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Research Article

Investigation of Light Baryons in Hot QCD

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ARTICLE INFO ABSTRACT Keywords: Light Baryons Thermal QCD Sum Rules Mass Residue Article History: Received: 27.04.2023 Accepted: 23.11.2023 Online Available: 27.02.2024 We investigate the behavior of light baryons in hot QCD. To this aim, we evaluate the light baryons mass and residue in hot medium using the thermal correlation function with two-point by means of the thermal QCD sum rule. In sum rule calculations, we consider the additional thermal condensates appearing in Wilson expansion at $T \neq 0$. We determine the thermal continuum threshold using obtained sum rules expressions to analyze numerically. We observe that the masses and residues of light baryons stay approximately the same until the temperature reaches a certain value and then they fall with the temperature increase. We see that vacuum values of parameters in our calculations are in good consistency with other studies in the literature. Also, we define the fit functions that show how the spectroscopic parameters for light baryons behave at $T \neq 0$.

1. Introduction

The determination of the behaviour of light baryons at $T\neq 0$ is among the important research topics of recent years in hadron physics. Such studies have a crucial role in commenting on the non-perturbative and perturbative nature of the hadronic matter in a hot medium. They can allow us to have more ideas about neutron stars' compact internal structure.

One of the most reliable and practicable phenomenological approaches to evaluate the spectroscopic parameters of light baryons at $T\neq 0$ is the thermal QCD sum rules method (TQCDSR). TQCDSR is the extended form to the finite temperature of QCD sum rules first proposed by Shifman, Vainshtein, and Zakharov for particles at $T=0$ [1, 2]. It was firstly given by Bochkarev and Shaposhnikov for particles in hot medium [3]. In TQCDSR, the Lorentz invariance is disrupted at T≠0 and some operators different

from vacuum operators arise in the operator product expansion (OPE) [4, 5]. Also, condensates in vacuum are changed placed by their thermal expectation values, and the fourvector velocity in the hot QCD is used.

In TQCDSR, the thermal correlation function (TCF), which forms the starting point of the method, is defined and the spectroscopic parameters of hadrons, such as mass and residue, are obtained by calculating this function in two different ways. The first calculation is to obtain the TCF in hadron language using the dispersion relationship.

The other calculation is to write the TCF in quark language using OPE. The coefficients with the same structures of these correlation functions obtained from two different ways are equalized to each other using quark-hadron duality. Finally, after the undesirable terms arising in the dispersion relationship are eliminated with the

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Borel transformation, the spectroscopic parameters of hadrons in a hot medium are extracted.

Spectroscopic parameters of mesons at non-zero temperature by the TQCDSR have been widely studied in the literature [6-31], but there are comparatively lesser works about the thermal behaviour of light baryons via different phenomenological methods [32-49]. In the present paper, we calculate the N , Σ , Λ , Σ , Σ , Σ^* , Ω^- light baryons mass and residue at $T \neq 0$ using the thermal light quark propagator by means of TQCDSR. In numerical calculations, we use the energy density and thermal versions of quark and gluon condensates obtained by lattice QCD.

This work is planned as follows: In Section 2, the thermal QCD sum rules for the masses and residues of light baryons at non-zero temperature are obtained. In Section 3, numerical calculations are presented for the considered light baryon. Section 4 is devoted to our discussions and conclusions.

2. Material and Method

In this part, QCD sum rules for light baryons mass and residue at non-zero temperature are presented. The starting point of the calculations is the following the TCF introduced as

$$
\Pi_{(\mu\nu)}(p,T) = i \int d^4 x e^{ip \cdot x}
$$

$$
\times \langle \mathcal{T} \left(\eta_{(\mu)}^{O(D)}(x) \eta_{(\nu)}^{+O(D)}(0) \right) \rangle,
$$
 (1)

where p is the four-momentum, T represents temperature and T denotes time ordering product. Also, $\eta^0(x)$ and $\eta^D_\mu(x)$ are interpolating currents for N, Σ, Λ, Ξ octet and Δ , Σ^* , Σ^* , Ω^- decuplet light baryons, respectively. We take the following expressions for these interpolating currents:

$$
\eta^{O}(x) = N \varepsilon_{abc} \sum_{i=1}^{2} \left[K \left(q_1^{T,a}(x) C A_1^i q_2^b(x) \right) \right]
$$

$$
\times A_2^i q_3^c(x) + L \left[\left(q_2^{T,a}(x) C A_1^i q_3^b(x) \right) A_2^i q_1^c(x) \right] + M \left[\left(q_1^{T,a}(x) C A_1^i q_3^b(x) \right) A_2^i q_2^c(x) \right],
$$
 (2)

$$
\eta_{\mu}^{D}(x) = N \varepsilon_{abc} \left[\left(q_{1}^{T,a}(x) C \gamma_{\mu} q_{2}^{b}(x) \right) \times q_{3}^{c}(x) + \left(q_{2}^{T,a}(x) C \gamma_{\mu} q_{3}^{b}(x) \right) q_{1}^{c}(x) \right. \\ \left. + \left(q_{3}^{T,a}(x) C \gamma_{\mu} q_{1}^{b}(x) \right) q_{2}^{c}(x) \right]. \tag{3}
$$

Here *a*, *b*, *c* denote color indices, $A_1^1 = I$, $A_1^2 =$ $A_2^1 = \gamma^5$ and $A_2^2 = t$. *C* and *t* are the charge conjugation operator and arbitrary auxiliary parameter, respectively. We take as $t = -1$ that accords to Ioffe current. The normalization constant N and the coefficients K, L, M as well as quark fields q_1 , q_2 and q_3 for $N, Σ, Λ, Ξ, Δ, Σ^*, Ξ^*, Ω$ light baryons are presented in Table 1.

Table 1. N , K , L , M coefficients as well as quark fields q_1 , q_2 and q_3 for N , Σ , Λ , Σ , Δ , Σ^* , Σ^* , Ω^- light baryons

	N	K	L	М	q_{1}	q_{2}	q_{3}
N	2	1	0	0	и	d	$\boldsymbol{\mathcal{u}}$
Σ	$-1/\sqrt{2}$	1	-1	0	\boldsymbol{u}	d	S
Λ	$1/\sqrt{6}$	1	1	2	и	d	S
Ξ	-2	1	0	0	\boldsymbol{u}	d	S
л	$1/\sqrt{3}$				d	d	u
Σ^*	2/3				\boldsymbol{u}	d	S
Ξ^*	$1/\sqrt{3}$				S	S	u
Ω	1/3				S	S	S

To obtain desired sum rules, TCF given in Eq. (1) is evaluated in two different forms: In a first form, the hadronic side, the obtained expression includes observable hadronic states like mass and residue. In a second form, the QCD side, TCF is evaluated in the way of light quark fields by Wick's theorem. The sum rules for hadronic states are eventually reached by equating the determined structures' coefficients from both sides help of the dispersion relation. To suppress the terms arising from continuum and the higher states, it is necessary to apply the Borel transformation to on both parts of the equation.

The hadronic part of TCF is determined by inserting a full set of hadronic states into Eq. (1) and performing the four-dimensional integral. Therefore, we can write the hadronic part of TCF as

$$
\Pi_{(\mu\nu)}^{HAD}(p,T) = -\langle 0 | \eta_{(\mu)}^{O(D)}(0) | L(p,s) \rangle_T
$$

$$
\times \frac{\langle L(p,s) | \bar{\eta}_{(\nu)}^{O(D)}(0) | 0 \rangle_T}{p^2 - m_{O(D)}^2(T)} + \cdots,
$$
 (4)

where $m_{O(D)}(T)$ is the thermal mass of octet or decuplet light baryon at $T \neq 0$, $|L(p, s)\rangle$ is the light baryon state and the dots represents the contributions of the higher states and continuum. The matrix elements for octet and decuplet baryon can be taken respectively as

$$
\langle 0|\eta^{0}(0) |L(p,s)\rangle_{T} = \lambda_{0}(T)u(p,s), \qquad (5)
$$

$$
\langle 0 | \eta_{\mu}^{D}(0) | L(p, s) \rangle_{T} = \lambda_{D}(T) u_{\mu}(p, s), \tag{6}
$$

Here, $\lambda_0(T)$ and $\lambda_0(T)$ are thermal residues of octet and decuplet baryon at $T \neq 0$, respectively. As $u(p, s)$ represents Dirac spinor, $u_{\mu}(p, s)$ denotes the Rarita–Schwinger spinor. After we insert the matrix elements given in Eqs. (5), (6) into Eq. (4) and sum over the spins for considered light baryon, we find the hadronic part of TCF with respect to the Borel parameter in two different Lorentz structures as

$$
\hat{B}\Pi_{1(\mu\nu)}^{HAD}(p^2, p_0, T) = -\lambda_{O(D)}(T)e^{-\frac{m_{O(D)}^2(T)}{M^2}}, \quad (7)
$$

$$
\hat{B} \Pi_{2(\mu\nu)}^{HAD}(p^2, p_0, T) = -\lambda_{O(D)}(T)
$$

$$
\times m_{O(D)}(T) e^{-\frac{m_{N,H(D)}^2(T)}{M^2}},
$$
 (8)

where M^2 indicates the Borel parameter, p_0 denotes the quasi-particle energy, Π_1 and Π_2 are coefficients of Lorentz structures \mathcal{D} and *I* for nucleon/hyperon as they are coefficients of pg_{uv} and g_{uv} for decuplet baryons, respectively. The next aim is to obtain the QCD part of TCF with respect to quark fields by operator product expansion (OPE). TCF in this representation can be separated to the different Lorentz structures as hadronic side. We select p and I Lorentz structures for nucleon/hyperon as well as pg_{uv} and $g_{\mu\nu}$ structures for decuplet baryons. By inserting the interpolating currents given in Eqs. (2) and (3) for considered light baryon into TCF and then contracting out all quark pairs by Wick's theorem, we find the QCD part of the TCF in connection with the thermal light quark propagators $S_q(x)$:

$$
\Pi^{QCD,N}(p,T) = 4i\varepsilon_{abc}\varepsilon_{\hat{a}\hat{b}\hat{c}} \int d^4x \, e^{ip.x}
$$
\n
$$
\times \left\{ \left\{ \left((\gamma_5 S_u^{cb}(x) \dot{S}_d^{b\hat{a}}(x) S_u^{a\hat{c}}(x) \gamma_5 \right. \right. \\ \left. - \gamma_5 S_u^{c\hat{c}}(x) \gamma_5 Tr \left[S_u^{ab}(x) \dot{S}_d^{b\hat{a}}(x) \right] \right) + \right. \\ \left. t \left(\gamma_5 S_u^{cb}(x) \gamma_5 \dot{S}_d^{b\hat{a}}(x) S_u^{a\hat{c}}(x) + \right. \\ \left. S_u^{cb}(x) \dot{S}_d^{b\hat{a}}(x) \gamma_5 S_u^{a\hat{c}}(x) \gamma_5 - \gamma_5 S_u^{cc}(x) Tr \left[S_u^{ab}(x) \gamma_5 \dot{S}_d^{b\hat{a}}(x) \right] - \right. \\ \left. S_u^{cc}(x) \gamma_5 Tr \left[S_u^{ab}(x) \dot{S}_d^{b\hat{a}}(x) \gamma_5 \right] \right) + \right. \\ \left. t^2 \left(S_u^{cb}(x) \gamma_5 \dot{S}_d^{b\hat{a}}(x) \gamma_5 S_u^{a\hat{c}}(x) - \right. \\ \left. S_u^{cc}(x) Tr \left[S_d^{b\hat{a}}(x) \gamma_5 \dot{S}_u^{a\hat{b}}(x) \gamma_5 \right] \right) \right\}_{T'} \right\} \tag{9}
$$

$$
\Pi^{QCD,\Sigma}(p,T) = \frac{i}{2} \varepsilon_{abc} \varepsilon_{\hat{a}\hat{b}\hat{c}} \int d^4x \, e^{ip.x}
$$
\n
$$
\langle \{ (\gamma_5 S_d^{c\hat{c}}(x) \gamma_5 Tr \left[S_u^{ab}(x) \dot{S}_s^{b\hat{a}}(x) \right]
$$
\n
$$
+ \gamma_5 S_d^{c\hat{a}}(x) \dot{S}_s^{b\hat{b}}(x) S_u^{a\hat{c}}(x) \gamma_5
$$
\n
$$
+ \gamma_5 S_u^{c\hat{b}}(x) \dot{S}_s^{a\hat{a}}(x) S_d^{b\hat{c}}(x) \gamma_5
$$
\n
$$
+ \gamma_5 S_u^{c\hat{c}}(x) \gamma_5 Tr \left[S_s^{a\hat{b}}(x) \dot{S}_a^{b\hat{a}}(x) \right]
$$
\n
$$
+ t \left(\gamma_5 S_d^{c\hat{a}}(x) \gamma_5 \dot{S}_s^{b\hat{b}}(x) S_u^{a\hat{c}}(x) \right]
$$
\n
$$
+ \gamma_5 S_d^{c\hat{c}}(x) Tr \left[\gamma_5 \dot{S}_s^{b\hat{a}}(x) S_u^{a\hat{b}}(x) \right]
$$
\n
$$
+ S_u^{c\hat{b}}(x) \dot{S}_s^{a\hat{a}}(x) \gamma_5 S_d^{b\hat{c}}(x) \gamma_5
$$
\n
$$
+ S_u^{c\hat{c}}(x) \gamma_5 Tr \left[\dot{S}_d^{b\hat{a}}(x) \gamma_5 S_s^{a\hat{b}}(x) \right]
$$
\n
$$
+ \gamma_5 S_u^{c\hat{c}}(x) Tr \left[\gamma_5 \dot{S}_d^{b\hat{a}}(x) S_s^{a\hat{b}}(x) \right]
$$

 $+ \gamma_5 S^{c\acute{c}}_u(x) \gamma_5 Tr\big[\dot{S}^{b\acute{a}}_d(x) S^{a\acute{b}}_s(x)\big]\big)$

$$
\frac{1}{2} + y_5 S_{4}^{cb}(x)y_5 S_{7}^{ab}(x) S_{5}^{ab}(x) S_{8}^{ab}(x) S_{9}^{ac}(x) S_{1}^{ac}(x) S_{1}^{ac}(x) S_{1}^{ac}(x) S_{1}^{ac}(x) S_{2}^{ac}(x) S_{3}^{ac}(x) S_{1}^{ac}(x) S_{2}^{ac}(x) S_{3}^{ac}(x) S_{3}^{ac}(x)
$$

 (12)

$$
\times \left\langle \left\{ 2S_d^{c\dot{a}}(x)\gamma_{\nu}\dot{S}_d^{a\dot{b}}(x)\gamma_{\mu}S_u^{b\dot{c}}(x) \right. \right. \\ \left. - 2S_d^{c\dot{b}}(x)\gamma_{\nu}\dot{S}_d^{a\dot{a}}(x)\gamma_{\mu}S_u^{b\dot{c}}(x) \right. \\ \left. + 4S_d^{c\dot{b}}(x)\gamma_{\nu}\dot{S}_u^{b\dot{a}}(x)\gamma_{\mu}S_d^{a\dot{c}}(x) \right. \\ \left. + 2S_u^{c\dot{a}}(x)\gamma_{\nu}\dot{S}_d^{a\dot{b}}(x)\gamma_{\mu}S_d^{b\dot{c}}(x) \right. \\ \left. - 2S_u^{c\dot{a}}(x)\gamma_{\nu}\dot{S}_d^{b\dot{b}}(x)\gamma_{\mu}S_d^{a\dot{c}}(x) \right. \\ \left. - S_u^{c\dot{c}}(x)Tr\left[S_d^{b\dot{a}}(x)\gamma_{\nu}\dot{S}_d^{a\dot{b}}(x)\gamma_{\mu}\right] \right. \\ \left. + S_u^{c\dot{c}}(x)Tr\left[S_d^{b\dot{b}}(x)\gamma_{\nu}\dot{S}_d^{a\dot{a}}(x)\gamma_{\mu}\right] \right. \\ \left. - S_u^{c\dot{c}}(x)Tr\left[S_d^{b\dot{b}}(x)\gamma_{\nu}\dot{S}_d^{a\dot{a}}(x)\gamma_{\mu}\right] \right\}_T \right) \tag{13}
$$

$$
\Pi_{\mu\nu}^{QCD,\Omega^{-}}(p,T) = \varepsilon_{abc}\varepsilon_{\acute{a}\acute{b}\acute{c}} \int d^4x \, e^{ip.x}
$$
\n
$$
\times \langle \{ S_s^{c\acute{a}}(x) \gamma_{\nu} \dot{S}_s^{a\acute{b}}(x) \gamma_{\mu} S_s^{b\acute{c}}(x) - S_s^{c\acute{a}}(x) \gamma_{\nu} \dot{S}_s^{a\acute{a}}(x) \gamma_{\mu} S_s^{a\acute{c}}(x) - S_s^{c\acute{b}}(x) \gamma_{\nu} \dot{S}_s^{a\acute{a}}(x) \gamma_{\mu} S_s^{b\acute{c}}(x) - S_s^{c\acute{c}}(x) \gamma_{\nu} \dot{S}_s^{b\acute{a}}(x) \gamma_{\mu} S_s^{a\acute{c}}(x) - S_s^{c\acute{c}}(x) Tr \left[S_s^{b\acute{a}}(x) \gamma_{\nu} \dot{S}_s^{a\acute{b}}(x) \gamma_{\mu} \right] + S_s^{c\acute{c}}(x) Tr \left[S_s^{b\acute{b}}(x) \gamma_{\nu} \dot{S}_s^{a\acute{a}}(x) \gamma_{\mu} \right] \rangle_T.
$$
\n(14)

$$
\Pi_{\mu\nu}^{QCD,\Xi^{*}}(p,T) = \frac{i}{3} \varepsilon_{abc} \varepsilon_{\hat{a}\hat{b}\hat{c}} \int d^{4}x \ e^{ip.x}
$$
\n
$$
\times \left\{ \{ 2S_{s}^{c\hat{a}}(x)\gamma_{\nu}\dot{S}_{s}^{a\hat{b}}(x)\gamma_{\mu}S_{u}^{b\hat{c}}(x) \right. \\ \left. - 2S_{s}^{c\hat{b}}(x)\gamma_{\nu}\dot{S}_{s}^{a\hat{a}}(x)\gamma_{\mu}S_{u}^{b\hat{c}}(x) \right. \\ \left. + 4S_{s}^{c\hat{b}}(x)\gamma_{\nu}\dot{S}_{u}^{b\hat{a}}(x)\gamma_{\mu}S_{s}^{a\hat{c}}(x) \right. \\ \left. + 2S_{u}^{c\hat{a}}(x)\gamma_{\nu}\dot{S}_{s}^{a\hat{b}}(x)\gamma_{\mu}S_{s}^{b\hat{c}}(x) \right. \\ \left. - 2S_{u}^{c\hat{a}}(x)\gamma_{\nu}\dot{S}_{s}^{b\hat{b}}(x)\gamma_{\mu}S_{s}^{a\hat{c}}(x) \right. \\ \left. - S_{u}^{c\hat{c}}(x)Tr[S_{s}^{b\hat{a}}(x)\gamma_{\nu}\dot{S}_{s}^{a\hat{b}}(x)\gamma_{\mu}] \right. \\ \left. + S_{u}^{c\hat{c}}(x)Tr[S_{u}^{b\hat{b}}(x)\gamma_{\nu}\dot{S}_{s}^{a\hat{a}}(x)\gamma_{\mu}] \right)_{T}, \qquad (15)
$$

$$
\Pi_{\mu\nu}^{QCD,\Sigma^*}(p,T) = -\frac{2i}{3}\varepsilon_{abc}\varepsilon_{\dot{a}\dot{b}\dot{c}}\int d^4x \;e^{ip.x}
$$

$$
\times \left\langle \left\{ S_d^{c\dot{a}}(x) \gamma_{\nu} \dot{S}_u^{b\dot{b}}(x) \gamma_{\mu} S_s^{a\dot{c}}(x) \right. \right. \\ \left. + S_d^{\dot{c}\dot{b}}(x) \gamma_{\nu} \dot{S}_s^{a\dot{a}}(x) \gamma_{\mu} S_u^{b\dot{c}}(x) \right. \\ \left. + S_s^{c\dot{a}}(x) \gamma_{\nu} \dot{S}_d^{b\dot{b}}(x) \gamma_{\mu} S_u^{a\dot{c}}(x) \right. \\ \left. + S_c^{\dot{c}\dot{b}}(x) \gamma_{\nu} \dot{S}_a^{a\dot{a}}(x) \gamma_{\mu} S_d^{b\dot{c}}(x) \right. \\ \left. + S_u^{\dot{c}\dot{a}}(x) \gamma_{\nu} \dot{S}_s^{b\dot{b}}(x) \gamma_{\mu} S_d^{a\dot{c}}(x) \right. \\ \left. + S_u^{\dot{c}\dot{b}}(x) \gamma_{\nu} \dot{S}_a^{a\dot{a}}(x) \gamma_{\mu} S_s^{b\dot{c}}(x) \right. \\ \left. + S_c^{\dot{c}\dot{c}}(x) Tr \left[S_d^{b\dot{a}}(x) \gamma_{\nu} \dot{S}_a^{a\dot{b}}(x) \gamma_{\mu} \right] \right. \\ \left. + S_u^{\dot{c}\dot{c}}(x) Tr \left[S_s^{b\dot{a}}(x) \gamma_{\nu} \dot{S}_a^{a\dot{b}}(x) \gamma_{\mu} \right] \right\rangle_T, \qquad (16)
$$

Here $\acute{S} = C S^T C$ and $S_q(x)$ is given as

$$
S_q^{ij}(x) = i \frac{x_\mu \gamma_\mu}{2\pi^2 x^4} \delta_{ij} - \frac{m_q}{4\pi^2 x^2} \delta_{ij} - \frac{\langle \overline{q}q \rangle}{12} \delta_{ij}
$$

$$
- \frac{x^2}{192} m_0^2 \langle \overline{q}q \rangle \left[1 - i \frac{m_q}{6} x_\mu \gamma_\mu \right] \delta_{ij}
$$

$$
+ \frac{i}{3} \left[x_\mu \gamma_\mu \left(\frac{m_q}{16} \langle \overline{q}q \rangle - \frac{1}{12} \langle u \Theta^f u \rangle \right) \right]
$$

$$
+ \frac{1}{3} \left(u \cdot x u_\mu \gamma_\mu \langle u \Theta^f u \rangle \right) \delta_{ij}
$$

$$
- \frac{i g_s \lambda_A^{ij}}{32\pi^2 x^2} G_{\mu\nu}^A(x_\mu \gamma_\mu \sigma^{\mu\nu} + \sigma^{\mu\nu} x_\mu \gamma_\mu), \qquad (17)
$$

where $\langle \bar{q}q \rangle$ shows the thermal quark condensate, m_q represents the light quark mass, $G_{\mu\nu}^A$ is the gluon field strength tensor at non-zero temperature. To proceed, we insert the abovegiven $S_q(x)$ into the QCD part of TCF for each light baryon.

After we perform the standard Borel transformation and continuum subtraction, we obtain the QCD side of TCF in the Borel system for each light baryon in terms of the functions Π_1 and Π_2 as the hadronic side. In the final step, we match the coefficients of these two different forms of TCF with the same structures and obtain the thermal sum rules in terms of spectroscopic parameters of considered light baryon:

$$
\hat{B}\Pi_1^{QCD}(p_0, T) = -\lambda_{O(D)}^2(T)e^{-m_{O(D)}^2(T)/M^2}, (18)
$$

\n
$$
\hat{B}\Pi_2^{QCD}(p_0, T) = -\lambda_{O(D)}^2(T) m_{O(D)}
$$

\n
$$
\times e^{-m_{O(D)}^2(T)/M^2}.
$$
\n(19)

From Eqs. (18) and (19), we extract masses and residues for considered light baryons.

3. Numerical Results

The starting point of the numerical analysis is the determination of some input parameters required in calculations. These input parameters which the vacuum values of quark masses, quark condensates and gluon condensates are gathered in Table 2.

Aside from these input parameters presented for $T = 0$, there are three more that the thermal quark condensate $\langle \overline{q}q \rangle$, the thermal gluon condensate $\langle G^2 \rangle$ and energy density of hot medium $\langle \Theta_{00}^f \rangle$. For $\langle \bar{q}q \rangle$, we use the following expression [50]

$$
\langle \overline{q}q \rangle = \langle 0|\overline{q}q|0 \rangle \frac{1}{1 + e^{A\left(BT^2 + C\left[\frac{1}{GeV}\right]T - 1\right)}},\tag{20}
$$

where fit parameters are $A = 18.10042$, $B =$ 1.84692 $\frac{1}{GeV^2}$, $C = 4.99216 \frac{1}{GeV}$, and this expression is consistent with the Lattice QCD studies presented in [51, 52].

 $\langle G^2 \rangle$ is given by

$$
\langle G^2 \rangle = \langle 0 | G^2 | 0 \rangle \left[1 - 1.65 \left(\frac{T}{T_c} \right)^{8.735} + 0.04967 \left(\frac{T}{T_c} \right)^{0.7211} \right],\tag{21}
$$

where T_c is the critical temperature and $\langle 0|G^2|0\rangle$ being gluon condensate at $T = 0$. Using the graphics drawn in the framework lattice QCD [52], we can describe the following fit formula for energy density of hot medium

$$
\langle \Theta_{00}^f \rangle = \langle \Theta_{00}^g \rangle = T^4 e^{(DT^2 - ET)} - FT^5, \tag{22}
$$

where D, E and F are fit parameters that $D =$ $113.867 \frac{1}{GeV^2}$, $E = 12.190 \frac{1}{GeV}$, $F = 10.141 \frac{1}{GeV}$, and this formula is valid at temperatures up to 130 MeV .

To complete the numerical calculations, we need to determine the three auxiliary parameters, the Borel parameter M^2 , the thermal version of the continuum threshold $s_0(T)$ and the parameter x $(x = cos\theta, t = tan\theta)$. According to the QCD sum rule principle, physical quantities should be roughly independent with respect to these parameters in suitable working ranges. To determine the suitable working ranges for M^2 and x , we investigate the behaviour of the physical parameters of the corresponding light baryon according to these auxiliary parameters in vacuum. We realize that they weakly depend on these parameters for selected regions.

Therefore, we take the working ranges of x $[\pm 0.6 \pm 0.2]$ and $[\pm 0.8, \pm 0.4]$ for nucleon and hyperon, respectively. The working ranges of M^2 are taken as $[0.8 GeV^2 - 1.2 GeV^2]$, $[1.0 GeV^2 - 1.6 GeV^2]$, $[1.0 GeV^2 - 1.6 GeV^2]$,

 $[1.2 GeV^2 - 1.8 GeV^2]$, $[1.5 GeV^2 - 3.0 GeV^2]$, $[1.7 \text{GeV}^2 - 3.5 \text{GeV}^2], [2.0 \text{GeV}^2 - 3.8 \text{GeV}^2],$ $[2.2 GeV^2 - 4.0 GeV^2]$ for the $N, \Sigma, \Lambda, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega^-,$ respectively.

At the end of this section, we would like to show the thermal behaviour of spectroscopic parameters of light baryons and residue at nonzero temperatures. For this aim, we display the mass and residue at selected s_0 and M^2 values versus temperature for the corresponding light baryon in Figure 1 and Figure 2, respectively. Selected s_0 and M^2 values in these graphs are given in Table 3. It is seen that masses and residues of N , Σ , Λ , Σ , Δ , Σ^* , Σ^* , Ω^- light baryons remain approximately unchanged until the temperature reaches $T \approx 0.13 - 0.15 \text{ GeV}$ and then they start to rapidly fall with the temperature increases. By using Figs. 1 and 2, we also obtain the following fit functions for the thermal mass and residue as

$$
m_L[\lambda_L](T) = \Gamma(1 - \delta T^n), \tag{23}
$$

Here, Γ, δ and *n* are fitting parameters and their values are given in Table 4 and 5 for mass and residue of the corresponding light baryon, respectively. At $T \rightarrow 0$ limit, it is shown that above our fit function for both mass and residue of corresponding light baryon are consistent with other studies and experimental results in vacuum.

4. Discussion and Conclusion

In this article, we wanted to determine the thermal behaviour of light baryons at $T \neq 0$. To this aim, we evaluated the mass and residue of light baryons by means of the TQCDSR in hot medium. In these calculations, we used the selected interpolating current for the corresponding light baryon and determined working ranges of auxiliary parameters partaken sum rules. We also obtained the thermal continuum threshold for considered light baryon in terms of their vacuum values.

Using the additional thermal condensates at $T \neq 0$, we numerically analyzed for N , Σ, Λ, Ξ, Δ, Σ^{*}, Ξ^{*}, Ω⁻ light baryons masses and residues. We see that the physical parameters of light baryons stay approximately the same until the temperature reaches $T \approx 0.13 - 0.15 \text{ GeV}$ and then they fall with the temperature increases. If we compare this thermal reduction behavior with other studies in the literature, we see that the behaviour with respect to temperature on the mass and residue of N , Σ , Λ , Σ octet baryons is rapport with studies given in [32, 33, 35, 47].On the other hand, the behaviour of the Δ , Σ^* , Σ^* , $\Omega^$ decuplet baryon's mass with respect to the temperature in this work is agreed with results given in [44,45].

As a result, this melting of physical parameters of light baryons with increasing temperature may be interpreted as a transition to the quark-gluon plasma phase from the hadron phase. Also, it is seen that the vacuum values of physical parameters in this study are in good conform with other studies in the literature.

We obtained the fit functions at non-zero temperature for the mass and residue of light baryons in this article. These fit functions may be used to investigate other spectroscopic parameters of light baryons in a hot medium. We hope that our results may help analyses of heavy ion collision experiments in later times.

Figure 1. Mass-temperature graph for the corresponding light baryon at selected s_0 and M^2

particle	$s_0(Gev^2)$	M^2 (GeV ²)
N	1.5	1.0
	2.8	1.3
	2.6	1.3
Ξ	3.2	1.5
	2.9	1.5
Σ*	3.5	1.7
Ξ^*	4.1	2.0
	4.7	3.0

Table 3. Selected s_0 and M^2 values in Figs. 1 and 2

Table 4. The values of parameters Γ , δ and n in the fit function defined for $m_L(T)$

particle	$\Gamma(GeV)$	n \overline{GeV}	\boldsymbol{n}				
N	0.81	3.67×10^{11}	16				
Σ	1.37	1.37×10^{7}	16				
Λ	1.24	1.29×10^{7}	10				
Ξ	1.38	1.27×10^{7}	10				
Λ	1.24	2.23×10^{11}	10				
Σ^*	1.39	2.32×10^{11}	16				
Ξ^*	1.52	2.34×10^{11}	16				
Ω.	1.69	2.31×10^{11}	16				

Table 5. The values of parameters Γ , δ and n in the fit function defined for $\lambda_L(T)$

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