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BEHAVIOR OF *Q*-DEFORMED QUANTUM PARTICLE STATISTICS ON THE HOLOGRAPHIC SCREEN

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Abstract

In this study, we propose an approach to investigate the relationship between MOND theory and the holographic principle by incorporating q-deformed theory. We first present a brief overview of Verlinde's entropic gravity assumption, which suggests that gravity can be interpreted as an entropic force arising from the statistical mechanics of quantum fields. Some thermo-statistical properties of q-deformed fermion gas model in two spatial dimensions are introduced. At the low-temperature limit, we derive the q-deformed thermal energy and analyze the impacts of fermionic q-deformation on MOND theory. Specifically, we consider the q-deformed Fermi-Dirac statistics of the bits on the holographic screen and examine MOND theory depending on q-deformed acceleration scale. Deformed Friedmann equation is studied by taking into account Friedmann–Robertson–Walker (FRW) universe. This equation shows a modified identification of the evolution of the universe that is compatible with both MOND theory and the holographic principle.

Keywords: Entropic Gravity, MOND Theory, *q*-Deformed Fermion

HOLOGRAFİK EKRANDA *Q*-DEFORME KUANTUM PARÇACIK İSTATİSTİĞİNİN DAVRANIŞI

Özet

Bu çalışmada, MOND teorisi ile holografik prensip arasındaki ilişkiyi araştırmak için q-deforme teorisi göz önüne alınarak bir yaklaşım önerilmiştir. İlk olarak Verlinde'nin, kütle çekimin kuantum alanlarının istatistiksel mekaniğinden kaynaklanan entropik bir kuvvet olarak yorumlanabileceğini öne sürdüğü entropik kütle çekim önerisi verilmiştir. İki boyutlu uzayda q-deforme fermiyon gaz modelinin bazı termo-istatistiksel özellikleri tanıtılmıştır. Düşük sıcaklık limitinde, q-deforme termal enerji türetilmiş ve fermiyonik q-deformasyonunun MOND teorisi üzerindeki etkileri araştırılmıştır. Özel olarak, holografik ekrandaki bitlerin q-deforme Fermi-Dirac istatistikleri ele alınmıştır ve q-deforme ivme ölçeğine bağlı olarak MOND teorisi araştırılmıştır. Friedmann–Robertson–Walker (FRW) evreni dikkate alınarak deforme Friedmann denklemi çalışılmıştır. Bu denklem, hem MOND teorisi hem de holografik prensip ile uyumlu olup, evrenin evriminin değiştirilmiş bir izahını göstermektedir.

Anahtar Kelimeler: Entropik Kütle Çekim, MOND Teori, *q*-Deforme Fermiyon

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1. Introduction

Dark matter is a hypothetical form of matter that does not interact with light or other forms of electromagnetic radiation, hence the term "dark." It is postulated to have gravitational effects on visible matter, which can explain the observed rotation curves of galaxies. The idea is that there is additional mass in the form of dark matter that contributes to the gravitational forces and keeps the rotation curves flat. While the concept of dark matter has been successful in explaining a wide range of astronomical observations, such as the rotational dynamics of galaxies, the nature of dark matter particles remains unknown. Despite extensive research and experimental efforts in particle physics, no direct evidence for dark matter particles has been found so far. Given the absence of direct evidence, researchers continue to investigate alternative explanations for the observed discrepancy between the Newtonian dynamical mass (based on visible matter) and the luminous mass in galaxies. These alternative explanations include modifications to the laws of gravity on galactic scales (modified gravity theories) or alternative theories of gravity that can reproduce the observed rotation curves without the need for dark matter [1-4].

There are various theoretical models that attempt to explain the nature of dark matter and dark energy including modifying Einstein's general relativity, introducing new fields or matter into the equations, or proposing entirely new physical theories [5-7]. One of the theoretical models is MOND theory [8-10] which is a theory of gravity that was suggested by physicist Mordehai Milgrom in the 1980s as an alternative to the concept of dark matter. According to MOND, the observed discrepancies between the predicted and observed motions of galaxies and galaxy clusters can be explained by modifying the laws of gravity, rather than invoking the existence of unseen matter.
Furthermore, the holographic approach has been used as a tool to modify gravity in various ways, such as in theories like entropic gravity [11, 12] and holographic dark energy [13, 14]. Modified gravity theories aim to provide explanations for phenomena such as the observed acceleration of the universe's expansion without relying on dark energy or dark matter. Instead, these theories seek to modify the fundamental principles of gravity itself.
Moreover, the holographic principle has been applied to derive both Newton's law of gravitation [15] and Einstein's field equations [16, 17], suggesting that these fundamental laws of gravity can emerge from the underlying principles of entropy and information. Also, the Friedmann equation, which describes the expansion of the universe in cosmology, has also been derived using the holographic approach [18, 19].
The letter is organized as follows. In Section 2, we give a summary of the Verlinde approach. In Section 3, we represent some low-temperature thermo-statistical behaviour of the q-deformed fermion gas model by considering two-dimensional space. In Section 4, we obtain the q-deformed Friedmann equation of MOND cosmology by applying the entropic force method using the q-deformed thermal energy instead of the equipartition energy. We conclude this letter in the last section.

2. Verlinde’s Approach on the Gravitation

Verlinde's idea fundamentally redefines the nature of gravity by asserting that it is not a fundamental force but rather a manifestation of an entropic force. According to this concept, the appearance and behavior of gravity arise from the underlying dynamics of entropy and the associated microscopic degrees of freedom. [11]. Newton's gravitational law can be derived by the following thermodynamic and holographic principle. We consider a test particle with mass m and distance Δx from the holographic screen. When this particle approaches the holographic screen, the change of entropy on the screen is given as

$$\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x \tag{1}$$

where c is the speed of light and k_B is the Boltzmann constant. According to Unruh's proposal [20], there is a relationship between temperature and acceleration such as

$$T = \frac{1}{2\pi}\frac{\hbar a}{k_B c}. \tag{2}$$

Now we assume that the holographic screen has a radius R and its area is $A = 4\pi R^2$. The relation between the number of bits N and the area of screen A is given by

$$N = \frac{Ac^3}{G\hbar} \tag{3}$$

where G is the Newton's gravitational constant. The total energy of the screen is expressed with the equipartition law of energy

$$E = \frac{1}{2}Nk_B T. \tag{4}$$

If we assume that the total energy of the holographic screen is linked to the mass M, it implies that this mass would manifest within the region of space bounded by the screen, such that

$$E = Mc^2. \tag{5}$$

Newton's law of gravitation can be obtained by using the Eqs. (2)-(5) as

$$a = G\frac{M}{R^2}. \tag{6}$$

The validity of the equipartition law of energy, as derived from the theory of statistical mechanics, is limited to temperature ranges that are not exceedingly low. By considering quantum statistical approach, the low temperature effects on the gravitation can be investigated. Therefore, in the next section, we will give some algebraic properties of q-deformed fermion gas model and obtain its thermo-statistical functions.

3. q-Deformed Fermi-Dirac Statistics

The q-deformed fermion oscillators algebra is defined by the following relations [21-24]

$$ff^* + qf^*f = 1 \tag{7}$$

$$[\hat{N}, f^*] = f^*, \qquad [\hat{N}, f] = -f \tag{8}$$

where f and f^* are the deformed fermionic annihilation and creation operatör, respectively, q is the real deformation parameter in the interval $0 < q < 1$ and $\hat{N}$ is the deformed fermionic number operator. The basic q-deformed quantum number is defined as [24,25]

$$[n] = \frac{1-(-1)^n q^n}{1+q}. \tag{9}$$

In order to determine the thermo-statistical behavior the q-deformed fermion gas model, a fermionic Jackson derivative operator is employed in place of the standard derivative. It is given

$$D_x^{(q)} f(x) = \frac{1}{x}\left[\frac{f(x)-f(-qx)}{1+q}\right] \tag{10}$$

for any function $f(x)$. The mean occupation number of the q-deformed fermion gas model is given as

$$n_i = \frac{1}{|lnq|}\left|ln\left(\frac{|e^{\beta(\varepsilon_i-\mu)}-1|}{e^{\beta(\varepsilon_i-\mu)}+q}\right)\right| \quad (11)$$

where μ is the chemical potential, ε_i is the kinetic energy of a particle in the one-particle energy state i, and $\beta = (1/k_BT)$. The deformed total number of particles N and the deformed total energy of the system E are defined as [25]

$$N = \int_0^\infty g(\varepsilon)n(\varepsilon,T,q)d\varepsilon \quad (12)$$

$$E = \int_0^\infty \varepsilon g(\varepsilon)n(\varepsilon,T,q)d\varepsilon \quad (13)$$

respectively. Here $g(\varepsilon) = mA/2\pi\hbar^2$ is the density of state in two dimensions, m is the mass of particle, and A is the area. When the temperature is very low, the above integrals can be expanded with the help of the Sommerfeld expansion method [26, 27]. So, we derive the following relations for the q-deformed fermion gas model in two dimension

$$N = \frac{mA\varepsilon_F}{2\pi\hbar^2} \quad (14)$$

$$E = E_0 - \frac{mAI(q)}{16\pi\hbar^2}T^2 \quad (15)$$

where ε_F is the Fermi energy, $E_0 = (mA\varepsilon_F^2)/(4\pi\hbar^2)$ is the ground state energy, and $I(q)$ ise defined as

$$I(q) = \int_{-\infty}^{\infty} \frac{x^2}{lnq}\left[\frac{1}{1-e^{-x}} - \frac{1}{1+qe^{-x}}\right]dx \quad (16)$$

where $x = \beta(\varepsilon - \mu)$. The funciton $I(q)$ takes always negative values in the interval $0 < q < 1$. From the definition $E = E_0 + E_{th}$, the deformed thermal energy of the system can be defined as

$$E_{th} = -\frac{NI(q)}{8\varepsilon_F}T^2. \quad (15)$$

Verlinde's approach, as outlined in reference [11], arises a connection between gravitational effects and the thermal excitations occurring on the holographic screen. For this reason, when taking into account gravitational effects, the ground state energy of the system can be ignored. By using the q-deformed thermal energy in Eq. (15), we will focus on the modification of Newton's gravitational law by following Pazy's way [28] in the next section.

4. *q*-Deformed MOND Theory

To obtain Newton's gravitational law, Verlinde considered the equipartition law of energy given in Eq. (4). We now want to replace Eq. (4) with the q-deformed thermal energy defined in Eq. (15) because of arising gravitational effects with the thermal excitations of the holographic screen. If the deformed thermal energy is equated with Einstein's mass-energy relation given Eq. (5), the temperature of the system can be found as

$$T = -\frac{8Mc^2\varepsilon_F}{I(q)N}. \quad (16)$$

Using the Unruh relation in Eq. (2), the acceleration can be obtained as

$$a^2 = -\frac{32\pi^2Mc^2\varepsilon_F}{I(q)N\hbar^2}. \quad (17)$$

In two-dimensional space, the number of degrees of freedom N can be defined as

$$N = \frac{Ac^3}{2G\hbar} = \frac{4\pi R^2c^3}{2G\hbar}. \quad (18)$$

Inserting the Eq. (18) in Eq. (17), we obtain

$$a^2 = \left(-\frac{16\pi c\varepsilon_F}{\hbar I(q)}\right)G\frac{M}{R^2} \quad (19)$$

and compare it with Milgrom's MOND equation [8-10], we reach

$$a\left(\frac{a}{(a_0)_q}\right) = G\frac{M}{R^2} \quad (20)$$

where modified acceleration scale $(a_0)_q$ is defined as

$$(a_0)_q = -\frac{16\pi c\varepsilon_F}{\hbar I(q)}. \quad (21)$$

Now, we want to study the cosmology corresponding to the above obtained MOND theory by considering both approaches introduced in Refs. [29, 30]. Consider the line element of FRW determined by

$$ds^2 = c^2dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} - r^2d\Omega^2\right). \quad (22)$$

In the last equation, the variable *a(t)* represents the scale factor of the universe and k is a constant that relates to the curvature of the FRW universe. The values *k = 1*, *0*, and *-1* correspond to a closed, flat, and open FRW universe, respectively. Additionally, Ω represents the line element of a unit sphere, which is relevant to the geometry of the spatial sections in the FRW metric. In the context of the FRW universe, we assume that the matter source can be described as a perfect fluid. The stress-energy tensor for this perfect fluid is given by:

$$T_{\mu\nu} = \left(\rho + \frac{p}{c^2}\right)u_\mu u_\nu - pg_{\mu\nu} \quad (23)$$

where ρ, p, and u_μ represent the mass-energy density, the pressure of the fluid, and the fluid's four vectors, respectively. We consider the holographic screen as a spherical surface with the radius $R = a(t)r$. The mass inside the holographic screen is given by

$$M = \frac{4\pi}{3}a^3r^3\left(\rho + \frac{3p}{c^2}\right). \quad (24)$$

From the Eq. (20), the acceleration can be obtained as

$$a = -\left[(a_0)_q\frac{GM}{R^2}\right]^{1/2} \quad (25)$$

combined with Eq. (24) to reach

$$a = -\left[\frac{4\pi G}{3}(a_0)_q\left(\rho + \frac{3p}{c^2}\right)R\right]^{1/2}. \quad (26)$$

Bearing $\ddot{R} = \ddot{\mathcal{R}}r = a$ in mind, we finally get

$$\frac{\ddot{\mathcal{R}}}{\mathcal{R}} = -\left[\frac{4\pi G}{3}(a_0)_q\left(\rho + \frac{3p}{c^2}\right)\right]^{1/2}. \quad (27)$$

which is the modified version of the Friedmann equations under q-deformed theory. Due to our assumption that the matter within the holographic screen is responsible for the acceleration, the negative sign in equation (27) arises.

5. Conclusions

In this paper, we use the holographic principle and q-deformed thermal energy in order to obtain the MOND theory and a modified version of the Friedmann equation compatible with MOND theory. We gave a brief review of Verlinde's entropic gravity assumption. Then we introduced a q-deformed fermion gas model in two dimensional spaces. By taking into account the low-temperature termo-statistical properties of the q-deformed fermion gas model, we obtained q-deformed thermal energy in Eq. (15).

By considering the q-deformed Fermi-Dirac statistics of the bits residing on the holographic screen, we have derived the q-deformed MOND theory, represented by Eq. (20). This q-deformed formulation takes into account the effects of q-deformation on the MOND theory, providing a modified description of the gravitational behavior within this framework. It can be seen that the q-deformed acceleration term $(a_0)_q$ corresponds to the Fermi energy of excitations on the holographic screen and the function $I(q)$. In the last, using FRW metric, the q-deformed Friedmann equation in Eq. (27) was obtained in terms of q-deformed acceleration scale. Our result in Eq. (27) is the same form in the Refs. [29, 30].

Consequently, our approach provides insights into the interplay between q-deformed fermion statistics, entropic gravity, and modifications to the MOND framework. This approach may provide new insight into the understanding of dark matter and the evolution of the universe.

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