	0,9306	0,9696	0,9877	0,999466	1,0000000	0,0000000
LUKSK	0,5962	0,7738	0,8639	0,9219	0,9688	0,9840	0,997122	0,9991776	0,0008224
MAALT	0,5205	0,7513	0,8654	0,9356	0,9730	0,9863	0,996400	1,0000000	0,0000000
MERKO	0,4946	0,7495	0,8830	0,9431	0,9747	0,9848	0,996205	0,9987350	0,0012650
MILYT	0,4736	0,7481	0,8743	0,9385	0,9703	0,9876	0,999461	1,0000000	0,0000000
MIPAZ	0,4688	0,7456	0,8691	0,9400	0,9760	0,9874	0,998856	1,0000000	0,0000000
MMART	0,4719	0,7387	0,8710	0,9393	0,9777	0,9898	0,998538	1,0000000	0,0000000
MRDIN	0,4975	0,7664	0,8887	0,9443	0,9812	0,9937	0,998040	0,9992160	0,0007840
MRSHL	0,4774	0,7601	0,8702	0,9351	0,9747	0,9873	0,999208	1,0000000	0,0000000
NETAS	0,4856	0,7471	0,8730	0,9363	0,9727	0,9899	0,998483	1,0000000	0,0000000
NIGDE	0,5154	0,7755	0,8945	0,9510	0,9798	0,9913	0,997628	0,9992095	0,0007905
NTTUR	0,5832	0,7625	0,8605	0,9261	0,9684	0,9854	0,997629	0,9992098	0,0007902
OLMKS	0,4740	0,7431	0,8626	0,9401	0,9751	0,9923	0,998538	1,0000000	0,0000000
PARSN	0,4747	0,7271	0,8581	0,9495	0,9782	0,9883	0,999611	1,0000000	0,0000000
PETKM	0,4532	0,7311	0,8608	0,9397	0,9795	0,9932	0,999242	1,0000000	0,0000000
PIMAS	0,4971	0,7408	0,8516	0,9377	0,9753	0,9904	0,999164	1,0000000	0,0000000
PINSU	0,5063	0,7400	0,8524	0,9381	0,9787	0,9891	0,999164	1,0000000	0,0000000
PNSUT	0,4799	0,7249	0,8579	0,9402	0,9787	0,9891	1,000000	1,0000000	0,0000000
PTOFS	0,5021	0,7753	0,8926	0,9626	0,9881	0,9947	0,998765	0,9991770	0,0008230
RANTL	0,4843	0,7666	0,8863	0,9371	0,9706	0,9839	0,995987	1,0000000	0,0000000
SARKY	0,4560	0,7450	0,8737	0,9382	0,9712	0,9954	0,998677	1,0000000	0,0000000
SISE	0,4422	0,7206	0,8664	0,9481	0,9809	0,9935	0,998920	0,9996399	0,0003601
TATKS	0,4877	0,7511	0,8769	0,9339	0,9717	0,9867	0,997335	0,9994670	0,0005330
THYAO	0,4797	0,7681	0,8740	0,9573	0,9846	0,9968	0,999210	0,9996049	0,0003951
TIRE	0,4975	0,7504	0,8599	0,9377	0,9732	0,9870	0,999164	1,0000000	0,0000000
TOASO	0,4866	0,7462	0,8788	0,9356	0,9724	0,9883	0,998746	1,0000000	0,0000000
TRKCM	0,4911	0,7501	0,8790	0,9403	0,9739	0,9876	0,995441	0,9995856	0,0004144
TUBORG	0,5247	0,7617	0,8708	0,9352	0,9691	0,9870	0,998328	1,0000000	0,0000000
TUDDF	0,4834	0,7419	0,8674	0,9409	0,9748	0,9904	0,999336	0,9996678	0,0003322
TUKAS	0,5107	0,7775	0,8842	0,9421	0,9655	0,9831	0,996747	0,9993494	0,0006506
TUPRS	0,5021	0,7605	0,8926	0,9626	0,9881	0,9947	0,998765	0,9991770	0,0008230
UCAK	0,5204	0,7798	0,8796	0,9335	0,9646	0,9858	0,997820	0,9989101	0,0010899
UNYEC	0,5154	0,7755	0,8945	0,9510	0,9798	0,9913	0,997628	0,9992095	0,0007905
VAKFN	0,5035	0,7572	0,8729	0,9344	0,9719	0,9862	0,998371	0,9995927	0,0004073
VESTL	0,4668	0,7396	0,8726	0,9397	0,9740	0,9887	0,998493	1,0000000	0,0000000
VKFYT	0,4935	0,7571	0,8591	0,9384	0,9730	0,9861	0,998735	1,0000000	0,0000000
VKING	0,5155	0,7748	0,8918	0,9431	0,9633	0,9836	0,994940	0,9993675	0,0006325
YASAS	0,4662	0,7379	0,8593	0,9406	0,9784	0,9910	0,998560	1,0000000	0,0000000
YKBNK	0,4846	0,7672	0,9007	0,9538	0,9773	0,9893	0,997993	0,9993311	0,0006689
YUNSA	0,4941	0,7742	0,8696	0,9369	0,9728	0,9904	0,998328	0,9995819	0,0004181
Averages of individual Stocks	0,4948	0,7542	0,8746	0,9417	0,9753	0,9891	0,997962	0,9996163	0,0003837

Table-2: Comparison of Empirical Frequency Distributions with Unit Normal

COMPARISON OF EMPIRICAL FREQUENCY DISTRIBUTIONS WITH UNIT NORMAL									
STOCK	**INTERVALS**								
	0.5S	**1.0S**	**1.5S**	**2.0S**	**2.5S**	**3.0S**	**4.0S**	**5.0S**	**>5.0S**
ISECI-100	0,0818	0,0702	0,0154	-0,0140	-0,0163	-0,0087	-0,002577	-0,0014657	-0,0000006
AEFES	0,1267	0,0589	0,0011	-0,0186	-0,0146	-0,0085	-0,001738	-0,0003594	0,0007471
AKBNK	0,1380	0,0747	0,0043	-0,0216	-0,0141	-0,0110	-0,001834	-0,0003786	0,0003732
AKGRT	0,0965	0,0705	0,0255	-0,0086	-0,0163	-0,0123	-0,003847	0,0000006	-0,0000006
AKSA	0,1245	0,0810	0,0188	-0,0099	-0,0076	-0,0046	-0,002124	-0,0007280	0,0004450
ALARK	0,0934	0,0447	-0,0144	-0,0169	-0,0084	-0,0032	-0,000978	0,0000006	-0,0000006
ALCAR	0,1002	0,0599	0,0029	-0,0137	-0,0119	-0,0053	-0,001705	-0,0004411	-0,0000006
ALCTL	0,0979	0,0430	-0,0062	-0,0129	-0,0071	-0,0036	-0,001257	-0,0003292	0,0004138
ALTIN	0,1222	0,0755	0,0072	-0,0206	-0,0138	-0,0100	-0,004312	0,0000006	0,0013759
ANACM	0,1015	0,0519	0,0020	-0,0096	-0,0113	-0,0092	-0,001257	-0,0003292	-0,0000006
ARCLK	0,0806	0,0607	0,0107	-0,0133	-0,0105	-0,0073	-0,002927	-0,0006636	0,0007701
ASELS	0,1076	0,0719	0,0162	0,0038	-0,0006	-0,0046	-0,001085	-0,0007642	-0,0000006
AYGAZ	0,1476	0,0939	0,0267	-0,0017	-0,0049	-0,0067	-0,001018	-0,0007193	-0,0000006
BAGFS	0,0708	0,0521	0,0059	-0,0074	-0,0079	-0,0066	-0,002032	-0,0002985	-0,0000006
BFREN	0,1156	0,0767	0,0164	-0,0134	-0,0185	-0,0116	-0,002225	0,0000006	-0,0000006
BRISA	0,0903	0,0521	-0,0011	-0,0142	-0,0107	-0,0046	-0,001258	-0,0003294	-0,0000006
BSPRO	0,1188	0,0707	0,0038	-0,0112	-0,0114	-0,0105	-0,001400	-0,0003649	0,0007642
BTCIM	0,1221	0,0798	0,0257	-0,0088	-0,0154	-0,0136	-0,005369	-0,0013572	-0,0000006
BURCE	0,1149	0,0625	-0,0019	-0,0170	-0,0103	-0,0068	-0,001833	0,0000006	0,0003292
DEVA	0,0891	0,0492	-0,0027	-0,0136	-0,0104	-0,0062	-0,000928	0,0000006	-0,0000006
DITAS	0,1458	0,0773	-0,0029	-0,0195	-0,0119	-0,0100	-0,002405	-0,0004106	0,0004067
DNZCM	0,0771	0,0487	-0,0051	-0,0111	-0,0069	-0,0066	-0,001013	0,0000006	-0,0000006
DOHOL	0,1287	0,0735	-0,0008	-0,0212	-0,0194	-0,0082	-0,001501	0,0000006	0,0006876
ECILC	0,0699	0,1720	0,0040	-0,0152	-0,0102	-0,0045	-0,001445	0,0000006	0,0017015
ECZYT	0,0756	0,0427	0,0004	-0,0133	-0,0097	-0,0071	-0,001389	0,0000006	-0,0000006
EGGUB	0,1089	0,0866	0,0209	-0,0035	-0,0063	-0,0063	-0,001018	-0,0007196	0,0006500
EGPRO	0,1187	0,0627	0,0018	-0,0171	-0,0139	-0,0057	-0,002731	-0,0011167	-0,0000006
ENKA	0,0854	0,0507	-0,0036	-0,0144	-0,0076	-0,0078	-0,001752	0,0000006	0,0022365
EPLAS	0,1013	0,0723	0,0229	-0,0125	-0,0154	-0,0097	-0,003490	0,0000006	0,0006636
ERBOS	0,1063	0,0793	0,0214	-0,0130	-0,0172	-0,0111	-0,004067	-0,0013759	-0,0000006
EREGL	0,1133	0,0647	0,0008	-0,0188	-0,0124	-0,0066	-0,001867	-0,0003208	-0,0000006
FENIS	0,2209	0,1074	0,0130	-0,0262	-0,0281	-0,0166	-0,006489	-0,0007701	-0,0000006
FRIGO	0,1222	0,0800	0,0180	-0,0109	-0,0206	-0,0042	-0,004067	-0,0006876	-0,0000006
FROTO	0,0846	0,0447	0,0004	-0,0093	-0,0122	-0,0073	-0,000602	0,0000006	0,0003594
GARAN	0,1250	0,0972	0,0364	0,0044	-0,0033	-0,0040	-0,000686	-0,0003732	-0,0000006
GLBYO	0,1622	0,1157	0,0500	0,0155	-0,0003	-0,0023	-0,000392	-0,0004537	0,0014657
GOODY	0,1117	0,0702	0,0087	-0,0187	-0,0177	-0,0104	-0,002479	-0,0003624	0,0005918
GUBRF	0,0728	0,0570	-0,0062	-0,0184	-0,0100	-0,0060	-0,001021	0,0000006	-0,0000006
GUSGR	0,1183	0,0793	0,0245	-0,0123	-0,0158	-0,0172	-0,002505	-0,0012831	0,0006319
HURGZ	0,1259	0,0678	0,0021	-0,0138	-0,0148	-0,0125	-0,001275	-0,0004450	0,0002985
INTEM	0,0838	0,0414	-0,0094	-0,0120	-0,0064	-0,0054	-0,001043	-0,0003679	0,0003294
ISCTR	0,1000	0,0819	0,0237	0,0014	-0,0048	-0,0040	-0,001807	-0,0007471	0,0003624
IZOCM	0,0826	0,0531	0,0022	-0,0146	-0,0088	-0,0063	-0,001378	0,0000006	-0,0000006
KARTN	0,1206	0,0733	0,0053	-0,0176	-0,0169	-0,0083	-0,000530	-0,0005918	0,0003316
KCHOL	0,1045	0,0766	0,0217	-0,0033	-0,0075	-0,0036	-0,000934	-0,0006636	0,0004411
KENT	0,2315	0,1208	0,0345	-0,0158	-0,0276	-0,0216	-0,009725	-0,0017015	0,0011436
KONYA	0,1156	0,0815	0,0088	-0,0144	-0,0136	-0,0117	-0,004602	-0,0003881	-0,0000006
KORDS	0,1130	0,0643	0,0056	-0,0127	-0,0099	-0,0079	-0,002133	-0,0007309	0,0008224
KOTKS	0,0999	0,0711	0,0022	-0,0239	-0,0180	-0,0096	-0,000472	0,0000006	0,0008224
LUKSK	0,2132	0,0912	-0,0025	-0,0326	-0,0189	-0,0133	-0,002816	-0,0008218	-0,0000006
MAALT	0,1375	0,0687	-0,0010	-0,0189	-0,0146	-0,0110	-0,003538	0,0000006	-0,0000006
MERKO	0,1116	0,0669	0,0166	-0,0114	-0,0129	-0,0125	-0,003733	-0,0012644	-0,0000006
MILYT	0,0906	0,0655	0,0079	-0,0160	-0,0173	-0,0097	-0,000477	0,0000006	-0,0000006

Table-2: Comparison of Empirical Frequency Distributions With Unit Normal(continued)

COMPARISON OF EMPIRICAL FREQUENCY DISTRIBUTIONS WITH UNIT NORMAL									
STOCK	**INTERVALS**								
	0.5S	**1.0S**	**1.5S**	**2.0S**	**2.5S**	**3.0S**	**4.0S**	**5.0S**	**>5.0S**
MIPAZ	0,0858	0,0630	0,0027	-0,0145	-0,0116	-0,0099	-0,001082	0,0000006	0,0012831
MMART	0,0889	0,0561	0,0046	-0,0152	-0,0099	-0,0075	-0,001400	0,0000006	0,0007899
MRDIN	0,1145	0,0838	0,0223	-0,0102	-0,0064	-0,0036	-0,001898	-0,0007834	0,0007834
MRSHL	0,0944	0,0775	0,0038	-0,0194	-0,0129	-0,0100	-0,000730	0,0000006	0,0003881
NETAS	0,1026	0,0645	0,0066	-0,0182	-0,0149	-0,0074	-0,001455	0,0000006	0,0007899
NIGDE	0,1324	0,0929	0,0281	-0,0035	-0,0078	-0,0060	-0,002310	-0,0007899	0,0013572
NTTUR	0,2002	0,0799	-0,0059	-0,0284	-0,0192	-0,0119	-0,002309	-0,0007896	-0,0000006
OLMKS	0,0910	0,0605	-0,0038	-0,0144	-0,0125	-0,0050	-0,001400	0,0000006	-0,0000006
PARSN	0,0917	0,0445	-0,0083	-0,0050	-0,0094	-0,0090	-0,000327	0,0000006	-0,0000006
PETKM	0,0702	0,0485	-0,0056	-0,0148	-0,0081	-0,0041	-0,000696	0,0000006	0,0004106
PIMAS	0,1141	0,0582	-0,0148	-0,0168	-0,0123	-0,0069	-0,000774	0,0000006	-0,0000006
PINSU	0,1233	0,0574	-0,0140	-0,0164	-0,0089	-0,0082	-0,000774	0,0000006	-0,0000006
PNSUT	0,0969	0,0423	-0,0085	-0,0143	-0,0089	-0,0082	0,000062	0,0000006	-0,0000006
PTOFS	0,1191	0,0927	0,0262	0,0081	0,0005	-0,0026	-0,001173	-0,0008224	0,0008218
RANTL	0,1013	0,0840	0,0199	-0,0174	-0,0170	-0,0134	-0,003951	0,0000006	-0,0000006
SARKY	0,0730	0,0624	0,0073	-0,0163	-0,0164	-0,0019	-0,001261	0,0000006	-0,0000006
SISE	0,0592	0,0380	0,0000	-0,0064	-0,0067	-0,0038	-0,001018	-0,0003595	-0,0000006
TATKS	0,1047	0,0685	0,0105	-0,0206	-0,0159	-0,0106	-0,002603	-0,0005324	0,0004175
THYAO	0,0967	0,0855	0,0076	0,0028	-0,0030	-0,0005	-0,000728	-0,0003945	0,0028123
TIRE	0,1145	0,0678	-0,0065	-0,0168	-0,0144	-0,0103	-0,000774	0,0000006	0,0008007
TOASO	0,1036	0,0636	0,0124	-0,0189	-0,0152	-0,0090	-0,001192	0,0000006	-0,0000006
TRKCM	0,1081	0,0675	0,0126	-0,0142	-0,0137	-0,0097	-0,004497	-0,0004138	-0,0000006
TUBORG	0,1417	0,0791	0,0044	-0,0193	-0,0185	-0,0103	-0,001610	0,0000006	0,0075752
TUDDF	0,1004	0,0593	0,0010	-0,0136	-0,0128	-0,0069	-0,000602	-0,0003316	0,0003679
TUKAS	0,1277	0,0949	0,0178	-0,0124	-0,0221	-0,0142	-0,003191	-0,0006500	-0,0000006
TUPRS	0,1191	0,0779	0,0262	0,0081	0,0005	-0,0026	-0,001173	-0,0008224	-0,0000006
UCAK	0,1374	0,0972	0,0132	-0,0210	-0,0230	-0,0115	-0,002118	-0,0010893	-0,0000006
UNYEC	0,1324	0,0929	0,0281	-0,0035	-0,0078	-0,0060	-0,002310	-0,0007899	-0,0000006
VAKFN	0,1205	0,0746	0,0065	-0,0201	-0,0157	-0,0111	-0,001567	-0,0004067	-0,0000006
VESTL	0,0838	0,0570	0,0062	-0,0148	-0,0136	-0,0086	-0,001445	0,0000006	0,0003945
VKFYT	0,1105	0,0745	-0,0073	-0,0161	-0,0146	-0,0112	-0,001203	0,0000006	-0,0000006
VKING	0,1325	0,0922	0,0254	-0,0114	-0,0243	-0,0137	-0,004998	-0,0006319	-0,0000006
YASAS	0,0832	0,0553	-0,0071	-0,0139	-0,0092	-0,0063	-0,001378	0,0000006	-0,0000006
YKBNK	0,1016	0,0846	0,0343	-0,0007	-0,0103	-0,0080	-0,001945	-0,0006683	-0,0000006
YUNSA	0,1111	0,0916	0,0032	-0,0176	-0,0148	-0,0069	-0,001610	-0,0004175	0,0004118
Averages	0,1118	0,0713	0,0082	-0,0128	-0,0122	-0,0082	-0,001977	-0,0003831	0,0004430

Table-3: Analysis of Extreme Tail Areas in Terms of Number of Observations

		INTERVAL							
		>2S		>3S		>4S		>5S	
STOCK	**N**	Expected	Actual	Expected	Actual	Expected	Actual	Expected	Actual
ISECI-100	3410	155,2	203	9,2	39	0,21	9	0,0020	5
AEFES	2778	126,4	178	7,5	31	0,18	5	0,0017	1
AKBNK	2637	120,0	177	7,1	36	0,17	5	0,0016	1
AKGRT	1535	69,8	83	4,1	23	0,10	6	0,0009	0
AKSA	2745	124,9	152	7,4	20	0,17	6	0,0016	2
ALARK	2886	131,3	180	7,8	17	0,18	3	0,0017	0
ALCAR	2264	103,0	134	6,1	18	0,14	4	0,0014	1
ALCTL	3032	138,0	177	8,2	19	0,19	4	0,0018	1
ALTIN	2286	104,0	151	6,2	29	0,14	10	0,0014	0
ANACM	3032	138,0	167	8,2	36	0,19	4	0,0018	1
ARCLK	3011	137,0	177	8,1	30	0,19	9	0,0018	2
ASELS	2615	119,0	109	7,1	19	0,16	3	0,0016	2
AYGAZ	2778	126,4	131	7,5	26	0,18	3	0,0017	2
BAGFS	3343	152,1	177	9,0	31	0,21	7	0,0020	1
BFREN	1749	79,6	103	4,7	25	0,11	4	0,0010	0
BRISA	3030	137,9	181	8,2	22	0,19	4	0,0018	1
BSPRO	2736	124,5	155	7,4	36	0,17	4	0,0016	1
BTCIM	1473	67,0	80	4,0	24	0,09	8	0,0009	2
BURCE	2111	96,1	132	5,7	20	0,13	4	0,0013	0
DEVA	3031	137,9	179	8,2	27	0,19	3	0,0018	0
DITAS	2432	110,7	158	6,6	31	0,15	6	0,0015	1
DNZCM	2791	127,0	158	7,5	26	0,18	3	0,0017	0
DOHOL	1919	87,3	128	5,2	21	0,12	3	0,0012	0
ECILC	2654	120,8	161	7,2	19	0,17	4	0,0016	0
ECZYT	2756	125,4	162	7,4	27	0,17	4	0,0017	0
EGGUB	2777	126,4	136	7,5	25	0,17	3	0,0017	2
EGPRO	1790	81,4	112	4,8	15	0,11	5	0,0011	2
ENKA	2756	125,4	165	7,4	29	0,17	5	0,0017	0
EPLAS	1689	76,8	98	4,6	21	0,11	6	0,0010	0
ERBOS	1453	66,1	85	3,9	20	0,09	6	0,0009	2
EREGL	3111	141,6	200	8,4	29	0,20	6	0,0019	1
FENIS	2595	118,1	186	7,0	50	0,16	17	0,0016	2
FRIGO	1453	66,1	82	3,9	10	0,09	6	0,0009	1
FROTO	3011	137,0	165	8,1	30	0,19	2	0,0018	0
GARAN	2675	121,7	110	7,2	18	0,17	2	0,0016	1
GLBYO	2201	100,1	66	5,9	11	0,14	1	0,0013	1
GOODY	2755	125,4	177	7,4	36	0,17	7	0,0017	1
GUBRF	2769	126,0	177	7,5	24	0,17	3	0,0017	0
GUSGR	1558	70,9	90	4,2	31	0,10	4	0,0009	2
HURGZ	2244	102,1	133	6,1	34	0,14	3	0,0013	1
INTEM	2714	123,5	156	7,3	22	0,17	3	0,0016	1
ISCTR	2675	121,7	118	7,2	18	0,17	5	0,0016	2
IZOCM	2777	126,4	167	7,5	25	0,17	4	0,0017	0
KARTN	3376	153,6	213	9,1	37	0,21	2	0,0020	2
KCHOL	3011	137,0	147	8,1	19	0,19	3	0,0018	2
KENT	2350	106,9	144	6,3	57	0,15	23	0,0014	4
KONYA	2573	117,1	154	6,9	37	0,16	12	0,0015	1
KORDS	2734	124,4	159	7,4	29	0,17	6	0,0016	2
KOTKS	1872	85,2	130	5,1	23	0,12	1	0,0011	0
LUKSK	2432	110,7	190	6,6	39	0,15	7	0,0015	2
MAALT	2778	126,4	179	7,5	38	0,18	10	0,0017	0
MERKO	1581	71,9	90	4,3	24	0,10	6	0,0009	2
MILYT	1854	84,4	114	5,0	23	0,12	1	0,0011	0
MIPAZ	1749	79,6	105	4,7	22	0,11	2	0,0010	0
MMART	2736	124,5	166	7,4	28	0,17	4	0,0016	0

Table-3: Analysis of Extreme Tail Areas in Terms of Number of Observations (Continued)

STOCK	N	INTERVAL							
		>2S		>3S		>4S		>5S	
		Expected	Actual	Expected	Actual	Expected	Actual	Expected	Actual
MRDIN	2551	116,1	142	6,9	16	0,16	5	0,0015	2
MRSHL	2526	114,9	164	6,8	32	0,16	2	0,0015	0
NETAS	1977	90,0	126	5,3	20	0,12	3	0,0012	0
NIGDE	2530	115,1	124	6,8	22	0,16	6	0,0015	2
NTTUR	2531	115,2	187	6,8	37	0,16	6	0,0015	2
OLMKS	2736	124,5	164	7,4	21	0,17	4	0,0016	0
PARSN	2572	117,0	130	6,9	30	0,16	1	0,0015	0
PETKM	2637	120,0	159	7,1	18	0,17	2	0,0016	0
PIMAS	2392	108,8	149	6,5	23	0,15	2	0,0014	0
PINSU	2392	108,8	148	6,5	26	0,15	2	0,0014	0
PNSUT	2392	108,8	143	6,5	26	0,15	0	0,0014	0
PTOFS	2430	110,6	91	6,6	13	0,15	3	0,0015	2
RANTL	1495	68,0	94	4,0	24	0,09	6	0,0009	0
SARKY	3024	137,6	187	8,2	14	0,19	4	0,0018	0
SISE	2777	126,4	144	7,5	18	0,17	3	0,0017	1
TATKS	1876	85,4	124	5,1	25	0,12	5	0,0011	1
THYAO	2531	115,2	108	6,8	8	0,16	2	0,0015	1
TIRE	2392	108,8	149	6,5	31	0,15	2	0,0014	0
TOASO	2392	108,8	154	6,5	28	0,15	3	0,0014	0
TRKCM	2413	109,8	144	6,5	30	0,15	11	0,0014	1
TUBORG	2392	108,8	155	6,5	31	0,15	4	0,0014	0
TUDDF	3010	137,0	178	8,1	29	0,19	2	0,0018	1
TUKAS	1537	69,9	89	4,1	26	0,10	5	0,0009	1
TUPRS	2430	110,6	91	6,6	13	0,15	3	0,0015	2
UCAK	1835	83,5	122	5,0	26	0,12	4	0,0011	2
UNYEC	2530	115,1	124	6,8	22	0,16	6	0,0015	2
VAKFN	2455	111,7	161	6,6	34	0,15	4	0,0015	1
VESTL	2654	120,8	160	7,2	30	0,17	4	0,0016	0
VKFYT	2371	107,9	146	6,4	33	0,15	3	0,0014	0
VKING	1581	71,9	90	4,3	26	0,10	8	0,0009	1
YASAS	2778	126,4	165	7,5	25	0,18	4	0,0017	0
YKBNK	2990	136,0	138	8,1	32	0,19	6	0,0018	2
YUNSA	2392	108,8	151	6,5	23	0,15	4	0,0014	1
TOTALS	210802	9591	12254	569,2	2226	13,28	406	0,1265	77

Another tool for detecting departures from normality is to construct (draw) normal probability graphs. Figure 4 is the normal probability graphs of individual stocks and ISECI-100 exhibiting S shaped curves. Normal probability graph shapes would be straight lines if the samples were distributed normally. As figure 4 illustrates none of the graphs has straight line, instead all graphs have elongated S shapes implying departures from normality. An analysis of the normal probability graphs shows fat tails and excess around centre, it can easily be seen from the normal probability graphs that the slopes of extreme tail areas of normal probability graphs are lower than the central parts of the graphs. Another fact is that empirical frequency distributions have higher probabilities in the center than normal distribution. Figure 4 illustrates that normal probability graphs of empirical evidence has steeper centers than the case of normal distributions. It is clearly seen from the Figure 4 that empirical frequency distributions centered around zero, implying peakedness around the means.

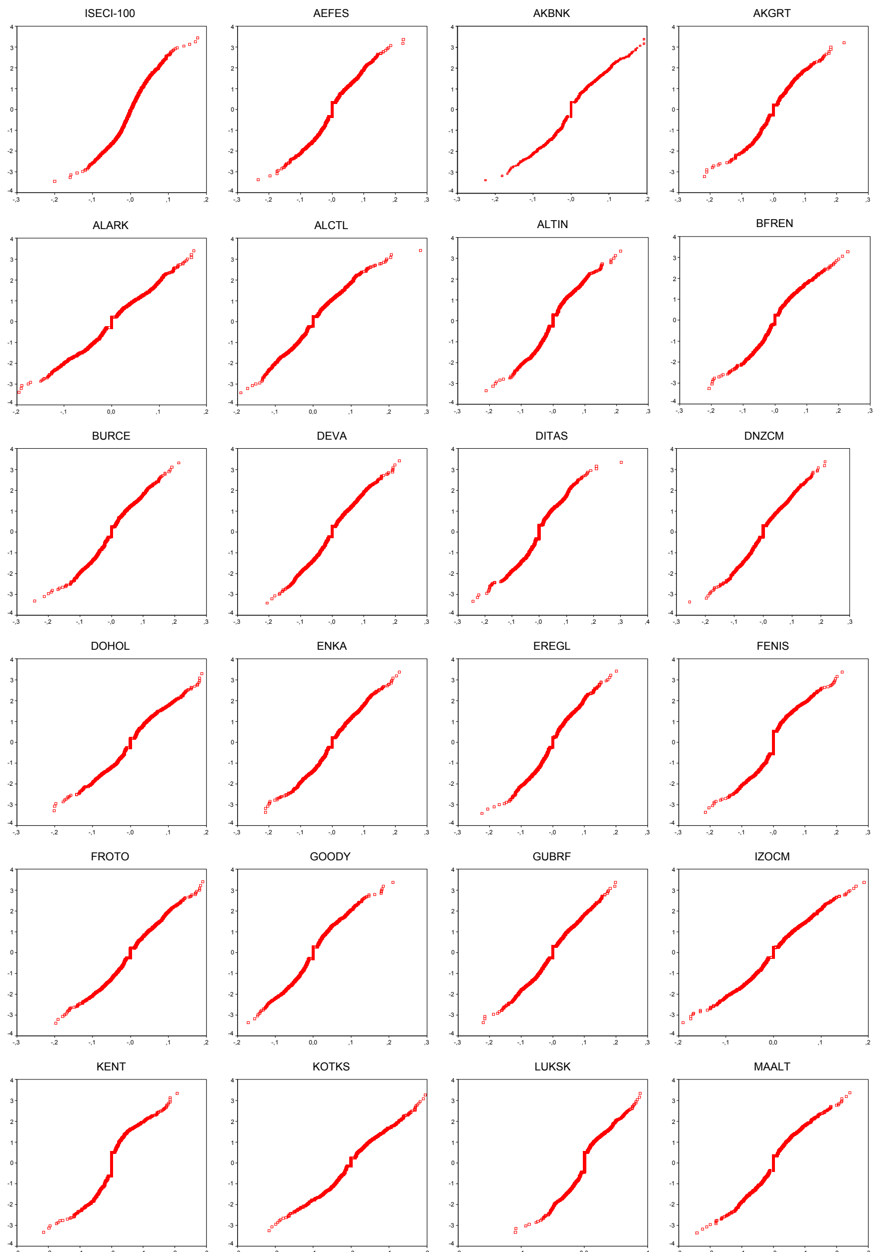


Figure-4: Normal Probability Graphs for Dialy Prices in Log Prices of ISECI-100 and of Selected Stocks

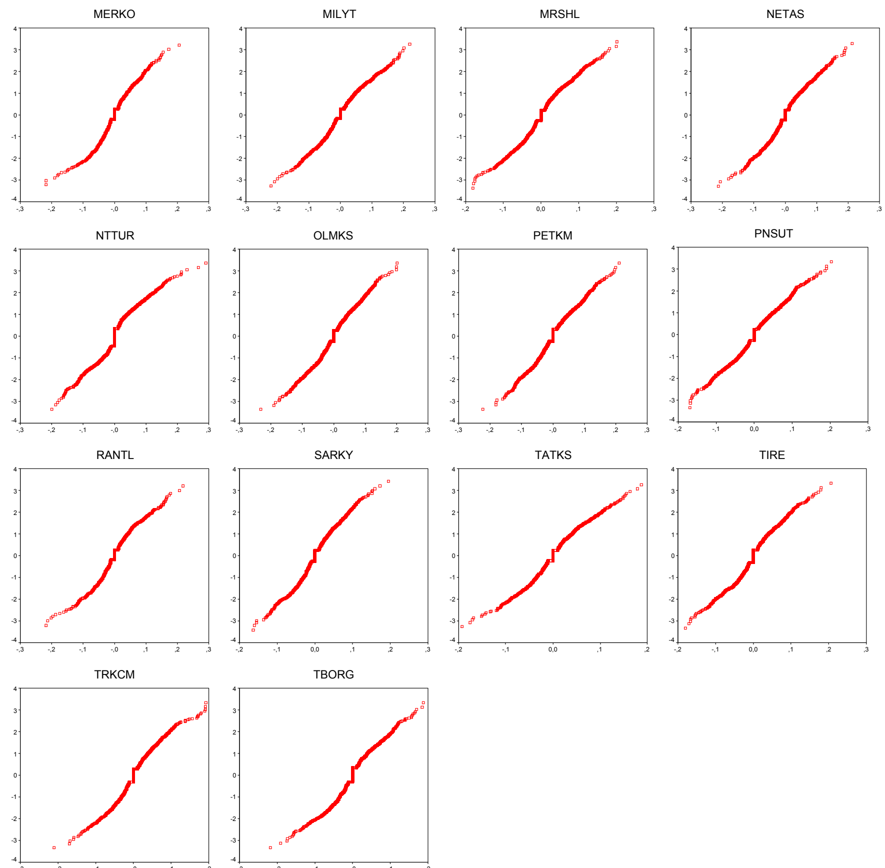


Figure-4: Normal Probability Graphs for Dialy Prices in Log Prices of ISECI-100 and of Selected Stocks (continued)

V. INVARIANCE UNDER ADDITION

Stable distributions, after their adjustment for the scale, should retain their statistical properties if they are added together. For instance, if the series of daily price changes were normally distributed, with a mean μ and a variance (σ^2), then 4-day price changes should also be normally distributed, with mean 4 μ and variance $4\sigma^2$. If the daily distribution were stable paretian, then 4-day price changes would have a mean of 4 μ, but the variance would be unstable.

To test this possibility , I created 4-day, 9-day, 16-day, and 25-day log price changes series using the daily observations of individual stocks and of ISECI-100. To check how mean and variance are scaled I have constructed Table 4.

Each cell of the Table 4 is calculated by actual variance divided by expected variance. We know that if the distribution of observations of each sample were Normal or Gaussian, the expected mean and variance for 4-day logarithmic price changes would be 4μ and $4\sigma^2$ (μ is the mean and σ^2 is the variance of daily logarithmic price changes), respectively. Thus, by multiplying variance of daily logarithmic price changes by 4, 9, 16, 25, expected variance of logarithmic price changes could be calculated for different intervals for each sample. Now it is easy to calculate actual variance of logarithmic price changes for different intervals from daily logarithmic price changes. For example, 4-day logarithmic price changes could be computed from daily logarithmic price changes by summing daily logarithmic price changes in groups of 4. Expected mean and variance of new observations should be 4μ, and $4\sigma^2$ respectively if the distribution of observations is strictly Normal or Gaussian. Then the *actual variance/expected variance* ratio should be equal to one or almost one. This computation can be done for 9-day, 16-day, 25-day and other intervals.

As can be seen from Table 4 that is not the case, which confirms unstable variance for empirical evidence.

Table-4: Comparison Of Behavior Of Variances For Different Intervals With Expected Values (Actual/Expected)

Invariance under addition (Actual/Expected)					Invariance under addition (Actual/Expected)				
STOCK	$4\sigma^2$	$9\sigma^2$	$16\sigma^2$	$25\sigma^2$	**STOCK**	$4\sigma^2$	$9\sigma^2$	$16\sigma^2$	$25\sigma^2$
ISECI-100	1,19	1,41	1,46	1,65	**KCHOL**	1,03	1,02	1,16	1,21
AEFES	1,07	0,98	1,72	0,99	**KENT**	1,05	1,11	1,51	1,51
AKBNK	0,97	0,94	0,84	0,92	**KONYA**	1,09	1,11	1,14	1,26
AKGRT	1,09	0,93	1,16	1,17	**KORDS**	1,17	1,13	1,34	1,10
AKSA	1,05	1,07	1,49	1,20	**KOTKS**	1,09	1,18	1,11	1,28
ALARK	1,17	1,45	1,99	1,54	**LUKSK**	1,05	1,28	1,32	1,35
ALCAR	1,04	1,23	1,38	1,32	**MAALT**	0,93	0,96	1,07	1,04
ALCTL	1,13	1,09	1,35	1,34	**MERKO**	0,99	0,84	0,97	0,89
ALTIN	1,08	1,38	1,23	1,33	**MILYT**	1,11	1,28	1,34	1,40
ANACM	1,01	1,17	1,42	1,28	**MIPAZ**	1,21	1,31	1,60	1,43
ARCLK	1,07	1,07	1,14	1,25	**MMART**	0,90	0,94	1,02	1,06
ASELS	0,99	1,05	1,13	1,02	**MRDIN**	0,93	0,95	1,33	1,21
AYGAZ	1,01	0,93	0,99	0,79	**MRSHL**	0,92	0,86	1,18	1,02
BAGFS	1,10	1,12	1,88	1,46	**NETAS**	1,32	1,21	1,90	1,25
BFREN	1,26	1,04	1,73	1,19	**NIGDE**	1,09	1,04	1,14	1,01
BRISA	1,05	1,04	1,16	1,18	**NTTUR**	1,18	1,32	1,64	1,43
BSPRO	1,05	1,10	1,08	1,10	**OLMKS**	1,04	1,08	1,08	1,12
BTCIM	1,01	0,86	0,94	1,21	**PARSN**	1,11	1,17	1,26	1,38
BURCE	1,01	1,14	1,33	1,34	**PETKM**	1,19	1,12	1,42	0,99
DEVA	1,15	1,37	1,17	1,32	**PIMAS**	1,12	1,25	1,26	1,42
DITAS	1,12	1,06	1,35	1,28	**PINSU**	1,13	1,21	1,20	1,43
DNZCM	1,13	1,23	1,33	1,49	**PNSUT**	1,15	1,20	1,32	1,42
DOHOL	1,36	1,37	1,88	1,94	**PTOFS**	1,08	0,88	1,03	0,81
ECILC	1,17	1,34	1,49	1,20	**RANTL**	0,95	1,15	1,30	1,52
ECZYT	1,16	1,27	1,49	1,50	**SARKY**	1,14	1,09	1,36	1,32
EGGUB	0,97	1,09	1,30	1,06	**SISE**	1,10	1,09	1,52	1,32
EGPRO	1,13	1,16	1,46	1,44	**TATKS**	1,00	1,01	1,22	1,03
ENKA	1,06	1,04	1,15	0,91	**THYAO**	1,11	1,00	1,15	0,96
EPLAS	1,11	1,06	2,38	1,21	**TIRE**	1,03	1,21	1,49	1,49
ERBOS	0,87	0,87	0,88	0,70	**TOASO**	1,15	1,16	1,39	1,32
EREGL	1,18	1,33	1,27	1,35	**TRKCM**	1,06	1,20	1,16	1,15
FENIS	0,96	0,93	1,00	0,97	**TUBORG**	1,02	1,18	0,96	1,09
FRIGO	1,14	1,08	1,10	1,03	**TUDDF**	1,11	1,09	1,24	1,37
FROTO	1,12	1,13	1,24	1,49	**TUKAS**	1,28	1,23	1,30	1,10
GARAN	1,02	0,91	1,00	1,21	**TUPRS**	1,08	0,88	1,03	0,81
GLBYO	1,03	1,17	1,10	1,29	**UCAK**	1,01	1,16	1,54	1,40
GOODY	1,07	1,13	1,22	1,21	**UNYEC**	1,09	1,04	1,14	1,01
GUBRF	1,19	1,23	1,20	1,62	**VAKFN**	0,94	1,15	1,15	1,25
GUSGR	1,02	1,14	1,36	1,43	**VESTL**	1,09	1,11	1,04	1,12
HURGZ	1,14	1,17	1,27	1,51	**VKFYT**	1,09	1,09	1,01	1,00
INTEM	1,17	1,26	1,49	1,40	**VKING**	1,20	1,24	1,23	1,04
ISCTR	1,09	1,18	1,43	1,55	**YASAS**	1,03	1,26	1,65	1,47
IZOCM	1,00	0,98	1,31	1,09	**YKBNK**	1,21	1,00	1,15	1,11
KARTN	1,03	1,08	1,44	1,42	**YUNSA**	0,94	1,06	1,09	1,35

VI. ESTIMATING CHARACTERISTIC EXPONENT ALPHA

The characteristic exponent alpha is the measurement of tail thickness. Estimating α is important to understand the behavior of the empirical observations. When the characteristic exponent (α) of stable Paretian distribution is exactly equal to 2, then the Normal distribution is obtained. When it is exactly equal to 1, then Cauchy distribution is obtained. When it is between 1 and 2 then the distribution can be classified as stable Paretian distribution.

Table-5: α Estimates for ISECI-100 and Individual Stocks by Range Analysis

STOCK	α_4	α_9	α_{16}	α_{25}	STOCK	α_4	α_9	α_{16}	α_{25}
ISECI-100	1,77	1,73	1,76	1,73	**KCHOL**	1,96	1,98	1,90	1,89
AEFES	1,90	2,01	1,67	2,01	**KENT**	1,94	1,91	1,74	1,77
AKBNK	2,04	2,06	2,13	2,06	**KONYA**	1,88	1,91	1,91	1,87
AKGRT	1,88	2,06	1,90	1,91	**KORDS**	1,80	1,90	1,81	1,94
AKSA	1,94	1,94	1,75	1,89	**KOTKS**	1,88	1,86	1,93	1,86
ALARK	1,79	1,71	1,60	1,76	**LUKSK**	1,93	1,80	1,82	1,83
ALCAR	1,94	1,83	1,79	1,84	**MAALT**	2,11	2,04	1,95	1,98
ALCTL	1,84	1,92	1,80	1,83	**MERKO**	2,01	2,18	2,02	2,07
ALTIN	1,90	1,74	1,86	1,84	**MILYT**	1,86	1,80	1,81	1,81
ANACM	1,99	1,87	1,78	1,86	**MIPAZ**	1,75	1,78	1,71	1,80
ARCLK	1,91	1,94	1,91	1,87	**MMART**	2,16	2,05	1,99	1,96
ASELS	2,02	1,96	1,91	1,99	**MRDIN**	2,11	2,05	1,82	1,89
AYGAZ	1,99	2,07	2,00	2,16	**MRSHL**	2,13	2,15	1,89	1,99
BAGFS	1,87	1,90	1,63	1,79	**NETAS**	1,67	1,84	1,62	1,87
BFREN	1,72	1,96	1,67	1,90	**NIGDE**	1,89	1,97	1,91	1,99
BRISA	1,93	1,96	1,90	1,90	**NTTUR**	1,79	1,78	1,70	1,80
BSPRO	1,93	1,92	1,95	1,94	**OLMKS**	1,95	1,93	1,95	1,93
BTCIM	1,99	2,15	2,04	1,89	**PARSN**	1,86	1,87	1,85	1,82
BURCE	1,98	1,89	1,81	1,83	**PETKM**	1,77	1,90	1,77	2,00
DEVA	1,82	1,75	1,89	1,84	**PIMAS**	1,85	1,82	1,85	1,80
DITAS	1,85	1,95	1,80	1,86	**PINSU**	1,84	1,84	1,87	1,80
DNZCM	1,84	1,83	1,82	1,78	**PNSUT**	1,81	1,85	1,82	1,80
DOHOL	1,63	1,75	1,63	1,66	**PTOFS**	1,89	2,12	1,98	2,14
ECILC	1,79	1,77	1,75	1,89	**RANTL**	2,08	1,88	1,83	1,77
ECZYT	1,81	1,80	1,75	1,77	**SARKY**	1,83	1,92	1,80	1,84
EGGUB	2,05	1,93	1,83	1,96	**SISE**	1,88	1,92	1,74	1,84
EGPRO	1,84	1,88	1,76	1,80	**TATKS**	2,00	1,99	1,86	1,98
ENKA	1,92	1,97	1,90	2,06	**THYAO**	1,86	2,00	1,90	2,03
EPLAS	1,86	1,95	1,52	1,89	**TIRE**	1,96	1,84	1,75	1,78
ERBOS	2,22	2,13	2,10	2,25	**TOASO**	1,82	1,87	1,79	1,84
EREGL	1,79	1,77	1,90	1,83	**TRKCM**	1,92	1,85	1,90	1,92
FENIS	2,07	2,07	2,00	2,02	**TUBORG**	1,98	1,86	2,03	1,95
FRIGO	1,82	1,93	1,94	1,98	**TUDDF**	1,86	1,92	1,86	1,82
FROTO	1,85	1,90	1,86	1,78	**TUKAS**	1,69	1,83	1,83	1,95
GARAN	1,97	2,09	2,00	1,89	**TUPRS**	1,89	2,12	1,98	2,14
GLBYO	1,95	1,87	1,93	1,86	**UCAK**	1,98	1,88	1,73	1,81
GOODY	1,91	1,89	1,87	1,89	**UNYEC**	1,89	1,97	1,91	1,99
GUBRF	1,78	1,83	1,88	1,74	**VAKFN**	2,09	1,88	1,90	1,87
GUSGR	1,97	1,89	1,80	1,80	**VESTL**	1,88	1,91	1,97	1,93
HURGZ	1,82	1,87	1,84	1,77	**VKFYT**	1,88	1,93	1,99	2,00
INTEM	1,80	1,81	1,75	1,81	**VKING**	1,77	1,82	1,86	1,98
ISCTR	1,88	1,86	1,77	1,76	**YASAS**	1,95	1,81	1,69	1,79
IZOCM	2,00	2,02	1,82	1,95	**YKBNK**	1,76	2,00	1,91	1,94
KARTN	1,95	1,94	1,77	1,80	**YUNSA**	2,09	1,95	1,94	1,83

Range Analysis is the one of the methods[8] to estimate this characteristic exponent. Taking from Fama (1965), characteristic exponent (α) can be estimated by the following equation,

$$R_n = n^{1/\alpha} R_1 , \quad (5)$$

$$\alpha = \frac{\log n}{\log R_n - \log R_1} \quad (6)$$

where R_n is the interquartile range of the distribution of sums of n independent observations, and R_1 is the interquartile of the distribution of individual observations.

By taking different summing intervals (i.e. different values of n), and different interfractile ranges, the equation can be used to estimate alpha from the same set of data. Table 5. is the estimates of alpha under different summing intervals with interquartile. The results of the Table 5 are also consistent with the findings of the previous sections. Almost 95% of the samples has characteristic exponent (α) less than 2 implying heavy tails.

CONCLUSION

While investigating the Turkish stock price changes, this study puts emphasis on the form or shape of the distribution. In section IV, histograms with normal curves of empirical evidence were drawn, and frequency distributions were computed for each sample stock and the ISECI-100. The results of this section exhibited that empirical distributions had peakedness around means when compared to Normal or Gaussian distribution. Another important result of section IV was that extreme tails of the empirical distributions contained more relative frequencies than would be expected in a Normal (Gaussian) distribution. Also, for each stock and the ISECI-100 normal probability graphs were drawn to test the departures from normality. Each graph had the shape of an S rather than a straight line implying departures from normality.

Section V included the analysis of stability of the variance of observations for each sample stock and ISECI-100. The actual variances of different observation intervals (4-day, 9-day, 16-day, 25-day observation of price changes) were calculated in order to compare to expected variances of the same observation intervals. The expected variances were computed by using the property of Normal (Gaussian) distribution such that variances of T-day price changes can easily be calculated by just multiplying the variance of daily price changes by time interval T ($T\sigma^2$). Then the actual variance/expected variance ratio was computed. It was expected that the values of the ratio were equal to 1 or almost 1 if the distributions of price changes were strictly Normal (Gaussian) and the value of the ratio should have converged to some number if the variance of the distribution of price changes were stable. In the analyses, almost for all cases the values of the ratio were significantly different than one and did not converge to a number implying unstable variances.

In section VI characteristic exponent α was estimated by using range analysis. We know that when characteristic exponent α is exactly equal to 2 the Normal distribution is obtained, and when it is exactly 1 the Cauchy distribution is obtained. When it is between 1 and 2 the distribution can be classified as stable Paretian. The results of this section presented that estimates of α is less than 2 in most cases.

The findings of the study are not consistent with the general assumption that stock price changes have Normal or Gaussian distribution. Empirical evidence from an emerging market (Istanbuk Stock Exchange) confirms peakedness around means in the unconditional distributions with heavy tails. The results are parellel with the findings of the well-known article of Fama (1965). I have also concluded that empirical evidence from Istanbul Stock Exchange fits the Mandlebrot's (1963) stable Paretian distribution with characteristic exponent less than 2.

The economical implications of the findings of this study can be summarized as follows:

In a market that price changes are characterized by Normal or Gaussian distribution, chances of occurrence of very large price changes across a very long period are so small or negligible when compared to the total change. However, in a market that price changes are characterized by stable Paretian distribution with characteristic exponent less than 2, it is most likely that large changes occur in very short periods. This implies that a stable Paretian market is more risky than a Normal (Gaussian) market.

In a stable Paretian market, due to the very large and rapid changes in short periods, investors may not protect themselves from exposing large losses.

In the study it is also concluded that empirical data confirm unstable variace under addition. In other words, the sample variances of empirical data show erratic behavior. This implies that the variances are infinite meaning that sample variances do not converge to a constant variance. In terms of investment decisions, infinite variance has significant implications. For instance, the classical Markowitz analysis of efficient portfolios model considers finite variance in determining efficient portfolios. However, a distribution with infinite variance invalidates this model. Thus, the usual statistical tools may be badly misleading from the point of financial decision making.

In sum, behavior of stock prices can be characterized as nonlinear dynamic systems. Thus, in investment decisions, this feature of stock price changes should be taken into account by investors. Further studies should focus on fractal, and chaotic models in order to have a better understanding of the stock prices.

NOTES

[1]The theory of random walks in stock prices is based on two hypotheses, i) successive price changes in an individual security are independent, and ii) the price changes conform to some probability distribution. Hence, If stock prices follow a random walk, then stock returns should be i.i.d. and, if enough i.i.d. returns are collected, the central limit theorem implies that the limiting distribution of these returns should be Normal.
[2]This is due to the fact that variance exhibits a very erratic behavior and it does not converge any given value as the sample size increases.
[3]When the characteristic exponent of a stable Paretian distribution is exactly equal to 2, then the Normal distribution is obtained. When it is exactly equal to 1, then cauchy distribution is obtained.
[4] This parameter is an index of skewness which determines the relative size of the two tails and can only take values in the interval [-1,1]. When it is equal to zero then distribution is symmetric. When it is greater (less) than zero, distribution is skewed to right (left).
[5]Adjusted daily closing prices of each stock. I would like to thank Egemen Menkul Kıymetler A.Ş. for their support in providing the data.
[6] ISECI-100 is a wieghted index using closing prices of stocks.
[7] Unit normal distributions within given standard deviations of mean changes are the expected distributions if the empirical distributions of observations are Normal (Gaussian).
[8] There are several other methods to estimate alpha, such as estimating from serial variance, or by plotting double log graphs of the empirical observations. (see Fama, E., 1965)

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