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On the core of cooperative grey games under bubbly uncertainty

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Abstract

This paper introduces a novel category of cooperative games called cooperative bubbly grey games, in which the value of each coalition is represented by a grey bubble rather than a numerical value. To be exact, rather than using actual numbers, the coalition values are represented as bubbles. Accordingly, we use the cooperative grey game model to overcome the bubbly uncertainty. Both the idea of the bubbly core and the cooperative bubbly game concept are introduced. Grey data may be used as a tool in cooperative grey games to address profit or cost sharing challenges. When considering cooperative grey bubbly games, we can take advantage of the fact that each bubble is characterized exactly by grey numbers, where the logarithmic grey price process is represented with bubbles. Additionally, the bubbly core, a novel solution concept, is presented. In addition, a numerical example is provided along with a necessary criterion for the non-emptiness of the bubbly grey core of such a game. The paper ends with a conclusion and an outlook on future studies.

1. Introduction

In classical cooperative game theory, the rewards for player coalitions are known with certainty; however, when uncertainty is included, the characteristic functions are not real-valued as in the crisp situation. In this case, they represent the uncertainty surrounding the outcomes of collaboration in a variety of ways, including stochastic, fuzzy, interval, and ellipsoidal uncertainty (Alparslan Gök et al., 2023; Granot, 1977; Mallozzi et al., 2011; Özcan et al., 2024; Özcan and Alparslan Gök, 2021;2022;2024; Özcan and Aytar, 2022; Özcan et al., 2023; Suijs et al., 1999; Tirkolaee et al., 2020). This study considers a brand-new type of uncertainty called bubbly grey uncertainty.

A novel approach that concentrates on the analysis of issues involving small samples and incomplete information was developed by Deng, 1982. By creating, exploring, and extracting usable knowledge from what is already accessible, this approach works with uncertain systems with imperfectly understood information. Additionally, the notion of grey systems and information is powerful and has potential real-world applications to benefit society. The notion of grey numbers gave rise to several new mathematical theories, including grey system theory. Grey numbers, one of the cornerstones of grey system theory, have been used by numerous academics to address this ambiguity. Games are more meaningful when a grey number provides decision information. In fact, grey numbers, a cornerstone of grey system theory, have been widely used by academics to handle ambiguity. Games are referred to as cooperative grey games when decision information is provided by grey numbers. The use of grey uncertainty has various applications in cooperative game theory (Olgun et al., 2016; Özcan and Alparslan Gök, 2024; Palancı et al., 2015; Palancı et al., 2017; Ylmaz et al., 2018; Zhang et al., 2005).

In operational research, climate negotiations and policy, environmental management, and pollution control, among other applications, optimization, pattern recognition, and machine learning can all serve as different solution approaches to detect, forecast, and control bubble size, shape, and location. We present a novel mathematical approach for depicting the logarithmic grey pricing process using intervals. More specifically, interval bubbles rather than real numbers are used to symbolize the values of the coalitions. As a result, we employ the cooperative grey game model to address bubbly uncertainty. In cooperative grey games, grey data may serve as a tool to address profit and cost-sharing concerns. We propose a novel and innovative mathematical method in which intervals of the logarithmic pricing process are used to describe this process. Specifically, interval bubbles rather than actual numbers represent the coalition values. In light of this, we develop and use a novel model of cooperative interval games to handle bubbly uncertainty. We benefit from the fact that each bubble is precisely described by a grey number when considering cooperative grey bubbly games, as bubbles are used to represent the logarithmic grey pricing process.

This paper is arranged as follows: Basic concepts and information from the theory of cooperative grey games and grey calculus are presented in Section 2. In Section 3, the cooperative bubbly game concept and the idea of the bubbly core are introduced. Section 4 presents a numerical example and a necessary condition for the non-emptiness of the bubbly grey core of such a game. Finally, Section 5 concludes the paper.

2. Preliminaries

Basic information about cooperative grey game theory and grey calculus is provided in this section (Deng, 1982).

In this case, the precise value of the number is unknown, but it is known within what range it lies. A grey number in applications is typically an interval or a broad range of values. In this study, we take into account interval grey numbers.
A grey number with both a lower limit $\underline{a}$ and a upper limit $\overline{a}$ is called an interval grey number, denoted as $w' \in [\underline{a}, \overline{a}]$.

A cooperative grey game is an ordered pair $< N, w' >$, where $N = \{1, \dots, n\}$ is the set of players, and $w': 2^N \to \mathcal{G}(\mathbb{R})$ is the characteristic function such that $w'(\emptyset) \in [0,0]$, grey payoff function $w'(S) \in [\underline{A_S}, \overline{A_S}]$ refers to the value of the grey expectation gain from coalition cooperation $S \in 2^N$, where $\overline{A_S}$ and $\underline{A_S}$ indicate the coalition of S highest and lowest feasible profits. As a result, a cooperative grey game can be considered of as a crisp cooperative game with grey profits w'. We denote by $\mathcal{G}(\mathbb{R})^N$ the set of all such grey payoff vectors. The collection of all cooperative grey games is represented by $\mathcal{G}G^N$.

Now we address a few concepts from the theory of cooperative grey games.

For $w_1, w_2 \in IG^N$ and $w_1', w_2' \in \mathcal{G}G^N$ we say that $w_1' \in w_1 \leq w_2' \in w_2$ if $w_1'(S) \leq w_2'(S)$, where $w_1'(S) \in w_1(S)$ and $w_2'(S) \in w_2(S)$ and, for each $S \in 2^N$.

For $w_1', w_2' \in \mathcal{G}G^N$ and $\lambda \in \mathbb{R}_+$ we define $< N, w_1' + w_2' >$ and $< N, \lambda w' >$ by $(w_1' + w_2')(S) = w_1'(S) + w_2'(S)$ and $(\lambda w')(S) = \lambda w'(S)$ for each $S \in 2^N$. For $w_1', w_2' \in \mathcal{G}G^N$, where $w_1' \in w_1', w_2' \in w_2$ with $|w_1'(S)| \geq |w_2'(S)|$ for each $S \in 2^N$, $< N, w_1' - w_2' >$ is defined by $(w_1' - w_2')(S) = w_1'(S) - w_2'(S) \in w_1(S) - w_2(S)$.

We call a game $< N, w' >$ *grey size monotonic* if $< N, |w'| >$ is monotonic, i.e., $|w'|(S) \leq |w'|(T)$ for all $S, T \in 2^N$ with $S \subset T$. For further use we denote by $SM\mathcal{G}G^N$ the class of grey size monotonic games with player set N.

We call a game $w \in \mathcal{G}(\mathbb{R})^N$ is said to be superadditive if for all $S, T \subset N$ with $S \cap T = \emptyset$ the next two criteria are hold:

i) $w'(S \cup T) \succcurlyeq w'(S) + w'(T)$,
ii) $|w'|(S \cup T) \geq |w'|(S) + |w'|(T)$.

We call a game $w' \in IG^N$ convex if $w'(S) + w'(T) \preccurlyeq w'(S \cup T) + w'(S \cap T)$ and $|w'|(S) + |w'|(T) \leq |w'|(S \cup T) + |w'|(S \cap T)$ for all $S, T \in 2^N$.

Let $< N, w' >$ be a cooperative grey game. Its grey core looks as follows:

$$\mathcal{C}(w') = \left\{ (w'_1, \dots, w'_n) \in \mathcal{G}(\mathbb{R})^N \mid \sum_{i \in N} w'_i = w'(N), \sum_{i \in S} w'_i \succcurlyeq w'(S), \forall S \in 2^N \setminus \{\emptyset\} \right\}.$$

The interval core, as we observe, consists of those interval payoff vectors that ensure the distribution of the grand coalition's uncertain worth such that each coalition of players could anticipate a marginally increased interval payoff than that coalition could independently anticipate, indicating that no coalition has any reason to disintegrate.

3. On cooperative bubbly grey games and their core

A *bubble* is defined as the difference between the company's fundamental logarithmic price and the risky asset logarithmic price, denoted as $\mathcal{B} = \mathcal{P} - \mathcal{P}^*$. Here $\mathcal{P}$ denotes the risky asset logarithmic price, whereas $\mathcal{P}^*$ represents the fundamental logarithmic price. Furthermore, $\mathcal{P}, \mathcal{P}^* \in \mathcal{G}(\mathbb{R})$. As previously stated, an interval is a specific type of bubble and $\mathcal{B} \in \mathcal{G}(\mathbb{R})$ (Palancı et al, 2014).

A *cooperative grey game under bubbly uncertainty* is an ordered pair $< N, w' >$. Here $N = \{1, \dots, n\}$ is the set of companies, $w': 2^N \to \mathcal{G}(\mathbb{R})$ is the characteristic function which assings to each coalition $S \in 2^N$ a bubble such that $w'(\emptyset) \in [0,0]$, where $\mathcal{G}(\mathbb{R})$ is the set of all nonempty, closed and bounded grey numbers in $\mathbb{R}$.
A coalition's worth S in this research is taken to be the total of gains that the coalition S may realize through an acceptable rearrangement, from which we get the associated bubbly grey game, $< N, w' >$, as follows:

$$w'(S) \in \sum_{i \in S} \mathcal{B}_i.$$

Note that here $\mathcal{B}_i$ is the grey bubble of the i −th company.

With the use of bubbly solution ideas, which link each cooperative bubbly game to a collection of bubble vectors, benefits or price issues in scenarios depicted by cooperative bubbly grey games may be solved. These methods assist disperse the grand coalition's outcome among the businesses once the bubbly uncertainty has been addressed by informing the companies about the ranges of various payoffs produced the grand coalition's collaboration.

Next, the cooperative bubbly grey game core is then introduced.

Definition 1. *Let $< N, w' >$ be a cooperative bubbly grey game. Its bubbly grey core is defined by*

$$\mathcal{C}(w') = \{(\mathcal{B}_1, \dots, \mathcal{B}_n) \in \mathcal{G}(\mathbb{R})^N \mid \textstyle\sum_{i \in N} \mathcal{B}_i = w'(N), \sum_{i \in S} \mathcal{B}_i \succcurlyeq w'(S), \forall S \in 2^N \setminus \{\emptyset\}\}.$$

The example that follows shows how a cooperative bubbly grey game may be derived from a real-world circumstance.

Example 3.1. *We take into account three companies, namely companies 1, 2, and 3, which take collaboration into account. The risky asset logarithmic grey prices of the three companies are* $\mathcal{P}_1 \in [12,17]$, $\mathcal{P}_2 \in [12,17] \in [15,19]$ *and* $\mathcal{P}_3 \in [20,28]$ *million euros, the fundamental logarithmic grey prices of the three companies are* $\mathcal{P}_1^* \in [6,9]$, $\mathcal{P}_2^* \in [11,14]$ *and* $\mathcal{P}_3^* \in [12,18]$, *respectively. Then, the bubbles of these companies are* $\mathcal{B}_1 \in [6,8]$, $\mathcal{B}_2 \in [4,5]$ *and* $\mathcal{B}_3 \in [8,10]$. *Cooperative bubbly grey games may be used to simulate this scenario, where* $N = \{1,2,3\}$ *represents the collection of firms, and the characteristic functions are as follows:*

N	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
w'	$[0,0]$	$[6,8]$	$[4,5]$	$[8,10]$	$[10,13]$	$[14,18]$	$[12,15]$	$[18,23]$

where

$$
\begin{aligned}
w'(\{1\}) &\in \mathcal{P}_1 - \mathcal{P}_1^*, \\
w'(\{2\}) &\in \mathcal{P}_2 - \mathcal{P}_2^*, \\
w'(\{3\}) &\in \mathcal{P}_3 - \mathcal{P}_3^*, \\
w'(\{1,2\}) &\in w'(\{1\}) + w'(\{2\}), \\
w'(\{1,3\}) &\in w'(\{1\}) + w'(\{3\}), \\
w'(\{2,3\}) &\in w'(\{2\}) + w'(\{3\}), \\
w'(\{1,2,3\}) &\in w'(\{1\}) + w'(\{2\}) + w'(\{3\}).
\end{aligned}
$$

Remark 3.1. *One can see that* $([6,8],[4,5],[8,10])$ *is an element of the bubbly grey core and gives* $[6,8]$ *grey payoff to company 1,* $[4,5]$ *grey payoff to company 2 and* $[8,10]$ *grey payoff to company 3.*

We provide the following brief explanations to help you better understand the previously introduced new configurations and options.

Examining logarithmic pricing rather than actual prices is a popular technique in the financial industry and in economics. The geometric evolution of many price processes, which is typically described and approximated by exponential growth mixed with random variation, is one key feature in this. The use of the natural logarithm may be seen as a linearization that converts the geometric progression into a arithmetic one. The analysis is then greatly eased. By removing the slope, or switching to incremental changes in the logarithmic prices, we can show that even this linearity frequently becomes more simple in the presence of leaps, or additional impulsiveness in the dynamics. Then, random variation and leaps are positioned at the level 0 constant value. In our setting of cooperative grey games, the so called Lévy processes are left for further research (Savku and Weber, 2018;2021;2022).

Remark 3.2. *The gap between the lower and higher boundaries of the asset logarithmic grey price is always greater than the difference between the lower and upper bounds of the fundamental logarithmic grey price, which is a fundamental truth in both economics and finance. Since cooperative bubbly grey games fall within this category, the partial subtraction operator used throughout this work is always provided for this class.*

4. Some results on the bubbly grey core

In this section, we provide a crucial requirement for a cooperative bubbly-grey game's core not being empty.

Consider the map $\lambda: 2^N \backslash \{\emptyset\} \to \mathbb{R}_+$ is called a *balanced map* if $\sum_{S \in 2^N \backslash \{\emptyset\}} \lambda(S) e^S = e^N$. Here, e^S is the characteristic vector for coalition S with

$$
e_i^S = \begin{cases} 1, & \text{if } i \in S, \\ 0, & \text{if } i \in N \backslash S. \end{cases}
$$

Definition 4.2. *Let* $< N, w' >$ *be a cooperative bubbly grey game. We say that* $< N, w' >$ *is grey bubbly-balanced (in short:* $\mathcal{GB}$*-balanced) if for each balanced map* $\lambda: 2^N \backslash \{\emptyset\} \to \mathbb{R}_+$ *we have* $\sum_{S \in 2^N \backslash \{\emptyset\}} \lambda(S) w'(S) \preccurlyeq w'(N)$.

Proposition 4.1. *If a cooperative bubbly grey game has a non-empty bubbly core then it is* $\mathcal{GB}$*-balanced.*

We now give an example of how Proposition 4.1 might be applied in a situation with bubbly uncertainty in the reality.

Example 4.2. *Consider the game in Example 3.1. The game features a non-empty bubbly grey center, as shown above. We determine that the game is* $\mathcal{GB}$*-balanced based on Proposition 4.1.*

Before we conclude Section 4, we look into some fundamental characteristics of the bubbly grey core $\mathcal{C}(w')$.

Proposition 4.2. *Let* $< N, w' >$ *be a cooperative bubbly grey game. Then,* $\mathcal{C}(w')$ *is a convex set.*

Proposition 4.3. *Let* $< N, w' >$ *be a cooperative bubbly grey game. Then,* $\mathcal{C}(w')$ *is a superadditive map.*
Proof. We have to prove that $\mathcal{C}(w_1') + \mathcal{C}(w_2') \subseteq \mathcal{C}(w_1' + w_2')$. First, we note that the inclusion holds if $\mathcal{C}(w_1') = \emptyset$ or $\mathcal{C}(w_2') = \emptyset$. Otherwise, we take $(\mathrm{I}_1, \ldots, \mathrm{I}_n) \in \mathcal{C}(w_1')$ and $(\mathcal{J}_1, \ldots, \mathcal{J}_n) \in \mathcal{C}(w_2')$. Then,

$$\sum_{k\in N} \mathrm{I}_k + \sum_{k\in N} \mathcal{J}_k \quad \in \quad w_1'(N) + w_2'(N),$$
$$\sum_{k\in N} (\mathrm{I}_k + \mathcal{J}_k) \quad \in \quad (w_1' + w_2')(N).$$

For each $S \in 2^N\backslash\{\emptyset\}$, $\sum_{k\in S} \mathcal{J}_k \succcurlyeq w_1'(S)$ and $\sum_{k\in S} \mathcal{J}_k \succcurlyeq w_2'(S)$ implying that $\sum_{k\in S} \overline{\mathrm{I}}_k \geq \overline{w}_1'(S)$ and $\sum_{k\in S} \overline{\mathcal{J}}_k \geq \overline{w}_2'(S)$. Then, for every $S \in 2^N\backslash\{\emptyset\}$,

$$\sum_{k\in S} \overline{\mathrm{I}}_k + \sum_{k\in S} \overline{\mathcal{J}}_k \quad \geq \quad \overline{w}_1'(S) + \overline{w}_2'(S),$$
$$\sum_{k\in S} \overline{\mathrm{I}}_k + \overline{\mathcal{J}}_k \quad \geq \quad (\overline{w}_1' + \overline{w}_2')(S).$$

Similarly,

$$\sum_{k\in S} \underline{\mathrm{I}_k} + \underline{\mathcal{J}_k} \geq (\underline{w_1'} + \underline{w_2'})(S).$$

Hence, the bubbly grey core $\mathcal{C}(w')$ is a superadditive map.

5. Conclusions and outlook

We propose a new category of cooperative grey games with bubbly uncertainty in this study. Our work is driven by the requirement for cooperation as well as several real-world sources of uncertainty. Additionally, we present a fresh idea for a solution: the core of cooperative bubbly grey games. This solution approach for cooperative interval games is an extension of the interval core since bubbles make it possible to take player correlations into account and go beyond intervals (Alparslan Gök, 2009;2010, Alparslan Gök et al., 2009).

The Log Periodic Power Law (LPPL), a theory created by D. Sornette and his colleagues, may be used to predict the speculative financial bubbles (Kürüm et al., 2014) described in the book chapter. Here, the crash's most likely timing is determined. Each parameter is predicted using a genetic technique for optimization. The signals given by LPPL before to the crisis in 1987 may be seen through analysis of a 1987 S&P1500 time series. S&P stands for Standard & Poor's in this context, which is an index of dangerous stock prices and benchmark value to contrast with one potentially risky price, for example, evaluating or optimizing a portfolio. Additionally, we show and explore antibubbles, which likewise follow a log-periodic power law but oscillate at an accelerating rate and are typically likely to be bearish. A novel alternative geometrical approach to financial bubbles is also introduced by authors in (Kürüm et al., 2014), and it is supported by machine learning, current optimization, and the theory of inverse issues (Kürüm et al., 2018).

The class of cooperative bubbly grey games is applicable to a variety of operations research and economic challenges, including sequencing issues, cost allocation issues resulting from connection circumstances, supply chain, inventory, and manufacturing processes. It would be intriguing to expand on our findings to investigate collaboration in various Operations Research game scenarios where there is grey bubble ambiguity.

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Conflicts of Interest

The authors declared that there is no conflict of interest.

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