Necatibey Eğitim Fakültesi Elektronik Fen ve Matematik Eğitimi Dergisi
Cilt 17, Özel Sayı, Ekim 2023, sayfa 666-694. ISSN: 1307-6086
Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education
Vol. 17, Special Issue, October 2023, pp. 666-694. ISSN: 1307-6086

Research Article

Students' Spatial Abilities, Attitudes towards Geometry, and Van Hiele Geometric Thinking Levels*

Zeynep Büşra UZUN [1], Gülcan ÖZTÜRK [2]

[1] Ministry of National Education, Beyoğlu Kadı Mehmet Middle School, İstanbul, Turkey, zeynepbusrauzun@gmail.com, https://orcid.org/0000-0002-7005-6064

[2] Balıkesir University, Necatibey Faculty of Education, Department of Computer Education and Instructional Technology, Balıkesir, Turkey, ozturkg@balikesir.edu.tr, https://orcid.org/0000-0003-4399-1329

Received : 17.04.2023 Accepted : 25.07.2023
https://doi.org/10.17522/balikesirnef.1284875

Abstract – The aim of this study was to investigate the relationships between eighth grade students' spatial abilities, attitudes towards geometry and Van Hiele geometric thinking levels. We conducted the study using exploratory correlational research model with 429 students. The students' spatial ability and attitudes towards geometry were moderate, and their Van Hiele geometric thinking levels were extremely low. We discovered that the students' spatial ability scores and Van Hiele geometric thinking levels differed depending on their pre-school attendance status and did not differ according to their gender and that their attitudes towards geometry were independent of gender and pre-school attendance status. The students' spatial abilities and Van Hiele geometric thinking levels were positively associated with their attitudes towards geometry.

Key words: spatial ability, attitude towards geometry, Van Hiele geometric thinking levels, eighth grade students

----

Corresponding author: Gülcan ÖZTÜRK, Balıkesir University, Necatibey Faculty of Education, Department of Computer Education and Instructional Technology, Balıkesir, Turkey.

Introduction

One of the most essential concepts in curriculum of mathematics is geometry. It is not only in course programs but also in all areas of life. Geometry helps to understand abstract concepts in the context of problem solving and mathematical forms, and everyone needs geometry to describe the World and solve problems, regardless of their profession (Hannula

---

* This study is part of the master's thesis prepared by the first author under the supervision of the second author.

& Toivanen, 2019). Students have difficulties to understand and connect simple geometrical concepts (Watan & Sugiman, 2018). Reasons for this are teaching geometry with traditional methods, introducing geometric shapes and objects only superficially without making a connection between properties of shapes and objects, students not trying to understand geometry, and having a negative attitude and fear of geometry (Jones & Tzekaki, 2016; Sinclair et al., 2016).

Spatial ability, spatial visualization ability, and spatial or visuospatial reasoning are some of the most crucial factors of geometry success (Ben-Chaim et al., 1986; Bruce et al., 2017; Owens, 2015; Woolcott et al., 2020). The ability to perceive three-dimensional objects, construct them by imagining in mind, not divert attention away from the object in the face of any stimuli, manipulate, move, and rotate objects, and perceive objects from differing viewpoints is known as spatial ability. Spatial ability is defined as the ability to recognize and combine objects by breaking them down into smaller pieces (Owens, 2015). The definition of the concept of spatial ability used in this study was made as describing a construction called "buildings" made of small cubes by using types of representations that are two-dimensional flat view, three-dimensional corner view, and map plan (Ben-Chaim et al., 1986).

In literature, there were studies examining relationships between spatial ability with variables such as mathematics success, attitude towards mathematics, geometry achievement, attitude towards geometry, Van Hiele geometric thinking levels, gender, age, school type, pre-school attendance status, early toy experience, interest in music, frequency of playing computer games, mathematical thinking skill, linguistic situations, cultural settings and ecocultural experiences as well as studies examining students' spatial abilities (Battista, 1990; Fitriyani et al., 2021; Ganley & Vasilyeva, 2011; Gutiérrez et al., 1991; McCoun, 1993; Okamoto, 2014; Owens, 2014, 2015, 2020a; Resnick et al., 2020; Turğut, 2007; Turğut & Yılmaz, 2012; Xie et al., 2019). There were also experimental studies that focused on effect of instructional methods on improving students' spatial abilities (Bofferding & Kocabas, 2021; Choo et al., 2021; Conceição & Rodrigues, 2021; Hannula & Toivanen, 2019; Batdal Karaduman & Davaslıgil, 2019; Lusyana & Setyaningrum, 2018; Newman et al., 2016; Owens, 2020b; Pujawan et al., 2020; Septia et al., 2018; Topraklıkoğlu & Öztürk, 2021; Wahab et al., 2017).

Another crucial factor in geometry success is attitude (Al-ebous, 2016; Sunzuma et al., 2012). Attitude is defined as the affective characteristic of behavior of individuals towards situations they encounter (Al-ebous, 2016; McCoun, 1993). Attitude towards geometry is defined as an orientation that includes all of one's thoughts, feelings and behaviors towards geometry, activities related to geometry, geometry teachers, and individual effects of geometry on students (Al-ebous, 2016).

In literature there were studies examining students' attitudes towards geometry, and studies examining relationships between attitude towards geometry and variables such as geometry readiness, geometry self-efficacy, geometric proof knowledge, geometry achievement, spatial ability, Van Hiele geometric thinking levels, and academic success (Abdelfatah, 2011; Bal, 2012; Cansız Aktaş & Aktaş, 2013; Sevgi & Gürtaş, 2020; Sunzuma et al., 2012; Topraklıkoğlu & Öztürk, 2019). There were also experimental studies investigating the effect of various instructional methods on students' attitudes towards geometry (Al-ebous, 2016; Duatepe, 2004).

Geometric thinking levels of students are also effective in geometry success (Duatepe, 2004). Van Hiele geometric thinking levels are the most well-known (Al-ebous, 2016; Duatepe, 2000, 2004; Gutiérrez et al., 1991; Kılıç et al., 2007; Ma et al., 2015; Misnasanti & Mahmudi, 2018; Pujawan et al., 2020; Van Putten, 2008; Watan & Sugiman, 2018; Wu & Ma, 2005, 2006). Van Hiele geometric thinking levels are associated with early life experiences as well as the quality of someone's education. Age has an insignificant effect on the development of levels (Usiskin, 1982). Throughout their educational careers, students remain at least one Van Hiele geometric thinking level. There is progression between levels, in that students are unable to advance to the next level without completing the previous one (Usiskin, 1982).

Usiskin (1982) defined Van Hiele geometric thinking levels as Level 0 (visual level), Level 1 (analysis), Level 2 (informal deduction), Level 3 (formal deduction, deduction), and Level 4 (seeing relationships, most advanced level, rigor).

Level 0 (Visual Level)

Students perceive objects and models as a whole at the visual level (Usiskin, 1982). Students are unable to recognize properties of objects; instead, they attempt to make sense of what they perceive by observing and imitating from daily life. For example, the definition of a rectangle is a quadrilateral whose opposite sides are equal and parallel to each other, and whose adjacent sides are perpendicular to each other is not particularly useful for students to recognize objects. The more frequently students encounter an object in their daily lives, the more meaningful object becomes to them.

Level 1 (Analysis)

Students compare, categorize, and analyze properties of geometric objects at this level (Usiskin, 1982). Students at this level evaluate object as a whole rather than understanding properties of objects separately. For example, students are unable to know that a rectangle is also a parallelogram.

Level 2 (Informal Deduction)

Students are able to make informal inferences and make connections between objects at this level. For example, students might understand that opposite sides of a rectangle are parallel, and so a rectangle is a special type of parallelogram. Students are able to understand a proof, but they are unable to construct one (Usiskin, 1982). Students understand concepts, but they are unable to make inferences on their own (Usiskin, 1982).

Level 3 (Deduction)

The most significant difference of deduction level from other levels is that students are able to do proof themselves. Students are able to make induction and perceive properties of objects separately (Usiskin, 1982).

Level 4 (Rigor)

Students recognize similarities and differences in different geometric systems at this level. They are able to also transfer and apply Euclidean geometry theorems to non-Euclidean geometry (Usiskin, 1982).

Van Hiele geometric thinking levels range from Level 1 to Level 5, according to Senk (1989). She used Level 1 for visual level, Level 2 for analysis, Level 3 for informal deduction, Level 4 for formal deduction, Level 5 for rigor, and level 0 for students who were not at visual level. We used Senk’s (1989) definition in this study.

In literature, there were studies examining students’ Van Hiele geometric thinking levels and studies examining relationships between Van Hiele geometric thinking levels and variables such as geometry achievement, attitude towards geometry, spatial ability, learning styles, gender, visual proof skills, geometric concept, teachers’ instructional practices and mathematical thinking skills (Duatepe, 2000; Gutiérrez et al., 1991; Guven & Okumus, 2011;

Ma et al., 2015; Polat et al., 2019; Özsoy et al., 2004; Tso & Liang, 2001; Turğut, 2007; Usiskin, 1982; Van Putten, 2008; Watan & Sugiman, 2018; Wu & Ma, 2005, 2006). In addition, there were studies investigating effect of teaching practices to improve students' Van Hiele geometric thinking levels (Duatepe, 2004; Duatepe-Paksu & Ubuz, 2009; Forsythe, 2015; Gal & Lew, 2008; Kılıç et al., 2007).

The relationship between geometric objects such as a construction made of small cubes and their properties should be understood. Visualization, rotation, movement, and remembering when required are relationships in consideration. The development of spatial ability is directly correlated with these (Newman et al., 2016; Topraklıkoğlu & Öztürk, 2021). Ability to construct geometric relationships increases as Van Hiele geometric thinking levels increase (Jones & Tzekaki, 2016; Sinclair et al., 2016).

Attitude towards geometry is one of the most crucial factors in developing geometric relationships and increasing geometry success (Al-ebous, 2016; Sinclair et al., 2016; Sunzuma et al., 2012). Balacheff (1990) stated that spatial ability and Van Hiele geometric thinking levels were critical issues for learning and teaching geometry and suggested that relationships between these concepts and other mathematical abilities should be investigated. Since then, studies have been conducted to reveal relationships between spatial ability and attitudes towards mathematics (Ganley & Vasilyeva, 2011; McCoun, 1993; Yıldırım Gül & Karataş, 2015); relationships between attitudes towards geometry and Van Hiele geometric thinking levels (Bal, 2012), relationships between spatial ability and Van Hiele geometric thinking levels (Gutiérrez et al., 1991; Kösa & Kalay, 2018; Misnasanti & Mahmudi, 2018; Tso & Liang, 2001). Although there were studies in literature examining relationship between spatial ability and attitude towards mathematics (Ganley & Vasilyeva, 2011; McCoun, 1993; Yıldırım Gül & Karataş, 2015), we found one study in literature that reveals relationship between spatial ability and attitude towards geometry (Topraklıkoğlu & Öztürk, 2019). Bal (2012) stated that attitude towards geometry related to geometry achievement and Van Hiele geometric thinking levels. The reason we would like to investigate attitudes towards geometry instead of attitudes towards mathematics was that we thought that attitudes towards mathematics are broader compared to attitudes towards geometry. We also thought that spatial ability, attitude towards geometry and Van Hiele geometric thinking levels had a more direct relationship. Accordingly, we hypothesized that there was a relationship between spatial ability, attitude towards geometry, and Van Hiele geometric thinking levels, therefore we designed this study to investigate relationship between these variables. Researching the relationship between spatial ability, attitude towards geometry and Van Hiele geometric thinking levels will contribute to the field of geometry teaching. In addition, teachers' attention to this relationship while planning their lessons is able to positively affect geometry success (Jones & Tzekaki, 2016). Thus, by taking necessary precautions for the instruction of teachers and teacher candidates, teaching practices regarding relationships between these variables can be planned and implemented (Jones & Tzekaki, 2016; Newcombe, 2010). Investigating whether the relationship between spatial ability, attitude towards geometry, and Van Hiele geometric thinking levels is meaningful in terms of different variables and making suggestions on this subject through interpreting the results may also contribute to field of geometry teaching (Sinclair et al., 2017). In the study, we aimed to investigate what eighth grade students' spatial abilities, attitudes towards geometry and Van Hiele geometric thinking levels were and whether these variables differ according to the students' gender, mathematics success grade and pre-school attendance status. We expressed the study's sub-problems as follows:

1. What were the students' spatial abilities, attitudes towards geometry and Van Hiele geometric thinking levels?

2. Did gender, mathematics success grade and pre-school attendance status affect the students' spatial abilities, attitudes towards geometry scores, and Van Hiele geometric thinking levels?

3. What was the relationship between the students' spatial abilities, attitudes towards geometry scores and Van Hiele geometric thinking levels?

Method

Research Model

We used the exploratory correlational model as the research model in this study. The exploratory correlational research model is used to make explanations by examining relationships between variables (Fraenkel & Wallen, 2012). We investigated relationship between the students' spatial abilities, attitudes towards geometry and Van Hiele geometric thinking levels in the study.

Sample

The study's sample included 429 eighth grade students from seven secondary schools in two cities in Turkey, including 221 females and 208 males. The convenience sampling method was used to select the participants. The convenience sampling method is formed by selecting participants easily to avoid wasting time, money, and labor (Fraenkel & Wallen, 2012). Table 1 summarizes the students' demographic characteristics in the sample.

Table 1 Students' Demographic Characteristics

		Gender		
Variables		Female	Male	Total
Mathematics success grade	0-44	22	20	42
	45-54	33	32	65
	55-69	33	41	74
	70-84	42	36	78
	85-100	91	79	170
Pre-school attendance status	Yes	132	119	251
	No	89	89	178

Data Collection Tools

We used four data collection tools to collect the study's data: personal information form, Spatial Ability Test [SAT] (Turğut, 2007), Attitude Towards Geometry Scale [ATGS] (Cansiz Aktaş & Aktaş, 2013) and Van Hiele Geometry test [VHGT] (Duatepe, 2000). The personal information form to determine the students' demographic characteristics included questions about gender, mathematics success grade and pre-school attendance status. We interrogated the mathematics success scores of students from the previous academic year to determine mathematics success grade.

SAT was Turkish version of MGMP Spatial Visualization Test, which was developed by Ben-Chaim et al. (1986). Turğut (2007) adapted SAT into Turkish. MGMP Spatial Visualization Test consisted of 32 questions with five options (Ben-Chaim et al., 1986). Turğut (2007) modified MGMP Spatial Visualization Test by adding items based on expert opinions instead of using items that were above the ability of secondary school students. Turğut (2007) named this new test as SAT and ensured its validity and reliability. SAT had 31 items with four options before the validity and reliability study, the test included 29 items after the study (Turğut, 2007). The lowest score was 0 while the highest score was 29 from SAT. Turğut (2007) found SAT's KR-20 reliability coefficient as .83. We presented six samples of SAT items in appendix A.

ATGS, which was developed by Cansız Aktaş and Aktaş (2013), had a total of 23 items, 11 of which were negative whereas 12 of which were positive. Responses to positive items were scored as follows: 1- strongly disagree, 2- disagree, 3- undecided, 4- agree, 5- completely agree. Answers given to negative items were vice versa. The lowest score was 23 while the highest score was 115 from ATGS. The scores on ATGS were found by dividing total scores by number of items. 1.00-1.80: "strongly disagree", 1.81-2.60: "disagree", 2.61-3.40: "undecided", 3.41-4.20: "agree", 4.21-5.00: "completely agree" scale was used to evaluate scores on ATGS. Because scores range from 1 to 5, we assumed that students had a strong positive attitude towards geometry as the scores approach 5, and a low positive attitude as the scores approach 1. Cansız Aktaş and Aktaş (2013) found ATGS's Cronbach's alfa reliability coefficient as .89. We gave ATGS items in appendix B.

VHGT was developed by Usiskin (1982) and adapted to Turkish by Duatepe (2000). VHGT consisted of 25 items. Items in range of 1-5 were level 1, items in range 6-10 were level 2, items in range 11-15 were level 3, items in range 16-20 were level 4, and items in range 21-25 were level 5. Students must answer at least three questions at the level correctly to reach the next level (Duatepe, 2000; Usiskin, 1982). We made scoring for a student who answered VHGT as follows: student got 0 point if s/he did not answer at least three questions correctly at any level. Student got 1 point if s/he correctly solved at least three of questions 1-5. Student got 2 points if s/he correctly solved at least three of questions 6-10. Student got 4 points if s/he correctly solved at least three of questions 11-15. Student got 8 points if s/he correctly solved at least three of questions 16-20 correctly. Student got 16 points if s/he correctly solved at least three questions correctly (Usiskin, 1982). The lowest score was 0, while the highest score was 31 from VHGT. Accordingly, if the sum of scores was 0 point, the level was 0. If the sum of scores was 1 point, the level was 1. If the sum of scores was 3 points, the level was 2. If the sum of scores was 7 points, the level was 3; if the sum of scores was 15 points, the level was 4. If the sum of scores was 31 points, the level was 5 (Usiskin, 1982). Duatepe (2000) found VHGT's KR-20 reliability coefficient as .82 for level 1; as .51 for level 2; and .70 for level 3. We presented six samples of VHGT's items from each level in appendix C.

Data Analysis, Validity and Reliability of Data

We coded the study's data and analyzed using statistical package program (SPSS 24). We calculated SAT's KR-20 reliability coefficient as .85, ATGS's Cronbach's alfa reliability coefficient as .82, VHGT's KR-20 reliability coefficients as .73, .75, and .23, respectively for level 1, level 2, and level 3. Because the number of students at level 3 (2 students) was too low, we thought that the reliability coefficient for this level was low. The reliability coefficients found were similar to reliability coefficients found in studies that the tests and scale were developed, and reliability coefficients larger than .70 indicate that data is reliable (Kline, 2016).

We calculated scores on SAT and ATGS, and levels from VHGT and descriptive statistics. In addition, we examined whether the scores demonstrated normal distribution according to independent variables to choose whether to use parametric or non-parametric tests in data analysis (Ghasemi & Zahediasl, 2011; Tabachnick & Fidell, 2013). We examined skewness and kurtosis values to see whether data had a normal distribution. The skewness and kurtosis values should be in range of -1.5 and +1.5 so that data distribution does not deviate from normal distribution (Tabachnick & Fidell, 2013).

Table 2 presents skewness and kurtosis values of scores on SAT and ATGS, and levels from VHGT, as well as skewness and kurtosis values distribution by gender, mathematics success grade, and pre-school attendance status.

Table 2 Skewness and Kurtosis Values

Tests and Scale	Variables		Skewness	SE	Kurtosis	SE
SAT	-		0.028	.118	-0.873	.235
	Gender	Female	-0.024	.164	-0.690	.326
		Male	0.017	.169	-1.072	.336
	Mathematics success grade	0-44	0.889	.365	0.744	.717
		45-54	0.885	.297	0.485	.586
		55-69	-0.090	.279	-0.960	.552
		70-84	-0.075	.272	-0.138	.538
		85-100	-0.440	.186	-0.475	.370
	Pre-school attendance status	Yes	-0.105	.154	-0.775	.306
		No	0.194	.182	-0.900	.362
ATGS	-		-0.160	.118	0.617	.235
	Gender	Female	-0.326	.164	0.504	.326
		Male	0.139	.169	0.617	.336
	Mathematics success grade	0-44	-0.730	.365	1.223	.717
		45-54	-0.853	.297	2.674	.586
		55-69	0.440	.279	2.605	.552
		70-84	-0.716	.272	1.330	.538
		85-100	-0.106	.186	-0.245	.370
	Pre-school attendance status	Yes	-0.323	.154	0.499	.306
		No	0.080	.182	1.043	.362
VHGT	-	-	0.332	.118	-0.793	.235
	Gender	Female	0.321	.164	-0.684	.326
		Male	0.344	.169	-0.888	.336
	Mathematics success grade	0-44	0.457	.365	-0.642	.717
		45-54	0.950	.297	-0.133	.586
		55-69	0.757	.279	0.129	.552
		70-84	-0.454	.272	-1.014	.538
		85-100	-0.213	.186	-1.155	.370
	Pre-school attendance status	Yes	0.300	.154	-0.824	.306
		No	0.334	.182	-0.839	.362

Note. SE: standard error, SAT: Spatial Ability Test, ATGS: Attitude Towards Geometry Scale, VHGT: Van Hiele Geometry test

The data had a normal distribution, according to values in Table 2. We used independent samples t-test to see whether scores on SAT and ATGS, levels from VHGT differed according to students' gender and pre-school attendance status; one-way analysis of variance [ANOVA] to see whether they differed according to their mathematics success grades because data had a normal distribution. T-test is used to evaluate the significance of the difference between the mean scores of two independent groups and ANOVA is used to test the significance of the difference between the mean scores of more than two independent groups. The variables compared in both tests should be continuous and normally distributed in the group to which they belong (Kline, 2016).

We determined the relationship between scores on SAT and ATGS, and levels from VHGT using Pearson's correlation coefficient. The correlation coefficient is used to determine the degree of relationship between two variables with a continuous and normal distribution (Kline, 2016).

We presented the results and interpretations for sub-problems in the following section.

Results

The study's first sub-problem was "What were the students' spatial abilities, attitudes towards geometry, and Van Hiele geometric thinking levels?". We calculated descriptive statistics for scores on SAT and ATGS, and levels from VHGT to answer this question. Table 3 presents descriptive statistics.

Table 3 Descriptive Statistics

Tests	N	NoQ	Minimum score	Maximum score	$\bar{x}$	SD
SAT	429	29	3	29	15.45	5.930
ATGS	429	23	1.35	5	3.24	0.596
VHGT	429	25	0	3	0.84	0.724

Note. N: number of students, NoQ: number of questions, $\bar{x}$: mean, SD: standard deviation, SAT: Spatial Ability Test, ATGS: Attitude Towards Geometry Scale, VHGT: Van Hiele Geometry test

The mean score on SAT was 15.45 (Table 3). Considering that maximum score on SAT available was 29, the students' spatial abilities were moderate. Similarly, Table 3 demonstrated that mean score on ATGS was 3.24. We evaluated this score as 2.61-3.40: "undecided" according to the scale described in the Method section. Accordingly, we thought that the students' attitudes towards geometry were moderate. The mean score on VHGT was 0.84 as Van Hiele geometric thinking levels (Table 3). The students' geometric thinking levels were extremely low according to this result. 151 students were at level 0, 199 students were at level 1 (visual level), 77 students were at level 2 (analysis), and two students were at level 3 (informal deduction) according to VHGT answers. Table 1 presents that, with exception of VHGT test, standard deviation values were not remarkably close to mean scores. The standard deviation of levels from VHGT was close to mean score, which we interpreted as levels from VHGT distributed far from the mean score.

When we examined the students' answers to the SAT one by one, we discovered that the majority of students correctly answered the 17th question (N=338). The 17th question asked how many unit cubes there were in the structure (see Appendix A). The majority of students answered the question correctly because it was an easy question that did not require viewing different perspectives. We discovered that the majority of students (N=352) answered incorrectly in the 28th question, this question was about the appearance of objects from different perspectives (see Appendix A). As a result, it was possible that students struggled to see objects from different perspectives in this question.

When we examined the students' answers on ATGS one by one, we observed that the majority of students marked item "geometry only helps me in exams" (3rd item) (see Appendix B) as "strongly disagree" (N=222) and "disagree" (N=86). The majority of students marked item "geometry helps my perception of objects in my environment" (15th item) (see Appendix B) as "completely agree" (N=70) and "agree" (N=117). We interpreted the students' answers as they consider geometry important and associate it with daily life.

When we examined students' answers on VHGT one by one, the first question (see Appendix C) that the majority of students (N=305) answered correctly includes simple basic geometric thinking. The majority of students (N=384) answered incorrectly in the 25th question (see Appendix C) because the 25th question required advanced geometric thinking.

The study's second sub-problem was "Did gender, mathematics success grade, and pre-school attendance status affect the students' spatial abilities, attitudes towards geometry scores, and Van Hiele geometric thinking levels?" We firstly analyzed female and male students' scores on SAT and ATGS, levels from VHGT to answer this question. Female students' mean score on SAT was 15.14; male students' mean score was 15.17. Female students' mean score on ATGS was 3.23, and male students' mean score was 3.26. Female students' mean level from VHGT was 0.84, male students' mean score was 0.84. We used t-test for independent samples to determine significance of differences in scores (Table 4). We controlled Levene's test for equality of variance for t-test result.

Table 4 T-test Results according to Gender

Tests	Gender	N	x̄	SD	df	t	p
SAT	Female	221	15.14	5.418	427	1.085	.279
	Male	208	15.77	6.428			
ATGS	Female	221	3.23	.636	427	.484	.629
	Male	208	3.26	.552			
VHGT	Female	221	.84	.708	427	.008	.994
	Male	208	.84	.744			

Note. N: number of students, x̄: mean, SD: standard deviation, df: degree of freedom, SAT: Spatial Ability Test, ATGS: Attitude Towards Geometry Scale, VHGT: Van Hiele Geometry test

The mean scores on SAT and ATGS, and levels from VHGT did not differ significantly according to gender (Table 4) (Kline, 2016).

We analyzed the students' scores on SAT and ATGS, and levels from VHGT according to their mathematics success grade (Table 5).

Table 5 Student scores according to the Mathematics Success Grade

Tests	Mathematics success grade	N	x̄	SD
SAT	0-44	42	11.83	5.046
	45-54	65	11.23	4.749
	55-69	74	13.74	5.432
	70-84	78	15.17	4.897
	85-100	170	18.82	5.274
ATGS	0-44	42	3.15	0.576
	45-54	65	3.19	0.490
	55-69	74	3.13	0.495
	70-84	78	3.27	0.554
	85-100	170	3.32	0.683
VHGT	0-44	42	0.57	0.590
	45-54	65	0.49	0.640
	55-69	74	0.78	0.781
	70-84	78	0.68	0.497
	85-100	170	1.13	0.742

Note. N: number of students, x̄:mean, SD: standard deviation, SAT: Spatial Ability Test, ATGS: Attitude Towards Geometry Scale, VHGT: Van Hiele Geometry test

The students' SAT, ATGS and VHGT mean scores differed according to their mathematics success grades (Table 5). We used ANOVA to determine the significance of differences in scores according to their mathematics success grade and we used Scheffe test or Dunnett C test according to the homogeneity of variances for paired comparisons. Scores on SAT met homogeneity of variations criterion, but scores on ATGS and levels from VHGT did not meet the homogeneity of variances criterion. As a result, we used Scheffe test for scores on SAT, and we used Dunnett C test for scores on ATGS and levels from VHGT in paired comparisons (Table 6).

Table 6 ANOVA Results According to Mathematics Success Grade

Tests		Sum of Squares	df	Mean Square	F	p	Significant difference	η^2
SAT	Between groups	3863.037	4	965.759	36.59	**.000**	A-E, B-E, C-E, D-E,	**.291**
	Within Groups	11189.033	424	26.389	7		A-D, B-D	
	Total	15052.070	428					
ATGS	Between groups	2.609	4	0.652	1.849	.119	-	-
	Within Groups	149.595	424	0.353				
	Total	152.205	428					
VHGT	Between groups	27.366	4	6.841	14.70	**.000**	A-E, B-E,	**.163**
	Within Groups	197.213	424	0.465	9		C-E, D-E	
	Total	224.578	428					

Note. df: degree of freedom, A: 0-44, B: 45-54, C: 55-69, D: 70-84, E: 85-100, SAT: Spatial Ability Test, ATGS: Attitude Towards Geometry Scale, VHGT: Van Hiele Geometry test

There was a statistically significant difference between the students' mean score on SAT according to their mathematics success grade, $F(4\text{-}424)=36.597$, $p<.05$, $\eta^2=.291$ (Table 6). We calculated Cohen's d value using highest mean, lowest mean, and pooled standard deviation obtained from ANOVA results (d=1.28). We interpreted Cohen's d value of 1.28 as a large effect size (Cohen, 1988; Lenhard & Lenhard, 2016). The eta square (η^2) value indicated that 29% of variance in scores on SAT was due to mathematics success grade. According to this finding, we were able to state that there was a significant relationship between spatial ability scores and mathematics success grade at a large effect size. When we examined results for pairs of groups, we found that students with higher mathematical achievement had better spatial ability than students with lower mathematical achievement. There was no significant difference between the students' mean score on ATGS according to their mathematics success grade, $F(4\text{-}424)=1.849$, $p>.05$ (Table 6). There was a statistically significant difference between the students' mean level from VHGT according to their mathematics success grades, $F(4,424)=14.709$, $p<.05$, $\eta^2=.163$. We calculated Cohen's d value for levels from VHGT (d=0.884). We interpreted Cohen's d value of 0.884 as a large effect size (Cohen, 1988; Lenhard & Lenhard, 2016). The eta square (η^2) value indicated that 16.3% of variance in levels from VHGT was due to mathematics success grade. According to this finding, there was a significant relationship between Van Hiele geometric thinking levels and mathematics success grade at a large effect size. When we evaluated results for pairs of groups, we discovered that students in the highest mathematical achievement group had higher Van Hiele geometric thinking levels than students in lower mathematical achievement groups.

Finally, we analyzed the students’ scores on SAT and ATGS, and levels from VHGT according to their pre-school attendance status to answer the second sub-problem. We used t-test for independent samples to determine the significance of differences in scores (Table 7). We controlled Levene’s test for equality of variance for the t-test result.

Table 7 T-test Results according to Pre-school Attendance Status

Tests	Pre-school attendance status	N	$\bar{x}$	SD	df	t	p	η^2
SAT	Yes	251	16.46	5.894	427	4.275	**.000**	**.042**
	No	178	14.02	5.700				
ATGS	Yes	251	3.29	0.628	427	1.760	.072	-
	No	178	3.18	0.545				
VHGT	Yes	251	0.89	0.751	427	1.757	.800	-
	No	178	0.76	0.681				

Note. N: number of students, $\bar{x}$: mean, SD: standard deviation, df: degree of freedom, SAT: Spatial Ability Test, ATGS: Attitude Towards Geometry Scale, VHGT: Van Hiele Geometry test

The students’ mean score on SAT differed significantly according to their pre-school attendance status, $t(427)=4.275$, $p<.05$, $\eta^2=0.042$ (Table 9). We calculated Cohen’s d value as .419. We interpreted Cohen’s d value of 0.419 as a small effect size (Cohen, 1988; Lenhard & Lenhard, 2016). The eta square (η^2) value indicated that 4% of variance in scores on SAT was due to pre-school attendance status. As a result, we concluded that spatial ability scores and pre-school attendance status had a statistically significant relationship with a small effect. The students’ mean score on ATGS did not differ significantly according to their pre-school attendance status, $t(427)=1.760$, $p>.05$. We were able to conclude from this result that there was no significant relationship between attitudes towards geometry scores and pre-school attendance status. The students’ mean score on VHGT did not differ significantly according to their pre-school attendance status, $t(427)=1.757$, $p>05$. Accordingly, we found that there was no significant relationship between Van Hiele geometric thinking levels and pre-school attendance.

The third sub-problem was "What was the relationship between the students’ spatial abilities, attitudes towards geometry scores and Van Hiele geometric thinking levels?" We performed simple and partial linear correlation analysis to determine relationships between scores on SAT and ATGS, and levels from VHGT for the answer to this question. Table 8 presents the results of correlation analysis.

Table 8 Correlations Between scores on SAT and ATGS, and levels from VHGT

Variables	N	r	p	η^2
SAT*ATGS	429	.146	.002	-
SAT*VHGT	429	.413	**.000**	**.171**
VHGT*ATGS	429	.140	.004	-

Note. N: number of students, r: Pearson's correlation coefficient, SAT: Spatial Ability Test, ATGS: Attitude Towards Geometry Scale, VHGT: Van Hiele Geometry test

The scores on SAT and levels from VHGT were moderately and significantly related, r=.413, p<.05 (Table 8). Considering determination coefficient r^2=.171 and eta square (η^2) value η^2=.171, we were able infer that 17.1% of variability in Van Hiele geometric thinking levels was due to spatial ability scores. Or we were able to infer that 17.1% of variability in spatial ability scores was due to Van Hiele geometric thinking levels (Lenhard & Lenhard, 2016).

Since we found the highest correlation between scores on SAT and levels from VHGT, we used scores on ATGS as control variable in partial correlation analysis. Table 9 presents findings of partial correlation analysis.

Table 9 Partial Correlation between Scores on SAT and levels from VHGT

Control Variable	Variables	N	r	p	η^2
ATGS	SAT*VHGT	429	.401	**.000**	**.161**

Note. N: number of students, r: Pearson's correlation coefficient, SAT: Spatial Ability Test, ATGS: Attitude Towards Geometry Scale, VHGT: Van Hiele Geometry test

When we kept the scores on ATGS constant, scores on SAT and levels from VHGT were moderately and significantly correlated, r=.401, p<.05, η^2=.161 (Table 9). This correlation (r=.401) between scores on SAT and levels from VHGT was not markedly different from correlation (r=.413) calculated without scores on ATGS controlled. The correlation coefficient decreased slightly. We were able to explain decrease in correlation coefficient by relationship between scores on ATGS and SAT, and by relationship between scores on ATGS and levels from VHGT.

Discussion, Conclusions and Suggestions

A variety of variables such as spatial ability, attitude towards geometry, and geometric thinking levels affect students' ability to work geometrically or learn geometrical reasoning (Jones & Tzekaki, 2016). We examined and compared the students' scores on SAT and ATGS, levels from VHGT in terms of different variables, and investigated relationship between the students' spatial abilities, attitudes towards geometry and Van Hiele geometric thinking levels in this study. We discussed the study's results in this section by comparing them to results of studies in literature and made suggestions for further research.

We found the students' spatial abilities and attitudes towards geometry to be moderate, and their geometric thinking levels were extremely low in the study. There were studies in literature that had similar and different results to those found in this study. Kılıç et al. (2007), Ma et al. (2015), Turğut (2007), Usiskin (1982), and Wu and Ma (2006) found that participants' geometric thinking levels were low. Polat et al. (2019) found that geometric thinking levels were moderate. Topraklıkoğlu and Öztürk (2019) found that students' spatial abilities were moderate and their attitudes towards geometry were positive. As a conclusion, we recommended that similar studies in different samples should be conducted in order to clarify situation.

We reported that the eighth-grade students' scores on SAT and ATGS, and levels from VHGT did not differ according to their gender. The results from SAT and VHGT were similar to studies conducted by Fitriyani et al. (2021), Ma et al. (2015), Turğut (2007) and, Turğut and Yılmaz (2012). However, studies (Battista, 1990; Ganley & Vasilyeva, 2011; Yıldırım Gül & Karataş, 2015) revealed that spatial ability scores differed according to gender. As a result, we were unable to clearly interpret that relationship between gender and spatial ability, as well as Van Hiele geometric thinking levels, and we recommended further studies to examine these relationships.

The scores on SAT and levels from VHGT differed according to mathematics success grades in the study, however scores on ATGS did not differ according to mathematics success grade. Topraklıkoğlu and Öztürk (2019), and Turğut and Yılmaz (2012) found that spatial ability scores differed according to the mathematics success grade. Topraklıkoğlu and Öztürk (2019) also found that attitude towards geometry scores differed according to the mathematics success grade which was similar to this study's findings. According to the results, scores on ATGS and levels from VHGT did not differ according to pre-school attendance status, however scores on SAT did. We concluded that spatial ability and pre-school attendance status had a small relationship. Similar to this study, Turğut (2007) found a relationship between eighth grade students' spatial abilities and their pre-school attendance status. Similarly, Turğut and Yılmaz (2012) found a relationship between seventh and eighth grade students' spatial abilities and their pre-school attendance status, and those who attended to pre-school were more successful than those who did not attend. As a result, we thought that pre-school attendance status might have an effect on students' spatial abilities. But further research is needed to reach a definitive conclusion. In other words, we suggested that a type of research such as a survey, a longitudinal study or experimental research to demonstrate a straight link between pre-school attendance status and development of students' spatial abilities.

There was no study found in literature that examined the relationship between spatial abilities, attitudes towards geometry and Van Hiele geometric thinking levels. We discovered that the strongest relationship was between spatial ability and Van Hiele geometric thinking levels. According to this result, we concluded that students with increased spatial abilities had high Van Hiele geometric thinking levels. Tso and Liang (2001) and Kösa and Kalay (2018) also reported a positive significant relationship between spatial ability and Van Hiele geometric thinking levels, which was similar to this study. Gutierrez et al. (1991) found that teaching using questions in three-dimensional spatial geometry test caused an increase in ninth grade students' Van Hiele geometric thinking levels. Unlike this study, study conducted by Misnasanti and Mahmudi (2018) revealed that there was no significant relationship between spatial ability and Van Hiele geometric thinking levels. As a result, further studies are needed to understand the relationship between students' spatial abilities and their Van Hiele geometric thinking levels. We found a weak and significant relationship between attitudes towards geometry and Van Hiele geometric thinking levels in the study. Bal (2012) revealed a weak relationship between attitudes towards geometry and Van Hiele geometric thinking levels, which was similar to the study.

We also found a weak relationship between spatial ability and attitude towards geometry in the study. Topraklıkoğlu and Öztürk (2019) found a weak relationship between seventh-grade students' spatial ability and attitude toward geometry. Similarly, Ganley and Vasilyeva (2011), and Yıldırım Gül and Karataş (2015) found a statistically insignificant relationship between spatial ability and attitude towards mathematics.

Investigating the relationship between eighth grade students' spatial abilities, attitudes towards geometry and Van Hiele geometric thinking levels might contribute to the field of geometry teaching, considering problems and developments in the field of geometry teaching. Investigating the relationship between students' spatial abilities, attitudes towards geometry and Van Hiele geometric thinking levels will be beneficial as teachers will pay more attention to this relationship while planning their lessons, and students' spatial abilities, attitudes towards geometry and Van Hiele geometric thinking levels will increase.

Eighth grade students participated in the study. Similar studies might be conducted at different grade levels. We used spatial ability test, attitude towards geometry scale and Van Hiele geometric thinking test to collect data for the study. Researchers might use different data collection tools in different studies. Effects of teaching practices aimed at improving students' spatial abilities, attitudes towards geometry, and Van Hiele geometric thinking levels might be investigated by conducting research on relationships between these three variables.

Compliance with Ethical Standards

Disclosure of potential conflicts of interest

This study is part of the master's thesis prepared by the first author under the supervision of the second author.

The authors declare that they have no conflict of interest.

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

CRediT author statement

Zeynep Büşra Uzun: Conceptualization, Validation, Investigation, Resources, Data Curation, Writing - Original Draft, Visualization

Gülcan Öztürk: Conceptualization, Methodology, Validation, Formal analysis, Writing - Review & Editing, Supervision

Research involving Human Participants and/or Animals

Informed consent to participate in the study was obtained from participants and their parent or guardian. Also, legal permission was obtained from the Ministry of National Education in Turkey to conduct the study. Anonymity of the participants was provided in the reporting. The authors signed the declaration stating that there was no need for ethics committee approval and submitted it to the journal because the manuscript is a study produced from a master's thesis using research data before 2020.

Öğrencilerin Uzamsal Yetenekleri, Geometriye Yönelik Tutumları ve Van Hiele Geometrik Düşünme Düzeyleri

Özet:

Bu çalışmanın amacı sekizinci sınıf öğrencilerinin uzamsal yetenekleri, geometriye yönelik tutumları ve Van Hiele geometrik düşünme düzeyleri arasındaki ilişkileri incelemektir. Araştırma keşfedici korelasyonel araştırma modeli ile 429 öğrencinin katılımıyla yürütüldü. Çalışmada öğrencilerin uzamsal yetenekleri ve geometriye yönelik tutumlarının orta düzeyde ve Van Hiele geometrik düşünme düzeylerinin çok düşük olduğu bulunmuştur. Çalışmada ayrıca öğrencilerin uzamsal yetenek puanlarının ve Van Hiele geometrik düşünme düzeylerinin okul öncesi devam durumlarına göre farklılaştığı, cinsiyetlerine göre farklılaşmadığı, geometriye yönelik tutumlarının ise cinsiyet ve okul öncesi devam durumlarından bağımsız olduğu ortaya çıkmıştır. Öğrencilerin uzamsal yetenekleri ve Van Hiele geometrik düşünme düzeyleri, geometriye yönelik tutumlarının pozitif olarak anlamlı bir şekilde ilişkili olduğu da elde edilen sonuçlar arasındadır.

Anahtar kelimeler: uzamsal yetenek, geometriye yönelik tutum, Van Hiele geometrik düşünme düzeyleri, sekizinci sınıf öğrencileri

References

Abdelfatah, H. (2011). A story-based dynamic geometry approach to improve attitudes toward geometry and geometric proof. *ZDM – Mathematics Education, 43*, 441-450. https://doi.org/10.1007/s11858-011-0341-6

Al-ebous, T. (2016). Effect of the Van Hiele model in geometric concepts acquisition: The attitudes towards geometry and learning transfer effect of the first three grades students in Jordan. *International Education Studies*, *9*(4), 87-98. http://dx.doi.org/10.5539/ies.v9n4p87

Bal, A. P. (2012). Teacher candidates' geometric thinking levels and attitudes to geometry. *Journal of Educational Sciences Research*, *2*(1), 17-34. https://dergipark.org.tr/en/pub/ebader/issue/44650/554613

Balacheff, N. (1990). ICME-6 Report of the international group psychology of mathematics education. *Educational Studies in Mathematics, 21*(2), 193-197. http://www.jstor.org/stable/3482481

Batdal Karaduman, G., ve Davaslıgil, Ü. (2019). The effects of differentiated geometry teaching for gifted students on creative thinking, spatial ability level and achievement. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*, *13*(2), 1305-1337. https://doi.org/10.17522/balikesirnef.654451

Battista, M. T. (1990). Spatial visualization and gender differences in high school geometry. *Journal for Research in Mathematics Education, 21*(1), 47-60. https://doi.org/10.2307/749456

Ben-Chaim, D., Lappan, G., & Houang, R. T. (1986). Development and analysis of a spatial visualization test for middle school boys and girls. *Perceptual and Motor Skills, 63,* 659-669. https://doi.org/10.2466/pms.1986.63.2.659

Bofferding, L., & Kocabas, S. (2021). Elementary students' use of spatial thinking strategies in a layered puzzle task. In M. Inprasitha, N. Changsri & N. Boonsena (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education, Vol 1* (pp. 129). PME. https://pme44.kku.ac.th/home/uploads/volumn/pme44_vol1.pdf

Bruce, C. D., Davis, B., Sinclair, N., McGarvey, L., Hallowell, D., Drefs, M., ..., & Woolcott, G. (2017). Understanding gaps in research networks: Using "spatial reasoning" as a window into the importance of networked educational research. *Educational Studies in Mathematics*, *95*(2), 143-161. https://doi.org/10.1007/s10649-016-9743-2

Cansız Aktaş, A. ve Aktaş, Y. D. (2013). The development of a current attitude scale towards geometry. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 7*, 225- 247. https://dergipark.org.tr/tr/pub/balikesirnef/issue/3377/46608

Choo, S., Park, S., & Nelson, N. J. (2021). Evaluating spatial thinking ability using item response theory: Differential item functioning across math learning disabilities and geometry instructions. *Learning Disability Quarterly*, *44*(2), 68-81. https://doi.org/10.1177/0731948720912417

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Erlbaum.

Conceição, J., & Rodrigues, M. (2021). 3D decomposition as a spatial reasoning process: A window to 1st grade students' spatial structuring. In M. Inprasitha, N. Changsri & N. Boonsena (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education, Vol 1* (pp. 136). PME. https://pme44.kku.ac.th/home/uploads/volumn/pme44_vol1.pdf

Duatepe, A. (2000). *An investigation of the relationship between Van Hiele geometric level of thinking and demographic variable for pre-service elementary school teacher* [Unpublished master's thesis]. Middle East Technical University. https://tez.yok.gov.tr/UlusalTezMerkezi/

Duatepe, A. (2004). *The effects of drama based instruction on seventh grade students' geometry achievement, Van Hiele geometric thinking levels, attitude toward mathematics and geometry* [Unpublished doctoral dissertation]. Middle East Technical University. https://tez.yok.gov.tr/UlusalTezMerkezi/

Duatepe-Paksu, A., & Ubuz, B. (2009). Effects of drama-based geometry instruction on student achievement, attitudes, and thinking levels. *The Journal of Educational Research*, *102*(4), 272-285. https://doi.org/10.3200/JOER.102.4.272-286

Fitriyani, H., Kusumah, Y. S., & Turmudi (2021). Spatial reasoning: A survey on the 8th grader students' gain in online learning. *International Journal on Emerging Mathematics Education*, *5*(1), 51-60. http://dx.doi.org/10.12928/ijeme.v5i1.20140

Forsythe, S. K. (2015). Dragging maintaining symmetry: Can it generate the concept of inclusivity as well as a family of shapes? *Research in Mathematics Education*, *17*(3), 198-219. https://doi.org/10.1080/14794802.2015.1065757

Fraenkel, J. R., & Wallen, N. E. (2012). *How to design and evaluate research in education* (8th ed.). McGraw-Hill.

Gal, H., & Lew, H. C. (2008). *Is a rectangle a parallelogram? Towards a bypass of Van Hiele level 3 decision making.* Paper presented at Topic Study Group 18, ICME-11 Monterrey, Mexico, July 2008. https://www.researchgate.net/publication/344351933_Is_a_Rectangle_a_Parallelogram_-Towards_a_Bypass_of_Van_Hiele_Level_3_Decision_Making

Ganley, C. M., & Vasilyeva, M. (2011). Sex differences in the relation between math performance, spatial skills, and attitudes. *Journal of Applied Developmental Psychology, 32*(4), 235-242. https://doi.org/10.1016/j.appdev.2011.04.001

Ghasemi, A., & Zahediasl, S. (2011). Normality tests for statistical analysis: A guide for non-statisticians. *International Journal of Endocrinology Metabolism*, *10*(2), 486-489. https://doi.org/10.5812/ijem.3505

Gutiérrez, A., Jaime, A., & Fortuny, J. M. (1991). An alternative paradigm to evaluate the acquisition of the van Hiele levels. *Journal for Research in Mathematics Education, 22*(3), 237-251. https://doi.org/10.5951/jresematheduc.22.3.0237

Guven, B., & Okumus, S. (2011). 8th Grade Turkish students' van Hiele levels and classification of quadrilaterals. *Proceedings of PME 35, 2*, 473-480. https://www.igpme.org/wp-content/uploads/2019/05/PME35-2011-Ankara.zip

Hannula, M. S., & Toivanen, M. (2019). Making and observing visual representations during problem solving: An eye tracking study. In *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education*. https://researchportal.helsinki.fi/files/129964849/Hannula_RR.docx

Jones, K., & Tzekaki, M. (2016). *Research on the teaching and learning of geometry*. In: Á. Gutiérrez, G. C. Leder & P. Boero (Eds) The second handbook of research on the psychology of mathematics education. Sense Publishers. https://doi.org/10.1007/978-94-6300-561-6_4

Kılıç, Ç., Köse, Y. N., Tanışlı, D., & Özdaş, A. (2007). The fifth grade students' Van Hiele geometric thinking levels in tessellation. *Elementary Education Online, 6*(1), 1305-1337. http://www.ilkogretim-online.org/fulltext/218-1596620431.pdf?1635191185

Kline, R. B. (2016). *Principles and practice of structural equation modelling* (4th ed.). The Guilford Press.

Kösa, T., & Kalay, H. (2018). Evaluation of the learning environment designed for developing 7th grade students' spatial orientation skills. *Kastamonu Education Journal, 26*(1), 83-92. https://dergipark.org.tr/tr/pub/kefdergi/issue/31578/348100

Lenhard, W., & Lenhard, A. (2016). *Calculation of effect sizes*. Psychometrica. https://doi.org/10.13140/RG.2.2.17823.92329

Lusyana, E., & Setyaningrum, W. (2018). Van Hiele instructional package for vocational school students' spatial reasoning. *Beta: Jurnal Tadris Matematika, 11*(1), 79-100. https://doi.org/10.20414/betajtm.v11i1.146

Ma, H. L., Lee, D. C., Lin, S. H., & Wu, D. (2015). A study of Van Hiele of geometric thinking among 1st through 6th. *Eurasia Journal of Mathematics, Science & Technology Education, 11*(5), 1181-1196. https://doi.org/10.12973/eurasia.2015.1412a

McCoun, P. K. C. (1993). *Gender differences in attitudes, spatial visualization ability, and learning styles of remedial mathematics students* [Unpublished master's thesis]. Texas Tech University.

Misnasanti, & Mahmudi, A. (2018). Van Hiele thinking level and geometry visual skill towards field dependent-independent students in junior high school. *Journal of Physics: Conference Series, 1097*, 012133. https://doi.org/10.1088/1742-6596/1097/1/012133

Newcombe, N. S. (2010). Picture this: Increasing math and science learning by improving spatial thinking. *American Educator, 34*(2), 29-43. https://www.aft.org/sites/default/files/periodicals/Newcombe_1.pdf

Newman, S. D., Hansen, M. T., & Gutierrez, A. (2016). An fMRI study of the impact of block building and board games on spatial ability. *Frontiers Psychology*, *7*, 1278. https://doi.org/10.3389/fpsyg.2016.01278

Okamoto, Y., Weckbacher, L. M., & Hallowell, D. (2014). How is spatial reasoning related to mathematical thinking and how important is early exposure to spatial activities? In P. Liljedahl, C. Nicol, S. Oesterle & D. Allan, (Eds.). *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education, Vol 1* (pp. 177). PME. https://www.pmena.org/pmenaproceedings/PMENA%2036%20PME%2038%202014%20Proceedings%20Vol%201.pdf

Owens, K. (2014). Diversifying our perspectives on mathematics about space and geometry: An ecocultural approach. *International Journal of Science and Mathematics Education, 12*, 941-974. https://doi.org/10.1007/s10763-013-9441-9

Owens, K. (2015). *Visuospatial reasoning: An ecocultural perspective for space, geometry and measurement education*. Springer. https://doi.org/10.1007/978-3-319-02463-9

Owens, K. (2020a). Transforming the established perceptions of visuospatial reasoning: Integrating an ecocultural perspective. *Mathematics Education Research Journal, 32,* 257-283. https://doi.org/10.1007/s13394-020-00332-z

Owens, K. (2020b). Noticing and visuospatial reasoning. *Australian Primary Mathematics Classroom, 25*(1), 11-15. https://link.gale.com/apps/doc/A655088074/AONE?u=anon~385de04a&sid=googleScholar&xid=755b3235

Özsoy, N., Yağdıran, E., & Öztürk, G. (2004). Tenth grade students' learning styles and their geometric thinking levels. *Eurasian Journal of Educational Research, 16*, 50–63. http://dspace.balikesir.edu.tr/handle/20.500.12462/5203

Polat, K., Oflaz, G., & Akgün, L. (2019). The relationship of visual proof skills with Van Hiele levels of geometric thinking and spatial ability. *Erciyes Journal of Education, 3*(2), 105-122. https://doi.org/10.32433/eje.604126

Pujawan, I. G. N., Suryawan, I. P. P., & Prabawati, D. A. A. (2020). The effect of Van Hiele learning model on students' spatial abilities. *International Journal of Instruction, 13*(3), 461-474. https://doi.org/10.29333/iji.2020.13332a

Resnick, I., Harris, D., Logan, T., & Lowrie, T. (2020). The relation between mathematics achievement and spatial reasoning. *Mathematics Education Research Journal, 32,* 171-174. https://doi.org/10.1007/s13394-020-00338-7

Senk, S. L. (1989). Van Hiele levels and achievement in writing geometry proofs. *Journal for Research in Mathematics Education*, *20*(3), 309-321. https://doi.org/10.5951/jresematheduc.20.3.0309

Septia, T., Prahmana, R. C. I., Pebrianto, & Wahyu, R. (2018). Improving students spatial reasoning with course lab. *Journal on Mathematics Education, 9*(2), 327-336. https://doi.org/10.22342/jme.9.2.3462.327-336

Sevgi, S., & Gürtaş, K. (2020). Analysis of attitude and self-efficacy of middle school students towards geometry. *Ahi Evran University Journal of Kırşehir Education Faculty, 21*(1), 416-455. https://doi.org/10.29299/kefad.2020.21.01.012

Sinclair, N., Bartolini Bussi, M. G., de Villiers, M., Jones, K., Kortenkamp, U., Leung, A., & Owens, K. (2016). Recent research on geometry education: An ICME-13 survey team report. *ZDM – Mathematics Education, 48,* 1-29. https://doi.org/10.1007/s11858-016-0796-6

Sinclair, N., Bartolini Bussi, M. G., de Villiers, M., Jones, K., Kortenkamp, U., Leung, A., & Owens, K. (2017). Geometry education, including the use of new technologies: A survey of recent research. In G. Kaiser (Ed.). *Proceedings of the 13th International Congress on Mathematical Education* (pp. 277-287). Springer, Cham. https://doi.org/10.1007/978-3-319-62597-3_18

Sunzuma, G., Masocha, M., & Zezekwa, N. (2012). Secondary school students' attitudes towards their learning of geometry: A survey of Bindura urban secondary schools. *Greener Journal of Educational Research, 3*(8), 402-410. https://doi.org/10.15580/GJER.2013.8.051513614

Tabachnick, B. G., & Fidell, L. S. (2013). *Using multivariate statistics* (6th ed.). Pearson.

Topraklıkoğlu, K., & Öztürk, G. (2019). Seventh grade students' spatial ability and their attitudes towards geometry. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 13*(2), 564-587. https://doi.org/10.17522/balikesirnef.539402

Topraklıkoğlu, K., & Öztürk, G. (2021). Teaching geometry through augmented reality activities using three-dimensional modelling. *International Online Journal of Educational Sciences, 13*(5), 1325-1342. https://doi.org/10.15345/iojes.2021.05.003

Tso, T. Y., & Liang, Y. N. (2001). The study of interrelationship between spatial abilities and Van Hiele levels of thinking on geometry of eight-grade student. *Journal of Research in Education Sciences, 46*(1&2), 1-20. https://doi.org/10.6300/JNTNU.2001.46.01

Turğut, M. (2007). *Investigation of 6, 7 and 8 grade students' spatial ability* [Unpublished master's thesis]. Dokuz Eylül University. https://tez.yok.gov.tr/UlusalTezMerkezi/

Turğut, M., & Yılmaz, S. (2012). Investigation of 7. and 8. grade students' spatial ability. *Journal of Dicle University Ziya Gökalp Education Faculty, 19*, 69-79. https://dergipark.org.tr/tr/download/article-file/786964

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry.* CDASSG Project, Chicago University. https://eric.ed.gov/?id=ED220288

Van Putten, S. (2008). *Levels of thought in geometry of pre-service mathematics educators according to the van Hiele Model* [Unpublished master's thesis]. University of Pretoria.

Wahab, R. A., Abdullah, A H., Mokhtar, M., Atan, N. A., & Abu, M. S. (2017). Evaluation by experts and designated users on the learning strategy using SketchUp make for elevating visual spatial skills and geometry thinking. *Bolema: Boletim de Educação Matemática, 31*(58), 819-840. https://doi.org/10.1590/1980-4415v31n58a15

Watan, S., & Sugiman (2018). Exploring the relationship between teachers' instructional and students' geometrical thinking levels based on van Hiele theory. *Journal of Physics: Conference Series 1097 012122*. https://doi.org/10.1088/1742-6596/1097/1/012122

Woolcott, G., Logan, T., Marshman, M., Ramful, A., Whannell, R., Lowrie, T. (2020). The re-emergence of spatial reasoning within primary years mathematics education. In J. Way, C. Attard, J. Anderson, J. Bobis, H. McMaster & K. Cartwright (Eds). *Research in Mathematics education in Australasia 2016-2019*. Springer. https://doi.org/10.1007/978-981-15-4269-5_10

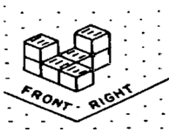
Wu, D., & Ma, H. (2005). A study of the geometric concepts of the elementary school students who are assigned to the Van Hiele level one. *Proceedings of PME 29, 4*, 329-336. https://www.emis.de/proceedings/PME29/PME29RRPapers/PME29Vol4WuMa.pdf

Wu, D., & Ma, H. (2006). The distributions of Van Hiele levels of geometric thinking among 1st through 6th graders. *Proceedings of PME 30, 5*, 409-416. https://www.emis.de/proceedings/PME30/5/409.pdf

Xie, F., Zhang, L., Chen, X., & Xin, Z. (2019). Is spatial ability related to mathematical ability: A meta-analysis. *Educational Psychology Review, 32*, 113-155. https://doi.org/10.1007/s10648-019-09496-y

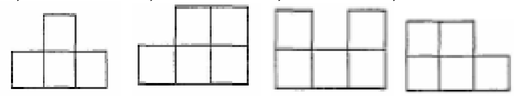
Yıldırım Gül, Ç., & Karataş, İ. (2015). Investigation of correlation among the 8th grade students' achievement on transformation geometry, spatial ability, levels of geometry understanding and attitudes towards mathematics. *Karaelmas Journal of Educational Sciences*, *3*(1), 36-48. http://ebd.beun.edu.tr/index.php/KEBD/article/view/63

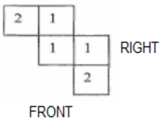
Appendix A. Four samples of SAT items (Turğut, 2007)

2. 

You are given a picture of a building drawn from the FRONT-RIGHT corner. Find the FRONT VIEW.

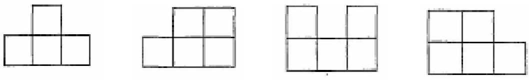
A) **B)** **C)** **D)**

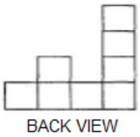


3. 

You are given the map plan of a building. Find the RIGHT VIEW.

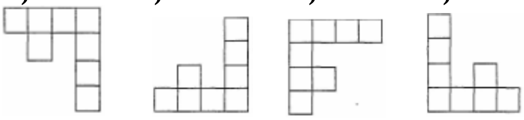
A) **B)** **C)** **D)**

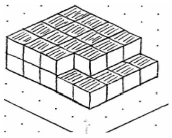


7. 

You are given the BACK VIEW o f a building. Find the FRONT VIEW.

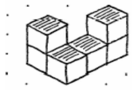
A) **B)** **C)** **D)**



17. 

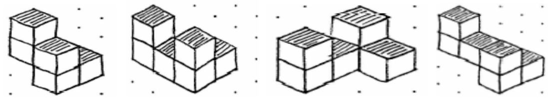
You are given a picture of a building. How many cubes were used in the construction of the building?

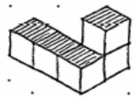
A) 17 **B)** 26 **C)** 35 **D)** 44

26. 

You are given a picture of a building. Find another view of the building.

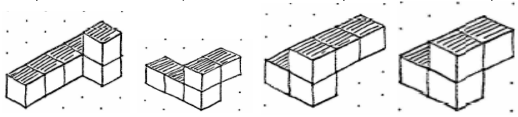
A) **B)** **C)** **D)**



28. 

You are given a picture of a building. Find another view of the building.

A) **B)** **C)** **D)**



Appendix B. English translations of ATGS items (Cansız Aktaş & Aktaş, 2013)

1. I like that a geometric problem can be solved a variety of ways.
2. Geometry is necessary for everyone.
3. Geometry only helps me in exams. (*)
4. I am unable to perform geometric proofs. (*)
5. Geometry helps in our understanding of the World.
6. I find it unnecessary to teach geometry to all students. (*)
7. I can make a geometric drawing of a figure that I see.
8. Geometric knowledge unrelated to real-life information. (*)
9. I can apply my geometric knowledge in daily life.
10. I consider myself successful in geometry.
11. I enjoy solving geometric problems in my spare time.
12. I am not confident in recognizing geometric relationships. (*)
13. I can solve a problem a variety of ways.
14. I am unable to create a geometry formula. (*)
15. Geometry helps my perception of objects in my environment.
16. I am unable to make a relationship between the subjects I learned in geometry. (*)
17. I think that geometry course should be taught only as an elective course. (*)
18. I do not feel comfortable in geometry courses. (*)
19. I can pose a solvable geometric problem.
20. I do not like to participate in conversations about geometry. (*)
21. I would like to increase the weekly course hours for the geometry class.
22. Even though it is a difficult geometric problem, I am confident that I will eventually find a solution.
23. I am unable to apply my geometry knowledge in other courses. (*)

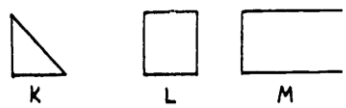
Note. *Item including negative expression

Appendix C. Five samples of VHGT items from each level (Usiskin, 1982)

Level 1

1. Which of these are squares?

(A) K only
(B) L only
(C) M only
(D) L and M only
(E) All are squares.

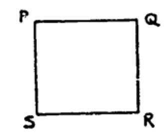


Level 2

6. PQRS is a square.

Which relationship is true in all squares?

(A) $\overline{PR}$ and $\overline{RS}$ have the same length.
(B) $\overline{QS}$ and $\overline{PR}$ are perpendicular.
(C) $\overline{PS}$ and $\overline{QR}$ are perpendicular.
(D) $\overline{PS}$ and $\overline{QS}$ have the same length.
(E) Angle Q is larger than angle R.



Level 3

11. Here are two statements.

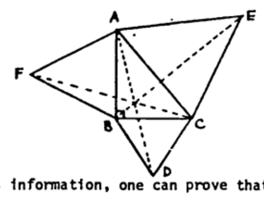
Statement 1: Figure F is a rectangle
Statement 2: Figure F is a triangle.

Which is correct?

(A) If 1 is true, then 2 is true.
(B) If 1 is false, then 2 is true.
(C) 1 and 2 cannot both be true.
(D) 1 and 2 cannot both be false.
(E) None of (A)-(D) is correct.

Level 4

16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.

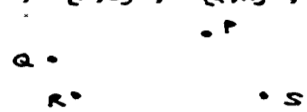


From this information, one can prove that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common. What would this proof tell you?

(A) Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.
(B) In some but not all right triangles, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.
(C) In any right triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.
(D) In any triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.
(E) In any equilateral triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

Level 5

21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R, and S, the lines are {P,Q} , {P,R} , {P,S} , {Q,R} , {Q,S} , and {R,S}



Here are how the words "intersect" and "parallel" are used in F-geometry. The lines {P,Q} and {P,R} intersect at P because {P,Q} and {P,R} have P in common.

The lines {P,Q} and {R,S} are parallel because they have no points in common.

From this information, which is correct?

(A) {P,R} and {Q,S} intersect.
(B) {P,R} and {Q,S} are parallel.
(C) {Q,R} and {R,S} are parallel.
(D) {P,S} and {Q,R} intersect.
(E) None of (A)-(D) is correct.