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Optimizing Truss Dynamics: A Multi-Objective Approach to Modify Natural Frequencies and Mode Shapes with Geometric Constraints

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Anahtar Kelimeler

Ters Modal Analiz
Modal Analiz
Optimizasyon
Kafes Yapılar

Graphical/Tabular Abstract (Grafik Özet)

In this article, a truss structure was studied. This truss structure was redesigned and optimized in terms of its dynamics without compromising structural integrity. To this end, an optimization problem was proposed and the objective is to move a subset of its eigenvalues to particular locations in the complex plane. / Bu makalede kafes yapısı incelenmiştir. Bu kafes yapı, yapısal bütünlükten ödün vermeden dinamik yönden yeniden tasarlanıp optimize edilmiştir. Bu amaçla, bir optimizasyon problemi önerilniş olup amacı, yapının özdeğerlerinin bir alt kümesini karmaşık düzlemdeki belirli konumlara taşımaktır.

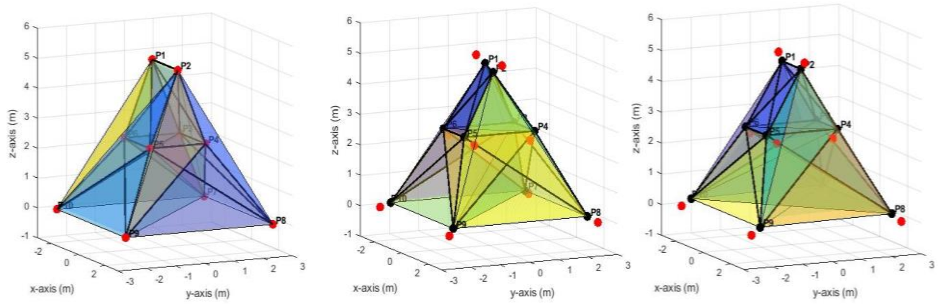

Figure A*: Optimized truss structures at different iteration steps: iteration step-II; iteration step-V; iteration step-X for optimization parameter set-I /* ***Şekil A:*** *(Farklı yineleme adımlarıyla optimize edilmiş kafes yapılar: yineleme adımı-II; yineleme adımı-V; optimizasyon parametre seti-I için yineleme adımı-X*

Highlights (Önemli noktalar)

- *The research advances the understanding of truss dynamics in addition to that provides a framework for approaching similar optimization challenges in mechanical engineering. / Araştırma kafes dinamiklerinin anlaşılmasında ve buna ek olarak makine mühendisliğindeki benzer optimizasyon zorluklarına yaklaşmada rehberlik sağlıyor.*
- *The study contributes optimizing multiple objective dynamic structures without compromising their geometric integrity. / Çalışma, çok amaçlı dinamik yapıların geometrik bütünlüğünden ödün vermeden optimize edilmesine katkıda bulunmaktadır.*
- *The core challenge of inverse modal analysis was addressed through the formulation. / Ters modal analizin temel zorlukları, analitik denklemler aracılığıyla ele alındı.*

Aim (Amaç): *Main aim of the study is improving dynamic behavior of truss structures, where the cost function is tailored to dynamically modify the structure while preserving specified geometric conditions. / Çalışmanın amacı geometrik kısıtı bulunan kafes yapılarında dinamik davranışının iyileştirilmesi ve bunu yaparken belirtilen geometrik koşulları korunmasının sağlanmasıdır.*

Originality (Özgünlük): *The study navigates the delicate balance between optimizing the dynamic aspects of the truss structure and respecting the essential geometry. / Çalışma, kafes kiriş yapısının dinamik yönlerini optimize etmek ile temel geometriye saygı duymak arasındaki hassas dengeyi sağlıyor.*

Results (Bulgular): *Numerical results have shown that, both analysis in Abaqus and Matlab softwares yield compatible natural frequency results. /Sayısal sonuçlar, hem Abaqus hem de Matlab yazılımlarındaki analizlerin uyumlu doğal frekans sonuçları verdiğini göstermiştir.*

Conclusion (Sonuç): *Across all optimization parameter sets and within the defined constraints, the primary objective of optimization was achieved. Notably, the preservation of truss structure topology to meet equality constraints underscores the success of the engineering redesign. / Tüm optimizasyon parametre setlerinde ve tanımlanan kısıtlamalar dahilinde optimizasyonun temel amacına ulaşıldı. Özellikle, eşitlik kısıtlamalarını karşılamak için kafes yapı topolojisinin korunması, mühendislik yeniden tasarımının başarısının altını çiziyor*

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Abstract

This article presents a comprehensive optimization approach to dynamically enhance a truss structure. The optimization problem addresses the systematic modification of the truss dynamics, focusing on achieving a specific set of natural frequencies without compromising the geometrical integrity. The truss structure is redesigned through the exploration of diverse cost functions, considering both minimization and maximization strategies for targeted subsets of natural frequencies and mode shape elements but also preserving essential geometric properties including dimensional intervals, symmetry conditions, and adherence to topological constraints. A dual-objective optimization paradigm is adopted; concurrently pursuing the minimization and maximization objectives together with various constraints are introduced to enforce geometric limits on each truss member, providing a holistic solution for effectively tailoring the dynamic characteristics of the truss structure. This study represents a nuanced understanding of dynamic optimization in truss design. The article's main contribution is improving balance between optimizing the dynamic requirements of the truss structure and considering the essential geometry constraints that ensures its practical utility. By doing so, the research not only advances the understanding of truss dynamics but also provides a framework for approaching similar optimization challenges in mechanical engineering.

Kafes Yapılarının Dinamik Optimizasyonu: Geometrik Kısıtlamalara Bağlı Olarak Doğal Frekansların ve Mod Şekillerinin Modifikasyonunda Çok Amaçlı Bir Yaklaşım

Makale Bilgisi

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Öz

Bu makale, bir kafes kiriş yapısını dinamik olarak geliştirmek için kapsamlı bir optimizasyon yaklaşımı sunmaktadır. Optimizasyon problemi, geometrik bütünlükten ödün vermeden belirli bir dizi doğal frekans elde etmeye odaklanarak kafes kiriş dinamiklerinin sistematik modifikasyonunu ele almaktadır. Kafes yapı, doğal frekansların ve mod şekli elemanlarının hedeflenen alt kümeleri dikkate alınarak, aynı zamanda boyut aralıkları, simetri koşulları ve topolojik kısıtlamalara bağlılık dahil olmak üzere temel geometrik özellikleri koruyarak çeşitli maliyet fonksiyonlarının araştırılması yoluyla yeniden tasarlanmıştır. Çift amaçlı bir optimizasyon paradigması benimsenmiştir; Minimize ve maksimize hedeflerini eşzamanlı olarak takip eden çeşitli kısıtlamalar, her bir kafes kiriş elemanı üzerinde geometrik sınırlamalar uygulamak için tanıtılmış ve kafes kiriş yapısının dinamik özelliklerini etkili bir şekilde uyarlamak için bütünsel bir çözüm sağlanmıştır. Makalenin ana katkısı, kafes yapılarının dinamik gereksinimlerinin optimize edilmesi ile pratik kullanımını sağlamak için temel geometri kısıtlamalarının dikkate alınmasını sağlamaktır. Bunu yaparak, araştırma sadece kafes dinamiklerinin anlaşılmasını geliştirmekle kalmıyor, aynı zamanda makine mühendisliğindeki benzer optimizasyon zorluklarına yaklaşmak için bir çerçeve oluşturmaktadır.

1. INTRODUCTION (GİRİŞ)

Mechanical components e.g. truss structures adhere to a predefined topology during their design, dictated by inherent geometric limitations that impact their functionality. Moreover, these components must conform to constraints imposed by neighbouring components, ensuring proper collaboration when assembled. Specifically, truss structures face the intricate task of not only

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withstanding dynamic forces but also preserving essential geometric properties. These properties include dimensional intervals, symmetry conditions, and adherence to topological constraints. Failure to uphold these requirements renders any optimization attempt futile, as the resulting truss structures would be physically impractical. Therefore, the presented article focuses on addressing the optimization of truss dynamics, distinguishing itself from previous studies by delving into the specific challenges associated with the design and performance of mechanical parts. The article also emphasizes the critical interplay of geometry, physical attributes, and dynamic constraints, highlighting their collective influence on the functionality of assembled mechanical structures.

The core of the research problem is formulated as a constraint optimization challenge, where the cost function is tailored to dynamically modify the structure while preserving specified geometric conditions. The article's main contribution lies in navigating the delicate balance between optimizing the dynamic aspects of the truss structure and respecting the essential geometry that ensures its practical utility. This study, therefore, contributes significantly to the broader field by offering insights and methodologies for optimizing dynamic structures without compromising their geometric integrity. Additionally, a set of geometric constraints tailored for truss-like structures is defined, presenting a comprehensive framework for the inverse modal analysis of truss structures.

As for methodology, we establish a general framework for the constrained optimization problem and outline a procedure for inverse modal analysis. Subsequently, we apply this methodology to a specific space truss structure examining a 25-bar space truss structure to illustrate the key concepts and solution steps of the constrained optimization problem. The remainder of the paper unfolds as follows: Section two provides a review of related works which is highlighting the distinctive contributions of this study. Section three delves into the theoretical foundation, commencing with background material necessary for setting up the constrained optimization problem. Following this, the mathematical formulation of the constrained optimization problem is presented. Section four showcases numerical results and various case studies, including the redesign of a 25-bar truss structure. Finally, section five concludes the paper with a summary and discussions on the implications of the findings.

2. RELATED WORKS (KONUYLA İLGİLİ ÇALIŞMALAR)

Engineering structural optimization encompasses three primary categories: size, shape, and topology optimization. Size optimization seeks to determine the optimal size parameters of components, while shape optimization assumes a fixed topology. Topology optimization, on the other hand, aims to find the optimal layout of a structure within a defined design domain. Typically, the design objective is formulated as a minimization problem, optimizing structural mass with multiple frequency constraints using various design variables, such as topology, sizing, and a combination of shape and size [1]. The design of truss structures poses a unique challenge due to the non-convex nature of the feasible designs, situated within highly non-linear boundaries. Meta-heuristic algorithms have emerged as effective tools for optimizing truss-like structures, often employing a population of design solutions to search for optima. Operators are strategically created and employed to achieve intensification and diversification during the search process [2].

Truss optimization problems commonly involve minimizing the structure's weight while adhering to a set of static constraints. Numerous meta-heuristic algorithms have been applied to address such problems, including artificial bee colony [3], differential evolution [4,5], firefly method [6], genetic algorithm [7], particle swarm optimization [8], and simulated annealing [9].

In addition to static constraints, researchers have explored dynamic constraints in truss optimization, particularly in handling frequency constraints. The penalty function approach has been commonly used to transform constrained optimization problems into unconstrained ones. Various meta-heuristic optimization methods have been employed for truss structures with frequency constraints, including charged system search [10], differential evolution method [11,12], firefly algorithm [13], harmony search [14], genetic algorithm [15,16,17], particle swarm optimization [18,19], colliding bodies method [18], ray optimization [19], simulated annealing [20], and teaching-learning based optimization method [21]. This extensive body of research reflects the ongoing exploration and refinement of optimization techniques for addressing the complexities associated with truss structures under diverse constraints. Recent studies have also shown that natural frequencies and mode shapes of a truss based structures can be optimized in terms of design parameters such as truss shape, length, thickness etc [22,23]. And yet, most of the studies are implementing different heuristic optimization algorithms to be able to improve the truss structures for a single objective [24-27]. Main contribution of the proposed study is offering insights to the reader for considering of multiple objectives and achieving structural behavior of truss structures by inverse modal analysis under geometric constraints.

3.PROBLEM FORMULATION (PROBLEM TANIMI)

The research problem addressed in this article revolves around the optimization of truss dynamics, specifically focusing on the modification of a 25-bar structure's frequency spectrum. Unlike previous studies, this research distinguishes itself by delving deep into the intricate challenges associated with the design and performance of mechanical parts, emphasizing the delicate interplay of geometry, physical attributes, and dynamic constraints.

To tackle the research problem, applied methodology is a constrained optimization approach. The cost function is tailored to dynamically modify the structure, seeking an optimized spectrum of natural frequencies. The preservation of topology is enforced through a set of constraints, which encapsulate symmetry conditions and special geometric considerations like square topology. The complexity of the problem necessitates a sophisticated optimization technique, and the Sequential Quadratic Programming (SQP) method is chosen as the solution approach. SQP is a powerful optimization algorithm that iteratively solves a sequence of quadratic subproblems, adjusting the decision variables to minimize the objective function while satisfying the constraints. In proposing a roadmap for solving the constrained optimization problem with SQP, the research can outline key steps. These may include formulating the mathematical model for the cost function and constraints, initializing the optimization process, iteratively solving subproblems using SQP, and validating the optimized solution against specified geometric conditions

3.1 Topology Optimization of Structural System with Mechanical and Dynamical Constraints

(Mekanik Ve Dinamik Kısıtlamalarla Yapısal Sistemin Topoloji Optimizasyonu)

In engineering design, topology optimisation has become potential tool that can be used to optimise material distribution within specified region of parts in order to achieve better structural performance. Static mechanical restrictions were the main focus of topology optimisation in the past in order to reduce material consumption and preserve structural integrity. However, more recent developments have expanded this method to include dynamic constraints, making it possible to optimise structural systems in a variety of dynamic settings and loading conditions [28].

The main goal of mechanical constraints in topology optimisation is to guarantee structural integrity under static loading scenarios. Techniques like finite element analysis (FEA) and mathematical optimisation algorithms like gradient-based approaches, meta–heuristic algorithms or evolutionary algorithms are frequently used in this procedure [29]. Integrating dynamical constraints into topology optimization expands the scope of design considerations beyond static loading scenarios. In dynamic environments, parts are subjected to time-varying loads, vibrations, and resonance phenomena, which can significantly affect their performance and durability [30]. The integration of mechanical and dynamical constraints in topology optimization has broad applications across various industries, including aerospace, automotive, civil engineering, and biomechanics. From our proposed study perspective, optimizing the shape of a truss structure involves finding the configuration that minimizes weight while satisfying certain geometric constraints such as structural integrity, symmetry conditions, stability etc. under static and dynamic conditions.

3.2. General Remarks (Genel Açıklamalar)

In this article, a truss structure was studied. This truss structure was redesigned and optimized in terms of its dynamics without compromising the structural integrity. The initial and final optimal design of the truss structure, is illustrated in Figure 1.To this end, an optimization problem was proposed where the design parameters are the geometry of the truss elements and the objective is to move a subset of its eigenvalues to particular locations in the complex plane. The coordinates of nodes are given below:

```
P1 = [-95.25 0 508]      ;      P2 = [+95.25 0 508]       ;
P3 = [-95.25 95.25 254]  ;      P4 = [+95.25 95.25 254]   ;
P5 = [+95.25 -95.25 254] ;      P6 = [-95.25 -95.25 254]  ;
P7 = [-254 +254 0]       ;      P8 = [+254 +254 0]        ;
P9 = [+254 -254 0]       ;      P10 = [-254 -254 0]       ;
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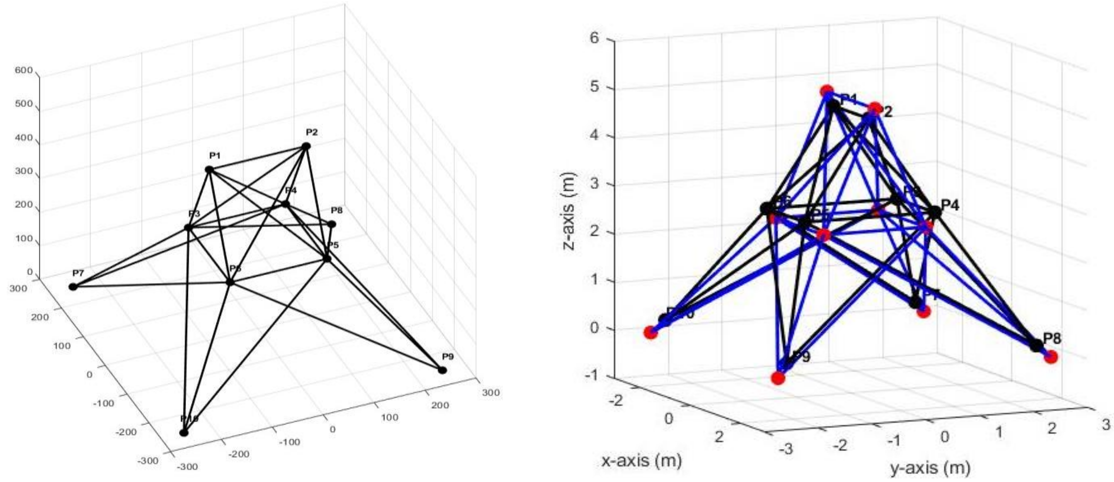

Figure 1. Final topology of the optimally designed 25 bar space truss structure for optimization parameter set-I: red dots and blue lines show the original truss structure nodal points and truss members, respectively; black dots and lines show the final truss structure design nodal points and truss members, respectively (Optimizasyon parametre seti-I için optimal olarak tasarlanmış 25 bar uzaylı kafes yapının son topolojisi: kırmızı noktalar ve mavi çizgiler sırasıyla orijinal kafes kiriş yapısının düğüm noktalarını ve kafes elemanlarını gösterir; siyah noktalar ve çizgiler sırasıyla son kafes yapı tasarımı düğüm noktalarını ve kafes kiriş elemanlarını gösterir.)

A typical element of connectivity matrix i.e. **LCpr = [a b];** implies that there is a truss element connecting nodal point *a* and nodal point *b*. Connectivity matrix of the truss structure is given below.

```
LC01 = [1 2]; LC02 = [1 4]; LC03 = [2 3];
LC04 = [1 5]; LC05 = [2 6]; LC06 = [2 4];
LC07 = [2 5]; LC08 = [1 3]; LC09 = [1 6];
LC10 = [6 3]; LC11 = [4 5]; LC12 = [3 4];
LC13 = [5 6]; LC14 = [10 3];LC15 = [7 6];
LC16 = [9 4]; LC17 = [8 5]; LC18 = [7 4];
LC19 = [8 3]; LC20 = [10 5];LC21 = [9 6];
LC22 = [10 6];LC23 = [7 3]; LC24 = [8 4];
LC25 = [9 5] ;
```

Considering the above constraints and objective, a constrained optimization was defined and solved to design a truss structure having optimal properties: Here, Θ is the set of design parameters, J is the cost function to be minimized. This cost function is subject to equality and inequality constraints.

A quick description of the optimal design of a mechanical system can be best made by the following minimization problem:

$$\begin{gathered} \min_{\Theta} J \\ s.t. \\ Geometric-constraints \end{gathered}$$

3.3 Geometric Constraints for Structural Systems (Yapısal Sistemlerin Geometrik Kısıtları)

The final shape of the structural system is optimized to meet certain dynamical design requirements. Each truss member's length may be confined to a range by the following inequality constraints Eq. (1) and Eq. (2).

$$\left\|\mathbf{P}_i - \mathbf{P}_j\right\| < L^{max}_{e,ij} \quad \forall ij \in \mathrm{N}^{max}_{eL} \tag{1}$$

$$\left\|\mathbf{P}_i - \mathbf{P}_j\right\| \geq L^{min}_{e,ij} \quad \forall ij \in \mathrm{N}^{min}_{eL} \tag{2}$$

3.4 Typical Cost Functions for Structural Systems (Yapısal Sistemlerin Tipik Maliyet Fonksiyonları)

Some typical cost functions were proposed as follows. The cost function in Eq. (3) aims to minimize a subset of natural frequencies.

$$J^1_{\omega} = \sum_{j \in J_1} (\omega_j - \omega_j \cdot \omega^{\times}_{n_j})^{2 \cdot \omega^{e}_{n_j}} \tag{3}$$

J_w^1 is the indices of the subset of natural frequencies to be minimized. $\omega_{n_j}^{\times}$ is multiplier constant that shifts the corresponding natural frequency by some fixed ratio. $\omega_{n_j}^{e}$ defines how important the associated natural frequency. If $\omega_{n_j}^{e}$ is set to $\omega_{n_j}^{e}$ then it implies that that natural frequency is removed from the cost function. In the cost function in Eq. (4), the target natural frequencies can be set explicitly:

$$J_\omega^1 = \sum_{j \in J_1} (\omega_j - \omega_{n_j}^{\oplus})^{2 \cdot \omega_{n_j}^{e}} \tag{4}$$

The following cost function aims to maximize a subset of natural frequencies in Eq. (5):

$$J_\omega^2 = \sum_{j \in J_2} \frac{1}{(\omega_j - \omega_j \cdot \omega_{n_j}^{\times})^{2 \cdot \omega_{n_j}^{e}}} \tag{5}$$

J_w^2 is the indices of the subset of natural frequencies to be maximized. These type of cost functions will try to move the natural frequencies from the target natural frequencies as described by either $\omega_j \cdot \omega_{n_j}^{\times}$ or $\omega_{n_i}^{\oplus}$. Cost functions can be obtained by combining the first and second cost function as shown in Eq.(6).

$$J_\omega^{1,2} = J_\omega^1 + J_\omega^2 \tag{6}$$

3.5 Verification of Matlab Fem Model by Abaqus Sofware (Matlab Sonlu Elemanlar Modelinin Abaqus Yazılımı ile Doğrulanması)

The accuracy and reliability of the finite element model used in this study were rigorously validated through verification in Abaqus software. Both the Matlab code and Abaqus software employed a lumped mass model for the truss element, with a stiffness matrix derived from a linear interpolating function. To ensure the robustness of the model, a comparison of natural frequencies calculated by Abaqus and the Matlab code was conducted, as presented in Table 1, revealing negligible differences that can be safely disregarded. The frequency analysis results for the first three modes, obtained through Abaqus, are depicted in Figures 2-4.

Table 1. Comparison of natural frequencies calculated by Abaqus software and code running on Matlab. (Abaqus yazılımı ve Matlab'da çalışan kod tarafından hesaplanan doğal frekansların karşılaştırılması.)

ω_n^a	ω_n^m	$\Delta\omega_n$ (%)	ω_n^a	ω_n^m	$\Delta\omega_n$ (%)
58.991	58.9912	+0.0001	231.34	231.34	+0.002
62.432	62.432	+0.0001	232.21	232.21	+0.003
76.385	76.385	+0.0002	248.87	248.87	-0.000
100.70	100.70	-0.0003	274.420	274.425	+0.002
102.43	102.43	-0.0001	284.77	284.77	+0.002
105.45	105.45	+0.0003	286.77	286.77	+0.000
110.56	110.56	-0.0001	307.35	307.35	+0.001
144.95	144.95	+0.0001	321.46	321.46	+0.002
219.64	219.64	+0.0002	395.40	395.40	+0.0003

4. NUMERICAL RESULTS FOR 25-BAR TRUSS STRUCTURE (25 BARLIK KAFES YAPISININ SAYISAL SONUÇLARI)

Numerical results for 25-bar truss structures can be seen in this section. The evolution of natural frequencies is depicted in Figure 5, which also showcases the time history of the cost function. Figure 6 provides insight into the truss structure's evolution at different iteration steps. Mode shapes of the final optimized truss structure are visualized in Figures 7-8. In total, three optimization simulations were executed. In simulations one and two, the first six natural frequencies excluded from the cost function, while the other higher frequencies were undergoing a constant positive and negative scale shift. Figure 9 and 10 shows natural frequency spectrum of the original and optimized truss structure for simulation one and two, respectively. In simulation three, the first six natural frequencies were constrained to approach 100 Hz, driving first six natural frequencies into a region with a minimum frequency of 100 Hz. Higher natural frequencies which are close to 300 Hz will tend to keep in the level. Figure 11 shows natural frequency spectrum of the original and optimized truss structure for simulation three.

$\mathbf{P}_i$ and $\mathbf{P}_j$ are nodal points attached to the end of a truss element. N_{eL}^{max} is two dimensional set holding pairs of nodes, maximum length constraint is imposed between the pairs of these nodes. N_{eL}^{min} is two-dimensional set holding pairs of nodes, minimum length constraint is imposed between the pairs of these nodes.

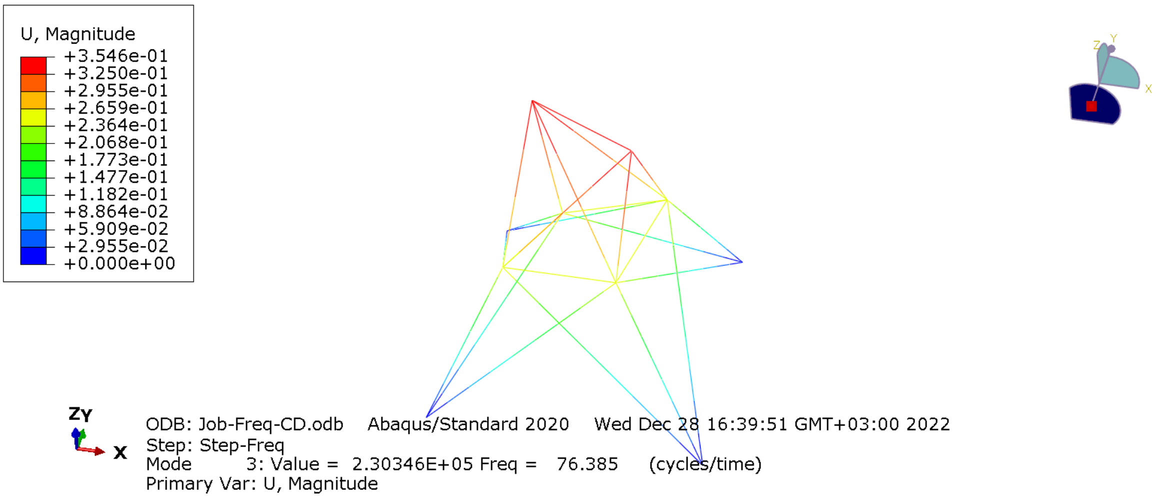

Figure 2. Abaqus frequency analysis results: Mode shape-I. (Abaqus frekans analizi sonuçları: Mod şekli-I.)

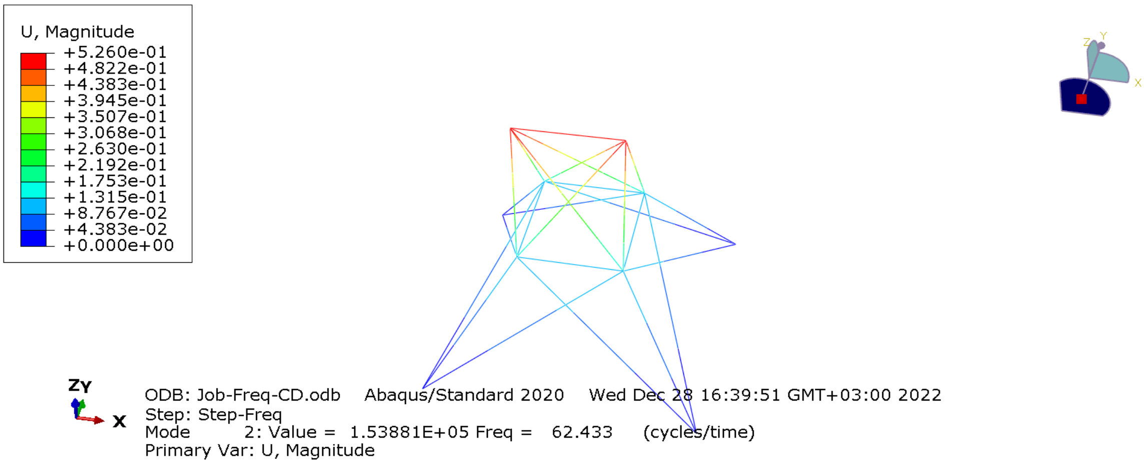

Figure 3. Abaqus frequency analysis results: Mode shape- II. (Abaqus frekans analizi sonuçları: Mod şekli-II.)

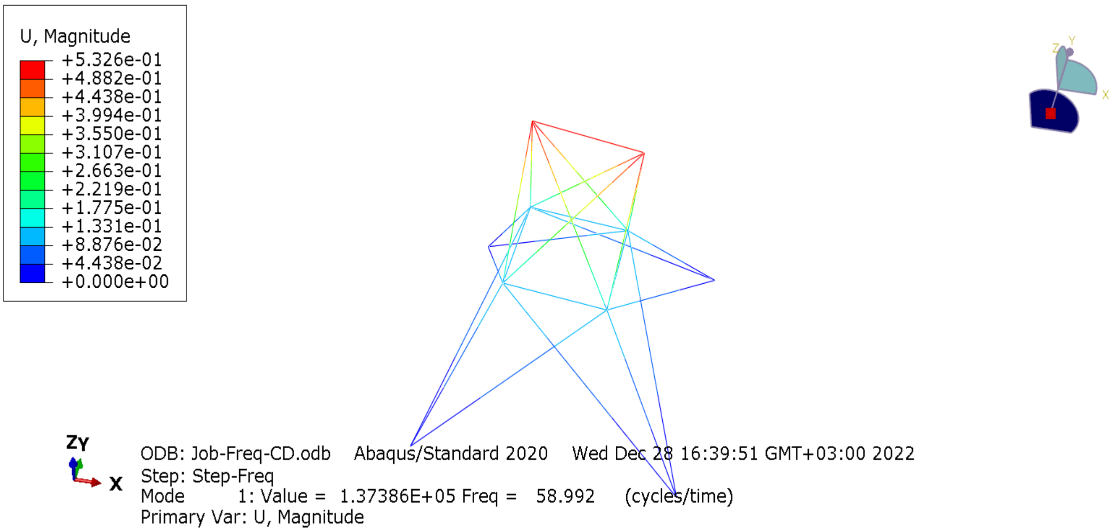

Figure 4. Abaqus frequency analysis results: Mode shape-III.(Abaqus frekans analizi sonuçları: Mod şekli-III.)

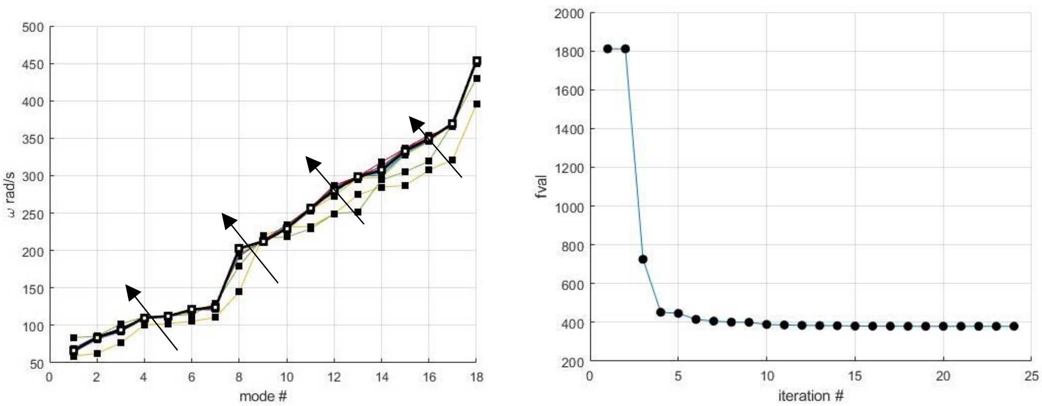

Figure 5. Evolution of the spectrum of natural frequencies of the 25 bar space truss structure. Final frequency spectrum is shown with a thick line. Time history of cost function for simulation made using optimization parameter set-I. (25 bar uzay kafes yapısının doğal frekans spektrumunun gelişimi. Son frekans spektrumu kalın çizgiyle gösterilmiştir. Optimizasyon parametre seti-I kullanılarak yapılan simülasyon için maliyet fonksiyonunun zaman geçmişi.)

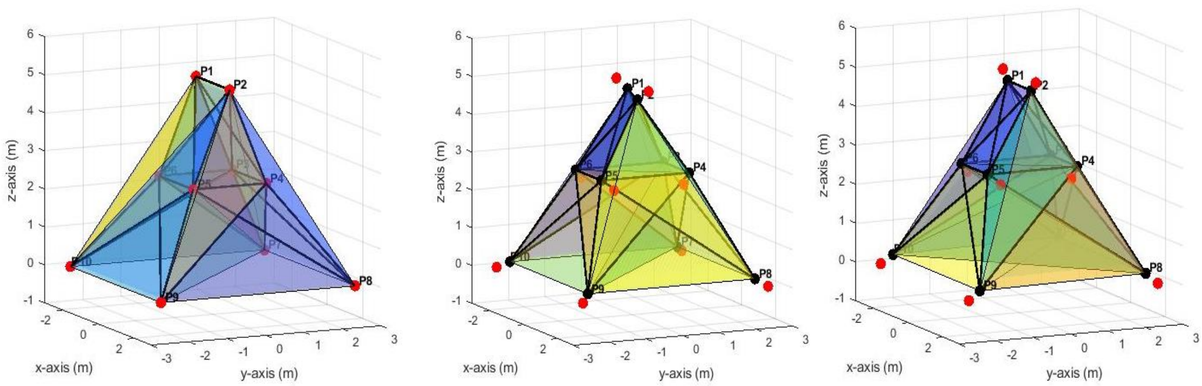

Figure 6. Optimized truss structures at different iteration steps: iteration step-II; iteration step-V; iteration step-X for optimization parameter set-I; red dots and blue lines show the original truss structure nodal points and truss members, respectively; black dots and lines show the final truss structure design nodal points and truss members, respectively. (Farklı yineleme adımlarıyla optimize edilmiş kafes yapılar: yineleme adımı-II; yineleme adımı-V; optimizasyon parametre seti-I için yineleme adımı-X; kırmızı noktalar ve mavi çizgiler sırasıyla orijinal kafes kiriş yapısının düğüm noktalarını ve kafes kiriş elemanlarını gösterir; siyah noktalar ve çizgiler sırasıyla son kafes yapı tasarımı düğüm noktalarını ve kafes kiriş elemanlarını gösterir.)

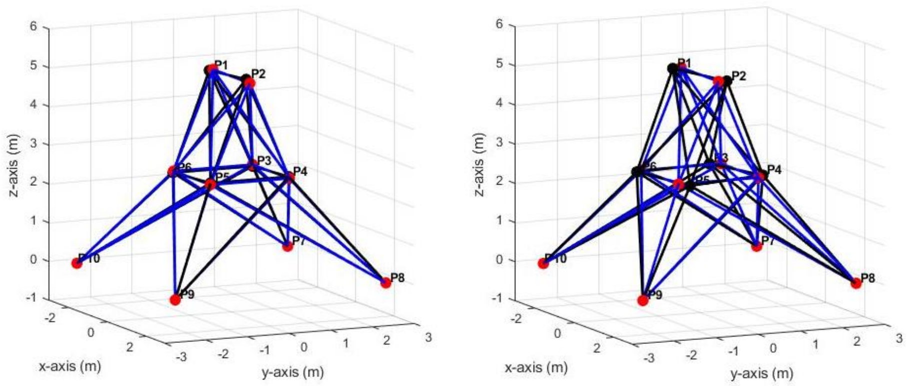

Figure 7. Mode shapes of truss structure for simulation made using optimization parameter set-I. Mode shape-I; Mode shape-III; red dots and blue lines show the original truss structure nodal points and truss members, respectively; black dots and blue lines show the final truss structure design nodal points and truss members, respectively. (Optimizasyon parametre seti-I kullanılarak yapılan benzetim için kafes yapının mod şekilleri. Mod şekli-I; Mod şekli-III; kırmızı noktalar ve mavi çizgiler sırasıyla orijinal kafes kiriş yapısının düğüm noktalarını ve kafes kiriş elemanlarını gösterir; siyah noktalar ve mavi çizgiler sırasıyla son kafes yapı tasarımı düğüm noktalarını ve kafes kiriş elemanlarını gösterir.)

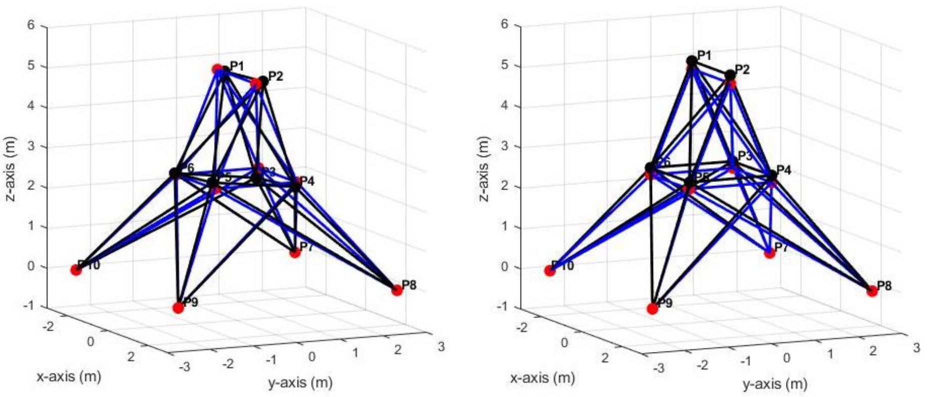

Figure 8. Mode shapes of truss structure for simulation made using optimization parameter set-I. Mode shape-V; Mode shape-VII; red dots and blue lines show the original truss structure nodal points and truss members, respectively; black dots and blue lines show the final truss structure design nodal points and truss members, respectively. (Optimizasyon parametre seti-I kullanılarak yapılan benzetim için kafes yapının mod şekilleri. Mod şekli-V; Mod şekli-VII; kırmızı noktalar ve mavi çizgiler sırasıyla orijinal kafes kiriş yapısının düğüm noktalarını ve kafes kiriş elemanlarını gösterir; siyah noktalar ve mavi çizgiler sırasıyla son kafes yapı tasarımı düğüm noktalarını ve kafes kiriş elemanlarını gösterir.)

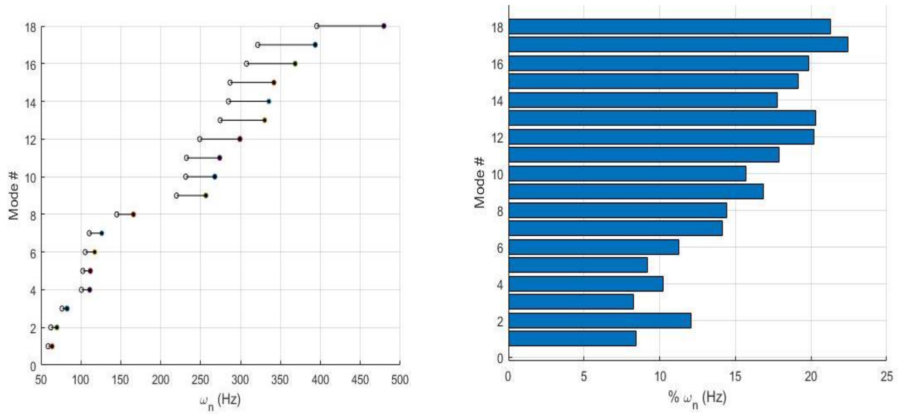

Figure 9. Natural frequency spectrum of the original and optimized truss structure. Circles and full circles show the original and the optimized natural frequencies of the truss structure, respectively for optimization parameter set-I. Percentage changes in the natural frequencies are also shown in the figure. (Orijinal ve optimize edilmiş kafes yapının doğal frekans spektrumu. Daireler ve içi dolu daireler, optimizasyon parametre seti-I için sırasıyla kafes yapının orijinal ve optimize edilmiş doğal frekanslarını gösterir. Doğal frekanslardaki yüzdesel değişimler de şekilde gösterilmiştir.)

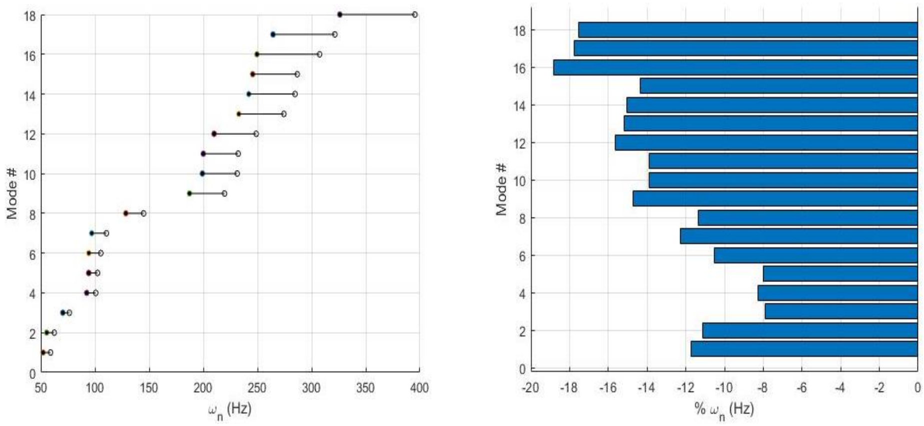

Figure 10. Natural frequency spectrum of the original and optimized truss structure. Circles and full circles show the original and the optimized natural frequencies of the truss structure, respectively for optimization parameter set-II. Percentage changes in the natural frequencies are also shown in the figure. (Orijinal ve optimize edilmiş kafes yapının doğal frekans spektrumu. Daireler ve içi dolu daireler, optimizasyon parametre seti-II için sırasıyla kafes yapının orijinal ve optimize edilmiş doğal frekanslarını gösterir. Doğal frekanslardaki yüzdesel değişimler de şekilde gösterilmiştir.)

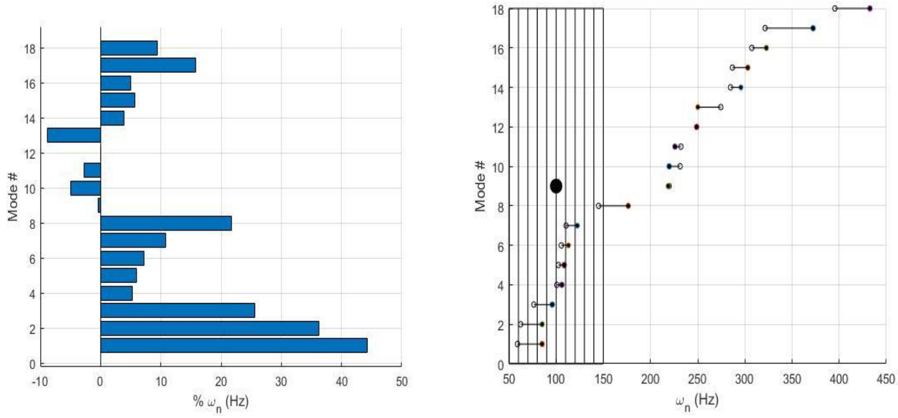

Figure 11. Natural frequency spectrum of the original and optimized truss structure. Circles and full circles show the original and the optimized natural frequencies of the truss structure, respectively for optimization parameter set-III. Percentage changes in the natural frequencies are also shown in the figure. (Orijinal ve optimize edilmiş kafes yapının doğal frekans spektrumu. Daireler ve içi dolu daireler, optimizasyon parametre seti-III için sırasıyla kafes yapının orijinal ve optimize edilmiş doğal frekanslarını gösterir. Doğal frekanslardaki yüzdesel değişimler de şekilde gösterilmiştir.)

Some geometric constraints are imposed on the coordinates of nodal points to preserve the topology of the truss structure. These geometric constraints are given below:

```
Ceq (1) =Pim(1,3)-Pim(2,3);
Ceq (2) =Pim(3,3)-Pim(4,3);
Ceq (3) =Pim(4,3)-Pim(5,3);
Ceq (4) =Pim(5,3)-Pim(6,3);
Ceq (5) =Pim(6,3)-Pim(3,3);
Ceq (6) =Pim(7,3)-Pim(8,3);
Ceq (7) =Pim(8,3)-Pim(9,3);
Ceq (8) =Pim(9,3)-Pim(10,3);
Ceq (9) =Pim(10,3)-Pim(7,3);
Ceq (10) =Pim(1,1)+Pim(2,1);
Ceq (11) =Pim(1,2)+Pim(2,2);
Ceq (12) =Pim(3,1)+Pim(5,1);
Ceq (13) =Pim(3,2)+Pim(5,2);
Ceq (14) =Pim(4,1)+Pim(6,1);
Ceq (15) =Pim(4,2)+Pim(6,2);
Ceq (16) =Pim(7,1)+Pim(9,1);
Ceq (17) =Pim(7,2)+Pim(9,2);
Ceq (18) =Pim(8,1)+Pim(10,1);
Ceq (19) =Pim(8,2)+Pim(10,2);
```

These type of equality constraints are defined in terms of **Pim**; **Pim** is a matrix of dimension 10x3 holding the coordinates of nodal points. **Ceq** is a vector of dimension N_e where N_e denotes the number of equality constraints. These **Ceq** constraints are imposed on the constrained optimization problem to preserve the symmetry original topology of truss structure so that final optimized truss structure will be a useful engineering design.

The pseudo-Matlab m-file code is explained and major points are discussed. g and h are vector functions representing all inequality and equality constraints res pectively (meaning bound, linear, and nonlinear constraints), so the minimization problem can be shown as in Eq. (7).

$$\begin{aligned} &\min_{\Theta} f(\Theta) \\ &subject \quad to \\ &g(\Theta) < 0 \\ &h(\Theta) = 0 \end{aligned} \tag{7}$$

5. CONCLUSIONS AND DISCUSSION (SONUÇLAR VE TARTIŞMA)

In conclusion, this article has delved into the realm of inverse modal analysis applied to truss structures, demonstrating the capability to strategically alter modal parameters to meet specific design criteria. The core challenge of inverse modal analysis was addressed through the formulation and solution of a constrained optimization problem. Leveraging the sensitivity of modal parameters to system characteristics and finite element modeling, we successfully configured the constrained optimization problem. The optimal incremental changes in the optimization parameters were computed, laying the foundation for a systematic and effective solution to the inverse modal analysis problem. Illustrating the methodology through the examination of a 25-bar truss structure, our findings substantiate the viability and efficacy of the proposed constrained optimization approach in the redesign of truss structures. By elucidating the sensitivity-driven optimization process, we provide valuable insights into the intricacies of truss structure dynamics and offer a robust framework for achieving desired modal characteristics.

Numerical results have shown that, both analysis in Abaqus and Matlab softwares yield compatible natural frequency results. Due to space constraints, detailed discussions focus on the results of the first optimization simulation set. Simulation results are presented in Figures 9-11, where circles represent the frequencies of the original truss structure and solid full circles indicate the natural frequencies of the final optimized truss structure. Across all optimization parameter sets and within the defined constraints, the primary objective of optimization was achieved. Notably, the preservation of truss structure topology to meet equality constraints underscores the success of the engineering redesign, affirming the feasibility of a purposeful and effective truss structure optimization.

This work contributes not only to the understanding of inverse modal analysis for truss structures but also highlights its practical application as a powerful tool for structural redesign. The success demonstrated in the illustrative example underscores the versatility of the proposed constrained optimization problem in addressing the complexities inherent in truss structures. As we navigate the intricate interplay between modal parameters and system characteristics, this study lays the groundwork for further advancements in the optimization-driven redesign of truss structures, fostering innovation and efficiency in structural engineering practices. As of future work, proposed study would be a candidate method to determine the dynamic integrity of truss based structures specifically for the big multiple objective industrial designs with preserving required pre-defined constraints. On the other hand, inverse modal analysis method would be applied to other geometric structures to compare the performance with meta-heuristic algorithms which can lead to handle design problems efficiently.

DECLARATION OF ETHICAL STANDARDS (ETİK STANDARTLARIN BEYANI)

The author of this article declares that the materials and methods they use in their work do not require ethical committee approval and/or legal-specific permission.

Bu makalenin yazarı çalışmalarında kullandıkları materyal ve yöntemlerin etik kurul izni ve/veya yasal-özel bir izin gerektirmediğini beyan ederler.

AUTHORS' CONTRIBUTIONS (YAZARLARIN KATKILARI)

Can Ulaş DOĞRUER**:** He introduced the general concept of article, created optimization model with geometric constraints and implemented into Matlab FEM Model.

Makalenin genel fikrini ortaya koydu, geometrik kısıtlamalarla optimizasyon modeli oluşturdu ve Matlab FEM Modeline uygulamasını gerçekleştirdi.

Can Barış TOPRAK**:** He contributed to literature survey of article and supported FEM model in Matlab software.

Makalenin literatür taramasına katkı sağladı ve Matlab yazılımında FEM modellemesine destek verdi.

Bora YILDIRIM**:** He carried out Abaqus simulations and verified numerical results.

Abaqus benzetim çalışmalarını gerçekleştirdi ve sayısal sonuçları doğruladı.

CONFLICT OF INTEREST (ÇIKAR ÇATIŞMASI)

There is no conflict of interest in this study.

Bu çalışmada herhangi bir çıkar çatışması yoktur.

REFERENCES (KAYNAKLAR)

[1] Zargham, S., Ward, T. A., Ramli, R., and Badruddin, I. A.: Topology optimization: a review for structural designs under vibration problems, Structural and Multidisciplinary Optimization, 53, 1157–1177, 2016. (Article)

[2] Pholdee, N. and Bureerat, S.: Comparative performance of meta-heuristic algorithms for mass minimisation of trusses with dynamic constraints, Advances in Engineering Software, 75, 1–13, 2014. (Article)

[3] Serra, M. and Venini, P.: On some applications of ant colony optimization metaheuristic to plane truss optimization, Structural and Multidisciplinary Optimization, 32, 499–506, 2006. (Article)

[4] Ho-Huu, V., Nguyen-Thoi, T., Nguyen-Thoi, M., and Le-Anh, L.: An improved constrained differential evolution using discrete variables (D-ICDE) for layout optimization of truss structures, Expert Systems with Applications, 42, 7057–7069, 2015. (Article)

[5] Ho-Huu, V., Nguyen-Thoi, T., Le-Anh, L., and Nguyen-Trang, T.: An effective reliability-based

improved constrained differential evolution for reliability-based design optimization of truss structures, Advances in Engineering Software, 92, 48–56, 2016. (Article)

[6] Miguel, L. F. F., Lopez, R. H., and Miguel, L. F. F.: Multimodal size, shape, and topology optimisation of truss structures using the Firefly algorithm, Advances in Engineering Software, 56, 23–37, 2013. (Article)

[7] Xu, T., Zuo,W., Xu, T., Song, G., and Li, R.: An adaptive reanalysis method for genetic algorithm with application to fast truss optimization, Acta Mechanica Sinica, 26, 225–234, 2010. (Article)

[8] Kaveh, A. and Talatahari, S.: A particle swarm ant colony optimization for truss structures with discrete variables, Journal of Constructional Steel Research, 65, 1558–1568, 2009. (Article)

[9] Lamberti, L.: An efficient simulated annealing algorithm for design optimization of truss structures, Computers & Structures, 86, 1936–1953, 2008. (Article)

[10] Jalili, S. and Talatahari, S.: Optimum design of truss structures under frequency constraints using hybrid CSS-MBLS algorithm, KSCE Journal of Civil Engineering, 22, 1840–1853, 2018. (Article)

[11] Ho-Huu, V., Nguyen-Thoi, T., Truong-Khac, T., Le-Anh, L., and Vo-Duy, T.: An improved differential evolution based on roulette wheel selection for shape and size optimization of truss structures with frequency constraints, Neural computing and applications, 29, 167–185, 2018. (Article)

[12] Jalili, S. and Hosseinzadeh, Y.: Combining migration and differential evolution strategies for optimum design of truss structures with dynamic constraints, Iranian Journal of Science and Technology, Transactions of Civil Engineering, 43, 289–312, 2019. (Article)

[13] Lieu, Q. X., Do, D. T., and Lee, J.: An adaptive hybrid evolutionary firefly algorithm for shape and size optimization of truss structures with frequency constraints, Computers & Structures, 195, 99–112, 2018. (Article)

[14] Miguel, L. F. F. and Miguel, L. F. F.: Shape and size optimization of truss structures considering dynamic constraints through modern metaheuristic algorithms, Expert Systems with Applications, 39, 9458–9467, 2012. (Article)

[15] Assimi, H. 185 and Jamali, A.: A hybrid algorithm coupling genetic programming and Nelder–Mead for topology and size optimization of trusses with static and dynamic constraints, Expert Systems with Applications, 95, 127–141, 2018. (Article)

[16] Zuo,W., Xu, T., Zhang, H., and Xu, T.: Fast structural optimization with frequency constraints by genetic algorithm using adaptive eigenvalue reanalysis methods, Structural and Multidisciplinary Optimization, 43, 799–810, 2011. (Article)

[17] Gholizadeh, S., Salajegheh, E., and Torkzadeh, P.: Structural optimization with frequency constraints by genetic algorithm using wavelet radial basis function neural network, Journal of Sound and Vibration, 312, 316–331, 2008. (Article)

[18] Kaveh, A. and Mahdavi, V.: A hybrid CBO–PSO algorithm for optimal design of truss structures with dynamic constraints, Applied Soft Computing, 34, 260–273, 2015. (Article)

[19] Kaveh, A. and Zolghadr, A.: A new PSRO algorithm for frequency constraint truss shape and size optimization, Struct Eng Mech, 52, 445–468, 2014. (Article)

[20] Tejani, G. G., Savsani, V. J., and Patel, V. K.: Modified sub-population teaching-learning-based optimization for design of truss structures with natural frequency constraints, Mechanics Based Design of Structures and Machines, 44, 495–513, 2016. (Article)

[21] Tejani, G. G., Savsani, V. J., Bureerat, S., Patel, V. K., and Savsani, P.: Topology optimization of truss subjected to static and dynamic constraints by integrating simulated annealing into passing vehicle search algorithms, Engineering with Computers, 35, 499–517, 2019. (Article)

[22] Salt, S. J., et al. "Layout optimization of pin-jointed truss structures with minimum frequency constraints." Engineering Optimization 55.8 (2023): 1403-1421.

[23] Sheng-Xue, He. "Truss optimization with frequency constraints using the medalist learning algorithm." Structures. Vol. 55. Elsevier, 2023.

[24] Millan-Paramo, Carlos, and João Elias Abdalla Filho. "Size and shape optimization of truss structures with natural frequency constraints using modified simulated annealing algorithm." Arabian Journal for Science and Engineering 45.5 (2020): 3511-3525.

[25] Lemonge, Afonso CC, et al. "Multi-objective truss structural optimization considering natural frequencies of vibration and global stability." Expert Systems with Applications 165 (2021): 113777.

[26] Carvalho, Érica CR, et al. "Solving multi-objective truss structural optimization problems considering natural frequencies of vibration and automatic member grouping." Evolutionary Intelligence (2022): 1-26.

[27] Khodadadi, Nima, and Seyedali Mirjalili. "Truss optimization with natural frequency constraints using generalized normal distribution

optimization." Applied Intelligence 52.9 (2022): 10384-10397.

[28] Fu, Yun-Fei, et al. "Smooth Topological Design of 3D Continuum Structures Using Elemental Volume Fractions." Computers & Structures, 2020.

[29] Zhu, Jihong, and Tong Gao. Topology optimization in engineering structure design. Elsevier, 2016.

[30] Li, Jianhongyu, Shenyan Chen, and Hai Huang. "Topology optimization of continuum structure with dynamic constraints using mode identification." Journal of Mechanical Science and Technology 29 (2015): 1407-1412.