An Examination of Turkish Middle School Students' Proportional Reasoning*

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Abstract
This study examined if middle school students were able to differentiate proportional and non-proportional situations, and whether the use of integer or non-integer ratios in proportional and non-proportional problems affected students' solution strategies. The analyses showed that students' success rates among the mentioned problem types significantly differed. They also tended to prefer the proportional solution method in non-proportional situations. In addition, in non-proportional problems, use of non-integer ratios evoked additive strategies while students preferred proportional solution methods in problems with integer ratios. However, contrary to the findings reported in the literature, students' use of erroneous strategies was not significantly affected by the use of integer or non-integer ratios in proportional problems.

Keywords: Proportional reasoning, proportional reasoning profiles, non-proportional problems, middle school

Introduction

Proportional reasoning has a wide range of applications in primary and secondary mathematics as well as in the following years of education (Modestou & Gagatsis, 2007; Van Dooren, De Bock, & Verschaffel, 2010). It is essential in understanding basic scientific concepts and handling everyday problems and situations (Spinillo & Bryant, 1999). The National Council of Teaching Mathematics (NCTM) puts a higher emphasis on the importance of developing students' proportional reasoning, stating that no matter how much time and effort is needed, anything required for the development of proportional reasoning must be provided (NCTM, 1989). Despite its significance, however, students have serious problems in understanding proportional relations. Most students graduate from high school without acquiring fluency in proportional reasoning (Capon & Kuhn, 1979; Lawton, 1993; Modestou & Gagatsis, 2010). Middle school is accepted as the most critical period for learning ratio and proportional relations (Lamon, 1994; Lo & Watanabe 1997; Lobato, Ellis, Charles, & Zbiek, 2010; Van Dooren et al., 2010; Van Dooren, De Bock, Vluegels, & Verschaffel, 2010). Thus, understanding middle school students' levels of proportional reasoning is important to design instruction to better meet students' instructional needs.

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Proportional reasoning is a special form of multiplicative reasoning that requires considering the covariation between variables, comparing the multiple variables at the same time, and using information as a whole (Lesh, Post, & Behr, 1988). When problems that students encounter in school are considered, there is a tendency to limit proportional reasoning as the ability to solve certain set of problems or use of some algorithms (Modestou & Gagatsis, 2008). However, in addition to solving proportional problems correctly, students should be able to differentiate proportional and non-proportional situations and use the appropriate method for each situation (Modestou & Gagatsis, 2008; Van Dooren et al., 2010).

Research shows that students tend to overuse proportional strategies even when the relation among the variables in not proportional. They may also use additive reasoning in proportional situations. The main reason for that is they identify some clues in word problems that have little to do with the nature of the problem while deciding the solution strategy. In what follows, we discuss these research findings in detail that guided the present study

Overgeneralization of Proportional Strategies

Overgeneralization of proportional strategies in non-proportional situations is a frequently observed phenomenon in the literature (Fernandez et al., 2012; Fernandez et al., 2010). In general, the use of linear models in situations that do not include a proportional relation among variables is named as "the illusion of linearity" (Van Dooren, De Bock, Janssens, & Verschaffel, 2007). According to this phenomenon, people have a tendency to see linear relations in everywhere because of the frequent use of linear models for explaining different daily life situations. Linearity starts to be a panacea for nearly all problem situations like area and volume calculations of enlarged and reduced geometric shapes (Modestou & Gagatsis, 2007; Van Dooren, De Bock, Hessels, Janssens, Verschaffel, 2004) and probability (Van Dooren, De Bock, Depaepe, Janssens, & Verschaffel, 2003). Stavy and Tirosh (1996) explain the illusion of linearity in a simpler way, as an intuitive rule of 'more A, more B' relation. Murphy (2012) emphasizes the importance of developing an understanding in the topic of area in which a lack of conceptual understanding of the topic would result in tendencies to follow intuitive rules.

As Van Dooren et al. (2010) state, students' use of proportional strategies in additive situations shows an increase especially in middle school years. The reason for this change may be rooted from students' exposure to ratio and proportion in the upper grades of elementary school. However, the proportionality problems used in teaching proportionality are typically limited in variability. They are mostly formulated in missing-value form (Cramer, Post, & Currier, 1993) with numbers that enable easy calculations. In the middle grades they also come across with similarly structured problems with numbers enabling easy calculations, which may reinforce the use of superficial clues in problems rather than acquiring a deeper understanding of proportional relations (Van Dooren et al., 2004).

Overgeneralization of Additive Strategies

Another important observation reported in the literature is students' use of additive strategies in proportional situations (Van Dooren et al., 2010). This overgeneralization may be related with students' early expertise in addition and counting routines (Boyer, Levine, Susan, & Huttenlocher, 2008). Besides this, students' insufficient understanding of multiplication and division, and multiplicative relations between variables may result in an increased use of additive strategies. Especially focusing on particular solution strategies limits the understanding of proportional reasoning (Lo & Watanabe, 1997), and problem solving process becomes a rule-following procedure where students follow superficial clues (Van Dooren, De Bock, Vluegels, & Verschaffel, 2010). Researchers observe that as students face with a problem including numbers that can't be divided easily, they prefer to use an additive method, in which calculation of numbers is much easier and familiar for them (Clark & Kamii, 1996).

Reliance on Superficial Clues in Word Problems

In fact, students' use of superficial clues on word problems is frequently reported in the literature (Fernandez, Llinares, Van Dooren, De Bock, & Verschaffel, 2010, 2012). When students are asked for the reason to prefer a particular strategy, they indicate that they choose randomly, however they actually respond systematically through artificial correlations based on their prior experiences of solving word problems (Lannin, Barker, & Townsend, 2007; Perso, 1992). Each choice of strategy reveals some relations and connections that are meaningful and useful for students according to their previous experiences, including the linguistic structure of the problem, key expressions or typical situations described in the problem, or the name of the chapter in which the problem is written. One of the most prevalent superficial clues students use is the numbers used in word problems (Sowder, 1988). That is, students refer to the relation among numbers to decide which operation to perform, instead of the relation among variables and the situation in the problem. Especially, the numbers used in proportional missing-value problems has an influence on students' solution strategies. As the number in the problem changes in a way that unit ratio does not give a whole number, students show a tendency to change their reasoning and start to develop an additive strategy (Fernandez et al., 2010; Singh, 2000). Similarly, as the number in the task is changed from non-integer ratio to integer ratio, an increase in the use of proportional strategy is observed (Van Dooren et al., 2010).

Four Learner Profiles in Proportional Reasoning

Van Dooren et al. (2010) identified four learner profiles based on students' solution strategies in proportional and non-proportional situations. These are:
(a) *Additive reasoners*, who overuse additive methods in proportional and constant situations where it is inappropriate,
(b) *Proportional reasoners*, who overuse multiplicative (proportional) method in non-proportional situations, that is, in additive or constant situations,

(c) *Number-sensitive reasoners,* who overuse proportional methods if numbers in the problem involve integer ratios, and overuse additive methods if numbers in the problem form non-integer ratios,
(d) *Correct reasoners*, who can differentiate proportional and non-proportional situations and choose the appropriate solution strategy for each situation.

Proportional Reasoning Studies in Turkey

There exists a body of research in Turkey that examined proportional reasoning in Turkish schools. Çeken and Ayas (2010) investigated inter-disciplinary relations among mathematics, science and technology, and social studies curricula according to the common curricular objectives that require an understanding of proportional relations at the elementary grade level. The curricular objectives were examined by considering the sequence of the objectives in the 4^{th}, 5^{th}, and 6^{th} grade curricula. The findings revealed that these disciplines showed insufficient connection and correlation in the curricula regarding the timing of the objectives, which require an understanding of proportional relations. Çelik and Özdemir (2011) examined the relation between proportional reasoning skills and problem posing skills of 7^{th} and 8^{th} grade students. The researchers used Langrall and Swafford's (2000) four-level categorization of proportional reasoning skills (Level 0, Level 1, Level 2, and Level 3) in analyzing their data. Level 3 represented the most sophisticated level, as the use of algebraic expressions and different solution methods (e.g., equivalent fractions, cross-multiplication) when solving proportional problems. This study showed a significant relation between proportional reasoning skills and problem posing skills. These studies highlighted the importance of proportional reasoning in understanding further mathematics as well as other subjects in schools.

Kaplan, İşleyen, and Öztürk (2011) investigated 6^{th} grade students' misconceptions and erroneous strategies when solving proportional tasks. These tasks showed a wide range such as velocity problems, perimeter and area relations, and mixture problems but they were not set in the same context. The classification of students' common erroneous strategies included categories such as "unable to focus on the problem" and "unable to use ratio." Along the same lines, Duatepe, Akkuş-Çıkla, and Kayhan (2005) examined 6^{th}, 7^{th}, and 8^{th} grade students' solution strategies in proportional problems. The data collection instrument was a test that included three proportional missing-value problems, two proportional comparison problems, three qualitative comparison problems, one non-proportional problem, and one inverse-relation problem. The study showed that cross-multiplication strategy was the most preferred method among Turkish students, contrary to the international literature findings. This result was presented as expected since instruction about ratio and proportion in Turkish schools is heavily dependent on the use of cross-multiplication method. Furthermore, the findings revealed that students attempted to use cross-multiplication method in non-proportional situations (i.e., in comparison and additive problems), which implied that 6^{th}, 7^{th}, and 8^{th} grade students had difficulties in differentiating proportional and non-proportional situations. In another study, Akkuş-Çıkla and Duatepe (2002) looked into preservice teachers' reasoning on ratio and proportion tasks. The results showed that although preservice teachers could solve the ratio and proportion tasks, they had problems on explaining the concepts of ratio and proportion.

As evident in these studies, 6th to 8th grade students along with preservice teachers also have some difficulties with proportional reasoning. As the concepts of ratio and proportion develop in middle grades, improving instruction in middle grades is necessary (Sowder et al., 1998). Successful performance on proportional tasks reveals little about students' competencies as proportional reasoners as they may be simply applying a procedure, such as the cross-multiplication (Duatepe et al., 2005). Therefore, when assessing students' proportional reasoning, it is necessary to include non-proportional tasks along with proportional ones. Especially, presenting "pseudo-proportional" (Modestou & Gagatsis, 2007) problems (problems that show the linguistic structure of a proportional task but do not include multiplicative relations among the variables) is important. These problems should involve both additive and constant relations. In addition, any fair assessment of proportional reasoning should include proportional tasks other than (and along with) missing-value problems that dominate classroom instruction, such as proportional comparison problems. Modestou and Gagatsis (2010) state that use of only missing-value problems would give an insufficient picture of proportional reasoning. An equally important issue is identifying factors that lead students to follow certain procedures without actually thinking about the meaning of problems, such as use of integer or non-integer ratios.

In this study, a data collection instrument is developed including all these types of problems (proportional, constant, and additive) written with numbers including both integer and non-integer ratios (please see the Appendix). Furthermore, all problem tasks were set in the same context. Using this instrument, the purpose of this study was to examine whether 5th and 6th grade students could differentiate proportional and non-proportional situations, and whether the use of integer or non-integer ratios in proportional and non-proportional problems affected their solution strategies. The study aims to provide a comprehensive picture of how 5th and 6th graders think about tasks that involve both proportional and non-proportional relations and identify whether the use of different types of ratios in problems affect their reasoning.

Research Questions

More specifically, this study investigated the following research questions:

Research Question (RQ)1: Can 5th and 6th grade students differentiate between the proportional problems (missing-value and comparison) and non-proportional situations (additive and constant situations)? In order to answer this question, the following questions will be examined:

(a) Can 5th and 6th grade students solve proportional (missing-value and comparison) and non-proportional (additive and constant) problems with the same success rate?

(b) For each grade level, what are the distributions of students' erroneous strategies in the four problem types?

RQ2: Does the use of integer and non-integer ratios in proportional and non-proportional problems affect 5th and 6th grade students' solution strategies? In order to answer this question, the following questions will be examined:

(a) Can 5th and 6th grade students solve eight different problems that differ along two dimensions (proportional and non-proportional; integer and non-integer ratios) with the same success rate?
(b) For each grade level, what are the distributions of students' erroneous strategies in the eight problem types?

The Context

In Turkey, prior to a change in mathematics curriculum at the middle school in 2014, children used to start learning about ratio and proportion in the 5th grade (MEB, 2009). Starting with the 6th grade, students are expected to solve proportional tasks by using their knowledge on algebraic expressions, equations, and unknowns. They are also introduced the cross-multiplication method to solve missing-value proportional problems at the 6th grade. This was the context in which the data for this study were collected. Therefore, one could assume that compared to the 5th graders who participated in this study, the 6th grade students were more familiar with certain sets of problems involving ratio and proportion and use of some algorithms (Modestou & Gagatsis, 2008).

Participants

The participants were 5th (n = 120) and 6th grade (n = 101) students studying in a private middle school in Istanbul, Turkey. Convenience sampling was used in order to choose the participants in this study. Number of the males and females in the sample was approximately equal (113 males and 108 females).

Instrument and Data Collection

The data collection instrument included twelve word problems with four buffer items (please see the Appendix). The eight problems involved two types of proportional situations: (a) missing-value and (b) comparison, and two types of non-proportional situations: (c) constant and (d) additive. There were two problems for each type; one with integer and the other with non-integer ratios (see Table 1). The eight problems were all set in the same context, only differed in type. The four buffer problems were chosen from the topics different than ratio and proportion from the Ministry of National Education-sanctioned textbook for the 5th grade mathematics (MEB, 2011). These problems aimed to obscure the purpose of the study for participants so that the instrument provides a more reliable data on students' proportional reasoning.

Table 1. Problem types and order of each question in the data collection instrument

Non-proportional				Proportional			
Constant (C)		Additive (A)		Missing-Value (PM)		Comparison (PC)	
Integer (CI) Q6	Non-integer (CNI) Q9	Integer (AI) Q8	Non-integer (ANI) Q4	Integer (PMI) Q10	Non-integer (PMNI) Q2	Integer (PCI) Q11	Non-integer (PCNI) Q3

The eight proportional and non-proportional problems were adopted with some modifications from previous research studies. Proportional missing-value problems, constant problems, and additive problems were developed in parallel with the items written by Van Dooren et al. (2005). The following problem which was used in another study of Van Dooren et al. (2010) formed the context of our instrument; that is "books":

> Lien and Peter are reading the same book. They read at the same speed, but Peter started earlier. When Lien has read 4 pages, Peter has read 10 pages. When Lien has read 6 pages, how many has Peter read?

We formulated a proportional problem by changing an additive problem as following:

> Emre and Sıla are reading the same book. They started together, but Emre reads slower. When Emre has read 4 pages, Sıla read 20 pages. When Emre has read 12 pages, how many has Sıla read?

We have rewritten each problem according to the numbers used in the problem, forming integer ratio or non-integer ratio, in the same way that Van Dooren et al. (2010) manipulated the number characteristics of additive and proportional word problems.

The main differences between data collection instrument of the current study and the study of Van Dooren et al. (2005) lie on the variability of proportional problems, and context of the problems. In this study, in addition to proportional missing-value problems, there were also proportional comparison problems (Modestou & Gagatsis, 2010). Through including comparison problems, we aimed to reveal students' way of reasoning when the numbers in the problem is not appropriate to use the cross-multiplication strategy. In order to avoid any possible influence of the context, all problems were written in the "book" context. This way, any uncontrolled variance in our results due to prior knowledge (knowledge on volume for instance), reading difficulty (length of written word problems) and complexity of numbers (numbers larger than 100) were cancelled out.

The instrument was pilot tested and also examined by two primary math education experts. The wording in some problems revised based on the experts' comments. The KR-20 of the proportional and non-proportional items found to be 0.75 and 0.68, respectively.

Data were collected in the second semester of the 2012-2013 school year. In each grade level, instruction of ratio and proportion was completed at least one month before data collection. The instrument was administered in morning sessions at both grade levels. No time limit was set for working on the problems. Students were given as much time as they needed to respond to all problems. They were asked to show their work on the problems by providing explanations or drawings if they were not able to express their answers mathematically.

Data Analysis

The first step in data analysis was to tabulate students' solutions to each type of the problem as shown in the Table 2 for each grade level. In creating this table, students' answers in proportional and non-proportional situations were categorized according to the method they used in each problem. If the answer and strategy used to solve the problem is correct then the related cell is named as "Correct." In case of an incorrect answer, erroneous strategy used to solve the problem is indicated in the parenthesis. Three main categories of erroneous answers were identified in the data: (a) additive strategy (A), (b) proportional strategy (P), and (c) other (O). After the categorization of each solution strategy, 24 of the participants (approximately 10% of total number of participants) were selected randomly for interrater reliability. Another rater tabulated these students' answers as correct, incorrect and erroneous solution strategy for each incorrect answer independent from the researcher. An interrater analysis using the Kappa statistic was performed to determine consistency among raters. The interrater reliability for the raters was found to be Kappa = 0.84 ($p < 0.01$), 95% CI (0.772, 0.910). Discrepancies among the raters were identified and then resolved through discussion.

Table 2. Classification of each problem for each student

Problem	Non-proportional				Proportional			
	CI	CNI	AI	ANI	PMI	PMNI	PCI	PCNI
Student ID								
1	Incorrect(P)	Incorrect (P)	Correct	Correct	Correct	Correct	Correct	Incorrect (O)
2	Incorrect(P)	Incorrect (P)	Incorrect(P)	Incorrect(P)	Correct	Correct	Correct	Correct
3	Incorrect(P)	Incorrect (A)	Correct	Correct	Correct	Correct	Correct	Correct
…	…	…	…	…	…	…	…	…
N=221								

In order to answer the first part of the first research question, that is how students performed on proportional and non-proportional situations, one-way repeated measures ANOVA was conducted for each grade level. Adding the number of correct answers for integer and non-integer problems, four different scores were calculated for each student (additive, constant, proportional missing-value, proportional comparison). For the second part of the first research question, use of erroneous methods (additive, proportional, and other) was calculated in percentages for each problem type in both grade levels.

To answer the second research question, first Cochran Q tests were used to analyze whether there were significant differences in students' answers for the eight different types of problems (i.e., AI, ANI, CI, CNI, PMI, PMNI, PCI, and PCNI) for each grade level. However, the Cochran Q test does not reveal the pairwise comparisons with significance. To identify this, McNemar Tests that enabled testing pairwise comparisons were used. In order to control the Type I error rate, Bonferroni correction was applied when testing the four pairwise comparisons that were of interest in this study (i.e., AI-ANI; CI-CNI; PMI-PMNI; PCI-PCNI). A further analysis regarding the role of the use of integer or non-integer numbers was also performed tabulating the use of additive, proportional, constant, and other methods employed in solving the eight types of problems. Differences in the percentages of the use of erroneous strategies were examined using the Chi-square test.

Findings

The findings are presented under each research question below.

RQ1a: Can 5th and 6th grade students solve proportional (missing-value and comparison) and non-proportional (additive and constant) problems with the same success rate?

As explained above, in order to answer the first research question, four interval scores (A, C, PM, and PC) were calculated by adding 5th grade and 6th grade students' scores on integer and non-integer problems within each problem type. For example, additive score (A) is calculated by adding AI and ANI scores. Therefore, the maximum score was 2, while the minimum was 0. For each grade level one-way repeated measures ANOVA was used to investigate whether students' scores on these problems significantly differed.

From Table 3, one can see that, on average, both 5th and 6th graders solved constant problems with the lowest success rate and proportional missing-value problems with the highest success rate.

Table 3. Descriptive statistics for repeated measures ANOVA for the 5th and 6th grades

	5th grade			6th grade		
	Mean	Std. Deviation	N	Mean	Std. Deviation	N
Constant	.7833	.92748	120	.7426	.94502	101
Additive	1.2417	.86962	120	.9109	.87291	101
Proportional Missing	1.7167	.61060	120	1.8515	.45577	101
Proportional Comparison	1.7000	.66862	120	1.8317	.44876	101

For proportional situations, the mean of correct answers given by 5th grade students was 3.42 (*SD* = 1.11) and it was 3.68 (*SD* = 0.77) for the 6th grade students. For non-proportional situations, the mean of the correct answers for 5th grade students was 2.03 (*SD* = 1.37) while it was 1.62 (*SD* = 1.45) for the 6th grade students. Effect sizes were calculated as $d = 1.11$ (for the 5th grade) and $d = 1.77$ (for the 6th grade) respectively. These values of d indicated a large effect size.

Table 4. Mauchly’s Test of sphericity for the 5th and 6th grades

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Problem (for 5th Grade)	.612	57.870	5	.000	.821	.840	.333
Problem (for 6th Grade)	.516	65.391	5	.000	.792	.813	.333

As the Mauchly’s test indicated that the assumption of sphericity had been violated (Table 4), $X^2(5) = 57.870$, $p < .001$, the degrees of freedom were corrected using Greenhouse-Geisser estimatation of sphericity ($\varepsilon = .82$) (Field, 2009). The repeated measures ANOVA test showed that there was a significant effect of the problem type on 5th grade students’ performance, $F = 41.761$, $df = 2.5$, $p < .01$ (Table 5). Similarly, the repeated measures ANOVA test was significant for the 6th grade students, $F = 76.276$, $df = 2.4$, $p < .01$ (Table 6).

Table 5. Tests of within subject effects (5th grade)

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Problem	Sphericity Assumed	70.723	3	23.574	41.761	.000	.260
	Greenhouse-Geisser	70.723	2.463	28.718	41.761	.000	.260
	Huynh-Feldt	70.723	2.519	28.079	41.761	.000	.260
	Lower-bound	70.723	1.000	70.723	41.761	.000	.260
Error (Problem)	Sphericity Assumed	201.527	357	.565			
	Greenhouse-Geisser	201.527	293.060	.688			
	Huynh-Feldt	201.527	299.725	.672			
	Lower-bound	201.527	119.000	1.694			

Note. Since the assumption of sphericity had been violated, the degrees of freedom were corrected using Greenhouse-Geisser estimation of sphericity (Field, 2009).

Table 6. Tests of within subject effects (6th grade)

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Problem	Sphericity Assumed	105.473	3	35.158	76.276	.000	.433
	Greenhouse-Geisser	105.473	2.377	44.366	76.276	.000	.433
	Huynh-Feldt	105.473	2.439	43.243	76.276	.000	.433
	Lower-bound	105.473	1.000	105.473	76.276	.000	.433
Error (Problem)	Sphericity Assumed	138.277	300	.461			
	Greenhouse-Geisser	138.277	237.731	.582			
	Huynh-Feldt	138.277	243.907	.567			
	Lower-bound	138.277	100.000	1.383			

Note. Since the assumption of sphericity had been violated, the degrees of freedom were corrected using Greenhouse-Geisser estimation of sphericity (Field, 2009).

Examining the post-hoc pairwise comparisons for 5th graders (Table 7), we found that success in constant problems was significantly different than additive ($p < .01$), proportional missing-value ($p < .01$), and proportional comparison problems ($p < .01$). Success on additive problems showed a significant difference when compared with proportional missing-value and proportional comparison problems. A significant difference was not observed between proportional comparison problems and proportional missing-value problems.

Table 7. Pairwise comparisons of each problem types in the 5th grade

(I) Problems	(J) Problems	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Constant	Additive	-.458*	.108	.000	-.747	-.169
	PM	-.933*	.097	.000	-1.193	-.674
	PC	-.917*	.103	.000	-1.193	-.641
Additive	Constant	.458*	.108	.000	.169	.747
	PM	-.475*	.102	.000	-.750	-.200
	PC	-.458*	.105	.000	-.740	-.176
PM	Constant	.933*	.097	.000	.674	1.193
	Additive	.475*	.102	.000	.200	.750
	PC	.017	.058	1.000	-.139	.172
PC	Constant	.917*	.103	.000	.641	1.193
	Additive	.458*	.105	.000	.176	.740
	PM	-.017	.058	1.000	-.172	.139

** The mean difference is significant at the .05 level.*

Examining the pairwise comparisons for 6th graders (Table 8) we detected that the success in constant problems was significantly different than proportional missing-value (p < .01), and proportional comparison problems (p < .01). The success on additive problems showed a significant difference when compared with the proportional missing-value problems, and proportional comparison problems. A significant difference was not observed between additive and constant problems and between proportional missing-value and proportional comparison problems.

Table 8. Pairwise comparisons of each problem types in the 6th grade

(I) Problems	(J) Problems	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Constant	Additive	-.168	.110	.774	-.464	.128
	PM	-1.109*	.103	.000	-1.387	-.831
	PC	-1.089*	.100	.000	-1.357	-.821
Additive	Constant	.168	.110	.774	-.128	.464
	PM	-.941*	.100	.000	-1.211	-.671
	PC	-.921*	.097	.000	-1.182	-.659
PM	Constant	1.109*	.103	.000	.831	1.387
	Additive	.941*	.100	.000	.671	1.211
	PC	.020	.051	1.000	-.117	.156
PC	Constant	1.089*	.100	.000	.821	1.357
	Additive	.921*	.097	.000	.659	1.182
	PM	-.020	.051	1.000	-.156	.117

** The mean difference is significant at the .05 level.*

RQ1b. What are the distributions of students' erroneous strategies in each problem type?

In addition to examining students' performance in solving proportional and non-proportional problems, further analysis was performed at the solution strategy level. Table 9 shows 5th and 6th grade students' choice of the solution strategies including percentages of the use of proportional, additive, constant, and other solution strategies in each problem type.

Table 9. The distribution of solution strategies used by 5th and 6th grade students in each problem type

	Constant				Additive				Proportional Missing-value				Proportional Comparison			
Method	P	A	C	O	P	A	C	O	P	A	C	O	P	A	C	O
Use in % in 5th grade	36.3	17.9	39.2	6.7	32.5	62.1	0	5.4	85.8	7.9	0	6.3	85.0	0.8	0	14.2
Use in % in 6th grade	49.5	9.9	36.6	4.0	53.0	45.5	0.0	3.5	92.6	3.0	0.0	4.5	91.1	0.5	0.0	8.4

Note. Proportional strategy (P), Additive strategy (A), Constant strategy (C), Other (O)

As can be seen from the Table 9, 5th graders had a tendency to overuse proportional strategies in non-proportional problems (about 36% of the answers in constant problems and about 33% of the answers in additive problems involved the use of proportional methods). Compared to the 5th grade students, 6th graders had a higher tendency to use proportional methods in non-proportional situations. About 50% of the constant problems and 53% of additive problems were solved using proportional methods. Although both 5th and 6th graders showed tendency to solve non-proportional problems with proportional strategies, proportional situations did not elicit the overuse of additive methods. In the 5th grade only about 8% of the answers included additive methods in proportional missing-value problems, and about 1% of the answers included additive methods in proportional comparison problems. The corresponding percentages were lower in the 6th grade: 3.0% of the answers included additive methods in proportional missing-value problems, and only about 1% of the answers included additive methods in proportional comparison problems.

To sum up, these analyses showed that both 5th grade and 6th grade students` success rates among the four problem types significantly differed and they tended to prefer a proportional solution method in non-proportional situations, which implies that they had difficulty in differentiating the proportional and non-proportional situations.

RQ2a: Can 5th and 6th grade students solve eight different problems that differ along two dimensions (proportional and non-proportional; integer and non-integer ratios) with the same success rate?

As explained above, the Cochran Q test was used to test for the differences in students' scores on the eight different types of problems (i.e., AI, ANI, CI, CNI, PMI, PMNI, PCI, and PCNI) for each grade level.

Table 10. Cochran Q test for each problem type in the 5th and 6th grades

	Frequencies			
	5th Grades		6th Grades	
	Incorrect	Correct	Incorrect	Correct
Constant Integer	74	46	65	36
Constant Non-integer	72	48	62	39
Additive Integer	52	68	65	36
Additive Non-integer	39	81	45	56
Proportional Missing Integer	18	102	5	96
Proportional Missing Non-integer	16	104	10	91
Proportional Comparison Integer	15	105	4	97
Proportional Comparison Non-integer	21	99	13	88

The test statistic was significant for both 5th graders (Cochran Q = 184.961, *df* = 7, $p < .01$) and 6th graders (Cochran Q = 268.871, *df* =7, $p < .01$). Next, McNemar tests with Bonferroni correction were conducted to determine if the four pairwise comparisons that were of interest in this study (i.e., AI-ANI; CI-CNI; PMI-PMNI; PCI-PCNI) were significant. In the 5th grade, the test statistic revealed that students' success on only additive problems was significantly affected by the types of ratios in the problem, X^2 (1) = 6.261, $p < .01$. In the 6th grade, however, the types of ratios involved in the problem created a significant effect not only in additive problems, X^2 (1) = 15.042, $p < .01$, but also in proportional comparison problems, X^2 (1) = 5.818, $p < .01$. We detected significance neither in constant (CI-CNI) nor proportional missing-value (PMI-PMNI) problems at either grade levels.

RQ2b. What are the distributions of students' erroneous strategies in the eight problem types?

A further analysis on the percentages of solution strategies preferred for each problem type revealed the following significant results: In the 5th grade (see Table 11), there was a significant increase in the use of additive methods in constant problems when the ratio changed from integer to non-integer (from 8.33% to 27.50%; X^2 (1) = 14.988, $p < .01$). Also, when the types of ratios changed the decrease in the use of proportional methods in constant problems was significant (from 51.67% to 20.83%; X^2 (1) = 24.683, $p < .01$). In additive problems, there was a significant decrease in the use of proportional methods from integer to non-integer ratios (from 41.67% to 23.33%; X^2 (1) = 9.729, $p < .01$).

Table 11. Solution strategies used by 5th graders and 6th graders depending on the problems types (in %)

		Constant				Additive				Proportional MV				Proportional C			
Method Grade	5th	P	A	C	O	P	A	C	O	P	A	C	O	P	A	C	O
I		51.67	8.33	38.33	1.67	41.67	56.67	0.00	1.67	85.00	9.17	0.00	5.83	87.50	0.83	0.00	11.67
NI		20.83	27.50	40.00	11.67	23.33	67.50	0.00	9.17	86.67	6.67	0.00	6.67	82.50	0.83	0.00	16.67
Method Grade	6th	P	A	C	O	P	A	C	O	P	A	C	O	P	A	C	O
I		59.41	2.97	35.64	1.98	63.37	35.64	0.00	0.99	95.05	1.98	0.00	2.97	96.04	0.00	0.00	3.96
NI		39.60	16.83	37.62	5.94	38.61	55.45	0.00	5.94	90.10	3.96	0.00	5.94	86.14	0.99	0.00	12.87

Note. Proportional strategy (P), Additive strategy (A), Constant strategy (C), Other (O), Integer ratio (I), Non-integer ratio (NI)

In the 6th grade (see Table 11), there was a significant increase in the use of additive strategies in constant problems when the types of ratios changed from integer to non-integer (from 2.97% to 16.83%; $X^2(1) = 10.877$, $p < .01$). Another significant result was the decrease in the use of proportional methods in constant problems from integer to non-integer ratios (from 59.41% to 39.60%; X^2 (1) = 7.922, $p < .01$). In additive problems, when the ratios changed from integer to non-integer, there was a significant decrease in the use of proportional strategies (from 63.37% to 38.61%; X^2 (1) = 12.381, $p < .01$).

To sum up, considerable amount of students in both grade levels switched their solution strategies depending on the type of ratios used in the non-proportional problems. More specifically, in constant problems with integer ratios there was an increase in the use of proportional strategies in both grade levels. Besides, as the numbers in constant problem was changed to non-integer ratios, some students switched to additive strategies. In additive problems when the ratio changed from integer to non-integer there was a significant decrease in the use of proportional strategies. These findings show that the use of non-integer ratios evoked the overuse of additive strategies for some students while some others preferred proportional solution strategy in problems with integer ratios. Although there was a 5% decrease in 5th grades and 10% decrease in 6th grades in the use of proportional strategies when the numbers were changed from integer to non-integer ratios, these were not a statistically significant.

Discussion

The purpose of this study was to determine whether Turkish middle school students were able to differentiate proportional and non-proportional situations, and whether the use of integer or non-integer ratios in proportional and non-proportional problems had any role on their solution strategies. Regarding the first research question, the findings revealed that students had different success rates in proportional and non-proportional situations. 5th and 6th grade students` performance was significantly different in each problem type (C, A, PM, PC). At both grade levels, while proportional missing-value problems were solved with the highest success rate, the lowest success rate was with the constant problems. Although solving a constant type of problem does not require any calculations, why did this type of problems elicit the least success rate in both grade levels? One possibility is that the participants may have mistakenly perceived them as typical proportional word problems, which they mostly come across in math classrooms. This may have led them simply use an algorithm rather than trying to reason about the actual relation among the quantities in the problems. It is very likely that the participants had very limited chance, if any, to deal with constant problems before. In order to successfully distinguish between proportional and non-proportional situations, however, students should be provided instructional opportunities in which they can work different relations (Fernández, Llinares, Van Dooren, De Bock, & Verschaffel, 2012).

Compared to the 6th graders, 5th graders seemed to be slightly more successful in non-proportional problems. And compared to the 5th graders, 6th graders had more success with proportional problems. This finding was in congruence with previous studies, which found that from 3rd to 6th grade (Van Dooren et al., 2010) and from 4th to 10th grade (Fernández et al., 2012) students' success on proportional problems increase while their success in non-proportional problems (additive problems) decrease.

Previous research stated that from primary to secondary level students' tendency to apply proportional strategies to non-proportional word problems increases (Fernández, Llinares, & Valls, 2008; Fernández et al., 2012; Modestou & Gagatsis, 2007; Van Dooren, De Bock, Gillard, & Verschaffel, 2009; Van Dooren et al., 2007, 2010). When percentages of the use of solution strategies were analyzed with respect to the four problem types, in both grade levels, there was an overuse of proportional strategies in non-proportional situations. That is, the findings revealed a tendency to overuse proportional methods in additive and constant problems regardless of the grade level in this study.

Furthermore, the literature also stated that students would overuse additive strategies in proportional situations (Behr & Harel, 1990; Fernández, Llinares, Van Dooren, De Bock, & Verschaffel, 2010, 2010; 2012; Singh, 2000; Van Dooren et al., 2007, 2010). However, the overuse of additive strategies in both types of proportional problems (missing-value and comparison) was not observed in the present study. This situation can be explained by the participation of only two grade levels. Fernández et al. (2012) could not identify a significant difference in the overuse of additive strategies from 5th to 6th grades either. However, the difference from 4th to 10th grade found to be significant in the same study. Therefore, in order to observe the overuse of proportional and additive methods, data collection could include grades starting from elementary grades to high school grades.

We were also expecting to find a significant difference in students` solution strategies depending on the type of ratios involved in the problems. Both 5th and 6th grade students' success rates were significantly influenced by the use of integer and non-integer ratios in additive problems. As the ratios were changed from integer to non-integer, success rates significantly increased at both grade levels. Surprisingly, the use of non-integer ratios in additive problems improved students' performance. This situation suggests that the use of situations that do not automatically fit into expectations may have enforced students to read the problem more carefully and to abandon using the proportional methods.

In addition to success rate analysis, further analyses regarding the solution strategies used on the eight types of problems (i.e., AI, ANI, CI, CNI, PMI, PMNI, PCI, and PCNI) indicated that presence of integer or non-integer ratios also showed a significant difference in the types of mistakes students made when solving proportional and non-proportional problems. The findings were contradictory to the literature as both 5th and 6th graders seemed to be influenced by the type of ratio used only in non-proportional problems. More specifically, in constant problems, both 5th and 6th grade students switched to proportional strategies when numbers formed integer ratios, while non-integer ratio use increased the overuse of additive strategies. Also, in additive problems when ratios changed from integer to non-integer, there was a significant decrease in the use of proportional methods. Interestingly, the presence of non-integer ratios in non-proportional problems (additive and constant) decreased the overuse of proportional strategies in both grade levels.

These findings suggest that students perceive integer ratios as the major cue for using proportional strategies without further considering the actual relations among the variables in the problem. As suggested by Greer (1987) solving the same problem by changing integer ratios to non-integer ones can be an option for making students realize that numbers used in the problem cannot be indicative of the solution strategy. Therefore, students need to be presented with proportional problems with different types of ratios in schools.

While non-proportional situations elicited different types of mistakes depending on the use of different ratio types, the use of integer or non-integer ratios did not show significant differences in students' choice of the erroneous solution strategies in proportional problems. Previous research found that students tended to overuse additive strategies in proportional tasks when numbers in the problem involved non-integer ratios (Behr & Harel, 1990; Fernández et al., 2010; 2012; Singh, 2000; Van Dooren et al., 2010). This finding may be explained by students' stronger tendency to prefer proportional strategies in inappropriate situations. Proportional strategies were so dominant that their use was not influenced even by the use of different types of ratios used in problems. Modestou and Gagatsis (2007) suggest that when students always deal with similar contextual or numbered problems, they associate these characteristics with proportional solution method, which is the most prevailing in classroom instruction. Therefore, students should be presented with a range of proportional and non-proportional situations in math classrooms (Fernández et al., 2012). As they are offered these different problem types, they will have the chance to focus on the relevant relationships among variables rather than relying on superficial cues.

The findings in this study may not be necessarily generalizable to public or all private schools in Turkey, given that the samples were selected from one private school. However, the results suggest a reasonable picture that would be similar across other private schools and public schools. Nonetheless, more research is needed to validate the findings in different contexts.

Further research may examine how superficial cues in problem statements affect students' solution strategies. Researchers may also analyze the overuse of proportional and additive methods in a developmental way. That is, data collection could include grades starting from elementary to high school grades. Longitudinal studies can be useful to provide an understanding of the progress students make in their approach to additive and multiplicative relations. Besides this, the effect of the recent change in the middle school math curriculum in Turkey on students' understanding on proportional and non-proportional situations is worth investigating. A study comparing proportional reasoning of students who studied under the earlier and present curricula would also be worthwhile. In the present study, the problems were set in the same context that enabled us to control contextual cues students may derive from problem statements.

As Tourniaire and Pulos (1985) stated proportional reasoning is a multifaceted construct and it must be presented to students as such. Students' difficulties in understanding proportional relations may be a reflection of the learning and teaching process they experience. Therefore, more research is needed on both preservice and inservice teachers' understanding about proportional and non-proportional relations.

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Appendix

The Data Collection Instrument

1) Emre and Sila go to a book store to buy books at discount. All the books are on discount and their prices are the same. Emre buys 4 books while Sila buys 6 books from the store. If Emre pays 10 TL for the books he buys, how much does Sila have to pay for the books she buys? (Proportional Problem [Missing-Value Structure] with Non-integer Ratio)

2) Emre and Sila go to a book store to buy some books which are on discount for their friends. Emre buys 8 math books, and Sila buys 12 language books. If Emre pays 20 TL for the books, and Sila pays 45 TL, whose book is more expensive? (Proportional Problem [Comparison Structure] with Non-integer Ratio)

3) Sila and Emre want to buy a book series. They need to have 75 TL to buy this series. Emre spends 25 TL of his weekly allowance; that is 40 TL. Sila spends 20 TL of her weekly allowance; that is 30 TL. Then, how many weeks do Sila and Emre need to save money up to buy the book serial? (Distractor)

4) Emre and Sila read the same book. They read at the same speed. But, Sila started to read the book before Emre. When Sila read 10 pages, Emre read 4 pages. How many pages will Sila have read when Emre reads 18 pages? (Additive Problem with Non-integer Ratio)

5) Emre wants to buy a book for Sila's birthday. The book which he wants to buy is 30 TL. If there is a 20% discount in the bookseller, how much money does Emre have to pay after the discount? (Distractor)

6) Emre and Sila go to the library to borrow some books. Emre borrows 8 books and Sila borrows 16 books. The books must be returned to the library within 24 days. If Emre returns his books within 24 days, after how many days does Sila have to return the books to the library? (Constant Problem with Integer Ratio)

7) Emre gift-wrap a book in 6 minutes. Sila buys 15 books to give her friends who come to her New Year party. Sila asks for help and wants Emre to gift-wrap these books. If Emre starts to gift-wrap at 12:17, what time will be when Emre finishes his work? (Distractor)

8) Emre and Sila read the same book. They read at the same speed. But, Sila started to read the book before Emre. When Sila read 12 pages, Emre read 6 pages. How many pages will Sila have read when Emre reads 24 pages? (Additive Problem with Integer Ratio)

9) Emre and Sila go to the library to borrow some books. Emre borrows 10 books and Sila borrows 12 books. The books must be returned to the library within 25 days. If Emre retuns his books within 24 days, after how many days does Sila have to return the books to the library? (Constant Problem with Non-Integer Ratio)

10) Emre and Sila go to a book store to buy books at discount. All the books are on discount and their prices are the same. Emre buys 3 books while Sila buys 8 books from the store. If Emre pays 15 TL for the books he buys, how much does Sila have to pay for the books she buys? (Proportional Problem [Missing-Value Structure] with Integer Ratio)

11) Emre and Sila go to a book store to buy some books which are on discount for their friends. Emre buys 8 short story books, and Sila buys 4 fairy tale books. If Emre pays 32 TL for the books, and Sila pays 20 TL, whose book is more expensive? (Proportional Problem [Comparison Structure] with Integer Ratio)

12) Sila has 20 books in her bookshelf. The number of books that Sila has is 10% of the number of books that Emre has. Then, what is the sum of books that Sila and Emre have in total? (Distractor)

Ortaokul Öğrencilerinin Orantısal Akıl Yürütmelerine Dair Bir İnceleme

Öz

Bu çalışma, ortaokul öğrencilerinin orantısal ve orantısal olmayan durumları ayırt edip edemediklerini ve orantısal ve orantısal olmayan problemlerde tam sayı veya tam sayı olmayan oranların kullanımının öğrencilerin çözüm stratejilerini etkileyip etkilemediğini incelemektedir. Bulgular, öğrencilerin orantısal ve orantısal olmayan problemlerdeki başarılarının anlamlı ölçüde farklı olduğunu göstermektedir. Ayrıca, öğrenciler orantısal olmayan problemlerde orantısal çözüm yöntemlerini tercih etmektedirler. Orantısal olmayan problemlerde tam sayı olmayan oranlar toplamsal stratejilerin kullanımına sebep olurken, tam sayılı oranların kullanıldığı problemlerde öğrenciler orantısal yöntemlere yönelmişlerdir. Alan yazındaki bulguların aksine, orantısal problemlerde tam sayılı veya tam sayı olmayan oranların kullanımı öğrencilerin yanlış çözüm stratejilerinde anlamlı bir etkiye yol açmamıştır.

Anahtar kelimeler: Orantısal akıl yürütme, orantısal akıl yürütme profilleri, orantısal olmayan problemler, ortaokul