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UNCERTAINTY AROUND CURRENCY CRISIS: TYPE-2 FUZZY SYSTEM MODELLING

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Abstract:

Different analyses and methods have been employed to capture the timing and the effects of crises on foreign exchange markets. Fuzzy System Modelling (FSM) has been applied and successfully captures the crisis. In this paper, we further explore the uncertainty in the results of FSM findings. In this approach it is assumed that the only uncertainty resource is the level of fuzziness. Once optimum number of clusters and level of fuzziness are obtained by RMSE supervision, uncertainty component of level of fuzziness is analyzed. Results and reflection of this kind of uncertainty are shown.

Özet:

Döviz Krizlerinin Tahminindeki Belirsizlikler: Tip-2 Bulanık Sistem Modellemesi

Döviz piyasalarında gözlemlenen krizlerin zamanlanması ve etkilerini araştırmak için bir çok analiz ve metod denenmiştir. Bulanık Sistem Modeli (BSM) bu çerçevede denenmiş ve kriz başarılı olarak tahmin edilmiştir. Bu çalışmada, BMS bulguları içerisinde belirsizliğin yansımaları tetkik edilmiştir. Bu yaklaşımda belirsizliğin tek kaynağı bulanıklık seviyesi olarak varsayılmıştır. İlk olarak en uygun yığın sayısı RMSE danışımı altında bulunmuş ve bulanıklık seviyesindeki belirsizlik analiz edilmiştir. Bu tür belirsizliğin sonuçları ve yansımaları gösterilmiştir.

Keywords: Uncertainty, entropy, Fuzzy System Modelling, inference.
Anahtar Sözcükler: Belirsizlik, antropi, Bulanık Sistem Modellemesi, çıkarım.

INTRODUCTION

Uncertainty is a natural part of most of, if not all of, economical and financial research. As a matter of fact, uncertainty is the essential part of every business. Simply, without uncertainty it is difficult to speak about decision, opportunity, risk, etc. Uncertainty is the basis of freedom, or in other words, it makes us free to choose (Bernstein, 1998).

Uncertainty measures are still evolving and new tools are emerging. Fuzzy Logic is an example of the evolution of logic which creates a superset of crisp sets to capture uncertainty. In this sense, crisp sets become a special case of Fuzzy Sets. In this paper a type of uncertainty, specifically fuzziness is examined briefly. Furthermore, the reflection of this type of uncertainty in inference is shown by an application of a fuzzy model which is used to capture the currency crisis in Turkey (Ozkan, Turksen and Aktan, 2003).

I. UNCERTAINTY

There has been a growing debate about the meaning of uncertainty and its measures. Keynes (1937) stated that:

> "*By `uncertain' knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty...The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence...About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know.*" (J.M. *Keynes*, 1937)"

It is clear that uncertainty is a form of information deficiency and as Klir's (1995) definition:

> "*Uncertainty involved in any problem solving situation is a result of some information deficiency. Information may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other way.... Information measured solely by the reduction of uncertainty does not capture the rich notion of information in human communication and cognition.* (1995: 245)."

Probability theory has been long established and used widely to deal with uncertainty. Formulations can be linked to Jacob Bernoulli's (1713) "***Law of Large Numbers***" that may be seen as the basis of ***frequentist*** view. Another oldest classical view perhaps stated by Pierre Simon de Laplace (1795). Classical view of probabilities can be classified as '*objective*' probabilities or better to say probabilities exist '*inherently*' in nature. The "***relative frequency***" view argues that the probability of a particular event in a particular trial is the relative frequency of occurrence of that event in an infinite sequence of "*similar*" trials (Richard von Mises, 1928, Hans Reichenbach, 1949). The "***propensity***" view of objective probabilities argues that probability represents the disposition or tendency of Nature to yield a particular event on a single trial, without it necessarily being associated with long-run frequency. It is important to note that these "*propensities*" are assumed to *objectively* exist (Charles S. Peirce ,1910, Karl Popper, 1959). In "*epistemic*" view, probabilities are really a measure of the *lack of knowledge* about the conditions which might affect the coin toss and thus merely represent our *'beliefs'* about the experiment. Knight expressed that "*if the real probability reasoning is followed out to its conclusion, it seems that there is `really' no probability at all, but certainty, if knowledge is complete*." (Knight, 1921: 219). Epistemic view can be traced to T. Bayes (1763) and can be divided into "***logical relationist***" (Keynes, 1921; Carnap, 1950) and "***subjectivist***" (F. Ramsey, 1926, De Finetti, 1931) view.

"*Probabilistic uncertainty*" was first established by Claude Shannon (1948) which is known as Shannon's Entropy measures. Let x_i be a discrete random variable taking a finite number of possible values $x_1, x_2,.., x_n$ with probabilities $p_1, p_2,..., p_n$ respectively such that $p_i \geq 0$, i=1,2,...,n and $\sum p_i = 1$. We attempt to arrive at a number that will measure the amount of uncertainty. Let 'h' be a function defined on the interval (0,1] and h(p) be interpreted as the uncertainty associated with the event $X=x_i$,i=1,...,n or the information conveyed by revealing that X has taken on the value xi in a given performance of the experiment. For each *n*, we shall define a function H_n of the *n* variables $p_1,p_2,...,p_n$. The function $H_n(p_1,p_2,...,p_n)$ is to be interpreted as the average uncertainty associated with the event $\{X=x_i, i=1,...,n\}$ given by

$$H_n(p_1, p_2,.., p_n) = -C\sum_{i=1}^{n} p_i \log_b(p_i) \quad (1)$$

where, C>0, b>1 and (0)logb(0) is forced to be zero, and H_n is a form of Hartley function (Hartley, 1928). For C=1 and b=2 this function becomes:

$$H_n(p_1, p_2,.., p_n) = -\sum_{i=1}^{n} p_i \log_2 p_i \quad (2)$$

which is known as Shannon's entropy or measure of uncertainty.

Uncertainty measure can be extended to fuzzy sets beyond crisp sets. It is possible to define, simple uncertainty, joint uncertainty, conditional uncertainty and if uncertainty expressed by sets of possible alternatives then Hartley function is well characterized by term *nonspecifity* (Klir, 1995).

Fuzzy Logic is first introduced by L. A. Zadeh (1965). It can be seen as the first attempt to extend classic logic and set theory. Following Zadeh, Fuzzy System Modelling (FSM) has been increasingly applied to various problems in such areas as computer science, system analysis, electronic engineering, and pharmacology. There are at least two advantages of FSM that attracts researchers in social sciences. These advantages are; (i) its power of linguistic explanation with resulting ease of understanding, and (ii) its tolerance to imprecise data which gives it flexibility and stability for prediction. Although the number of its application is limited for the time being, it seems that its use is increasing. Desgupta and Deb (1996), Richardson (1998) are two examples in the field of social choice. A few examples can also be found in the field of Econometrics, fuzzy game and in economic analysis: Giles and Draeseke (2001), Lindstrom (1998), Tseng at al (1998), Ozawa et al (1997).

Fuzzy set involves a type of uncertainty that is called *fuzziness*. A sensible measure of fuzziness, f, must satisfy some requirements of the degree of fuzziness. Three essential requirements are (Klir 1995):

- $f(X)=0$ iff X is a crisp set
- $f(X)$ is maximum when fuzziness becomes highest
- $f(Y) \leq f(Z)$ if Y is sharper than set Z.

There are three basic types of uncertainty, specifically, *nonspecifity* (imprecision), fuzziness, strife. All these uncertainty types are involved in Fuzzy System Modelling approaches with different measures. These measures are not going to be given in detail because they are out of this papers scope.

II. REFLECTION OF UNCERTAINTY:

Sources of uncertainty may be classified as:

- Model Uncertainty
- Parameter Uncertainty
- Data Uncertainty

In this paper one type of parameter uncertainty, perhaps important one, namely level of fuzziness in FSM will be investigated and its affect on inference will be shown by using the model constructed for predicting the currency crisis experienced in Turkey (Ozkan, Turksen and Aktan, 2003).

Model is basically:

$$\Delta F_t = Q(\Delta R_{t-2}, \Delta R_{t-3}, \Delta FDT_{t-1}, \Delta FDT_{t-2}) \quad (3)$$

where ΔF_t is change in TL/USD exchange rate, R_t, is O/N (Overnight interest rate) and FDT is "foreign deposits in deposits money banks", where all are monthly data. In this investigation, a Rule Based Fuzzy System Modelling (RBFSM) method is applied (Ozkan, 2003):

- Set search values for cluster and level of fuzziness, cluster list $C=\{c_1, c_2, .., c_{nc}\}$, level of fuzziness list $M=\{m_1, m_2, \ldots, m_{nm}\}$ and α_{cut}
- Do FCM clustering by using (n+1) dimension (input+output, {X|Y} space). Apply Mahalonobis distance as similarity measure. Hence cluster centers can be represented as:

$$v^m_{x|y,c} = (x^m_{1,c}, x^m_{2,c}, \ldots, x^m_{n,c}, y^m_c), \quad (4)$$

where

m: the level of fuzziness

c: c^{th} cluster,

$x^m_{i,c}$: is the i^{th} dimension (input variable) of c^{th} cluster center for the level of fuzziness,

$v^m_{x|y,c}$: is the c^{th} cluster center in the input-output space.

The values of the cluster centers to input space are:

$$v^m_{x,c} = (x^m_{1,c}, x^m_{2,c}, \ldots, x^m_{n,c}) \quad (5)$$

- For each cluster and level of fuzziness

- Membership values for each input vector are re-calculated by using projected input space cluster centers, $v_{x,c}^{nc,m}$. Least Square Estimation of Regression coefficients for each cluster are generated by calculated membership values for each cluster and {X|Y}. This causes a separation of clusters (either by using alpha-cut or zero, where we use zero) and generates Linear Regression function for each cluster. As a mathematical representation for each cluster we have:

$$(\overline{d\mu_{x,c}^{m}})\overline{Y} = (\overline{d\mu_{x,c}^{m}})\overline{X} \tag{6}$$

and $\overline{d\mu_{x,c}^{nc,m}}$ is a diagonal matrix that has the form,

$$\overline{d\mu_{x,c}^{m}} = \begin{vmatrix} \mu_{1,c}^{m} & 0 & \dots & 0 \\ 0 & \mu_{2,c}^{m} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \mu_{k,c}^{m} \end{vmatrix} \tag{7}$$

Where,

k is the number of data vectors, c is the number of clusters,
$\overline{X}$: mxn Input data (explanatory variables),
$\overline{Y}$: mx1 Output (dependent variable)

Delete all vectors $\{X_k|Y_k\}$ where $\mu_{c,k} \le \alpha_{cut}$ and obtain a subset of the data.

Obtain a regression function for each cluster by using LSE such as:

$$y_c^m = \beta_0^m + \beta_1^m x_1 + ... + \beta_n^m x_n = f_c^m(x_1,...,x_n) \tag{8}$$

where c=1..nc, i.e., there is a regression equation for each fuzzy cluster.

- Calculate Root Mean Square Error by using Inference as:

$$Y_k^m = \sum_{c=1}^{nc} \mu_{c,k} y_c \text{ and} \tag{9}$$

$$RMSE_{nc,m} = \sqrt{\frac{1}{k}\sum_{j=1}^{k}(Y_j^m - Y_j)^2} \tag{10}$$

where, Y_j is the j[th] observation and $Y_j^{nc,m}$ is the j[th] prediction when m is the level of fuzziness.

- Select, $\{c^*, m^*\}$ pair based on minimum $RMSE_{nc,m}$, for c ∈ C and m ∈ M
- Use Test data to validate model performance.

This approach is a supervised learning approach where the number of clusters and the level of fuzziness are both determined by supervision of RMSE measure.

The optimum values of (m^*, c^*) are found as (1.3 and 5) in this investigation for currency crisis in TL/USD monthly values analysis. In order to generate Type-2 membership values and their affect on inference, number of cluster, c, is kept constant and list of level of fuzziness is used as m={1.01, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0}.

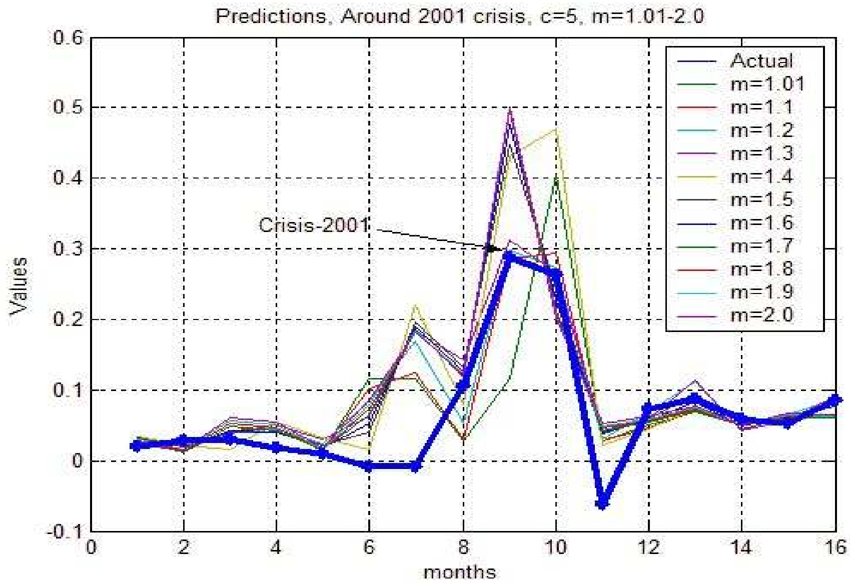

Figure 1. RBFSM Predictions for 2001 Currency Crisis.

The degrees of overlapping of clusters are changed by the level of fuzziness, m. Intuitively it is possible to analyze the uncertainty called fuzziness by changing the level of fuzziness. Similar works has been done in KIS laboratory at Toronto University[1] for different type of applications. Although other sources directly effect the parameters of the model it is very difficult to separate and quantify all sources of uncertainty. In estimating the overall uncertainty, it may be necessary to take each source of uncertainty and treat it separately to obtain the contribution from that source.

It can be seen in Figure 1, model predicts large currency change before crisis happens. Another striking point is that the predictions generated for different levels of fuzziness values are more spread around the crisis month. The difference between minimum and maximum predictions starts to increase two months before and reaches its maximum value at crisis month. This can be seen more accurately from the membership values of these data vectors.

Table 1. Membership Values of Data Vectors (Italic Bold ones are for the crisis month) for c=5 and m=1.3

Months	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	% USD/TRL Change
-4	0.984	0	0	0.013	0.003	1.02%
-3	0.238	0.574	0.005	0.088	0.094	-0.80%
-2	0.128	0.524	0.21	0.077	0.061	-0.97%
-1	0.281	0.072	0.161	0.354	0.132	10.48%
Crisis	***0.146***	***0.288***	***0.27***	***0.143***	***0.153***	***28.85%***
+1	0.161	0.167	0.368	0.155	0.149	26.26%
+2	0.233	0.013	0	0.369	0.384	-6.17%
+3	0.034	0.001	0	0.955	0.01	7.22%
+4	0.007	0	0	0.11	0.882	8.82%
+5	0.002	0	0	0.996	0.002	5.92%
+6	0.009	0	0	0.983	0.007	5.32%

In Table 1 membership values of the vectors for each month around crisis together with their cluster numbers when there are 5 clusters in the model, and level of fuzziness is 1.3. It is seen that these values are well spread for crisis month although the level of fuzziness is 1.3, eventhough, it is fairly close to the crisp clustering where m=1.

[1] Uncu (2003), Ozkan-Turksen (2003) have been working experimentally.

There are five clusters in this model. And each cluster has its own regression equation. Alpha-cut is used as zero. By changing the level of fuzziness, degree of overlapping is changed and hence it is possible to see the affect of fuzziness on regression coefficients. However it should be noted that crisis should generate a one specific cluster since it has a large change in currency and the methodology used for clustering is 'n+1' dimensional ({input|output}) clustering, i.e., means the effect of output is included during clustering. Therefore for the cluster that indicates the month of a crisis, it may not be possible to generate regression coefficients.

Figures 2, 3, 4, and figure 5 show the behaviour of each coefficient and Figure 6 shows 3-dimensional view of all predictions.

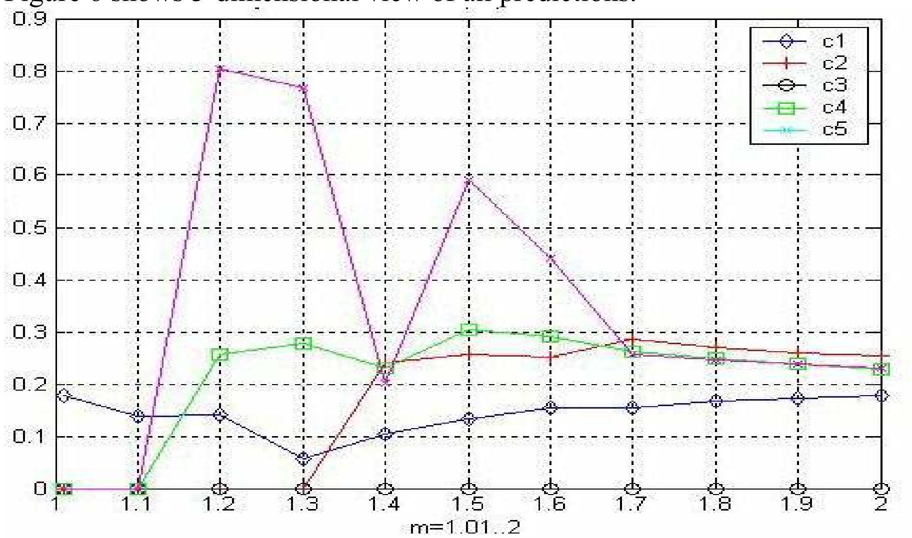

Figure 2. Δfdt_{t-1} Coefficient Values for Different Clusters and Level of Fuzziness

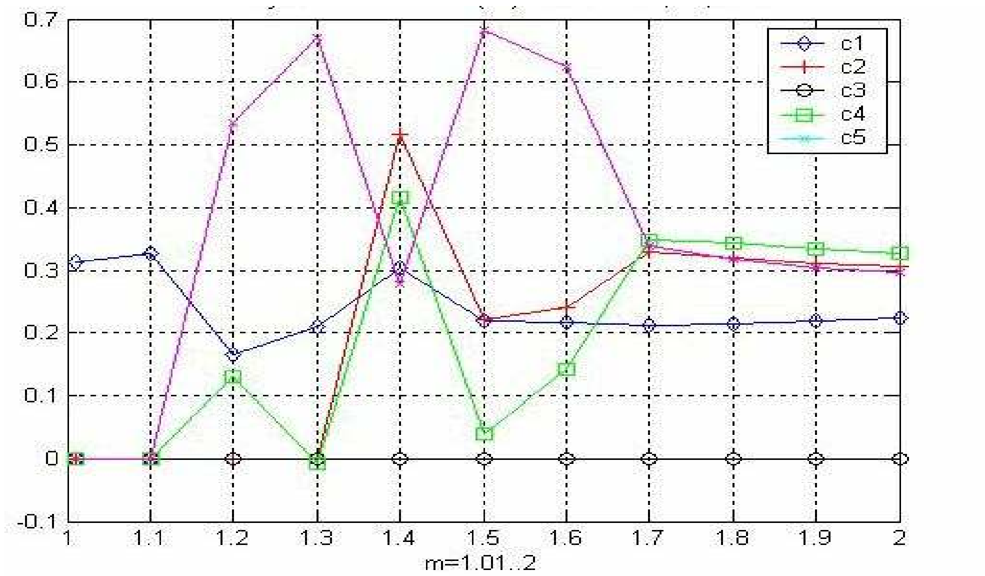

Figure 3. Δfdt_{t-2} Coefficient Values for Different Clusters and Level of Fuzziness

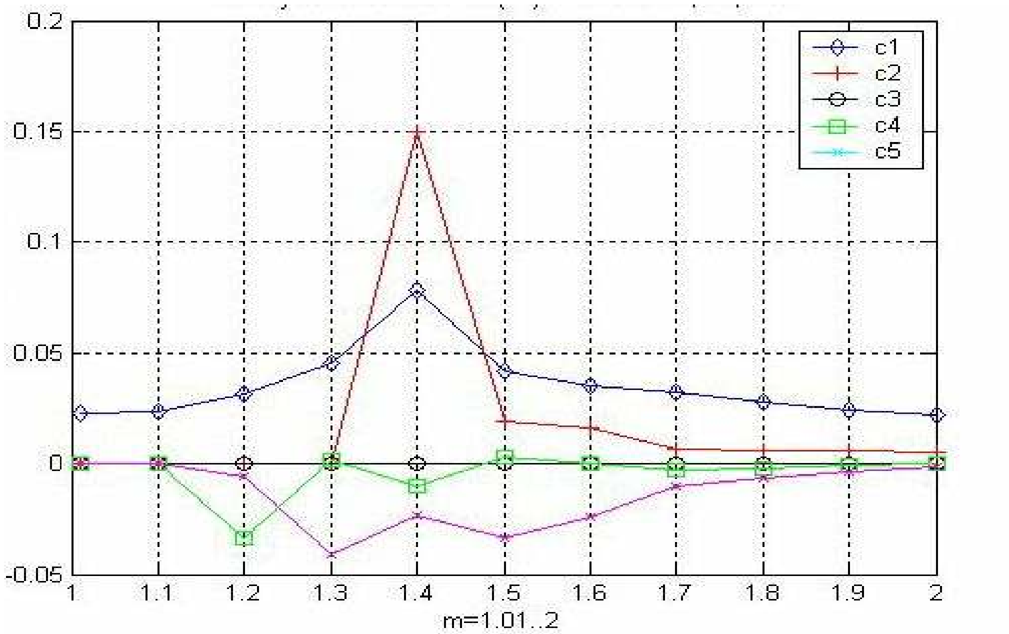

Figure 4. Δr_{t-2} Coefficient Values for Different Clusters and Level of Fuzziness

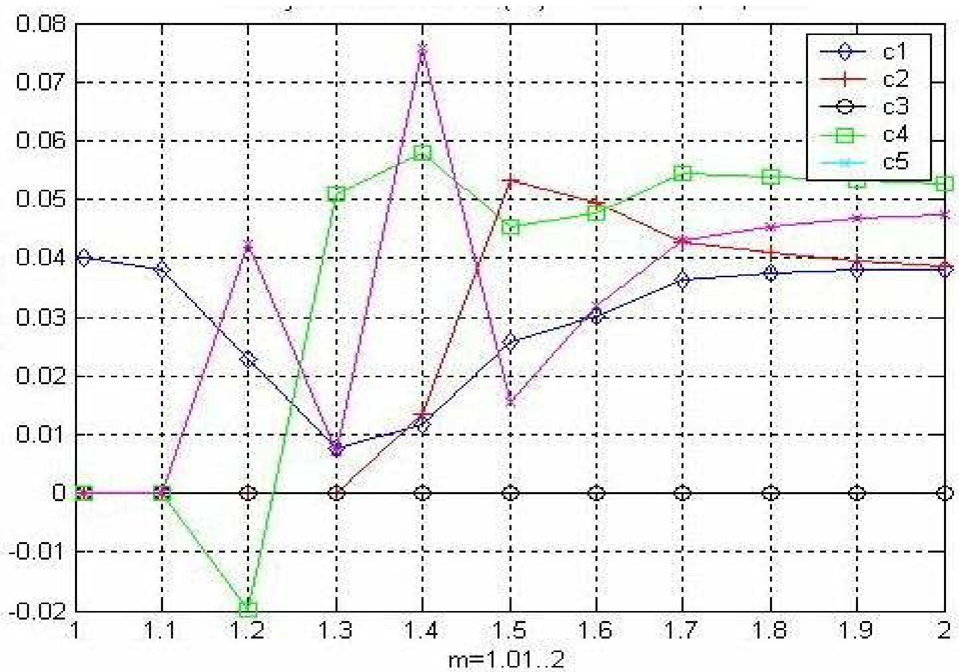

Figure 5. Δr_{t-3} Coefficient Values for Different Clusters and Level of Fuzziness

As it can be seen from above figures that coefficients of ΔFDT_{t-2} and ΔR_{t-3} are more varying with level of fuzziness. Coefficients calculated for second cluster are relatively more spread. The affect of uncertainty in level of fuzziness creates these results on fuzzy regressions coefficients. As the level of fuzziness increases the coefficient values become more stable. This might be expected since alpha-cut value is used as zero which means that any pattern becomes a member of the cluster even it has very little membership values for this cluster. In this manner, it affects the regression coefficients.

Table 2 shows the coefficient of variation of regression coefficients (standard deviation of coefficient/mean of coefficient). The regression functions for each cluster are in a linear regression form such as,

$$\Delta FT_c = \beta_0 + \beta_1 \Delta R_{t-2} + \beta_2 \Delta R_{t-3} + \beta_3 \Delta FDT_{t-1} + \beta_4 \Delta FDT_{t-2} \quad (13)$$

It is possible to calculate standard deviation and mean for each coefficient, β_i except cluster 3 which has $\beta_j=0$, j=1..4.

Table 2. Coefficient of Variation of Regression Coefficients in Each Cluster

Cluster	β_0	β_1	β_2	β_3	β_4
1	0.2003	0.4645	0.384	0.2544	0.2148
2	1.3304	2.318	0.8845	0.7959	0.8721
3	0.0149	-	-	-	-
4	1.4878	-2.528	0.7786	0.5076	0.8886
5	-8.0251	-1.0711	0.7409	0.8027	0.6523

.

Another way to calculate the effect of level of fuzziness, or uncertainty as fuzziness, is to calculate the movement of centers and their effect on inference. Information uncertainty based on cluster centers may lead to find unsupervised optimum number of clusters. Alternately if they are calculated as partial information entropy, a modified Bezdek (1981) cluster validity index can be obtained. This work is out of the scope of this paper and is left for future work that concludes all types of parameter uncertainty as uncertainty coefficients.

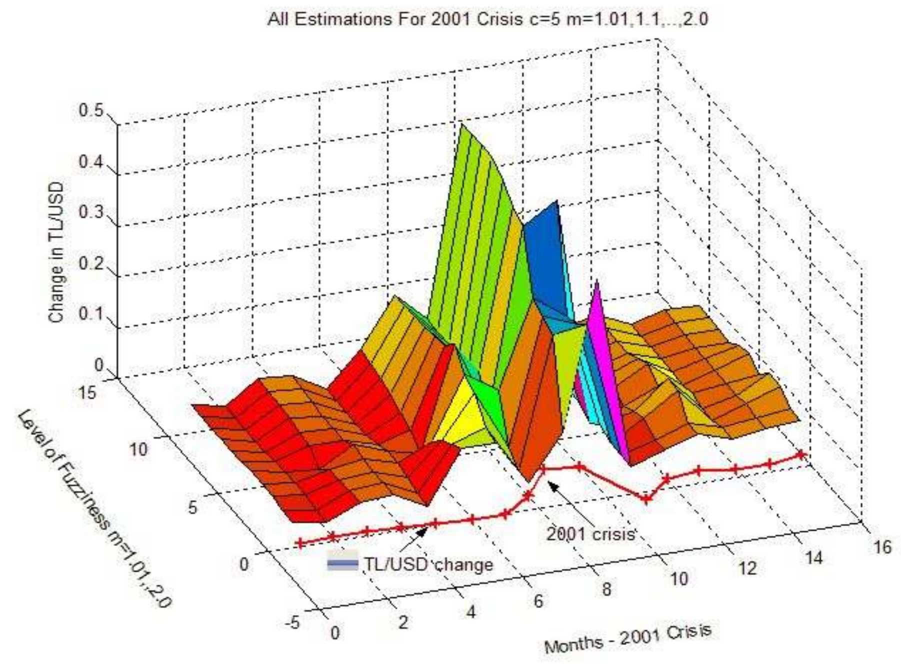

Figure 6. 2001 Crisis Prediction for C=5 And M=1.01,...,2.0

Figure 6 shows all predictions around 2001 crisis (±8 months). These predictions are calculated based on learning from '*training*' data which include 1994 currency crisis.

CONCLUSION

In this paper one type of uncertainty, fuzziness, and its affect on regression equations has been investigated by an application of the FSM created to capture currency crisis experienced in Turkey in 2001 (Ozkan 2003). In the model data has been split into '*training*' and '*test*' data sets. Training data set was used to learn model parameters and test data set was used to asses the model predictions. Optimum number of clusters and level of fuzziness was found to be (5, 1.3) under RMSE supervision.

Fuzziness and its effect were analyzed by changing level of fuzziness around optimum value and the effect of this change on regression coefficient was calculated. The degree of overlapping of clusters depends on the level of fuzziness and changing it effects directly on both rule bases and inferences. Changes in rule bases, or variation of centers, are not investigated in this paper. Variations in the value of regression coefficient give extra information, such as the contribution of input variables to uncertainty. In this example, ΔFDT_{t-2} and ΔR_{t-3} are more time-varying i.e., they contribute to uncertainty more then the others. It is shown that variations of the level of fuzziness exposes uncertainty associated with the exchange rates around crisis.

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