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Reseach Article

Quintic Trigonometric B-spline Algorithm for Numerical Solution of the Modified Regularized Long Wave Equation

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Abstract: This study introduces a new numerical algorithm for solving the modified regularized long-wave (MRLW) equation. To discretize the spatial variables and their derivatives, the collocation technique with quintic trigonometric B-spline functions is utilized and for the temporal derivative, the Adam's Moulton scheme is implemented. The performance and efficiency of the computational algorithm is tested on two sample problems. The error norm L_∞ and three conservation constants are computed and compared with some of those available in the literature. The computed results verify that the suggested algorithm has the advantage in obtaining a highly accurate approximate solution of the MRLW equation as compared to the existing methods. The advantage of the method is that it is easy to implement and requires the low computational cost.

Araştırma Makalesi

Modified Regularized Long Wave Denkleminin Nümerik Çözümü İçin Kuintik Trigonometrik B-spline Algoritması

Anahtar Kelimeler:
Kuintik trigonometrik B-spline,
modified regularized long-wave denklemi,
Kolokasyon metodu

Özet: Bu çalışma, modified regularized long-wave (MRLW) denklemini çözmek için yeni bir sayısal algoritma sunmaktadır. Konumsal değişkenleri ve türevlerini ayrıklaştırmak için kuintik trigonometrik B-spline kolokasyon tekniği kullanılır ve zamansal türev için, Adam's Moulton şeması uygulanır. Sayısal algoritmanın performans ve verimliliği iki örnek problem üzerinde test edilmiştir. L_∞ hata normu ve üç korunum sabitleri hesaplanır ve literatürde mevcut olanlardan bazıları ile karşılaştırılır. Hesaplanan sonuçlar, önerilen algoritmanın, mevcut yöntemlere kıyasla MRLW denkleminin yüksek derecede doğru yaklaşık çözümünü elde etmede avantaja sahip olduğunu doğrulamaktadır. Yöntemin avantajı, uygulanmasının kolay olması ve düşük hesaplama maliyeti gerektirmesidir.

1. INTRODUCTION

The modified regularized long-wave (MRLW) equation is an extension of the regularized long-wave (RLW) equation and expressed as follows:

$$w_t + w_x + \varepsilon w^2 w_x - \mu w_{xxt} = 0, \quad x \in [\alpha, \beta] \qquad (1)$$

with the boundary conditions (BCs)

$$\begin{aligned} w(\alpha,t) &= 0 \qquad w(\beta,t) = 0 \\ w_x(\alpha,t) &= 0 \qquad w_x(\beta,t) = 0 \end{aligned} \qquad (2)$$

$$t \in (0, T]$$

and the initial condition (IC)

$$w(x,0) = f(x), \quad x \in [\alpha, \beta] \qquad (3)$$

where w represents the wave amplitude and ε and μ are positive parameters.

The MRLW equation plays very important role in modelling phenomena that exhibit dispersion waves in combination with weak nonlinearity; such as, pressure waves in a liquid gas bubble mixture, phonon packets in

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nonlinear crystals and nonlinear transverse waves in shallow water. Since the analytical solutions of the MRLW equation are obtained only for restricted solution set of the IC and BCs, the numerical studies have become increasingly important in recent years. Therefore, several computational techniques have been proposed to get the approximate solutions of the MRLW equation. Some of those methods are finite difference technique [1,2,3], the homotopy perturbation approach [4], the second-order Fourier pseudospectral method [5], the new approach based on the homotopy analysis [6], the moving least square collocation approach [7], the meshless method [8], Petrov-Galerkin method [9], subdomain technique with quartic B-spline functions [10], Galerkin method based on various B-spline functions [11,12], collocation approach with various B-splines [13,14,15,16,17,18].
The main objective of the present work is to develop a numerical algorithm to obtain highly accurate numerical solutions of the MRLW equation. For this purpose, this new algorithm is formed by using quintic trigonometric B-spline collocation approach in space and Adam's Moulton scheme in time integration. The rest structure of the paper is as follows: Section 2 deals with the temporal and spatial integration of the MRLW equation. In section 3, two test problems are studied to justify the efficiency and accuracy of the present algorithm. A brief summary about main findings of the suggested algorithm is given in section 4.

2. CONSTRUCTION OF THE PROPOSED ALGORITHM

To set up the temporal and spatial integration of the MRLW equation, the domain $[\alpha,\beta]\times[0,T]$ is discretized by uniform grid points (x_r,t_n) where $x_r=\alpha+rh$, $r=0,1,\dots M$ and $t_n=n\Delta t$, $n=0,1,\dots N$. The quantities h and Δt are the spatial and temporal step widths respectively.

2.1. Temporal Discretization

Suppose that $v=w-\mu w_{xx}$. Then, the Eq. (1) can be rewritten of the form:

$$v_t=(w-\mu w_{xx})_t=-\varepsilon w^2 w_x-w_x \tag{4}$$

For the temporal discretization of Eq. (4), using the following two-step Adam's Moulton Scheme:

$$v^{n+1}=v^n+\Delta t\left(\frac{5}{12}v_t^{n+1}+\frac{2}{3}v_t^n-\frac{1}{12}v_t^{n-1}\right)+O(\Delta t^4) \tag{5}$$

the semi-integrated form of Eq. (4) becomes

$$w^{n+1}-\mu w_{xx}^{n+1}+\frac{5\Delta t\varepsilon}{12}((w^2)^{n+1}w_x^{n+1}+w_x^{n+1}) =w^n-\mu w_{xx}^n-\frac{2\Delta t\varepsilon}{3}((w^2)^n w_x^n+w_x^n)+\frac{\Delta t\varepsilon}{12}((w^2)^n w_x^n+w_x^n) \tag{6}$$

2.1. Spatial Discretization

To carry out the spatial integration of Eq (6), we split the spatial domain $[\alpha,\beta]$ into uniformly M finite elements at the knots

$$\alpha=x_0<x_1<\cdots<x_M=\beta$$

To construct the trigonometric quintic B-spline functions, we need 10 more knots outside the spatial domain $[x_0,x_M]$ as $x_{-5}<x_{-4}<x_{-3}<x_{-2}<x_{-1}<x_0$ and $x_{M+1}<x_{M+2}<x_{M+3}<x_{M+4}<x_{M+5}$. Then, trigonometric quintic B-splines $T_r^5(x)$, $r=-2,\dots,M+2$, at these knots are defined as

$$T_r^5(x)=\frac{1}{\theta}\begin{cases}\psi_1 & x_{r-3}\le x<x_{r-2},\\ \psi_2 & x_{r-2}\le x<x_{r-1},\\ \psi_3 & x_{r-1}\le x<x_r,\\ \psi_4, & x_r\le x<x_{r+1},\\ \psi_5, & x_{r+1}\le x<x_{r+2},\\ \psi_6, & x_{r+2}\le x<x_{r+3},\\ 0, & \text{otherwise}\end{cases} \tag{7}$$

where

$$\begin{aligned}\psi_1 &= \tau^5(x_{r-3}),\\ \psi_2 &= -\tau^4(x_{r-3})\tau(x_{r-1})-\tau^3(x_{r-3})\tau(x_r)\tau(x_{r-2})\\ &\quad-\tau^2(x_{r-3})\tau(x_{r+1})\tau^2(x_{r-2})-\tau(x_{r-3})\tau(x_{r+2})\tau^3(x_{r-2})\\ &\quad-\tau(x_{r+3})\tau^4(x_{r-2}),\\ \psi_3 &= \tau^3(x_{r-3})\tau^2(x_r)+\tau^2(x_{r-3})\tau(x_{r+1})\tau(x_{r-2})\tau(x_r)\\ &\quad+\tau^2(x_{r-3})\tau^2(x_{r+1})\tau(x_{r-1})+\tau(x_{r-1})\tau(x_{r+2})\tau^2(x_{r-2})\tau(x_r)\\ &\quad+\tau(x_{r-3})\tau(x_{r+2})\tau(x_{r-2})\tau(x_{r+1})\tau(x_{r-1})\\ &\quad+\tau(x_{r-3})\tau^2(x_{r-2})\tau^2(x_{r-1})+\tau(x_{r+3})\tau^3(x_{r-2})\tau(x_r)\\ &\quad+\tau(x_{r+3})\tau^2(x_{r-2})\tau(x_{r+1})\tau(x_r)\\ &\quad+\tau^2(x_{r+3})\tau(x_{r-2})\tau(x_{r+2})\tau^2(x_{r-1})+\tau^2(x_{r+3})\tau^3(x_{r-1}),\\ \psi_4 &= -\tau^2(x_{r-3})\tau^3(x_{r+1})-\tau(x_{r-3})\tau(x_{r+2})\tau(x_{r-2})\tau^2(x_{r+1})\\ &\quad-\tau(x_{r-3})\tau^2(x_{r+2})\tau(x_{r-1})\tau(x_{r+1})-\tau(x_{r-3})\tau^3(x_{r+2})\tau(x_r)\\ &\quad-\tau(x_{r+3})\tau^2(x_{r-2})\tau^2(x_{r+1})\\ &\quad-\tau(x_{r-3})\tau(x_{r+2})\tau(x_{r-2})\tau(x_{r+1})\tau(x_{r-1})\\ &\quad-\tau(x_{r+3})\tau(x_{r-2})\tau^2(x_{r+1})\tau(x_r)-\tau^2(x_{r+3})\tau^2(x_{r-1})\tau(x_{r+1})\\ &\quad-\tau^2(x_{r+3})\tau(x_{r-1})\tau(x_{r+2})\tau(x_r)-\tau^3(x_{r+3})\tau^3(x_r),\\ \psi_5 &= \tau(x_{r-3})\tau^4(x_{r+2})+\tau(x_{r+3})\tau(x_{r-2})\tau^3(x_{r+2})\\ &\quad+\tau^2(x_{r+3})\tau(x_{r-1})\tau^2(x_{r+2})+\tau^3(x_{r+3})\tau(x_r)\tau(x_{r+2})\\ &\quad+\tau^4(x_{r+3})\tau(x_{r+1}),\\ \psi_6 &= -\tau^5(x_{r+3}),\end{aligned}$$

and

$$\theta=sin\left(\frac{h}{2}\right)sin(h)\,sin\left(\frac{3h}{2}\right)sin(2h)\,sin\left(\frac{5h}{2}\right),$$

$$\tau(x_r) = sin\left(\frac{x - x_r}{2}\right).$$

Let $W(x,t)$ be approximate solution to the analytical solution $w(x,t)$. Then, $W(x,t)$ can be expressed in the following form:

$$W(x,t) = \sum_{j=-2}^{M+2} \delta_j T_j^5 \tag{8}$$

where $\delta_j(t)$ are time dependent variables which will be calculated by means of the BCs and collocation procedure. Since the each subinterval $[x_{r-1}, x_r]$ is covered by six quintic trigonometric B-spline functions, the unknown function W and its first two spatial derivatives at the knots x_r are obtained as

$$\begin{aligned} W_r &= a_1\delta_{r-2} + a_2\delta_{r-1} + a_3\delta_r + a_2\delta_{r+1} + a_1\delta_{r+2} \\ W_r' &= b_1\delta_{r-2} + b_2\delta_{r-1} - b_2\delta_{r+1} - b_1\delta_{r+2} \\ W_r'' &= c_1\delta_{r-2} + c_2\delta_{r-1} + c_3\delta_r + c_2\delta_{r+1} + c_1\delta_{r+2} \end{aligned} \tag{9}$$

where

$$\begin{aligned} a_1 &= \frac{sin^5\left(\frac{h}{2}\right)}{\theta} \\ a_2 &= \frac{2\,sin^5\left(\frac{h}{2}\right)cos\left(\frac{h}{2}\right)\left(16\,cos^2\left(\frac{h}{2}\right) - 3\right)}{\theta} \\ a_3 &= \frac{2\,sin^5\left(\frac{h}{2}\right)\left(1 + 48\,cos^4\left(\frac{h}{2}\right) - 16\,cos^2\left(\frac{h}{2}\right)\right)}{\theta} \\ b_1 &= \frac{\left(\frac{-5}{2}\right)sin^4\left(\frac{h}{2}\right)cos\left(\frac{h}{2}\right)}{\theta} \\ b_2 &= \frac{-5\,sin^4\left(\frac{h}{2}\right)cos^2\left(\frac{h}{2}\right)\left(8\,cos^2\left(\frac{h}{2}\right) - 3\right)}{\theta} \\ c_1 &= \frac{\left(\frac{5}{4}\right)sin^3\left(\frac{h}{2}\right)\left(5\,cos^2\left(\frac{h}{2}\right) - 1\right)}{\theta} \\ c_2 &= \frac{\left(\frac{5}{2}\right)sin^3\left(\frac{h}{2}\right)cos\left(\frac{h}{2}\right)\left(3 + 16\,cos^4\left(\frac{h}{2}\right) - 15\,cos^2\left(\frac{h}{2}\right)\right)}{\theta} \\ c_3 &= \frac{\left(-\frac{5}{2}\right)sin^3\left(\frac{h}{2}\right)\left(1 + 16\,cos^6\left(\frac{h}{2}\right) - 5\,cos^2\left(\frac{h}{2}\right)\right)}{\theta}. \end{aligned}$$

Substituting Eq. (9) into Eq. (6) gives the fully-integrated form of the MRLW as noted below:

$$\begin{aligned} &\delta_{r-2}^{n+1}\left(a_1 - \mu c_1 + \frac{5\Delta t\varepsilon}{12}b_1((W^2)_r^{n+1} + 1)\right) \\ &+\delta_{r-1}^{n+1}\left(a_2 - \mu c_2 + \frac{5\Delta t\varepsilon}{12}b_2((W^2)_r^{n+1} + 1)\right) \\ &+\delta_r^{n+1}(a_3 - \mu c_3) \\ &+\delta_{r+1}^{n+1}\left(a_2 - \mu c_2 - \frac{5\Delta t\varepsilon}{12}b_2((W^2)_r^{n+1} + 1)\right) \\ &+\delta_{r+2}^{n+1}\left(a_1 - \mu c_1 - \frac{5\Delta t\varepsilon}{12}b_1((W^2)_r^{n+1} + 1)\right) \\ &= W_r^n - \mu(W'')_r^n + \frac{2\Delta t\varepsilon}{3}(W')_r^n((W^2)_r^n + 1) \\ &\qquad - \frac{\Delta t\varepsilon}{12}(W')_r^n((W^2)_r^n + 1) \end{aligned} \tag{10}$$

From the above equation, we get the system consisting of $(M+1)$ equations and $(M+5)$ unknowns. For this system to be solvable, BCs (2) is used to eliminate the parameters

$$\delta_{-2}^{n+1}, \delta_{-1}^{n+1}, \delta_{M+1}^{n+1} \text{ and } \delta_{M+2}^{n+1}$$

from the system so that a solvable system of $(M+1) \times (M+1)$ dimension is achieved. So as to start time evolution of the parameter δ_r^{n+1}, the initial vectors

$$\delta^0 = (\delta_{-2}^0, \delta_{-1}^0, \ldots, \delta_{M+2}^0)^T$$

and

$$\delta^1 = (\delta_{-2}^1, \delta_{-1}^1, \ldots, \delta_{M+2}^1)^T$$

need to be found. The initial vector δ^0 is calculated using the following ICs and BCs

$$\begin{aligned} W'(\alpha,0) &= W''(\alpha,0) = 0 \\ W(x_r,0) &= f(x_r) \\ W'(\beta,0) &= W''(\beta,0) = 0 \end{aligned}$$

where $r = 0,1,..,M$. After the initial vector δ^0 is found, the other initial vector δ^1 is obtained by applying Crank-Nicolson Scheme to Eq (4). Thus, the unknown vector

$$\delta^{n+1} = (\delta_{-2}^{n+1}, \delta_{-1}^{n+1}, \ldots, \delta_{M+2}^{n+1})^T (n = 1,2,\ldots)$$

at any desired time can be iteratively found by utilizing the previous two δ^n and δ^{n-1} unknown vectors. Since the resulting system is an implicit nonlinear system with respect to the term δ, an inner iterative algorithm is employed. In this iteration, before moving the calculation of the next time step approximation for time parameter, we equalize the new δ^{n+1} vectors to the previous δ^{n+1} vectors three times at each time step.

3. RESULTS

In this section, the motion of the single solitary wave and the interaction of two solitary waves are studied to validate the efficiency and applicability of the suggested algorithm. Accuracy of solution is checked by evaluating error norm L_∞

$$L_\infty = \max_m |w_m - W_m|, \tag{11}$$

and the temporal order of convergence is worked out by the formula

$$\text{order} = \frac{log\left|\frac{(L_\infty)_{\Delta t_i}}{(L_\infty)_{\Delta t_{i+1}}}\right|}{log\left|\frac{\Delta t_i}{\Delta t_{i+1}}\right|} \tag{12}$$

where $(L_\infty)_{\Delta t_i}$ represents the error norm L_∞ for temporal step Δt_i. The three invariants corresponding to mass I_1, momentum I_2 and energy I_3 are computed with the help of the following formulas [19]

$$I_1 = \int_{-\infty}^{\infty} w dx \approx \int_{\alpha}^{\beta} W dx,$$

$$I_2 = \int_{-\infty}^{\infty} (w^2 + \mu(w_x)^2)dx \approx \int_{\alpha}^{\beta} (W^2 + \mu(W_x)^2)dx, \quad (13)$$

$$I_3 = \int_{-\infty}^{\infty} (w^4 - \mu(w_x)^2)dx \approx \int_{\alpha}^{\beta} \left(W^4 - \frac{6\mu}{\varepsilon}(W_x)^2\right)dx.$$

In the computation process, the numerical values of the invariants are evaluated by using the trapezoidal rule.

3.1. The Motion of The Single Solitary Wave

In this test problem, single solitary wave solution of the MRLW equation has of the form:

$$w(x,t) = \sqrt{\frac{6c}{\varepsilon}} sech[k(x - \tilde{x}_0 - (c+1)t)] \quad (14)$$

where $k = \sqrt{\frac{c}{\mu(c+1)}}$, the velocity of the solitary wave is $c+1$, $A = \sqrt{\frac{6c}{\varepsilon}}$ is amplitude of the wave and $\tilde{x}_0$ is the initial peak position. The BCs are taken zero at both ends and the IC is obtained as

$$w(x,0) = \sqrt{\frac{6c}{\varepsilon}} sech[k(x - \tilde{x}_0)] \quad (15)$$

The analytical values of invariants are computed using IC (15) in the integrals I_1, I_2, I_3 as follows:

$$I_1 = \frac{\pi A}{k},$$

$$I_2 = \frac{2A^2}{k} + \frac{2k\mu A^2}{3},$$

$$I_3 = \frac{4A^2}{3k\varepsilon}(A^2\varepsilon - 3\mu k^2).$$

The numerical simulations are performed in the spatial domain $[0,100]$ and the time period $[0,10]$ with the parameters $\varepsilon = 6$, $\mu = c = 1$, $\tilde{x}_0 = 40$. The solitary wave profile is displayed in Figure 1 at different time levels. From Figure 1, it is observed that the solitary wave propagates to the right keeping its original shape. A comparison of the results obtained by the proposed algorithm with the some existing techniques given in [2,3,7,11,12,13,14,15] is provided in Table 1. Comparison verifies that the suggested algorithm gives much better results than the other techniques given in Table 1. The conservation invariants, the temporal rate of convergence and the error norm L_∞ are given in Table 2. It can be noticed from Table 2 that for the fixed space step, when the temporal step size is decreased from 0.1 to 0.0125, the temporal order of convergence is almost three and the calculated invariants are in almost good agreement with their theoretical values. The plot of absolute error with $h = 0.2$, $\Delta t = 0.025$ is exhibited in Figure 2.
As seen from Figure 2, the obtained error is compatible with the result given in Table 1.

Table 1. Comparison of L_∞ and invariants at $t = 10$ with $h = 0.2$, $\Delta t = 0.025$

Method	I_1	I_2	I_3	$L_\infty \times 10^3$
Present	4.44288	3.30001	1.41440	0.12
[2]	4.44288	3.29874	1.41532	4.84
[3]	4.44288	3.29874	1.41531	1.65
[7]	4.44288	3.29979	1.41416	0.80
[11]	4.44318	3.30030	1.41469	1.08
[12]	4.44288	3.29983	1.41421	0.85
[13]	4.44288	3.29979	1.41415	9.06
[14]	4.44288	3.29983	1.41420	5.44
[15]	4.44518	3.30248	1.41741	1.25
Exact	4.44288	3.29983	1.41721	

Table 2. The error norm, invariants and temporal order of convergence at $t = 10$ with $h = 0.1$

Δt	L_∞	$Order$	I_1	I_2	I_3
0.1	6.92×10^{-3}		4.44279	3.31103	1.42543
0.05	8.96×10^{-4}	2.95	4.44287	3.30123	1.41562
0.025	1.15×10^{-4}	2.97	4.44288	3.30001	1.41439
0.0125	1.47×10^{-5}	2.96	4.44288	3.29985	1.41424
Exact			4.44288	3.29983	1.41421

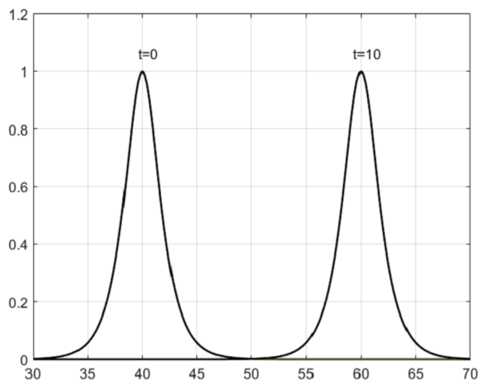

Fig. 1. $W(x,t)$ at $t = 0,10$ with $h = 0.2$, $\Delta t = 0.025$

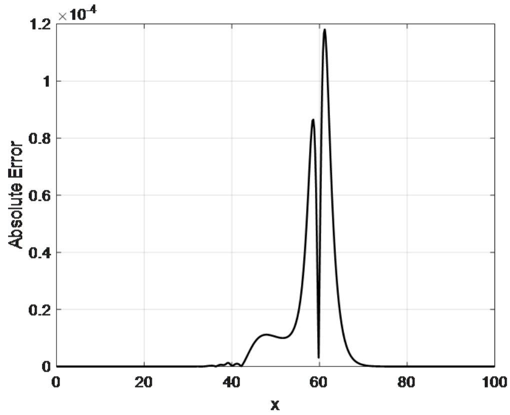

Fig. 2: Absolute error for the suggested method

3.2. The Interaction of Two Solitary Waves

In the problem of interaction of two solitary waves, the following IC is tackled

$$w(x,0) = A_1\, sech(k_1[x - \tilde{x}_1]) + A_2\, sech(k_2[x - \tilde{x}_2]) \quad (15)$$

where $A_i = \sqrt{\frac{6c_i}{\varepsilon}}$, $k_i = \sqrt{\frac{c_i}{\mu(c_i+1)}}$, $i = 1,2$ and c_i, x_i are arbitrary constants.

For computational work, the parameters are selected as $\varepsilon = 6$, $\mu = 1$, $c_1 = 4$, $c_2 = 1$, $x_1 = 25$, $x_2 = 55$, $\Delta t = 0.02$ and $h = 0.2$ in the space domain $[0,150]$ and the time period $0 \le t \le 20$. Using these parameters, two singular waves, which the initial peak positions are $x = 25$ and 55, are obtained as seen in Figure 3 . As the amplitude of the first solitary wave is small compared to the second one, the collision of those takes place around time $t = 9$. Also, It is seen from Figure 3 that these waves, which are separated from each other, then maintain their amplitudes.

The analytical values of the invariants are found as

$$I_1 = \frac{\pi}{k_1 k_2}(k_2 A_1 + k_1 A_2)$$

$$I_2 = \frac{2}{k_1 k_2}(k_2 A_1^2 + k_1 A_2^2) + \frac{2\mu}{3k_1 k_2}(k_1^2 k_2 A_1^2 + k_1 k_2^2 A_2^2)$$

$$I_3 = \frac{4}{3k_1 k_2 \varepsilon}(\varepsilon k_1 A_2^4 - 3\mu k_1 k_2^2 A_2^2 + \varepsilon k_2 A_1^4 - 3\mu k_1^2 k_2 A_1^2)$$

The computed invariants is provided in Tables 3 at various time levels. It can be obviously seen that the numerical values of the invariants are compatible with those of analytical values during the interaction process.

Table 3. Invariants for interaction of two solitary waves with $h = 0.2$, $\Delta t = 0.02$ at various time levels.

t	I_1	I_2	I_3
0	11.4677	14.6292	22.8805
4	11.4677	14.6292	22.8804
8	11.4677	14.6293	22.8804
9	11.4677	14.6293	22.8804
12	11.4677	14.6292	22.8805
16	11.4677	14.6292	22.8805
20	11.4677	14.6292	22.8805
Exact	11.4677	14.6292	22.8805

4. DISCUSSION AND CONCLUSION

In the present work, a new numerical algorithm has been introduced to get approximate solution of the MRLW equation. Third-order two-step scheme is first proposed to discretize the temporal variable and then collocation approach with quintic trigonometric B-splines is used to get the fully integrated form of the MRLW equation. To see the performance and efficiency of the suggested algorithm, the motion of the single solitary wave and interaction of two solitary waves are examined. The accuracy of the proposed algorithm is checked by working out error norm L_∞ and the obtained results are compared with the techniques available in the literature. The computed results confirm that the error norm L_∞ obtained by the suggested algorithm is markedly less than those of the earlier studies found in the literature. The conservation invariants and the rate of temporal convergence are numerically computed and seen to be consistent with their analytical values. In conclusion, the quintic trigonometric B-spline collocation approach together with Adam's Moulton scheme can be successfully applied to nonlinear problems.

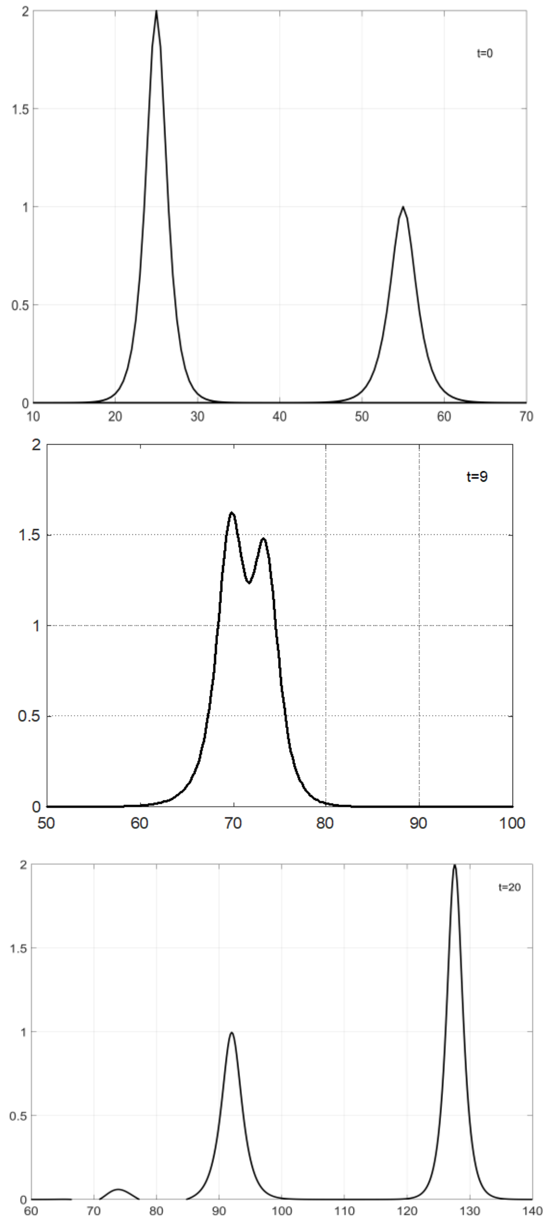

Fig. 3: The simulation of interaction process.

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