

## Modified Models of Bentler and Woodward Model of Confirmatory Factor Analysis to Analysis of Covariance

### Kovaryans Analizinde Bentler ve Woodward'un Doğrulayıcı Faktör Analizi Modelinin Değiştirilmiş Şekilleri

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#### ABSTRACT

*Analysis of covariance is a technique used to fix effects of covariates on dependent variables to test treatment effect in experimental studies. In the analysis of covariance it is assumed that covariates are measured perfectly reliable. The assumption of perfectly reliable covariates is almost impossible to meet. In reality, it is almost impossible to measure a covariate without error. In this study, a structural model suggested by Bentler and Woodward (1979) was modified both observed and latent and observed models for a reel data set. These modified models do not require perfectly reliable covariates. In these models the amount of errors in covariates are accounted for testing fit of models and significance of treatment effect*

**Keywords:** Analysis of covariance, structural modeling, covariate, path analysis

#### ÖZ

*Kovaryans analizi, deneysel çalışmalarda bağımlı değişkene kodeğişkenlerin etkisinin bertaraf edilmesinde kullanılan istatistiksel bir tekniktir. Kovaryans analizinde kodeğişkenlerin mükemmel güvenilirlikte ölçüldüğü varsayılır. Mükemmel güvenilir kodeğişkenler sayılısının sağlanması nerdeyse imkânsızdır. Gerçektende hatasız kodeğişken ölçümleri elde etmek pek mümkün değildir. Bu çalışmada Bentler ve Woodward(1979) tarafından önerilen yapısal modelleme uyarlanarak hem sadece gözlenen hem de gizil ve gözlenen değişkenler kullanılarak oluşturulan modeller gerçek bir veri seti için kullanılmıştır. Bu yapısal modeller mükemmel güvenilirliğe sahip kodeğişken sayılısını gerektirmemektedir. Bu önerilen modellerde, kodeğişkenlerdeki hata miktarları modelin doğruluğunu ve yapılan işlemin etkisinin manidarlığını test etmede kullanılmaktadır.*

**Anahtar Sözcükler:** Kovaryans analizi, yapısal modelleme, kodeğişken, yol analizi

## INTRODUCTION

Covariance analysis is a statistical technique commonly used in experimental research. As Bayram (2009) stated analysis of covariance was first put forward in 1935 by Fisher. As it is known, analysis of covariance is a statistical technique to test the presence of the experimental effects by taking into account the effect of the initial differences between the groups in terms of covariate variable or variables associated with the dependent variable used in the experimental design. A variable not included in a research design and affects dependent variable is called a covariate. As Raykov (2010) indicated analysis of covariance is a statistical method that may be viewed as an extension of analysis of variance when, in addition to one or more factors and it is required to account for possible differences due to a continuous variable(s), usually called covariate(s) or concomitant variable(s). The analysis of covariance is a statistical technique which is a combination of regression and analysis of variance (Lawal, 2014). Büyüköztürk (2006) stated that the purpose of covariance analysis is statistically controlling the effect of factors outside of the design related with the dependent variable in a research. Similarly, Hays also (1994) mentioned that the purpose of the analysis of covariance is estimating the results of analysis of covariance when effect of covariate is held constant. As Cox and McCullagh (1982) pointed out covariance analysis is a numerical technique for correcting the effect of covariate in experimental researches. To apply analysis of covariance to the data set there are some required assumptions to provide. As Stevens (2009) pointed out analysis of covariance rests on three additional assumptions regarding the regression part of the covariance analysis to ANOVA assumption, as follows:

1. A linear relationship between the dependent variable and the covariate(s).
2. Homogeneity of the regression slopes for one covariate is the same in each group.
3. The covariate is measured without error.

Glass, Peckham and Sanders (1972), have stressed that many of the assumptions in the provision of mathematical models that are always wrong in small or large sizes. The

assumption of covariate is measured without error is unlikely to meet. The presence of errors of measurement can produce misleading conclusions, either by inflating or by obscuring the true differences among the treatment levels Hays (1994: 836). Using unreliable covariates can lead to the erroneous conclusion that a treatment has an effect when it doesn't or that a treatment has no effect when it really does (Arbuckle, 2013: 145).

Another type of error related to covariance analysis is *error of specification*. According to Hays "An error of specification occurs when one measures a trait or concept that is actually different in some degree from what one hopes to measure. Thus, for example, if a test purports to measure academic achievement in some content area mainly reflects *test wiseness* on the part of subjects, an error of specification is made" (Hays, 1994: 836).

Some alternative methods of analysis of covariance are recommended in case of failure to meet the assumptions. According to Hays (1994) one alternative way to analysis of covariance is holding covariates at a constant value, but this approach is often difficult, expensive, and limits the generalizing of the conclusions. Another alternative way to analysis of covariance is matching or blocking approaches. Another alternative method of analysis of variance is rank analysis of covariance suggested by Lesaffre, and Senn, (2003); Huitema (1980). In this method normality, homogeneity of variance and/or regression slope of the linearity assumptions can be violated. Implementation of this technique requires that data must be at least ordinal scale or transformed to ordinal scale when scale of data is a higher-level than ordinal scale. Henson (1998) pointed out that in analysis of covariance, meeting the homogeneity of regression assumption is critical in determining its viability as a statistical tool. As D'Alonzo (2004) mentioned when the homogeneity of regression slopes assumption has been violated, the researcher needs to look for an alternative approach to the ANCOVA. Huitema (1980) when the hypothesis of equal (null hypothesis) slopes of the regression lines for different groups in the analysis of covariance ( $H_0: \beta_{\text{group1}} = \beta_{\text{group2}} = \dots = \beta_{\text{groupj}}$ ) is rejected analysis of covariance is not used, then as an alternative to analysis of covariance the Johnson-Neyman technique need to be used. In Johnson-Neyman technique, recommended to

use when regression slopes are heterogeneous for different groups, the values of X covariate which make significant difference or non-significant difference for the Y dependent variable are determined. So, which X values contribute significant differences for the X values are interpreted.

In this study, as an application to analysis of covariance with structural equation model suggested by Bentler and Woodward (1979) is modified to show the applicability of the proposed models. Two modified models were tested; the first modified method was a path analysis method, uses only observed variables with two covariates. The second modified method uses two observed covariates and one latent variable. In this study, applicability of modified models of structural equation modeling for analysis of covariance proposed by Bentler and Woodward (1979) to different data set is being investigated.

#### **Problem statement**

Are the modified models of covariance analysis with structural equation model that suggested by Bentler and Woodward (1979) applicable to path analysis and structural models for the real data set used in this study?

#### ***Sub Problems***

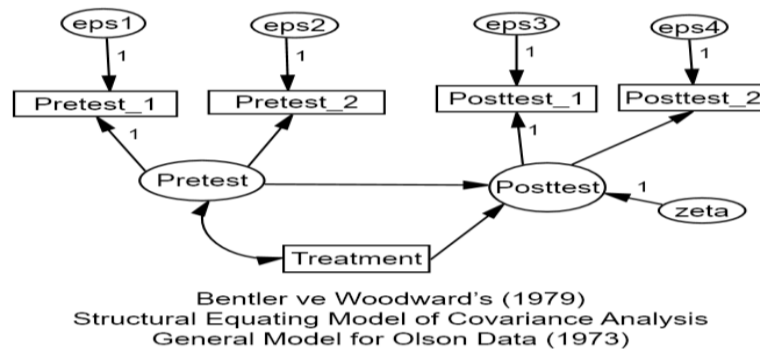
1. Is it possible to suggest a theoretically reasonable path analysis model for the research data by utilizing the Bentler and Woodward's (1979) modified structural equation model for covariance analysis?
2. Is it possible to suggest a theoretically reasonable structural equation model for the research data by utilizing the Bentler and Woodward's (1979) modified structural equation model for covariance analysis?

## METHOD

First, Bentler and Woodward's (1979) suggestion of analysis of covariance model with structural modeling, the main source of this study, was presented below. Then, two models to be tested in this study are described.

### Bentler and Woodward (1979) Model for Analysis of Covariance

Bentler and Woodward's (1979) model includes two experimental groups and two covariates (*Pretest\_1* and *Pretest\_2*). This model can be adjusted for more experimental groups and covariates. The model presented in Figure 1.



**Figure 1.** Bentler and Woodward's (1979) Structural Equating Model

In the model *Pretest\_1* and *Pretest\_2* are both imperfect measures of unobserved ability *Pretest*. The unique variables *eps1* and *eps2* represent errors of measurement in *Pretest\_1* and *Pretest\_2*, as well as any other influences on the two tests not represented elsewhere in the path diagram. Similarly, in the model *Posttest\_1* and *Posttest\_2* are both imperfect measures of unobserved ability *Posttest*. The unique variables *eps3* and *eps4* represent errors of measurement in *Posttest\_1* and *Posttest\_2*, as well as any other influences on the two tests not represented elsewhere in the path diagram. The unique variables *zeta* represents errors of measurement in *Posttest*, unobserved variable.

In the model significance of the regression weight associated with the arrow pointing from *Treatment* to *Posttest* determines whether experimental effect is significant. So, in

this analysis of covariance model, the model in Figure 1 first needs to be tested. If this model does not fit the data, some modifications should be performed to improve the model for having an acceptable model. After having a fit model, the regression weight associated with the arrow pointing from *Treatment* to *Posttest* is determined as zero and the model is tested. If the model with zero treatment effect is significant then it is concluded that experimental effect is not significant or else the experimental effect is significant.

### Data Set

In this study, the data from Aktas (2012) study was utilized. Some of the data from Aktas' study, achievement pretest, attitude pretest, achievement posttest and treatment groups (experimental and control) variables, were used in this study. The study includes 54 subjects, 28 in the experimental group and 26 in the control group.

In the published thesis of Aktas (2012), she used only the experimental group to test the posttest and pretest differences by using t-test for related two groups and she did not use analysis of covariance. However, she has given permission to me for using all the data set.

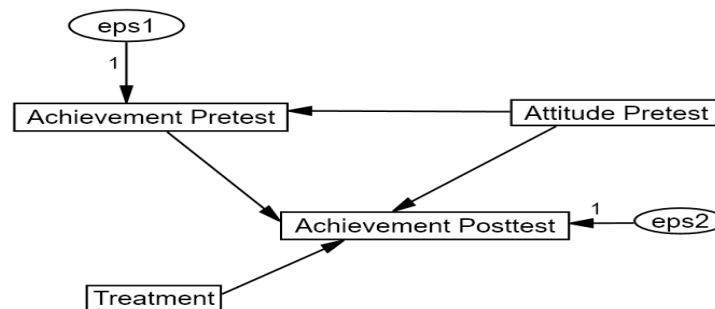
Some descriptive statistics are as follows: For the control group the arithmetic mean and standard deviations respectively were 73.54 and 16.63 for *Achievement Pretest*; 87.46 and 12.97 for *Achievement Posttest*; and finally, 155.04 and 12.62 for *Attitude Pretest*. For the experimental group the arithmetic mean and standard deviations respectively were 65.14 and 17.92 for *Achievement Pretest*; 81.14 and 18.21 for *Achievement Posttest*; and finally, 160.61 and 11.35 for *Attitude Pretest*. The Pearson correlation coefficients between *Achievement Pretest* and *Achievement Posttest* was 0.54 ( $p=.004$ ), *Achievement Pretest* and *Attitude Pretest* was 0.288 ( $p=.154$ ), *Achievement Posttest* and *Attitude Pretest* was 0.521 ( $p=.006$ ).

### Modified Analysis of Covariance Models

The Bentler and Woodward (1979) Model consists of 2 covariates (*Pretest\_1* and *Pretest\_2*) and two posttests (*Posttest\_1* and *Posttest\_2*). The original model includes a latent variable of Posttest. Because the original model has two observed posttest

measures, it allows latent variable to be created, as it given in the “Section of Bentler and Woodward (1979) Model for Analysis of Covariance, given above”. However, the data in this study has only one *Posttest* observed measure so data in this situation do not allow using a *posttest* latent variable. For this reason, the original model is modified. Also, this study has a modified model for path analysis method.

Two modified models of Bentler and Woodward (1979) were tested. Those methods are called as path analysis method and structural modeling with observed and latent variables. Those models are named as the “Model 1a” and the “Model 2a” and given at the Figure 2 and Figure 3 below. These models are taken as initial models. We had to start with a model that we believe is correct in order to use it as the basis for testing a stronger *no treatment effect* version of the model Arbuckle, (2013: 149). A reel data set was used in this study to modify Bentler and Woodward’s structural modeling for analysis of covariance for different situations.



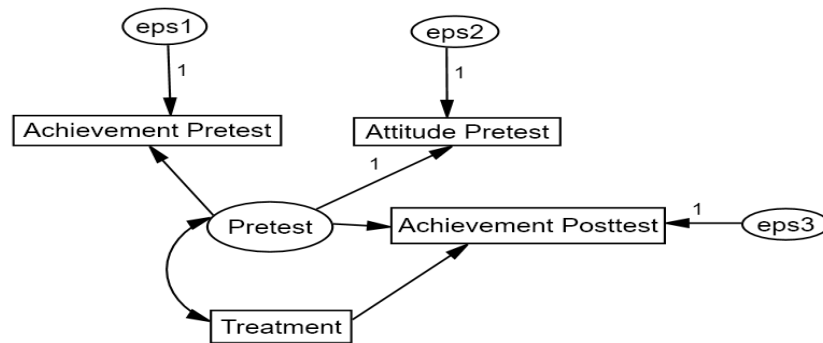
Model 1a: Initial Model of Covariance Analysis with Observed Varia

**Figure 2.** The Model 1a: Modified Initial Model of Covariance Analysis with Observed Variables (Path Analysis)

As it is seen Figure 2, in the “Model 1a” all of the variables are observed variables which makes this model a path analysis model. In the model the observed variable of *Achievement Posttest* is predicted from the observed variables of *Attitude Pretest*, *Achievement Pretest*, and *Treatment*. Also, in the “Model 1a” the observed variable of *Achievement Pretest* is predicted from the observed variable of *Attitude Pretest*. In the “Model 1a” covariates of *Attitude Pretest*, and *Achievement Pretest* are not perfectly

reliable. In this model variables of *eps1* and *eps2* represent errors of measurement in *Attitude Pretest*, and *Achievement Pretest*, as well as any other influences on the two tests not represented elsewhere in the path diagram.

In Figure 3, the “Model 2a”, initial model of covariance analysis with latent and observed variables, is presented.



Model 2a: Initial Model of Covariance Analysis with Latent and Observed Variables

**Figure 3.** The Model 2a: Initial Model of Covariance Analysis with Latent and Observed Variables (Structural Model)

As it is seen in Figure 3, the “Model 2a” is a model created by utilizing a pretest latent variable and other observed variables. In the model observed variables of *Attitude Pretest* and *Achievement Pretest* are predicted from the latent variable of *Pretest*. In this model observed variables of *Achievement Posttest* is predicted from the latent variable of *Pretest* and the observed variable of *Treatment*. In the model *Attitude Pretest*, and *Achievement Pretest* variables, predicted from the covariate *Pretest*, are not perfectly reliable variables. In the model variables of *eps1*, *eps2* and *eps3* represent errors of measurement in *Attitude Pretest*, *Achievement Pretest*, and *Achievement Posttest*, as well as any other influences on the related endogenous variables not represented elsewhere in the path diagram.



To apply analysis of covariance by using confirmatory factor analysis first, two modified theoretical models (Model 1a and Model 2a) will be tested. If tested models fit then we go to the second step. In second step, treatment effect will be removed from the model (the regression weight associated with the arrow pointing from *Treatment* to *Posttest* is determined as zero) then the models are tested. If the models still fit (after removing treatment effect) we will reach the conclusion that the experimental effect is not significant. If the models fit (after removing treatment effect) we will reach the conclusion that the experimental effect is significant. To use confirmatory factor analysis for covariance analysis first we need to have an initial model that the data fits. If initial model does not fit then model must be modified by using modification indices by the computer software to have a fit model.

## RESULTS

The followed procedures in analysis for each model are as follows: The first step in the analysis was testing the initial model. Second step was modifying the initial model if it was not fit to data for having a valid model. The final step was testing the model after removing treatment effect from the model. After the results of final step a decision was made about the significance of treatment effect. *First of all*, the assumptions of models were checked.

### Checking Assumptions for Covariance Analysis

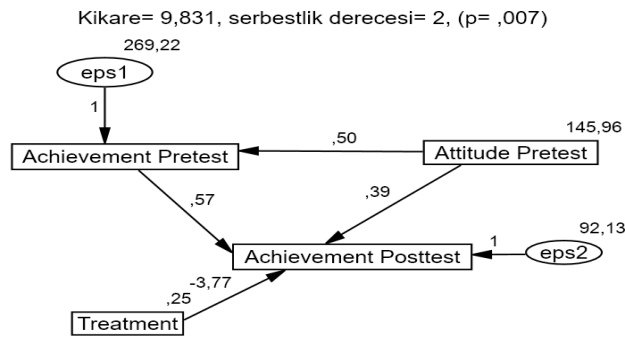
*Normality Assumption:* Kolmogorov-Smirnov tests were performed for normality assumptions (Tan, 2016). Kolmogorov-Smirnov Z values indicated that all of the three variables met the normality assumption. The significance level of Kolmogorov-Smirnov Z values are as follows:  $Z=1.009$  and  $p=0.261$  for *Achievement Pretest*,  $Z=1.327$  and  $p=0.059$  for *Achievement Posttest* and  $Z=1.074$  and  $p=0.199$  for *Attitude Pretest*.

*Homogeneity Assumption:* One sample Levene's tests have not found significant for homogeneity ( $F=0.563$  and  $p=0.456$ ). The significance level of Levene test higher than 0.05. This result shows that homogeneity of variances assumption was met.

*Homogeneity of the Regression Slopes Assumption:* If we consider one covariate this assumption means that the slope of the regression line is the same in each group. If we consider two covariates as Stevens (2009) pointed out the assumption is parallelism of the regression planes. In this study, to test the equality of the slopes (between independent variable and covariates) of the regression lines for experimental and control group significance of interaction effect, for *Treatment* by *Achievement Pretest* and *Treatment* by *Attitude Pretest*, were tested. There were no significant interaction effect both for *Treatment* by *Achievement Pretest* ( $F=1.089$  and  $p=.397$ ) and *Treatment* by *Attitude Pretest* ( $F=2.391$  and  $p=.069$ ). Thus, interaction tests for treatment and covariates were not found significant and homogeneity assumption of regression lines was met.

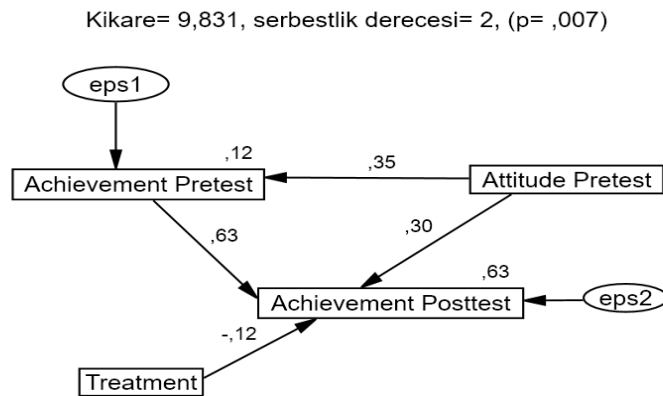
#### **The Model 1a: Results of Covariance Analysis with Observed Variables (Path Analysis) Model**

In the confirmatory factor analysis presented here AMOS 20.0 program was used. Maximum likelihood method was used as a parameter estimate method for the “Model 1a” presented in Figure 2 Results of analysis are presented below. In “Notes for Model” section of AMOS text output, it is presented that number of distinct sample moments as 10, number of distinct parameters to be estimated as 8 and degrees of freedom for chi square test as  $(10-8) 2$ . Results showed that the initial path analysis model was found significant with  $\text{chi-square}= 9.831$  and  $p=0.007$ . In Figure 4 standardized and unstandardized results as AMOS graphical outputs are given.



Model 1a: Initial Model of Covariance Analysis with Observed Variables

Unstandardized estimates



Model 1a: Initial Model of Covariance Analysis with Observed Variables

Standardized estimates

**Figure 4.** AMOS Graphical Outputs (Path diagrams) for Model 1a

Result showed that *first adapted model (the Model 1a) for covariance analysis did not fit the data*. In this case, we cannot test the significance of *Treatment* effect which is the next step. First, we need to have a fit model include experimental effect. To have a fit model modification indices recommended by AMOS, given in Table 1 below, can be used.

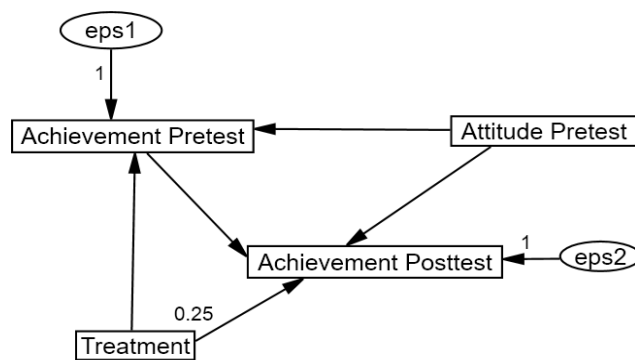
**Table 1.** Recommended Modification Indices in AMOS Text Output for the Model 1a

Covariances				
			Modification Indice	Estimated Change in Parameter
eps1	<-->	Treatment	6.162	-2.795
Regression Weights				
			Modification Indice	Estimated Change in Parameter
Ach_Pretest	---	Treatment	6.162	-11.197

AMOS program as shown in Table 1 suggest two modifications to improve the “Model 1a”. The first of these recommended indices is a covariance between *Treatment* variable and error variable *eps1*. This first recommendation is not considered because it does not make sense theoretically. The second of the recommended indices is a causal effect from *Treatment* variable to *Achievement Pretest* variable. In this modification, linking the *Treatment* variable to the *Achievement Pretest* scores is concerned. This recommendation seems feasible and logical. Then, the modification was made by adding a causal effect from *Treatment* to one of the covariates, *Achievement Pretest*. This modified model named as the Model 1b. In the “Model 1b” at least 6.162 decrease is expected in chi square model fit value which is given in AMOS text output in Table 1. As it is cited above in the Model 1b a causal effect from *Treatment* to *Achievement Pretest* is added. Also, in the model variance value of the *Treatment* variable was constrained not to have model identification problem. In the Model 1b, the variance of *Treatment* variable in the “Model 1a” was used as a constrained value. Thus, the number of parameters to be estimated in the “Model 1b” was reduced by constraining the variance of *Treatment* variable. AMOS results for the “Model 1b” are given below:

### AMOS Results for the Model 1b, Covariance Analysis Model with Observed Variables

As it is cited above, in the “Model 1b” a modification was made by adding a causal effect from *Treatment* to one of the covariates, *Achievement Pretest*. Also in the model, variance value of the *Treatment* variable was constrained by using 0.25 value, taken from the “Model 1a”. The “Model 1b” is shown via a graphic in Figure 5, below:



Model 1b: Modified Model of Covariance Analysis with Observed Variables

### Figure 5. AMOS Input Path Diagram (Model Specification) for Model 1b

In “Notes for Model” section of AMOS text output for the “Model 1b”, it is presented that number of distinct sample moments as 10, number of distinct parameters to be estimated as 8 and degrees of freedom for chi square test as  $(10-8) 2$ . Results showed that the modified path analysis model, the “Model 1b” was not found significant with chi-square= 2.889 and  $p=0.236$ . Some text outputs of AMOS are given in Table 2, below:

**Table 2.** The Model 1b, AMOS Text Outputs for Modified Model of Covariance Analysis with Observed Variables

Unstandardized Regression Weights

			Estimate	Standard Error	Critical Ratio	
Ach_Pretest	<---	Att_Pretest	.616	.175	3.524	***
Ach_Pretest	<---	Treatment	-11.824	4.222	-2.801	.005
Ach_Posttest	<---	Ach_Pretest	.565	.086	6.586	***
Ach_Posttest	<---	Treatment	-3.772	2.825	-1.335	.182
Ach_Posttest	<---	Att_Pretest	.395	.121	3.254	.001

Variiances

	Estimate	Standard Error	Critical Ratio	P
Treatment	.250			
Att_Pretest	145.957	28.353	5.148	***
eps1	236.165	45.877	5.148	***
eps2	92.127	17.896	5.148	***

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 Squared Multiple Correlations:
 

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	Estimate
Ach_Pretest	.277
Ach_Posttest	.665

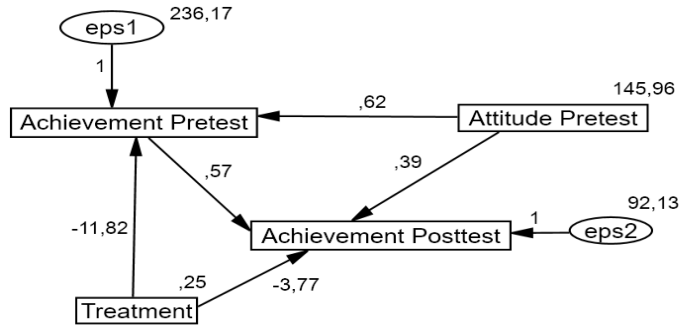
\*\*\* :p<.001

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As it is seen, in “unstandardized regression weights” section of AMOS text output in Table 2 the regression weights of *Attitude Pretest* and *Treatment* variables to *Achievement Pretest* variable was found significant. Similarly, the regression weights of *Achievement Pretest* and *Attitude Pretest* variables to *Achievement Posttest* variable was found significant. Only non-significant regression weight was found between *Treatment* variable and *Achievement Posttest* variable. As it is given in “variances” section of AMOS text output in Table 2, all of the estimated variances in the Model 1b” were found significantly different from zero. As it is shown in Table 2, squared multiple correlation was found 0.28 for *Achievement Pretest* variable and 0.67 for *Achievement Posttest* variable. In other words, accounted variance for *Achievement Posttest* was found approximately 67%. Some estimation of the model fit indices from text output of AMOS proves that the “Model 1b” is fit. Some of the model fit estimates are as follows: Chi square=2.889 with p=0.236, Chi square/df=1.444, GFI=0.974, AGFI=0.871, NFI=0.959, CFI=0.986, and RMSEA=0.092.

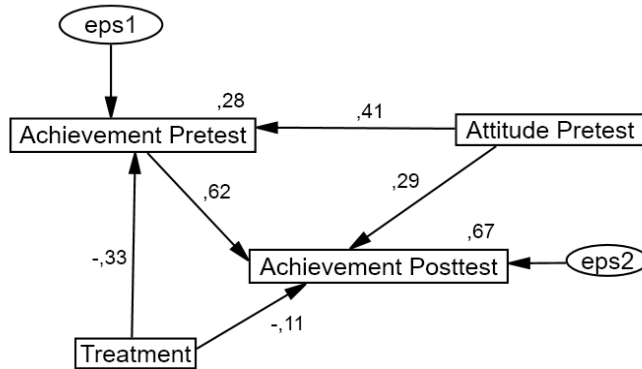
In Figure 6 standardized and unstandardized results as AMOS graphical outputs for the "Model 1b" are given.

Chi square= 2,889, Degrees of freedoms= 2, (p= ,236)



Model 1b: Modified Model of Covariance Analysis with Observed Variables Unstandardized estimates

Chi square= 2,889, Degrees of freedoms= 2, (p= ,236)



Model 1b: Modified Model of Covariance Analysis with Observed Variables Standardized estimates

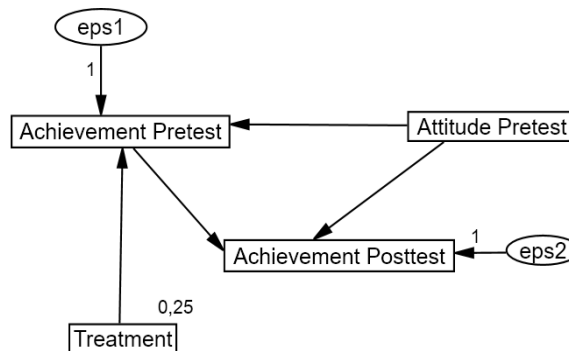
Figure 6. AMOS Graphical Outputs (Path diagrams) for Model 1b



As a result, the “Model 1b” provides the model data fit. In other words, now we have a valid model. As it seen in Figure 6, Model 1b the standardized path coefficient between *Treatment* to *Achievement Posttest* is -.11. In *Treatment* variable the control group coded as 1 and the experimental group is coded as 2. So being in experimental group has positive effect on *Achievement Posttest* variable. Similar situation is valid for the standardized path coefficient between *Treatment* to *Achievement Pretest* variable. Final step was performed for the Model 1b by removing treatment effect from the modified Model 1b to decide if model fit changes when the causal effect from the variable *Treatment* to the variable *Achievement Posttest* is removed and this model is named as the “Model 1c”. AMOS results for the “Model 1c” are given next:

**Model 1c: Covariance Analysis Model with Observed Variables When Experimental Effect is Removed**

The “Model 1c” is obtained from “Model 1b” by removing causal effect from *Treatment* variable to *Achievement Posttest* variable. The “Model 1c” is given in Figure 7, below:



Model 1c: Covariance Analysis with Observed Variables when Treatment Causal Effect is Removed from the Model 1b

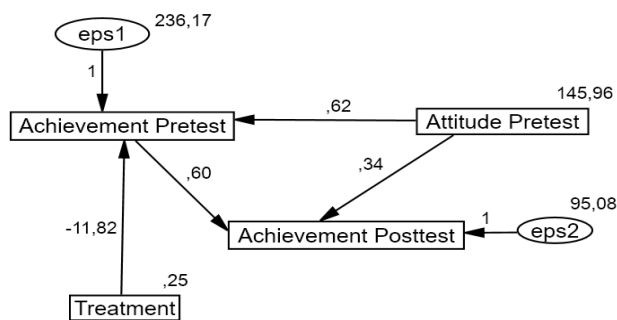
**Figure 7.** AMOS Input Path Diagram(Model Specification) for the Model 1c

**AMOS Results in Graphical Output for the Model 1c**

In “Notes for Model” section of AMOS text output for the “Model 1c”, it is presented that number of distinct sample moments as 10, number of distinct parameters to be estimated as 7 and degrees of freedom for chi square test as (10-7) 3. Results show that the “Model 1c” was not found significant with chi-square= 4.56 and p=0.207.

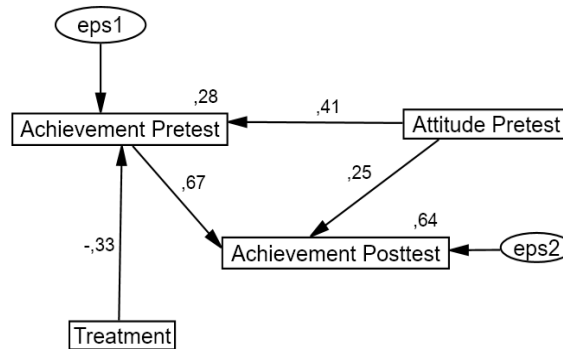
In Figure 7 standardized and unstandardized results as AMOS graphical outputs for the “Model 1c” are given:

Chi square= 4,560, Degrees of freedoms= 3, (p= ,207)



Model 1c: Covariance Analysis with Observed Variables when Treatment Causal Effect is Removed from the Model 1b  
Unstandardized estimates

Chi square= 4,560, Degrees of freedoms= 3, (p= ,207)



Model 1c: Covariance Analysis with Observed Variables  
when Treatment Causal Effect is Removed from the Model 1b  
Standardized estimates

**Figure 8.** AMOS Graphical Outputs (Path diagrams) for Model 1c

The final model, the “Model 1c” was not found significant with chi-square= 4.560 and  $p=0.207$  as it was prior model, the “Model 1b”. In other words, models with and without *Treatment* effect fit the data to estimate *Achievement Posttest* scores. Thus, it is concluded that there is no significant *Treatment* effect for the experiment. In term of covariance analysis the conclusion is as follow: There is no significant difference between the arithmetic means of *Achievement Posttest* for the experiment and control group when covariate variables of *Achievement Pretest* and *Attitude Pretest* are controlled.

### **The Model 2a: Results of Covariance Analysis with Observed and Latent Variables**

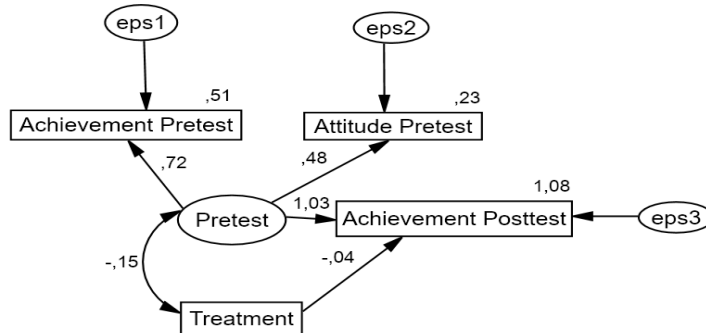
The initial structural model obtained by modifying the Bentler and Woodward’s (1979) model is given in Figure 3.

#### ***Results of Covariance Analysis with Observed and Latent Variables for the Model 2a***

In AMOS maximum likelihood method was used as a parameter estimate method for the “Model 2a” and analysis results are presented below. In “Notes for Model” section of AMOS text output, the number of distinct sample moments is presented as 10,

number of distinct parameters is estimated as 9 and degrees of freedom for chi square test as (10-9) 1. Results showed that initial structural analysis model, the “Model 2a” was found significant with chi-square= 9.195 and  $p=0.002$ . In Figure 9 standardized and unstandardized results as AMOS graphical outputs are given.

Chi square= 9,195, Degree of freedom= 1, ( $p= .002$ )



Model 2a: Initial Model of Covariance Analysis  
with Latent and Observed Variables  
Standardized estimates

**Figure 9.** AMOS Graphical Outputs (Path diagrams) for the Model 2a

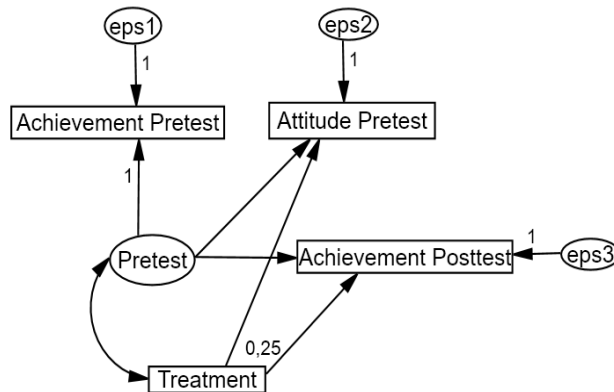
Another indication of model misfit is abnormal or irrational parameter estimation. The variance estimation for eps3 error term was found -21.254 in the “Model 2a”. This illogical estimation is an indication of model misfit. In this case, a model improvement or model modification study has to be made. To have a fit model, modification indices recommended by AMOS, given in Table 3 below, can be used.

**Table 3.** Recommended Modification Indices in AMOS Text Output for the Model 2a

Covariances				
			Modification Indice	Estimated Change in Parameter
eps2	<-->	Treatment	6.357	1.779
Regression Weights				
			Modification Indice	Estimated Change in Parameter
Att_Pretest	<--	Treatment	6.224	7.145

### The Model 2b: Modified Model of Covariance Analysis with Latent and Observed Variables

AMOS program as shown in Table 3 suggest two modifications to improve the “Model 2a”. The second of the recommended indices is a causal effect from *Treatment* variable to *Attitude Pretest* variable. This proposal seems feasible or logical. Then, the modification was made by adding a causal effect from *Treatment* to one of the covariates, *Attitude Pretest*. This modified model named as Model 2b. In the “Model 2b” at least 6.224 decrease is expected in chi square model fit value which is given in AMOS text output in Table 3. As it is cited above in the Model 2b a causal effect from *Treatment* to *Attitude Pretest* is added. By doing this one more parameter is added to the model. Also in the Model 2b variance value of the *Treatment* variable was constrained to not have model identification problem. In the Model 2b, the variance of *Treatment* variable in the “Model 2a” was used as a constrained value, 0.25. Thus, the number of parameters to be estimated in the “Model 2b” was reduced by constraining the variance of *Treatment* variable. The “Model 2b” is given in Figure 10.



Model 2b: Modified Model of Covariance Analysis  
with Latent and Observed Variables

**Figure 10.** AMOS Input Path Diagram(Model Specification) for Model 2b

**AMOS Text and Graphical Outputs for the Model 2b**

In “Notes for Model” section of AMOS text output for the “Model 2b”, it is presented that number of distinct sample moments as 10, number of distinct parameters to be estimated as 9 and degrees of freedom for chi square test as (10-9) 1. Results showed that the modified structural analysis model, the “Model 2b” was not found significant with chi-square= 0.000 and p=0.994. Some text outputs of AMOS are given in Table 4, below:

**Table 4.** The Model 2b: AMOS Text Output for Modified Model of Covariance Analysis with Latent and Observed Variables

Unstandardized Regression Weights

			Estimate	Standard Error	Critical Ratio	p
Ach_Posttest	<---	Pretest	1,206	,261	4,620	***
Ach_Posttest	<---	Treatment	3,806	4,367	,871	,384
Ach_Pretest	<---	Pretest	1,000			
Att_Pretest	<---	Treatment	9,950	3,276	3,037	,002
Att_Pretest	<---	Pretest	,522	,123	4,230	***

Variances

	Estimate	Standard Error	Critical Ratio	p
Treatment	,250			
Pretest	180,663	61,540	2,936	,003
eps2	93,804	19,888	4,717	***
eps3	6,611	42,561	,155	,877
eps1	125,512	38,083	3,296	***

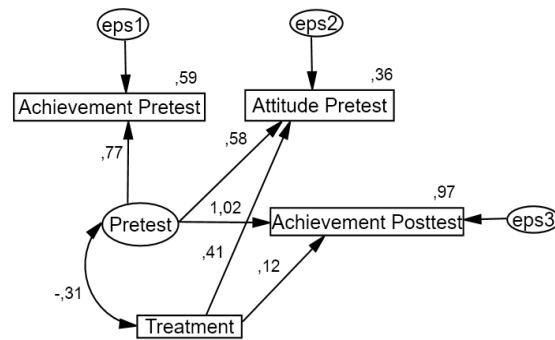
## Squared Multiple Correlations:

	Estimate
Ach_Pretest	,590
Ach_Posttest	,974
Att_Pretest	,357

As it is seen in “unstandardized regression weights” section of AMOS text output in Table 4 the regression weights of *Pretest* latent variable to *Attitude Pretest*, and *Achievement Posttest* variables have been found significant. Similarly, the regression weights of *Treatment* to *Attitude Pretest* variable has been found significant. Only non-significant regression weight has been found between *Treatment* variable to *Achievement Posttest* variable. As it is given in “variances” section of AMOS text output in Table 4, the estimated variances of *Pretest*, *eps1* and *eps3* in the “Model 2b” have been found significantly different from zero; however, the estimated variance of *eps2* in the “Model 2b” has not been found significantly different from zero. As it is shown in Table 4, squared multiple correlation was found 0.59 for *Achievement Pretest* variable, 0.36 for *Attitude pretest*, and 0.97 for *Achievement Posttest* variable. In other words, accounted variance for *Achievement Posttest* was found approximately 97%. Some estimation of the model fit indices from text output of AMOS proves that the “Model 2b” is almost perfectly fit. Some of the model fit estimates are as follows: Chi square=0.000 with p=0.994, Chi square/df=0.000, GFI=1.000, AGFI=1.000, NFI=1.000, and RMSEA=0.000. In Figure 11 standardized and unstandardized results as AMOS graphical outputs for the “Model 2b” are given.



Kikare= ,000, serbestlik derecesi= 1, (p= ,994)



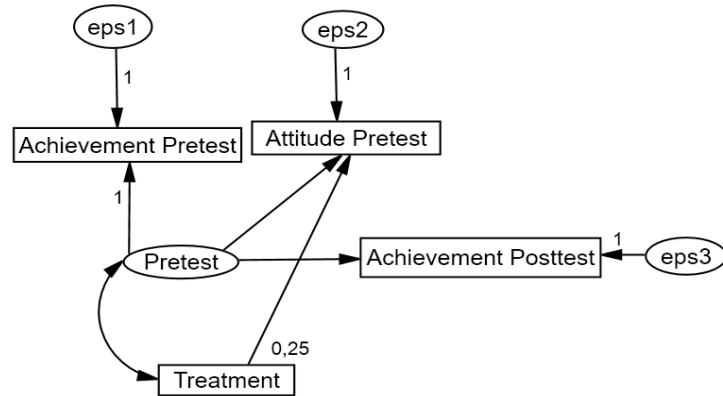
Model 2b: Modified Model of Covariance Analysis  
with Latent and Observed Variables  
Standardized estimates

**Figure 11.** AMOS Graphical Outputs (Path diagrams) for Model 2b

As a result, the “Model 2b” provides the model data fit. In other words, now we have a very valid model. Final step was performed for the Model 2b by removing treatment effect from the modified the Model 2b to decide if model fit changes when the causal effect from the variable *Treatment* to the variable *Achievement Posttest* is removed and this model is named as the “Model 2c”. AMOS results for the “Model 2c” are given next.

#### **The Model 2c: Covariance Analysis Model with Latent and Observed Variables When Experimental Effect is Removed**

The “Model 2c” is obtained from the Model 2b by removing causal effect from *Treatment* variable to *Achievement Posttest* variable. The “Model 2c” is given in Figure 12, below:



Model 2c: Covariance Analysis Model  
with Latent and Observed Variables when Experimental Effect is Removed

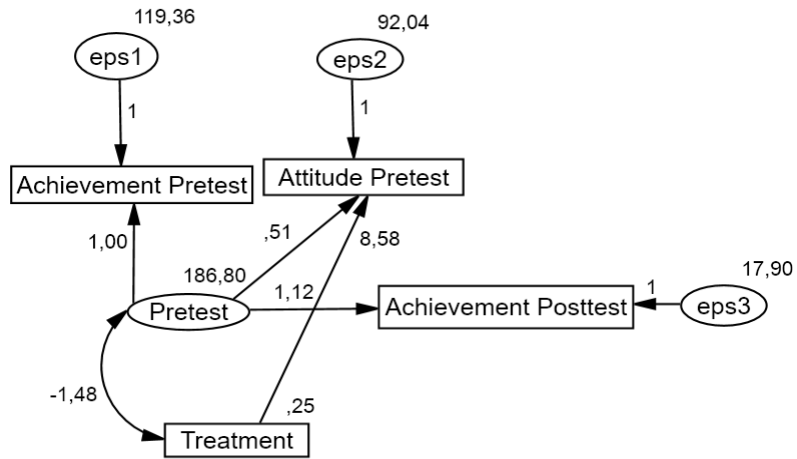
**Figure 12.** AMOS Input Path Diagram(Model Specification) for Model 2c

**AMOS Text and Graphical Outputs for the Model 2c**

In “Notes for Model” section of AMOS text output for the “Model 2c”, it is presented that number of distinct sample moments as 10, number of distinct parameters to be estimated as 8 and degrees of freedom for chi square test as (10-8) 2. Results showed that the “Model 2c” was not found significant with chi-square= 0.88 and p=0.644.

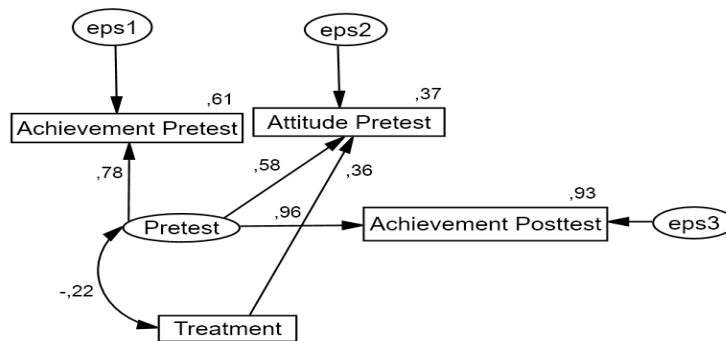
In Figure 13 standardized and unstandardized results as AMOS graphical outputs for the “Model 2c” are given:

Kikare= ,880, serbestlik derecesi= 2, (p= ,644)



Model 2c: Covariance Analysis Model  
with Latent and Observed Variables when Experimental Effect is Removed  
Unstandardized estimates

Kikare= ,880, serbestlik derecesi= 2, (p= ,644)



Model 2c: Covariance Analysis Model  
with Latent and Observed Variables when Experimental Effect is Removed  
Standardized estimates

**Figure 13.** AMOS Graphical Outputs (Path diagrams) for Model 2c

The final model of latent and observed variables, the “Model 2c” was not found significant with chi-square= 4.560 and p=0.207 as it was prior model of latent and

observed variables, the “Model 2b”. In other words, models with and without *Treatment* effect fit the data to estimate *Achievement Posttest* scores. Thus, it is concluded that there is no significant *Treatment* effect for the experiment. In term of covariance analysis, the conclusion is as follow: There is no significant difference between the arithmetic means of *Achievement Posttest* for the experiment and control group when covariate variables of *Achievement Pretest* and *Attitude Pretest* are controlled.

### **DISCUSSION and CONCLUSION**

One of the important assumptions in analysis of covariance is measuring covariates without error. However measuring covariate(s) without error is almost impossible to meet. As Hays (1994) and Arbuckle (2007) pointed out measurement errors of covariates can cause to have misleading research conclusions. So, it is important to use a statistical procedure taking account of measurement errors of covariates for analysis of covariance. One of the statistical procedures accounts for measurement error of covariates in analysis of covariance is confirmatory factor analysis. As Anderson and Gerbing (1984) pointed it out the development of confirmatory analyses for covariance structures (Bentler, 1983; Joreskog, 1969, 1970, 1971, 1978) has provided considerable means to test and modify theories. Anderson and Gerbing (1984: 155). In this study, applicability of confirmatory factor analysis to perform analysis of covariance, suggested by Bentler and Woodward (1979) was showed. Two modified models of Bentler and Woodward (1979) were tested. Those methods are called as path analysis method and structural modeling with observed and latent variables in analysis of covariance. A reel data set was used in this study to modify Bentler and Woodward’s structural modeling for analysis of covariance for different situations.

To apply confirmatory factor analysis, the first step is testing the initial model. Second step is modifying the initial model if it did not fit to data for having a valid model. The final step is testing the model after removing treatment effect from the model. After the results of final step a decision is made about the significance of treatment effect.

In conclusion, almost identical results were found for two modified models, path analysis and structural models. Two modified models, path analysis and structural models, to perform analysis of covariance show that structural modeling can be used for analysis of covariance. In other words, the model proposed by Bentler and Woodward (1979) can be used by adapting for different research designs and data set as done in this study. It is important to notice that no requirement of assuming errorless covariate measures and including measurement errors to the analysis makes advantageous use of structural models in the analysis of covariance. Finally, using structural modeling to perform analysis of covariance prevents researchers from reaching misleading research findings due to low reliability of covariate(s).

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## GENİŞ ÖZET

*Bu çalışmada deneysel bir araştırmada elde edilen verilere kovaryans analizinin uygulanmasında Bentler ve Woodward (1979) tarafından önerilen yapısal eşitlik modelinin uyarlanarak uygulanabilirliği gösterilmektedir. Çalışmada, gerçek araştırma verilerine öncelikle yol analizi modeli uygulanmıştır (Model 1). Bu modelde hiç gizil değişken kullanılmamış yani tamamen gözlenen değişkenlerle kovaryans analizi yapılmıştır. Uyarlanan ikinci model de ise iki tane gözlenen kodeğişkeninden bir tane gizil kodeğişken yapılandırılmıştır (Model 2). Bu modelde gizil ve gözlenen değişkenlerle kovaryans analizinin yapılmasına yönelik bir model uyarlaması yapılmıştır.*

*Bentler ve Woodward'un (1979) önerdikleri model Şekil 1'de sunulmuştur. Modelde Treatment(grup) ile son test arasındaki regresyon katsayısının sıfırdan farklı olup olmadığı, yapılan deneysel işlemin etkili olup olmadığını belirlemektedir. Yani bu kovaryans analizi modelinde, önce Şekil 1'deki modelin test edilmesi gerekir. Model kabul edilmez ise modelde iyileştirme yoluna gidilip kabul edilebilir bir modelin oluşturulması gerekir. Daha sonra bu kabul edilen modelde grupta son test arasındaki regresyon katsayısının sıfır değerine sabitlendiğinde, eğer model kabul ediliyorsa deneysel işlem etkisinin manidar olmadığı ve eğer model reddediliyorsa deneysel işlem etkisinin manidar olduğu sonucuna ulaşılır.*

*Bu çalışmada Aktaş'ın (2012) çalışmasındaki veri seti kullanılmıştır. Bu çalışmada önerilen modeller tamamen gözlenen değişkenlerle yapılan kovaryans analizi ve gizil ve gözlenen değişkenlerle yapılan kovaryans analizi modelleridir. Bu modeller başlangıç modeli olarak sırasıyla Model 1a ve Model 2a olarak adlandırılmıştır. Şekil 2'de sunulan Model 1a tamamen gözlenen değişkenlerden faydalanarak yapılan bir yol analizi modelidir. Şekil 3'te görüldüğü gibi, Model 2a bir ön test gizil değişkeni ve diğer gözlenen değişkenlerden faydalanarak oluşturulan bir modeldir.*

*Verilerin normallik, homojenlik ve regresyon doğrularının eğimlerinin eşitliği sayıltularını sağladığından kovaryans Analizi için uygun olduğu bulunmuştur. Model 1a veriye uyum göstermemiştir (Ki-kare= 9.831 ve p=0.007). Model 1a veri tarafından doğrulanmadığı için AMOS programının önerdiği Başarı ön test puanlarının Treatment (grup) değişkenine bağlı olması modifikasyonu uygulanıp Model 1b oluşturulmuştur. Model 1b veriye uyum göstermiştir (Ki-kare=2.889 ve p=0.236, Ki-kare/sd=1.444, GFI=0.974, AGFI=0.871, NFI=0.959, RMSEA=0.092). Model 1c, Model 1b'den Treatment(grup) ve Başarı son test arasındaki tek yönlü ok kaldırılarak elde edilmiştir. Model 1c veriye uyum göstermiştir (Ki-kare= 4.560 ve p=0.207). Sonuç olarak Başarı son test puanlarını yordamada deneysel işlem etkisi olduğunda ve olmadığına model veri uyumu sağlanmaktadır. Sonuç olarak deney veya kontrol grubunda olmanın Başarı Son test puanı üzerinde manidar bir etkisi yoktur.*

*Çalışmada önerilen ikinci model Gizil ve Gözlenen Değişkenlerle Kovaryans Analizi Model (Model 2a) Şekil 3'te sunulmuştur. Model 2a veriye uyum göstermemiştir (Ki-kare= 9.195 ve p=0.002). AMOS programı tarafından önerilen regresyon ağırlığı olarak Treatment (grup) değişkeninden Tutum Ön test değişkenine tek yönlü ok modifikasyonu yapıldığında Model 1b veriye uyum göstermiştir (Ki-kare=0.000 ve*

$p=0.994$ ,  $Ki-kare/sd=0.000$ ,  $GFI=1.000$ ,  $AGFI=1.000$ ,  $NFI=1.000$ ,  $RMSEA=0.000$ ). Model 2c, Model 2b'den Treatment (grup) ve Başarı son test arasındaki tek yönlü ok kaldırılarak elde edilmiştir. Model 2c'de veriye uyum göstermiştir ( $Ki-kare=0.880$  ve  $p=0.644$ ,  $Ki-kare/sd=0.440$ ,  $GFI=0.992$ ,  $AGFI=0.959$ ,  $NFI=0.987$ ,  $RMSEA=0.000$ ). Sonuç olarak deney veya kontrol grubunda olmanın Başarı son test puanı üzerinde manidar bir etkisi yoktur.

Bu çalışmada önerilen hem sadece gözlenen değişkenlerle yol analizi hem de gizil ve gözlenen değişkenlerle kovaryans analizi sonuçları göstermektedir ki önerilen yapısal eşitleme modelleri kovaryans analizi için kullanılabilir. Kodeğişkenlere ait ölçümlerin mükemmel olduğu varsayılmayıp kodeğişkenlere ait ölçme hatalarının da analize dâhil edilmesi kovaryans analizinde yapısal modellerin kullanımını avantajlı hale getirmektedir.